Recruitment and Selection in Organizations[*]

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Abstract

This paper studies employer recruitment and selection of job applicants when productivity is match-specific. Job seekers have private, noisy estimates of match value, while the firm performs noisy interviews. Job seekers' willingness to incur the application costs varies with the perceived hiring probability, while the firm considers the applicant pool's composition when setting hiring standards. I show that changes in the accuracy of job seekers' estimates, or the firm's interview, affect application decisions, and both can raise hiring costs when they discourage applications. Thus, the firm may favor noisier interviews or prefer to face applicants that are less informed of their person-organization fit.

Keywords: hiring, recruitment, selection, employer search.

JEL classifications: D21, D82, L23, M12, M51

1 Introduction

Attracting and selecting the most suitable workers is arguably one of the main challenges that organizations face. This challenge has become even more difficult due to a shift towards knowledge-intensive and team-oriented work practices that place a stronger emphasis on hiring the "*right*" worker *for the organization*.[1,2] The main obstacle to efficient matching comes from information costs: prior to reaching an employment agreement, firms and jobseekers must devote time and resources to identifying a potential match and evaluating its surplus (Pissarides, 2009). To improve matching, employers engage in a variety of recruitment and selection activities - the former to create an applicant pool composed of the most promising prospects, and the latter to identify the applicants that are the best fit for the organization.[3] For instance, a firm may advertise the characteristics of its workplace, showcase its particular culture, or rely on current employees to describe their work experience, in the hope of attracting workers who thrive in such an environment. Concurrently, firms can adopt new selection techniques to obtain more precise estimates of applicants' expected performance at the firm.[4] This paper is concerned with the equilibrium effects of recruitment and selection activities on matching in the presence of fit and a firm's incentive to improve these activities.

Despite the vast literature on jobseekers' search behavior, comparatively less is known about firm-level hiring strategies. Nevertheless, and focusing on selection, conventional wisdom states that a key factor limiting pre-employment screening is the additional resources needed to improve a firm's estimate of future performance beyond the estimates obtained from the applicant's credentials (Oyer

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[1] While the practical importance of hiring is underscored by the amount of resources that firms allocate to it, there is some evidence of its effect on firm performance. For instance, the importance of hiring practices in workplaces dominated by team structures can be traced back to Ichniowski, Shaw and Prennushi (1997). See, also, Bloom and Van Reenen (2010) for an analysis of HR practices in empirical studies of productivity effects of management practices.

[2] The importance of person-organization or person-job fit has been recently documented in the economic literature (for an overview, see Oyer and Schaefer (2011)). For instance, Lazear (2003) argues that workers' human capital is general and multidimensional, but firms differ in the value they attach to each dimension. Hayes, Oyer and Schaefer (2006) find strong evidence of co-worker complementarity, supporting the claim that the "right" worker for a firm may depend on the firm's current workforce. Oyer and Schaefer (2012) and Lazear et al. (forthcoming) provide further evidence of match-specific productivity derived from worker-boss match effects.

[3] The terms "recruitment" and "selection" here follow their usage in the Human Resource and Industrial Psychology literature. Following Barber (1998, pp 5-6), "recruitment includes those practices and activities carried on by the organization with the primary purpose of identifying and attracting potential employees." Selection is typically defined as the practices aimed at separating from a pool of applicants those who have the appropriate knowledge, skills and abilities to perform well on the job (Gatewood et al. 2010).

[4] Typical selection techniques involve direct evaluation of applicants through a series of interviews (structured or unstructured), testing (e.g., psychometric, personality, intelligence), background and resume checks, "trial" periods aimed at measuring on-the-job performance, or situational judgment tests (SJTs) that study the subjects' reaction to hypothetical business situations (see Gatewood, Feild and Barrick, 2010).

and Schaefer 2011). This is especially the case when low termination costs make probationary periods a viable alternative for learning about match value (Waldman 2013). Thus, this view suggests that firms would be more willing to adopt innovations in selection methods that provide more precise estimates of match value at the same cost. I show that this view is incomplete, as it does not consider the need for the firm to elicit applications in the first place. In effect, raising the precision of a firm's screening can raise or lower an applicant's perceived likelihood of receiving a job offer and, thus, her incentives to engage in a costly, time-consuming hiring process. Improved selection can then raise recruitment costs if the firm needs to offer a higher wage to keep the same applicant pool. As I show, this force can be sufficient for a firm to forgo adopting a perfect screening method, even if it is costless to do so.

This insight can shed some light on a recent debate in the Human Resources literature, in which several authors have lamented the lack of firms' adoption of "more informative" selection methods, such as structured interviews (Terpstra and Rozell 1997; Van der Zee, Bakker and Bakker 2002) or personality tests (Rynes et al. 2002, 2007), where this lack of adoption cannot be explained by implementation costs (Ones et al. 2007).[5] Such firm behavior is even more puzzling when match-specific productivity is important, as standard worker credentials may be poor predictors of fit.[6] One explanation is that applicants' perceptions of both their fit with the organization and the validity of the hiring process dictate their willingness to be recruited (Breaugh and Starke 2000; Ryan and Ployhart 2000; Chapman et al. 2005), and these new selection methods may have an adverse effect on such perceptions.[7] I develop a model along these lines, in which firms may rationally forgo more informative selection techniques or informative advertising of job/firm characteristics, even if such activities are costless. In the model, an applicant's private estimate of match value translates through the accuracy of a firm's screening to a perceived probability of receiving an employment offer. Improving the accuracy of the firm's screening may adversely affect an applicant's prediction and dissuade him from applying. This dissuasion effect indirectly raises recruitment costs, as the firm now needs to offer a higher wage premium to restore the previous

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[5] A survey poll carried out by the Society for Human Resource Management, a professional HR association, indicates that only 18% of firms use a personality test for hiring or employee promotions (SHRM 2011).

[6] There is evidence in the Personnel Economics literature of the productivity effect of pre-employment testing on new hires. Autor and Scarborough (2008) report that in their sample, median employee tenure increased by 12% after the introduction of an electronic application that included a computer-administered personality test. Hoffman, Kahn, and Li (2015) similarly report a 15% increase in mean employee tenure after the introduction of testing.

[7] Other explanations offered in the literature are: (i) poor predictive power and low validity of new screening tests, especially personality tests (Morgeson et al. 2007); (ii) practitioners' unawareness of the benefits of new selection methods or a failure to implement them for a variety of reasons -overwork, political considerations, or organizational inertia (Rynes et al. 2002); and (iii) legal impediments to the deployment of personality tests, as they may result in more adverse impact (however, see Autor and Scarborough (2008) for the effect of testing on minority hiring).

applicant pool. This effect on recruitment also explains why firms that are less concerned with recruitment -since they are "flooded" with job applications- would be more willing to adopt more informative screening technologies.

The aim of this paper is to clarify how the information available to each side of the market affects firms' hiring costs and the profitability of improving recruitment and selection. To this end, I study a hiring model with the following ingredients: (i) Match specificity: job seekers differ in their productivity when employed by different firms. To simplify the analysis, I assume that there is one firm for which each job seekers' productivity is initially unknown, while all job seekers have the same productivity when matched with a group of alternative firms. (ii) Bilateral asymmetric information: prior to applying, each job seeker obtains a noisy, private signal of her productivity when matched with the firm (her "type"), while the firm can subject her to an "interview" that produces a noisy signal of her productivity. (iii) Costly Search: both the applicants and the firm must devote resources during the hiring process. Applicants incur their costs at the time of application, while the firm incurs its costs when it interviews applicants. (iv) Incomplete Contracting: the firm can condition payments neither on the result of the interview nor on whether the job seeker actually incurred the application costs, but it can commit to a "posted wage" paid to every hired applicant. Finally, the base model assumes that generating a vacancy is costless, so the firm will hire any applicant whose expected productivity exceeds the posted wage.

In this model, the posted wage is a worker's sole employment benefit; thus, a job seeker's willingness to incur the application cost will depend on the announced wage premium and the perceived likelihood of receiving a job offer. Underlying the equilibrium is a simultaneous Bayesian inference problem that both job seekers and the firm must solve: prior to applying, each job seeker must predict her hiring probability, given her type and the firm's hiring rule, while an imperfect interview leads the firm to also consider the self-selected applicant pool when setting a hiring rule.[8] Therefore, both application decisions and the hiring rule are jointly determined in equilibrium. As match-specific value is uncorrelated across firms, only those job seekers that consider themselves to be a sufficiently good fit apply to the firm. Thus, the equilibrium exhibits positive assortative matching: all job seekers with an estimate of match value above a threshold apply to the firm, but only high interview performers are hired (Proposition 1).

Matching frictions in this setup stem from incomplete contracting. Indeed, the effect of ap-

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[8] Stanton and Thomas (2015) find evidence that employers' beliefs about the applicant pool affect firms' search intensity and hiring strategies, while Burks et al. (2015) find that workers hired through referrals have different characteristics than non-referred workers, consistent with the notion that referrals affect the information available to job seekers about match value.

plicants' perceptions on the efficiency of matching would disappear if application costs were contractible, as the firm would then compensate the applicant for her costs and offer a wage that matches her outside option, in which case the hiring outcome would be constrained efficient (Proposition 2). Thus, the need to attract applications leads the firm to consider the quality of the applicant pool when setting the wage (Proposition 3).

I assume that the effect of improving recruitment and selection activities is mainly informational: improved selection leads to a more precise interview, while improving recruitment leads job seekers to have a less noisy estimate of match value. As application and hiring decisions are jointly determined, improving screening or recruitment has subtle effects on the size and composition of the applicant pool. For instance, high-type applicants would expect higher scores, and low-type applicants lower scores, from a more discriminating interview. However, the firm will also weigh the interview score more heavily than the quality of the applicant pool when making hiring decisions. The equilibrium effect of improved screening is, then, to discourage applications when the average quality of the applicant pool is either high or low and to encourage more applications for a mediocre applicant pool (Proposition 4). In contrast, better informed job seekers are more likely to apply when the quality of the applicant pool is high, but are dissuaded if the quality of the applicant pool is low (Proposition 5).

When faced with an opportunity to improve the hiring process, the firm must consider both the direct effect of more informative signals and the indirect effect of a change in the size and quality of the applicant pool. For instance, while a more informative interview always reduces hiring mistakes, it can also discourage job seekers from applying. This reduces the incentives to improve screening, especially when the interview is already fairly informative. Indeed, when applicants may be dissuaded from applying, the firm never adopts a perfectly informative interview, even if it is costless (Proposition 6). Moreover, better informed job seekers also face less uncertainty over their interview score. This may prove costly for the firm, however, if it reduces the applicant pool. Perhaps surprisingly, the firm always avoids informative advertising when applicants are poorly informed about match quality (Proposition 7).

I show in Section 7 that these results are robust to variations in the characteristics of the hiring process. If the firm faces a binding slot constraint, then a smaller applicant pool does not change the level of employment but forces the firm to hire lower quality applicants. In this case, a more discriminating interview indirectly raises hiring costs as it always dissuades applications. Furthermore, the incentives to invest in screening workers are lower if there is on-the-job learning and easy separation for bad matches. However, the firm may also avoid advertising in this case.

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This paper is related primarily to the literature on hiring practices in firms. This literature has studied several methods for firms to profitably induce self-selection among privately informed applicants. For instance, the design of pay-for-performance schemes can be used to identify the workers that are most productive (Lazear 2001; Oyer and Schaefer 2005), more motivated (Delfgaauw and Dur 2007), more likely to stay with the firm (Salop and Salop 1976) or that share the firm's vision and values (Van den Steen 2005). I concentrate, however, on direct methods of screening: a firm selects workers by subjecting them to a noisy assessment.

Several papers in the labor literature have considered firms' evaluation of applicants in explaining hiring outcomes. Pries and Rogerson (2005) develop a matching model with both screening and on-the-job learning, and study the impact of different labor policies on the firm's hiring standard. However, the firm does not need to recruit workers, as matches are exogenously formed according to a fixed matching function. The role of applicants' perceptions of the hiring process is a central theme in Chade, Lewis and Smith (2014), who consider college admissions with heterogeneous students who can apply at a cost to, at most, two colleges. While colleges perform an imperfect interview, students are perfectly aware of their own caliber. This precludes the study of recruitment strategies that raise students' knowledge of their caliber.[9]

Wolthoff (2012) proposes a search model in which job seekers can apply to multiple firms while a firm can interview multiple applicants. Costly applications and costly interviews lead to matching frictions. However, all job seekers are ex ante homogeneous and face the same probability of being hired by a given firm, while every firm's interview perfectly identifies match quality.[10] Finally, DeVaro (2008) studies the role of recruitment choices on a firm's hiring outcome, where the firm can increase its applicant pool by increasing the wage premium and its quality by employing informal recruitment methods (e.g., word-of-mouth referrals). However, the application decisions of job seekers are taken as exogenous, implying that their perceptions of the hiring process do not affect the firm's recruiting.[11]

The rest of the paper is structured as follows. Section 2 describes the model. Sections 3 and 4 analyze the equilibrium application and hiring decisions, as well as the equilibrium wage. Section 5 provides the main comparative statics on the applicant pool, and Section 6 discusses the firm's incentives to improve recruitment and selection. Section 7 considers several extensions of the basic

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[9]Nagypal (2004) also considers the college application decision but assumes that students are imperfectly informed of their caliber. However, as the college interview is perfect, the model cannot study the effect of more informative screening on applicants' behavior.

[10]Wolthoff (2012), in a dynamic extension to the basic model, also considers the case with ex ante heterogeneous workers. However, workers' productivity is assumed to be publicly known.

[11]See, also, DeVaro (2005) for empirical evidence of the effect of recruitment choices on hiring outcomes.

analysis, and I conclude in Section 8. All proofs are in the Appendix.

2   The Model

Players: There is a continuum of job seekers of unit mass. Job seekers are risk-neutral, protected by limited liability, and can seek employment in firm A or in any firm within a group of alternative, identical firms. Firm A (henceforth "the firm") can create a continuum of vacancies of mass one at no cost. I relax this assumption in Section 7 by allowing for slot constraints so that the firm can hire, at most, a mass K of workers. A job seeker has known productivity $\underline{w} \in \mathbb{R}$ when employed by an outside firm, while her productivity θ when employed at the firm is a random variable that is i.i.d. across job seekers and normally distributed, $\theta \sim N(0, 1/h_0)$.[12] Competition for workers implies that a job seeker can find employment at any time in any of those firms at a wage $\underline{w}$.[13] The sources of match-specificity can range from the existence of worker-firm production complementarities (Hayes, Oyer and Schaefer 2006), to heterogeneity in firm valuations of worker attributes (Lazear 2009), or even to differences in beliefs and preferences of workers (Van den Steen 2005) (see Oyer and Schaefer (2011) for a general discussion). In this paper, I focus on the effect on hiring outcomes of variability in match productivity across applicants for a single firm. This leads to a tractable characterization of equilibrium and allows a clear characterization of the returns to recruitment and screening. Alternatively, different firms may value each worker's set of knowledge, skill and abilities similarly. In that case, a worker's outside option and her productivity when employed by the firm will be correlated, inducing the standard adverse selection effect under asymmetric information.[14]

Hiring Process: The hiring process is divided into three stages: application, evaluation, and hiring decision. At the application stage, job seekers decide whether to apply to the firm. Any job seeker that applies to the firm incurs a private cost c_A. Thus, if θ were commonly observed by all market participants, $\theta - \underline{w} - c_A$ would be the surplus generated by a θ-worker when employed at the firm, and efficient matching would have job seekers with $\theta \geq \underline{w} + c_A$ matching with the firm. Conversely, if parties cannot obtain any information regarding θ, efficient matching would have all job seekers matched with the firm if $\underline{w} + c_A < 0 (= E[\theta])$ and matched with outside firms if $\underline{w} + c_A > 0$. Prior to submitting her application, a job seeker receives a private signal s_A that is

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[12] I do not restrict the sources of match specificity, which can arise both from the characteristics of co-workers and the attributes of the firm/job that jointly shape the productivity of the worker in *that* firm. While one could further differentiate between worker-firm productivity and worker-job productivity (as in Kristof-Brown et al. (2005)), I will not explore this distinction here.

[13] Also, the value of leisure is strictly lower than $\underline{w}$ for all job seekers so that they all strictly prefer employment. This simplification is without loss of generality, as the role of the group of alternative firms is to provide a homogeneous outside option to all applicants to firm A.

[14] The effect of correlation in the job seekers productivity across firms is explored in Alonso (2015a).

informative of θ, where $s_A|\theta$ is normally distributed, $s_A|\theta \sim N(\theta, 1/h_A)$, with h_A the precision of a job seeker's private assessment of θ.[15]

The evaluation stage ("interview") can be thought of as a statistical experiment in which the firm obtains information about an applicant's θ through a series of tests. Interviews are costly for the firm since evaluating a measure m of applicants imposes a cost $c_F m$.[16] The result of each interview is summarized by a signal s_F, which is privately observed by the firm, and is correlated with θ according to $s_F|\theta \sim N(\theta, 1/h_F)$. Thus, h_F is the precision with which the firm can evaluate an applicant's match-specific productivity.

An important aspect of the model is that both applicants and the firm find it costly to generate a productive match. Following Pissarides (2009), these matching costs derive both from the value of the forgone opportunities and from the resources devoted to discovering match quality. Note that, while the firm uses resources to evaluate, train or bargain with applicants, applicants also need to invest time and resources in training for the firm's selection process, complying with the requisite credentials, covering the administrative application costs, and, ultimately, engaging actively in the interview process.[17] To simplify the exposition, I consider all these costs to be homogeneous across job seekers and equal to c_A.[18]

This model of the hiring process shares several similarities with the literature on employer search, in which employers can decide the number of applicants to evaluate (extensive margin) and the extent to which each applicant is evaluated (intensive margin) (see, e.g., Rees (1966) and Barron, Bishop and Dunkelberg (1985)). In this paper, the extensive margin is given by the measure of applicants evaluated and depends on c_F, while the intensive margin is represented by the precision of the firm's interview h_F. In the analysis, however, the firm is endowed with an evaluation technology characterized by (c_F, c_A, h_F). Therefore, only the extensive margin is determined in equilibrium, while some of the results in Section 6 concern the firm's returns to increasing the intensive margin.

Informational content of private signals: It will prove convenient to normalize the signals s_A

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[15] In some cases, a job seeker's assessment of her suitability for a job is fully embodied in certifiable credentials. In reality, however, an applicant's beliefs and views about her match productivity cannot be described in a verifiable fashion; that is, they are "soft" information. In general, "high bandwidth" information that is difficult to describe and encode is typically privately known by applicants (Autor 2001).

[16] The main focus will be on $c_F = 0$. I consider the impact of positive evaluation costs in Section 7.

[17] Applicants' evaluation costs during the interview phase range from psychic costs associated with intense scrutiny to the opportunity cost of time and the effort costs necessary to perform during the interview (for instance, when the "interview" is a probationary period).

[18] This assumption simplifies the firm's inference problem and allows a simple characterization of the equilibrium simultaneous Bayesian inference problem. Alonso (2015b) considers a model in which an applicant faces different (private) application costs, but her signal s_A is embodied in her credentials and, thus, certifiable. Even if the firm could pay each job seeker an "application fee," it would still be the case, as in this paper, that improved information may raise hiring costs if it discourages applications.

and s_F in terms of the posterior means that they induce. Thus, let v_i be

$$v_i = E[\theta|s_i] = \frac{h_i}{h_0 + h_i} s_i,$$

with ex-ante distribution $v_i \sim N\left(0, \sigma_{v_i}^2\right)$, where $\sigma_{v_i}^2 = \frac{h_i}{h_0(h_0+h_i)}$, $i \in \{A, F\}$. I will refer to v_A as the applicant's "type" and v_F as the interview "score."

This specification has two advantages. First, changes in h_i, $i \in \{A, F\}$, have no effect on how a given v_i is interpreted as a predictor of θ since $E[\theta|v_i] = v_i$. If the firm had no additional information, hiring decisions would depend solely on v_F, regardless of the interview's precision. Second, increases in the precisions h_i, $i \in \{A, F\}$, lead to a higher variance of the signals v_i, $i \in \{A, F\}$, which is consistent with the fact that more informative signals lead to a higher dispersion of posterior expectations.[19]

A key feature of the model is that the private evaluations v_A and v_F are correlated, thus allowing for both the estimation of the applicant's type from the interview score and the applicant's prediction of the interview score given her type. As the (linear) correlation coefficient ρ between v_A and v_F is

$$\rho^2 = \frac{h_F}{h_0 + h_F} \frac{h_A}{h_0 + h_A}, \tag{1}$$

we have the following mean and variance when estimating v_i from v_j, $i, j \in \{A, F\}, i \neq j$,

$$E[v_i|v_j] = \frac{h_i}{h_0 + h_i} v_j, \tag{2}$$

$$\sigma_{v_i|v_j}^2 \equiv Var[v_i|v_j] = (1 - \rho^2)\sigma_{v_i}^2 = \left(\frac{h_i}{h_0 + h_i}\right)^2 \left(\frac{1}{h_0 + h_j} + \frac{1}{h_i}\right). \tag{3}$$

Contracts: I take an incomplete-contracting view of the hiring process in that the firm can commit to payments based only on whether the applicant is hired. Implicit is the assumption that both the applicant's type and the interview score are privately observed (i.e., they are "soft" information), and contracts cannot be written directly on these values. This implies, for instance, that the firm cannot contractually commit to base hiring decisions on the interview score in arbitrary ways. Also, I assume that the firm cannot condition payments on whether the applicant has incurred the necessary application costs and is ready to be evaluated. Informally, if the firm pays each individual for simply "showing up," all individuals *will* "show up" to the firm, while some of them will not incur the application costs and would then immediately apply elsewhere. As applicants cannot be directly compensated for their costs, the firm must make employment sufficiently desirable in order

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[19]For instance, Ganuza and Penalva (2010) derive a series of informational orders based on the dispersion of conditional expectations, where, for the class of decision problems considered, a more informative signal induces a higher dispersion in posterior expectations.

to attract applications. To do so, I assume that the firm can ex-ante commit to a "posted wage" w_E to be paid to a hired applicant.

Timing and Equilibrium: The model is static and considers matching in a single period. The firm is endowed with a commonly known evaluation technology (c_F, c_A, h_F) and posts a wage w_E. Job seekers learn their type v_A and, after observing w_E, decide to apply to the firm. Given the mass of applicants, the firm decides whether to submit each applicant to an interview and whether to extend an employment offer, paying w_E to a hired applicant. Independent of whether or not they are evaluated, applicants that do not receive an employment offer, or that reject an employment offer, can instantaneously find employment at any of the identical firms that pay $\underline{w}$. Finally, payoffs are realized and the game ends. The notion of equilibrium is Perfect Bayesian Equilibrium.

3   Equilibrium Hiring and Application Decisions

I start the analysis by characterizing the application and hiring choices in a subgame in which the firm posts wage w_E. For simplicity, Sections 3-6 consider the case in which the firm incurs no costs of evaluation- i.e., $c_F = 0$. Section 7 considers the case of strictly positive evaluation costs.

I solve for an equilibrium by backward induction. I first derive the firm's sequentially rational hiring rule after evaluating an applicant. The firm optimally sets a "hiring standard," which depends on the composition of the applicant pool, and hires any applicant whose interview score exceeds it. Anticipating the firm's hiring standard and interview decision, I derive a job seeker's application decision as a function of her type.

3.1   Firm's Hiring Decision

Suppose that all job seekers with types v_A in the set A apply to the firm.[20] As v_A is correlated with θ, the firm has two informative signals of match-specific productivity after the interview: the interview score, v_F, and the fact that the job seeker chose to apply to the firm, $v_A \in A$. The firm's inability to contractually condition hiring outcomes on v_F implies that in any sequentially rational hiring rule, the firm offers employment only if an applicant's expected productivity, as given by $E[\theta|v_F, v_A \in A]$, does not fall short of the cost of hiring, as giveng by the wage w_E. The following lemma shows that this leads the firm to optimally adopt a threshold hiring rule.

Lemma 1. *For each measurable set A, there exists $\underline{v}_F(A)$ such that the firm extends an employment offer after interviewing an applicant of type $v_A \in A$ if and only if $v_F \geq \underline{v}_F(A)$. The hiring standard*

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[20]We need not worry about mixing by job seekers as, given our assumptions on the signal structure and optimal behavior by the firm, job seekers have a strict preference on applications with probability 1.

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$\underline{v}_F(A)$ *satisfies*

$$E\left[\theta|\underline{v}_F(A), v_A \in A\right] = w_E. \tag{4}$$

To understand the firm's updating in our setup with joint normality of match value and signals, suppose first that the applicant's type could be credibly disclosed (i.e., v_A is "hard" information). Then the firm would simply weigh each signal to obtain

$$E[\theta|v_F, v_A] = \frac{h_0 + h_F}{h_0 + h_F + h_A} v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} v_A. \tag{5}$$

When the applicant's type is "soft," however, the firm faces a filtering problem, as the interview score v_F can be used to refine the estimate of the applicant's actual type v_A given the "application signal" $\{v_A \in A\}$. Therefore, the firm's estimate of match value becomes

$$E[\theta|v_F, v_A \in A] = \frac{h_0 + h_F}{h_0 + h_F + h_A} v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} E[v_A|v_F, v_A \in A]. \tag{6}$$

That hiring decisions satisfy a cut-off rule then follows from the observation that as v_F and v_A satisfy the MLRP with θ, they also satisfy the same property among them (Karlin and Rubin 1956). Therefore, the filtering term $E[v_A|v_F, v_A \in A]$ is non-decreasing in the interview score for any set A- a better score leads to a more optimistic revision of the applicant's type. As a result, $E[\theta|v_F, v_A \in A]$ strictly increases in v_F both because a higher interview score implies a higher expected match value and a higher interview score identifies a higher applicant type. Finally, the existence of a "hiring standard" $\underline{v}_F(A)$ satisfying (4) is ensured by the unbounded support of $E[\theta|v_F, v_A \in A]$ for fixed A.

3.2 Job seeker's Application Decision

Given the firm's hiring standard (4), which job seekers are willing to apply if the firm interviews all applicants? As v_F and v_A are correlated, each job seeker faces a prediction problem: to estimate the likelihood of meeting the firm's hiring criteria given her type. In general, arbitrary hiring rules may deter applications from job seekers with a high estimate of θ, but may attract job seekers with lower estimates. However, since the firm's equilibrium hiring decision follows a cut-off rule, a job seeker's application decision will also be monotone in her type.

Lemma 2. *Suppose that $w_E > \underline{w} + c_A$ and that the firm evaluates all applicants. Then, for any threshold hiring standard $\underline{v}_F$ there exists a marginal type $\underline{v}_A(\underline{v}_F)$ such that a job seeker of type v_A applies to the firm iff $v_A \geq \underline{v}_A(\underline{v}_F)$, where $\underline{v}_A(\underline{v}_F)$ is the unique solution to*

$$(w_E - \underline{w}) \Pr\left[v_F \geq \underline{v}_F | \underline{v}_A(\underline{v}_F)\right] = c_A. \tag{7}$$

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Recall that any rejected applicant can immediately secure employment elsewhere at a wage $\underline{w}$. Thus, the left-hand side of (7) captures the expected incremental benefit for a type-v_A job seeker of gaining employment at the firm. To evaluate this benefit, an applicant needs to predict the likelihood of meeting the hiring standard after being interviewed- i.e., estimate $\Pr[v_F \geq \underline{v}_F | v_A]$. Since v_F and v_A satisfy the MLRP, $\Pr[v_F \geq \underline{v}_F | v_A]$ is increasing in the applicant's type and, as all applicants incur the same application cost, the expected gain from applying to the firm also increases in v_A. Therefore, the firm's threshold hiring rule induces a monotone application rule: All types $v_A > \underline{v}_A(\underline{v}_F)$ apply to the firm, where the marginal type $\underline{v}_A(\underline{v}_F)$ satisfies (7) and obtains no expected rent from applying.

3.3   Equilibrium Application and Evaluation

Contractual incompleteness of the hiring process constrains the firm's behavior in two ways. First, the firm cannot commit to arbitrary hiring rules; cf. Lemma 1. As a result, all interviewed applicants face a positive probability of being rejected. Second, non-contractibility of the interview itself implies that: (i) the firm cannot commit to skip the interview for some applicants; and (ii) the firm cannot pay a different wage to an applicant hired without an interview. As every worker receives the same wage, and the interview is costless for the firm, then the firm will interview all applicants. There are situations, however, in which the firm would benefit from not interviewing applicants. For instance, if job seekers have very precise estimates of match value (high h_A) and the firm's interview is very noisy (low h_F), the firm could post a wage $w_E = \underline{w} + c_A$ and hire all applicants without interviews. As job seekers are indifferent between applying to the firm or elsewhere, an equilibrium exists in which types $v_A \geq w_E$ apply and are hired without an interview. I explore this possibility in Section 7 when I consider the case of positive interview costs $c_F > 0$.

As the firm interviews all applicants, only those job seekers that are sufficiently confident of meeting the firm's equilibrum hiring standard will incur the application cost c_A. To describe the equilibrium, I introduce the following "reaction" functions for job seekers and the firm. First, define $b_A(v_A, p)$ as

$$\begin{aligned} b_A(v_A, p) &= \max\{\underline{v}_F : \Pr[v_F \geq \underline{v}_F | v_A] \geq p\} \\ &= E[v_F | v_A] + \sigma_{v_F|v_A} \Phi^{-1}(p). \end{aligned} \tag{8}$$

That is, $b_A(v_A, p)$ is the maximum hiring standard that a type-v_A job seeker would pass with probability at least p. Second, define $b_F(v_A, w)$ as the firm's optimal hiring standard when the

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applicant pool is $\{v'_A : v'_A \geq v_A\}$ and the wage is w; that is,

$$E\left[\theta|b_F(v_A, w), v'_A \geq v_A\right] = w. \tag{9}$$

Proposition 1 (Equilibrium Hiring and Applications). *For each $w_E > \underline{w} + c_A$, the unique sequentially rational continuation equilibrium is described by a type $\underline{v}_A$ such that all types $v_A \geq \underline{v}_A$ apply to the firm, while types $v_A < \underline{v}_A$ immediately gain employment elsewhere at wage $\underline{w}$. The firm evaluates all applicants and hires an applicant iff $v_F \geq \underline{v}_F$. The marginal applicant $\underline{v}_A$ and the hiring standard $\underline{v}_F$ are the unique solution to*

$$\underline{v}_F = b_F(\underline{v}_A, w_E), \tag{10}$$

$$\underline{v}_F = b_A(\underline{v}_A, \frac{c_A}{w_E - \underline{w}}). \tag{11}$$

Figure 1 depicts the equilibrium hiring standard and application decision of Proposition 1. The equilibrium (10-11) is given by the unique intersection of the functions $b_A(v_A, \frac{c_A}{w_E-\underline{w}})$ and $b_F(v_A, w_E)$. In this setup, match specificity leads to positive assortative matching: for any posted wage $w_E > \underline{w} + c_A$, all job seekers that believe themselves to be a good match apply to the firm ($v_A \geq \underline{v}_A$), and the top interview performers are hired ($v_F \geq \underline{v}_F$), where $\underline{v}_A$ and $\underline{v}_F$ are the unique solution to the simultaneous Bayesian inference problem (10-11). Figure 1 also depicts the optimal hiring rule if the applicant's type is certifiable. As is intuitive, unobservability of v_A raises the probability that lower types are hired but reduces that of higher types. Finally, uniqueness of equilibrium results from both a decreasing hiring standard in the quality of the applicant pool (and, hence, decreasing in $\underline{v}_A$), and an increasing maximum hiring standard that a job seeker is willing to beat as a function her type.

I now describe in more detail this inference problem by looking separately at the firm's filtering and applicant's prediction problems. The analysis of comparative statics with respect to the precision of signals is carried out in Section 5.

Filtering Problem. In our jointly normal framework, we have $v_A|v_F \sim N\left(E[v_A|v_F], \sigma^2_{v_A|v_F}\right)$, where $E[v_A|v_F]$ and $\sigma^2_{v_A|v_F}$ are given by (2). Therefore, an applicant randomly drawn from a pool $\{v_A : v_A \geq \underline{v}_A\}$ whose test result is v_F is expected to have a productivity

$$E[\theta|v_F, v_A \geq \underline{v}_A] = v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A}\sigma_{v_A|v_F}\lambda\left(\frac{\underline{v}_A - E[v_A|v_F]}{\sigma_{v_A|v_F}}\right), \tag{12}$$

where λ is the hazard rate of a standard Normal.[21] That is, the firm will correct its initial assessment of the candidate, as given by v_F, by an amount that depends on the difference between the marginal applicant and the firm's expectation of the applicant's type given v_F.

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[21] This expression follows from applying to (6) the fact that for a normal distribution of mean μ and variance σ, the truncated expectation is $E[x|x \geq a] = \mu + \sigma\lambda\left(\frac{a-\mu}{\sigma}\right)$.
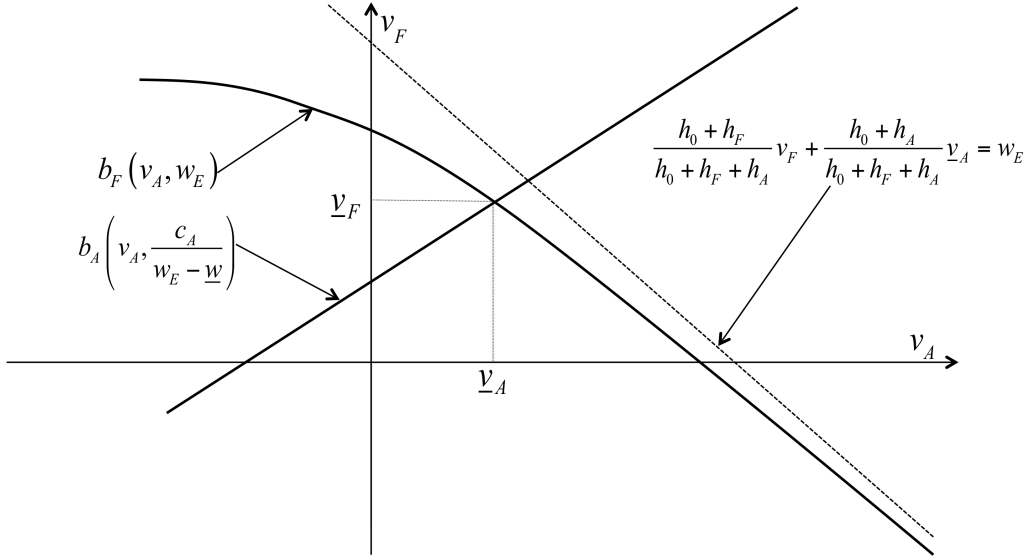
Figure 1: Equilibrium Hiring and Applications for wage w_E.

It is instructive to compare (12) to the case in which the applicant's type is observable by the firm, as given by (5). In this case, the sensitivity of the firm's posterior expectation with respect to v_F is independent of the type of applicant. This is no longer true when v_A is unobservable, as the firm tries to infer v_A from v_F. In fact, twice differentiating (12) establishes that both pieces of information act as substitutes, in the sense that

$$\frac{\partial^2 E[\theta|v_F, v_A \geq \underline{v}_A]}{\partial v_F \partial \underline{v}_A} \leq 0.$$

Thus, the firm's posterior expectation becomes less responsive to the interview score as the applicant pool becomes more selective. The intuition for this result is that a more selective applicant pool (higher $\underline{v}_A$) is also a "more informative" applicant pool, as the firm faces less uncertainty regarding the type of a randomly chosen applicant.[22] Thus, the firm puts more weight on the update term in (12) as $\underline{v}_A$ increases. In fact, if the applicant pool becomes very selective, so that $\underline{v}_A$ tends to ∞, then (12) converges to

$$E[\theta|v_F, v_A \geq \underline{v}_A] \approx \frac{h_0 + h_F}{h_0 + h_F + h_A} v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} \underline{v}_A.$$

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[22] This result is immediate in this case, as a normal distribution has an increasing and unbounded hazard rate. This implies that a randomly chosen applicant from a pool $\{v_A \geq \underline{v}_A\}$ is increasingly likely to be close to the marginal type $\underline{v}_A$ as $\underline{v}_A$ increases. This result would remain true to different distributional assumptions as long as the underlying distribution has an increasing and unbounded hazard rate.

That is, the firm updates as if it faces no uncertainty about the applicant's type (which approximately equals the type of the marginal applicant).

In summary, I can write (10) using (12) as

$$\underline{v}_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} \sigma_{v_A|v_F} \lambda \left(\frac{v_A - E[v_A|\underline{v}_F]}{\sigma_{v_A|v_F}} \right) = w_E. \tag{13}$$

Prediction Problem. I now turn to the applicant's prediction problem. Conditional on v_A, the interview score v_F is normally distributed, with $E[\underline{v}_A|\underline{v}_F]$ and $\sigma^2_{v_A|v_F}$ given by (2). Therefore, (10) translates to

$$\underline{v}_F - E[v_F|\underline{v}_A] = -\sigma_{v_F|v_A} \Phi^{-1}(\frac{c_A}{w_E - \underline{w}}). \tag{14}$$

That is, the difference between the firm's hiring standard and the expected score of the marginal applicant is proportional to the variance that the applicant faces over the interview score. This is intuitive: if $c_A/(w_E - \underline{w}) < 1/2$, so that the marginal applicant is more likely to fail the interview than to pass it, a "less predictable" interview (i.e., one with a higher perceived variance) increases the option value of applying and would attract a lower type, all else equal. Conversely, if $c_A/(w_E - \underline{w}) < 1/2$, so that the marginal applicant is more likely to pass the test, a more uncertain interview would increase $\underline{v}_A$ and, thus, result in fewer applications.

4 The Wage as a Recruitment and Selection Tool

The firm's recruitment efforts can be based on three dimensions: (i) more-intense advertising of its vacancies; (ii) more-informative advertising of job/firm characteristics; and (iii) increasing the job's appeal to prospective applicants. In this model, job appeal is embodied by the posted wage w_E. I now characterize the equilibrium wage w_E given the hiring and application decisions described in Proposition 1. To better understand the role of non-contractible application costs, I first study a benchmark case in which these costs can be contractually covered by the firm.

4.1 Benchmark: Contractible Applicant Costs

Suppose that the firm can condition payments on whether the applicant incurred the application costs. The firm then offers a contract (c, w_C) to each applicant, which pays c if the applicant incurred the costs c_A, and, additionally, a wage w_C if the candidate is hired. The following proposition describes the equilibrium in this case.

Proposition 2 (Contractible Application Costs) *There exists a unique PBE of the game in which application costs are contractible: the firm offers a contract $(c, w_C) = (c_A, \underline{w})$; all job seekers*

of type $v_A \geq \underline{v}_A^C$ apply to the firm; and only those with interview scores $v_F \geq \underline{v}_F^C$ are hired. The marginal type $\underline{v}_A^C$ and the hiring standard $\underline{v}_F^C$ solve

$$E[\theta - \underline{w}|\underline{v}_A^C, v_F \geq \underline{v}_F^C] \Pr\left[v_F \geq \underline{v}_F^C|\underline{v}_A^C\right] = c_A, \tag{15}$$

$$E[\theta|\underline{v}_F^C, v_A \geq \underline{v}_A^C] = \underline{w}. \tag{16}$$

If application costs are contractible, the firm will optimally cover them and pay a wage that matches the applicant's outside option $\underline{w}$. That is, match specificity will not translate into wage dispersion if the firm can directly cover the application costs. To see that the contract $(c_A, \underline{w})$ is optimal, note that all applicants obtain no rents from applying to the firm. The marginal applicant $\underline{v}_A^C$ and hiring standard $\underline{v}_F^C$ are then given by the joint solution to (15) and (16). First, (15) implies that the firm obtains a zero profit if it decided to evaluate the marginal applicant. This condition is necessary for an equilibrium- if expected profit exceeds application costs, the firm can raise the wage in order to attract more applicants, while if expected profit falls below the application costs, the firm can lower its application subsidy (and increase the wage) to discourage applications. Second, (16) is the sequentially rational hiring standard in which the firm makes a zero profit on the marginal hire. Importantly, there is no ex-post distortion in the hiring decision given the available information to the firm: the firm hires the applicant as long as the expected match value exceeds the applicant's outside option.[23]

4.2   Limits to the wage as a recruitment tool.

When application costs are not contractible, the wage plays a dual incentive-sorting role: it motivates job seekers to incur the applications costs, but it induces applications only from those that are themselves sufficiently confident of being a good match. The first role implies that low wages $w_E < \underline{w} + c_A$ are ineffectual in recruiting applicants. However, the firm's inability to commit to arbitrary hiring rules limits the efficacy of the wage in its second role. Indeed, Lemma 3 shows that high wages are undesirable, as increasing them may actually discourage applications.

Lemma 3.   *Let $\underline{v}_A(w_E)$ be defined by (10-11), and let $w_{\max}$ be the unique solution to*

$$d\underline{v}_A/dw_E|_{w_E=w_{\max}} = 0.$$

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[23]Unlike in Proposition 3 below, in the equilibrium described in Proposition 2, all applicants are indifferent between applying and being hired by the firm at wage $\underline{w}$ or securing their outside option. Thus, one would argue that the absence of incentive conflicts could lead applicants to truthfully disclose v_A. That is indeed the case: there is an equilibrium with costless communication in which the applicant truthfully reports her type to the firm. Moreover, there is no inefficiency in matching, as hiring decisions are ex-post optimal and make use of all available information. Therefore, non-contractibility of application costs also implies that information is lost since the applicant cannot credibly disclose it to the firm.

Then, the equilibrium posted wage w_E satisfies

$$\underline{w} + c_A < w_E < w_{\max}.$$

Increasing w_E has two countervailing effects on an applicants' behavior. To be sure, a higher wage makes employment more desirable. A higher wage, however, increases the firm's hiring cost, thus leading to a higher hiring standard and raising the probability that the marginal applicant fails the interview. The proof of Lemma 3 shows that the first effect dominates for low wages, while the second effect dominates for high wages. In other words, increasing the wage w_E above $w_{\max}$ actually increases the marginal type $\underline{v}_A$ and, thus, reduces the number of applications. This implies that there is a lower wage that attracts the same applicant pool at a lower cost; thus, wages above $w_{\max}$ are dominated and would never be posted in equilibrium.[24]

4.3 Equilibrium Wage

With a continuum of job seekers, the firm's expected profit is the product of the total mass of hired applicants and the expected match surplus of a hired applicant. If all applicants are evaluated, this is formally equivalent to

$$
\begin{aligned}
\Pi &= (1 - F(\underline{v}_A, \underline{v}_F)) E[\theta - w_E | v_F \geq \underline{v}_F, v_F \geq \underline{v}_A] = \int_{-\infty}^{\infty} \int_{\underline{v}_F}^{\infty} \int_{\underline{v}_A}^{\infty} (\theta - w_E)\, dF(\theta, v_A, v_F) \\
&= \int_{-\infty}^{\infty} (\theta - w_E)\, \Phi\left[z_A(\theta, \underline{v}_A)\right] \Phi\left[z_F(\theta, \underline{v}_F)\right] dF(\theta),
\end{aligned}
\tag{17}
$$

with Φ the cdf of a standard normal distribution, and

$$z_i(\theta, \underline{v}_i) = \sqrt{h_i} \left[\theta - \underline{v}_i \left(h_i + h_0\right)/h_i\right], i = A, F. \tag{18}$$

We can interpret (17) as the payoff from a decentralized sequential testing process (Wald 1945), where the firm obtains the benefit $\theta - w_E$ from a candidate of value θ only if a "double detection" occurs: when both the candidate applies (which occurs with probability $\Phi\left[z_A(\theta, \underline{v}_A)\right]$), and is hired (which, independently of the application decision, would occur with probability $\Phi\left[z_F(\theta, \underline{v}_F)\right]$).[25]

The firm behaves as a standard monopsonist over match-specific value when setting the wage: by raising the wage, it attracts more applicants but raises the wage bill per employee. The following proposition describes the properties of the optimal posted wage.

Proposition 3 (Optimal Posted Wage) *If the firm faces no direct costs of evaluating applicants, then:*

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[24] To be precise, this is true as $\underline{v}_A\left(w_E\right)$ is continuous and unbounded as $w_E \to \underline{w}$, for any wage above $w_E > w_E^{\max}$.

[25] See De Groot (1970, Chapter 12-14) for a discussion of sequential testing processes.

(i) the optimal wage w_E^* satisfies

$$\frac{\Pr\left[v_F \geq \underline{v}_F, v_A \geq \underline{v}_A\right]}{\Pr\left[v_F \geq \underline{v}_F, \underline{v}_A\right]} = E[\theta - w_E^* | v_F \geq \underline{v}_F, \underline{v}_A] \left(-\frac{d\underline{v}_A}{dw_E}\right)\Bigg|_{w_E = w_E^*}, \quad \text{and} \tag{19}$$

(ii) let $\underline{v}_A^0$ be such that

$$E[\theta - \underline{w}|v_F \geq \underline{v}_F, \underline{v}_A^0] = 0, \tag{20}$$

$$E[\theta - \underline{w}|v_A \geq \underline{v}_A^0, \underline{v}_F] = 0. \tag{21}$$

Then,

$$\lim_{c_A \to 0} \underline{v}_A = \underline{v}_A^0.$$

The optimality condition (19) follows from applying the envelope theorem to the firm's profits given its sequentially rational hiring rule. The firm will never set a wage such that the marginal applicant, conditional on being hired, is a bad match. Indeed, from (19), it readily follows that $E[\theta - w_E^* | v_F \geq \underline{v}_F, \underline{v}_A] > 0$. Also, comparing (19) to the case of contractible costs (15), it is clear that non-contractibility of costs leads to too few applicants applying to the firm.

Proposition 3-ii shows that vanishing evaluation costs would not lead the firm to attract and evaluate all job seekers. In particular, the firm does not attract any job seeker with $v_A < \underline{v}_A^0$ when $c_A > 0$. Establishing a finite marginal applicant for vanishing application costs has two effects. First, it reduces the probability that the firm benefits from a good match, as it lowers the probability of hiring. Second, however, it increases the information available to the firm since the applicant pool is more selective. The conditions (20-21) jointly determine the lowest type of applicant $\underline{v}_A^0$ that the firm would be willing to attract. In particular, $\underline{v}_A^0$ is such that the firm generates no profit when hiring type $\underline{v}_A^0$ following an optimal hiring rule performed under ignorance of the applicant's type (21).[26]

Equilibrium implications of match specificity This section ends with a discussion two important properties of this model of person-to-organization match specificity: equilibrium exhibits assortative matching and advantageous selection.

In this context, assortative matching means that better candidates (for the firm) apply and

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[26]If applications are truly costless- i.e., $c_A = 0-$ then, as in the case of contractible costs, the firm could offer a wage $\underline{w}$ and job seekers are indifferent between applying to the firm and applying elsewhere. As employment in the firm generates no rents, there is an equilibrium in which job seekers can truthfully communicate their private type v_A. In this equilibrium, moreover, the firm is willing to evaluate all job-seekers. However, truthful communication of v_A disappears for any $c_A > 0$, as the firm needs to pay a wage premium $w^* > \underline{w}$ to attract applicants.

better interview performers are hired.[27] This, of course, is a consequence of the assumption that all job seekers are homogeneous in their outside option. Furthermore, the model exhibits advantageous selection as a consequence of the independence of value across firms: worsening the terms of trade by reducing the wage[28] can only improve the quality of the applicant pool. This, of course, will not be true if matches that provide higher synergies also have greater outside options.[29]

5   The Equilibrium Effect of More-Informative Signals on the Applicant Pool.

One of the implications of match specificity is that a firm may underinvest in screening applicants or in informative advertising of job/firm characteristics if improving the information on either side of the market has an adverse effect on the applicant pool. To derive this result, in this section I analyze the equilibrium effect on the marginal applicant of a more precise interview (higher h_F) and of better informed applicants (higher h_A), for a fixed wage w.

5.1   Applicant's Prediction and Firm's Inference

I start by studying the effect of more precise signals on the reaction functions $b_A(v_A, p)$ and $b_F(\underline{v}_A, w)$ defined in (8) and (9), for a fixed hiring probability p and wage w.

Applicant's Prediction Problem   The reaction function $b_A(v_A, p)$ specifies the maximum hiring standard that a type v_A passes with probability at least p, and is affected by changes in h_F or h_A through its effect on the perceived mean and variance of the interview score (where $E[v_F|v_A]$ and $\sigma^2_{v_F|v_A}$ are given in (2) and (3)). The next lemma summarizes the effect on $b_A(v_A, p)$ of a marginal increase in h_F or h_A.

Lemma 5. *(i) There exists $\tilde{v}_A(p)$ such that $\partial b^A/\partial h_F > 0$ iff $v_A > \tilde{v}_A(p)$. Furthermore, $\partial \tilde{v}_A/\partial p > 0$ if and only if $\partial \sigma_{v_F|v_A}/\partial h_F > 0$. (ii) Finally, $\partial b^A/\partial h_A > 0$ if and only if $p > 1/2$.*

According to Lemma 5-i, better screening leads to a counterclockwise rotation of b^A around an invariant type $\tilde{v}_A$. That is, high types are more confident, while low types are less confident, of

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[27]Notice that we need to accommodate the standard notion of assortative matching to our decentralized, sequential screening process since it is possible that applicants with expected match value $E[\theta|v_A, v_F]$ are rejected (because $v_F < \underline{v}_F$), while applicants with $E[\theta|v'_A, v'_F] < E[\theta|v_A, v_F]$ with lower match value are accepted (because $v'_F > \underline{v}_F$). Following Smith (2011), the equilibrium is assortative in the sense that if (v_A, v_F) are hired and (v'_A, v'_F) are also hired, then $(\max\{v_A, v'_A\}, \max\{v_F, v'_F\})$ must also be hired, and if (v_A, v_F) are not hired (because they don't apply or, having applied, they don't meet the hiring standard) and (v'_A, v'_F) are also not hired, then $(\min\{v_A, v'_A\}, \min\{v_F, v'_F\})$ must also be hired.

[28]This is restricting attention to the range of undominated strategies given in Lemma 3.

[29]Alonso (2015a) explores the extent to which the presence of adverse selection affects the returns to recruiting and selection activities.

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beating a given hiring standard. Moreover, if one requires a higher passing probability, the invariant type $\tilde{v}_A$ increases when one requires a higher passing probability iff a better interview is also less predictable.

The intuition is as follows. First, applicants expect the interview score to be more responsive to match value -good ex ante matches $(v_A > 0)$ expect higher average scores while poor ex ante matches $(v_A < 0)$ expect lower average scores-. That is, the change in $E[v_F|v_A]$ accounts for the rotation of b^A. Second, applicant's payoffs follow a call-option as applicants with low interview scores are rejected and obtain their outside option. Thus, when a more informative interview is also less predictable $(\partial \sigma_{v_F|v_A}/\partial h_F > 0)$, it increases the hiring probability, and thus increases b^A, when $p < 1/2$ (i.e., the applicant is a "long shot"), but it will reduce his hiring probability if $p > 1/2$ (i.e., when the applicant is a "shoe-in" for the job). The effect of a more informative interview on $\sigma_{v_F|v_A}$ is however ambiguous: a higher h_F leads to a higher correlation between v_A and v_F but also increases the unconditional variance of v_F. The combined effect leads to a more predictable interview score iff both h_A and h_F are sufficiently high. More specifically we have that

$$\frac{\partial \sigma_{v_F|v_A}}{\partial h_F} < 0 \iff h_0 < \frac{h_A h_F}{h_0 + h_F + h_A}. \tag{22}$$

To understand Lemma 5-ii note that increasing h_A does not affect an applicant's expected interview score but reduces its variance. Therefore, if the applicant is a "shoe-in" $(p > 1/2)$, higher h_A increases her chances of being hired, and thus $\partial b^A/\partial h_A > 0$, while it makes hiring less likely if the applicant is a "long-shot" $(p < 1/2)$, in which case $\partial b^A/\partial h_A < 0$.

Firm's inference problem   The reaction function $b_F(\underline{v}_A, w)$ gives the hiring standard that the firm optimally sets when $\underline{v}_A$ is the lowest type in the applicant pool and the firm must pay w to every worker. The following lemma describes the effect on $b_F(\underline{v}_A, w)$ of increasing h_A or h_F.

Lemma 6. *For fixed $(\underline{v}_A, w)$ we have (i) $\partial b_F/\partial h_F > 0$, and (ii) $\partial b_F/\partial h_A < 0$ if $p \geq 1/2$, while for $p < 1/2$ there exists $\check{v}_A$ such that $\partial b_F/\partial h_A > 0$ for $\underline{v}_A < \check{v}_A$ iff $\left(\frac{h_A}{h_0 + h_A}\right)^3 \frac{h_F}{h_0 + h_F} > \frac{1}{4}$.*

In words, a better interview always leads the firm to raise its hiring standard, while the firm demands a higher hiring standard from better informed applicants if (i) the applicant pool is not sufficiently selective, and (ii) both applicants are sufficiently well informed and the interview is not too noisy. To understand Lemma 6-i, note that a more informative interview would lead to a revision of the firm's posterior expectation of match value (6) according to

$$\frac{\partial E[\theta|v_F, v_A \geq \underline{v}_A]}{\partial h_F} = \frac{h_A v_F - (h_0 + h_A) E[v_A|v_F, v_A \geq \underline{v}_A]}{(h_0 + h_A + h_F)^2} + \frac{h_0 + h_A}{h_0 + h_F + h_A} \frac{\partial E[v_A|v_F, v_A \geq \underline{v}_A]}{\partial h_F}. \tag{23}$$

This expression reflects the dual role of v_F in providing a direct estimate of θ and also refining the firm's estimate of the applicant's type. Looking at the rhs of (23), the first term represents the increase in the relative weight that the firm puts on the interview score compared to the "application signal" $\{v_A : v_A \geq \underline{v}_A\}$, while the second term captures the effect of a better interview on the firm's ability to "detect" which applicant type it is facing. As a better interview provides a less noisy estimate of v_A, it leads to a reduction in the truncated expectation $E[v_A|v_F, v_A \geq \underline{v}_A]$. That is, for the same signal realizations v_F and $\{v_A : v_A \geq \underline{v}_A\}$, the firm becomes less optimistic about the type of applicant it is evaluating. Combining these effects, Lemma 6-i states that improving the interview always makes the firm more skeptical of match value since (23) is always negative. As a result, the firm will demand a higher hiring standard when adopting a more informative interview, regardless of the composition of the applicant pool.

Following a similar decomposition on the effect of better informed applicants on the firm's posterior expectation, we have

$$\frac{\partial E[\theta|v_F, v_A \geq \underline{v}_A]}{\partial h_A} = \frac{h_F E[v_A|v_F, v_A \geq \underline{v}_A] - (h_0 + h_F) v_F}{(h_0 + h_A + h_F)^2} + \frac{h_0 + h_A}{h_0 + h_F + h_A} \frac{\partial E[v_A|v_F, v_A \geq \underline{v}_A]}{\partial h_A}.$$
(24)

Looking at the rhs of (24), the first term is the change in the relative weight of the application signal, while the second term is the change in the firm's ability to predict the applicant's type. If the applicant pool is selective (large $\underline{v}_A$), the firm puts more weight on the applicant signal and is better able to predict the applicant's type. These two effects imply that (24) is positive. Thus, the firm lowers the required hiring standard when applicants in a selective pool are better informed (cf. Lemma 6-ii). Lemma 6-ii also shows that advertising to a non-selective applicant pool (sufficiently negative $\underline{v}_A$) would actually lead to a higher hiring standard if applicants were sufficiently well-informed.

5.2 Equilibrium effects of improved screening and recruitment

How would applicants react to an interview process that imposes the same application costs but better identifies match value? When will better informed applicants be more willing to submit to the firm's hiring process? To answer these questions, I now study the combined change in the reaction functions described in Lemmas 5 and 6. Indeed, letting $\underline{v}_A$ be the equilibrium marginal applicant and $p = c_A/(w_E - \underline{w})$, whenever

$$\frac{\partial b_A(\underline{v}_A, p)}{\partial h_i} > \frac{\partial b_F(\underline{v}_A, w_E)}{\partial h_i},$$
(25)

increasing h_i lowers the equilibrium $\underline{v}_A$. This follows, as the marginal applicant would be willing to meet a strictly higher standard than the new one set by the firm. Conversely, if (25) does not hold, then increasing h_i would discourage applications and lead to a more selective applicant pool. I study, separately, the effect of a better interview and the effect of better informed job seekers on the applicant pool.

5.2.1 Effect of improved screening on the applicant pool

The following proposition summarizes the equilibrium change in $\underline{v}_A$ from increasing h_F.

Proposition 4 *Consider a fixed w_E. Then, there exist two cut-off levels $0 < \underline{p}^F \leq \overline{p}^F < 1$ such that $\partial \underline{v}_A / \partial h_F \geq 0$ if $p \leq \underline{p}^F$ or $p \geq \overline{p}^F$.*

Depending on the composition of the applicant pool, a more informative interview can either dissuade more job seekers from applying or encourage more applications. Lemma 5.i shows that a better interview will induce high types to beat a tougher hiring standard, while it will discourage low types. Furthermore, the firm always sets a higher hiring standard for a given applicant pool in response to a less noisy interview (cf. Lemma 6-i). It readily follows then that if the marginal applicant is weak (i.e., low $\underline{v}_A$)- or equivalently, when her probability of being hired is small- a better interview induces a more selective applicant pool, as the firm demands a higher hiring standard, but low types expect lower average scores. However, if the marginal applicant is strong (i.e., high $\underline{v}_A$), she is willing to beat a higher standard, but the firm also rationally raises the hiring standard. The proposition shows that this second effect dominates, and a less noisy interview also reduces applications from a selective applicant pool. Figure 2 summarizes these two cases of low $\underline{v}_A$ and high $\underline{v}_A$.

Finally, a better interview may actually induce more job seekers to apply. This is the case when the marginal applicant is "mediocre." Intuitively, the firm's hiring standard increases less in response to a better interview, as the applicant pool becomes less selective. If the marginal applicant expects higher average scores, however, then improving the interview can result in more applications and aid the firm's recruitment activity.

5.2.2 Effect of improved recruitment on the applicant pool

Suppose now that as a result of advertising, or of the choice of recruitment channel, job seekers are better informed of match value. What effect will it have on the equilibrium composition of the applicant pool? The following proposition provides comparative statics on the marginal applicant with respect to h_A.
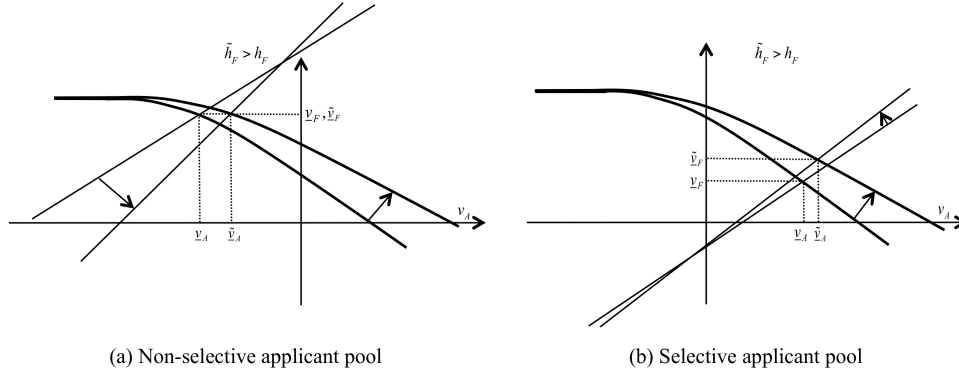
(a) Non-selective applicant pool   (b) Selective applicant pool

Figure 2: Effect of improved screening on marginal applicant.

Proposition 5. *Consider a fixed w_E. Then, there exist two cut-off levels $0 < \underline{p}^A \leq \overline{p}^A < \frac{1}{2}$ such that $\partial \underline{v}_A(w_E, p)/\partial h_A \leq 0$ if $p \geq \overline{p}^A$ and $\partial \underline{v}_A(w_E, p)/\partial h_A \geq 0$ if $p \leq \underline{p}^A$.*

Improving job seekers' information has opposite effects on applications depending on how selective the applicant pool is. To see this, note that a relatively high hiring probability (in particular, $p \geq 1/2$) also implies a "strong" marginal applicant. Lemma 5-ii shows that increasing h_A reduces the perceived variance of the interview, and, thus, a strong marginal applicant is willing to beat a higher hiring standard, while the firm reacts by lowering the hiring standard (cf. Lemma 6-ii). Both effects then lead to a reduction in $\underline{v}_A$ and, thus, an increase in the size of the applicant pool. Conversely, a low hiring probability also implies a "weak" marginal applicant. On the one hand, the firm may react to a higher h_A by demanding a higher or lower hiring standard (cf. Lemma 6-ii). On the other hand, the reduction in the option value of applying makes the marginal applicant unwilling to beat the previous hiring standard. Proposition 5 shows that this second effect always dominates when the probability that the marginal applicant gains employment is sufficiently low. Figure 3 shows graphically the effect of increasing on both a non-selective and a selective applicant pool.

5.2.3   Interpreting effects on the applicant pool as changes in the informativeness of the application signal.

Propositions 4 and 5 show that improving the information on either side of the market has different effects on the composition of the applicant pool. I now interpret these results in light of the changes in the *informativeness of the application signal* as the applicant pool becomes more selective. To this end, recall from Section 3 that a more selective applicant pool is also "more informative," as

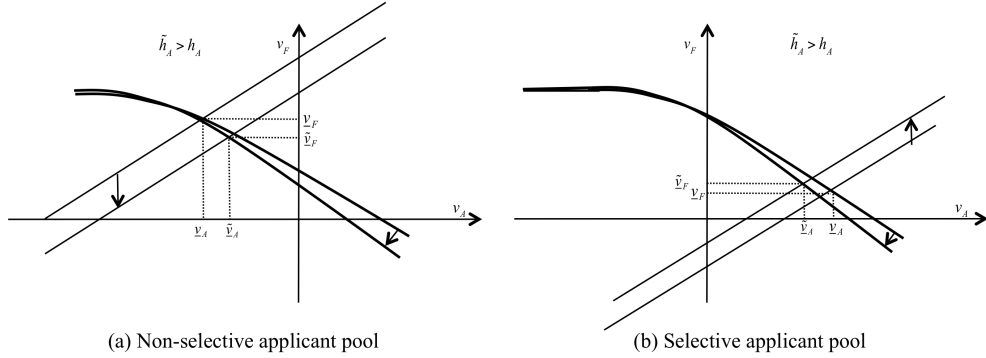(a) Non-selective applicant pool   (b) Selective applicant pool

Figure 3: Effect of improved recruitment on marginal applicant.

the firm faces less uncertainty about the identity of a randomly drawn applicant. This suggests two benchmarks: one in which the firm regards the application signal as uninformative, and one in which the firm perfectly observes each applicant's type.

Benchmark 1: Uninformative "application signal." Suppose that the firm does not take into account the self-selected nature of the applicant pool and believes that every applicant is a random draw from the job seekers' population. Then, the firm considers only the interview score to appraise the applicant's match value and sets a fixed hiring standard $\underline{v}_F = w$. In this case, changes in h_F or h_A do not alter the hiring standard, and, thus, the marginal applicant behaves according to Lemma 5. If the marginal applicant with a hiring probability p' is still willing to apply after increasing h_F, this will be true for any $p > p'$. That is, a better interview encourages applications when the marginal applicant is "strong" and discourages applications when the marginal applicant is "weak." Moreover, a higher h_A reduces the perceived variance of the interview score, and, thus, dissuades weak applicants but encourages strong applicants. In summary, improving the information on either side of the market always encourages applications when the applicant pool is selective and dissuades applicants when the applicant pool is non-selective.

Benchmark 2: Observable applicant's type. Now consider a setup where v_A is perfectly observed by the firm, because, for example, it is "hard" information and the applicant discloses it. The firm then sets a type-dependent hiring standard $\underline{v}_F(v_A)$ according to

$$\frac{h_0 + h_F}{h_0 + h_F + h_A}\underline{v}_F(v_A) + \frac{h_0 + h_A}{h_0 + h_F + h_A}v_A = w. \tag{26}$$

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The applicant's prediction problem is simplified in this case, as the law of iterated expectations implies that her estimated interview score is independent of the precision of the signals- i.e., $E[E[\theta|v_F, v_A]|v_A] = v_A$. Moreover, the conditional variance of $E[\theta|v_F, v_A]$ given v_A is simply

$$\sigma^2_{E[\theta|v_F, v_A]|v_A} = \frac{h_F}{(h_0 + h_A)(h_0 + h_F + h_A)},$$

which always increases in h_F and always decreases in h_A. In effect, when credentials are "hard information," a better interview makes the firm's final assessment noisier to the applicant, while a better informed applicant actually perceives the final assessment as being less noisy. This implies that improving the information on either side of the market now has opposing effects: if the applicant pool is very selective, a better interview discourages applicants and more informative advertising encourages applications; a non-selective applicant pool will be reduced if the firm engages in informative advertising, but will actually attract more applicants upon adoption of a more discriminating interview.

Equilibrium approximation for selective and non-selective applicant pools   These two benchmark cases exhibit opposing effects in the extreme situations when the marginal applicant has either a high or a low probability of being hired. Moreover, both cases provide good approximations of the equilibrium given by (10) and (11). On the one hand, as the hiring probability $p = c_A/(w_E - \underline{w})$ tends to zero, the applicant pool becomes indistinguishable from the general population of job seekers, and

$$E[\theta|v_F, v_A \geq \underline{v}_A] \approx E[\theta|v_F, v_A \in \mathbb{R}] = v_F.$$

In other words, when ex-ante sorting of applicants is muted, the firm rationally disregards the fact that an applicant is willing to be evaluated. Therefore, for non-selective applicant pools, both a better interview and more-informative advertising discourages applications.

On the other hand, for large p, the applicant pool is a fairly selective group of job seekers. Because the hazard rate of a normal distribution increases without bound, the expected match value can be approximated by

$$E[\theta|v_F, v_A \geq \underline{v}_A] \approx \frac{h_0 + h_F}{h_0 + h_F + h_A}v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A}\underline{v}_A.$$

In effect, a very selective applicant pool also provides a very informative signal of the applicant's type and the firm's hiring rule approximates one in which the applicant's type is observable to the firm and always equals $\underline{v}_A$. Therefore, for a selective applicant pool, a better interview also dissuades applicants while informative advertising actually attracts applicants.

6   Recruitment and Selection

I now consider the firm's incentives to engage in activities that improve the recruitment and selection phases of the hiring process. First, the firm could improve recruitment by reducing frictions in the job seekers' application, through, for example, activities that lower c_A. It is immediate that the firm always benefits from lower application costs, as it can then attract the same applicant pool with a lower wage.[30] Second, the firm could face better informed job seekers by either supplying information through informative advertising or by using recruitment channels associated with better-informed job seekers (e.g. relying on referrals). Third, the firm could improve its selection of applicants by adopting evaluation techniques that reduce the uncertainty surrounding match value. I restrict attention to the latter two cases and adopt a reduced-form approach by positing that improving recruitment leads to a marginal increase in h_A, while improving selection marginally raises h_F. In effect, improving recruitment increases the information available to job seekers, while improving selection increases the information available to the firm through an interview.

What are the firm's incentives to improve the information on each side of the market? Abstracting from the costs of implementation, an application of the envelope theorem to the firm's equilibrium profits leads to the following decomposition of the total effect of increasing h_i, $i \in \{A, F\}$, into a direct and an indirect effect:

$$\frac{d\Pi}{dh_i} = \underbrace{\frac{\partial \Pi}{\partial h_i}}_{\text{direct effect}} + \underbrace{\frac{\partial \Pi}{\partial \underline{v}_A} \frac{d\underline{v}_A}{dh_i}}_{\text{indirect effect}} . \tag{27}$$

Increasing h_i implies that matching would be performed with a less noisy appraisal of match value (direct effect), but would affect the firm's recruitment costs of the firm as a result of the change in the applicant pool (indirect effect). To analyze (27), let $\mu = \Pr[v_F \geq \underline{v}_F, v_A \geq \underline{v}_A]$ be the probability that a randomly chosen job seeker applies to the firm and is hired. Given the unit mass of job seekers, μ also describes the equilibrium employment by the firm. Also, let $\gamma_i = E\left[\theta | \underline{v}_i, v_j \geq \underline{v}_j\right], i, j \in \{A, F\}$ and $i \neq j$. In words, γ_A is the expected match productivity of the marginal applicant that passes the interview test, while γ_F is the expected match productivity of the marginal hire.

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[30]This argument relies on the assumption of advantageous selection, which is satisfied in this case. If a lower wage reduces the ex-ante quality of the applicant pool— for instance, if match-specific productivity is correlated with each applicant's outside option— then increasing frictions may actually improve hiring outcomes. See Horton (2013) for some experimental evidence and Alonso (2015a) for a theoretical analyisis of hiring in the presence of correlated match productivity.

Lemma 7. *The direct and indirect effect in (27) are given by*

$$\frac{\partial \Pi}{\partial h_i} = \underbrace{\frac{1}{2(h_0 + h_i)} Var\left[\theta | \underline{v}_i, v_j \geq \underline{v}_j\right]\left(-\frac{\partial \mu}{\partial \underline{v}_i}\right)}_{\text{sorting effect}} + \underbrace{(\gamma_i - w_E)\frac{\partial \mu}{\partial h_i}}_{\text{dispersion effect}}, \qquad (28)$$

$$\frac{\partial \Pi}{\partial \underline{v}_A} = (\gamma_A - w_E)\left(\frac{\partial \mu}{\partial \underline{v}_A}\right) < 0, \qquad (29)$$

for $i, j \in \{A, F\}, i \neq j$, where the change in employment following a more informative signal is

$$\frac{\partial \mu}{\partial h_i} = \frac{1}{2(h_i + h_0)}\left(\gamma_i - \frac{h_i - h_0}{h_i}\underline{v}_i\right)\left(-\frac{\partial \mu}{\partial \underline{v}_i}\right). \qquad (30)$$

To understand Lemma 7, first consider (28) which is the direct effect of a higher h_i. The first term in (28), the *sorting effect*, is proportional to the variance of match value at the margin of the relevant decision maker. This term captures the idea that a more precise signal better separates "the wheat from the chaff," as it would lead to a stochastically higher v_i for higher θ and, conversely, a stochastically lower v_i for lower θ. The second term in (28) is the *dispersion effect*: a higher h_i increases the unconditional variance of v_i and, thus, changes the likelihood that a random job seeker gains employment at the firm (by changing the likelihood of applying, or of being hired). The effect on profits, then, depends on whether raising h_i increases employment ($\partial \mu / \partial h_i$) and on the firm's profit on the marginal decision maker ($\gamma_i - w_E$).

Turning to the indirect effect in (27), $\partial \Pi / \partial \underline{v}_A$ is always strictly negative, as the firm's monopsonistic behavior implies a strictly profitable marginal applicant if hired— i.e., $\gamma_A > w_E$. Therefore, the sign of the indirect effect is given by the sign of $d\underline{v}_A / dh_i$— i.e., on whether a more precise signal dissuades or attracts applicants in equilibrium. I next study the total effect separately for the case of a more discriminating interview and for the case of informative advertising of job/firm characteristics.

6.1 Marginal Returns to Improved Selection.

How would the firm benefit from having access to a marginally more informative interview? It is easy to see that the direct effect of a more discriminating interview is always positive. This follows from two observations. First, sequentially rational hiring decisions require the firm to obtain a zero profit on the marginal hire; thus, $\gamma_F - w_E = 0$, and the dispersion effect in (28) is zero. That is, changes in total employment, as more or fewer applicants pass the more discriminating interview, have no effect on firm's profits when the firm makes no profit on the marginal hire. Second, the sorting effect in (28) is always positive: for the marginal hire $\underline{v}_F$, a better test would increase the probability that $v_F < \underline{v}_F$ if $\theta < \underline{v}_F$, while it would increase the probability that $v_F > \underline{v}_F$ if $\theta > \underline{v}_F$.

In effect, a bad match would be more likely to fail the interview, thus reducing type I errors in selection, while a good match would be more likely to pass it, thus reducing type II errors. As this sorting effect is proportional to the variance of the marginal hire, $\partial\Pi/\partial h_F$ decreases in h_F and vanishes as the interview becomes perfectly informative.

The indirect effect $\frac{\partial\Pi}{\partial \underline{v}_A}\frac{d\underline{v}_A}{dh_F}$, where $\partial\Pi/\partial\underline{v}_A$ is given by (29) and $d\underline{v}_A/dh_F$ is given in Proposition 5, captures the interdependence between recruitment and selection activities: a more discriminating interview affects hiring costs through the equilibrim effect on applicant recruitment. This effect is negative if and only if a better test discourages applications ($d\underline{v}_A/dh_F > 0$), as the firm would need to pay a higher wage to attract the same applicant pool.

It follows that the total effect is always positive if a better interview induces more applications. Following Proposition 5, this is the case when the marginal applicant has an intermediate chance of being hired. However, the total effect can be very small and even negative, so the firm actually benefits from a noisier interview. For instance, if the interview is very informative, the direct effect of further improvements is small. While the value of the marginal applicant can be quite high, especially if application decisions are made with poor information, the effect on the marginal applicant $d\underline{v}_A/dh_F$ as $h_F \to \infty$ is also negligible. Nevertheless, the following proposition shows that, if a better interview discourages applications, there is a threshold $\tilde{h}_F$ such that the total effect is always negative for $h_F > \tilde{h}_F$. In other words, for fear of discouraging applications, a firm would cease to improve its interview beyond $\tilde{h}_F$, even if it is *costless to perfectly evaluate* match quality.

Proposition 6. (Negative total effect of improved selection) *Given h_A and h_0, there exist t_H and t_L such that whenever $\underline{w}+c_A < t_L$ or $\underline{w}+c_A > t_H$, there exist $\tilde{h}_F$ such that $d\Pi/dh_F < 0$ for any $h_F > \tilde{h}_F$.*

6.2 Marginal Returns to Improved Recruitment.

Is the firm better off when recruiting from a population of better informed job seekers? From (28), the direct effect of higher h_A is

$$\frac{\partial\Pi}{\partial h_A} = \frac{1}{2(h_0 + h_A)}Var\left[\theta|\underline{v}_A, v_F \geq \underline{v}_F\right]\left(-\frac{\partial\mu}{\partial\underline{v}_A}\right) + (\gamma_A - w_E)\frac{\partial\mu}{\partial h_A}. \tag{31}$$

The first term in (31) is the sorting effect of higher h_A and is always positive: a less noisy v_A leads to a higher correlation between match value and the application decision, ultimately improving the quality of its applicant pool. The second term in (31) is the dispersion effect: a higher h_A, by increasing the unconditional variance of v_A, leads to changes in the size of the applicant pool and equilibrium employment. Noting that the marginal applicant that passes the interview is always

a profitable match— i.e., $\gamma_A > w_E$— the dispersion effect is negative if and only if the firm's employment is reduced when job seekers are better informed.

Can the direct effect (31) be negative? The answer is yes. To see this, note that combining (30) for $i = A$ with (31), we have

$$sign\left[\frac{\partial \Pi}{\partial h_i}\right] = sign\left[Var\left[\theta|\underline{v}_A, v_F \geq \underline{v}_F\right] + (\gamma_A - w_E)(\gamma_A - \frac{h_A - h_0}{h_A}\underline{v}_A)\right]. \tag{32}$$

If the marginal applicant is below the population average ($\underline{v}_A < E[\theta] = 0$), the second term of (32) becomes unbounded from below as h_A becomes arbitrarily small. That is, when job seekers have very poor information concerning their person-organization fit, but, nevertheless, the majority of them apply for a job, then informative advertising would actually reduce the firm's profits, holding application and hiring decisions constant. The intuition is that more informative signals can have an adverse impact under suboptimal decision rules, as too few applicants apply and, thus, $\gamma_A - w_E > 0$. Therefore, the standard monopsony inefficiency in firm recruitment then leads to a negative value of information (for the firm), holding application decisions constant. A general lesson in matching markets with dispersed information is that improved information leads to better matching (Shimer and Smith 2000). In this case, however, even absent the strategic impact on application and selection, better informed applicants can be detrimental to the firm.

The total effect of improved recruitment is positive whenever it leads to more applications because it increases the mass of applicants that believe themselves to be a good match and also attracts applications from lower types. Conversely, as the following proposition shows, the total effect of facing better informed job seekers can actually be negative.

Proposition 7. (Negative total effect of recruitment) *There exist $t_A < 0$ such that whenever $c_A + \underline{w} < t_A$, there exists a threshold $\tilde{h}_A$ such that $d\Pi/dh_A < 0$ for $h_A < \tilde{h}_A$.*

Interestingly, the proposition shows that if job seekers are poorly informed of fit, then the firm may never profit from reducing their uncertainty about fit. This is the case when the average job seeker is a good match for the firm (and application costs are low) and a majority apply to the firm (so that $\underline{v}_A < 0$). Providing some information to job seekers may lead applicants to apply elsewhere, although this decision is made with a very noisy assessment of match value.

7 Extensions

A feature of this hiring model is that, apart from information asymmetries, the only other friction hindering matching is the application cost c_A. In this section, I perform a robustness check of the main insights by allowing for alternative (and perhaps more realistic) matching frictions. First, I

extend the basic analysis by allowing for a costly interview— i.e., $c_F > 0$. I then consider the effect of slot constraints on the incentives to improve screening and recruitment, as well as the effect of the interaction between pre-hiring screening and post-hiring on-the-job learning on the applicant pool that a firm attracts. I show that the main insight of this paper— that improving the information of either side of the market may raise a firm's hiring costs when it discourages applications— continues to hold, although the equilibrium effect on the applicant pool can be noticeably more complex.

7.1   Costly Firm Evaluation

Suppose, now, that $c_F > 0$, so that interviewing a positive mass m of applicants imposes a cost $c_F m > 0$. While applicants still need to incur the applications costs, the firm must now decide whether or not to interview applicants. As both the benefit of interviewing a randomly chosen applicant and the marginal cost of an interview are constant, for a given applicant pool, the firm will interview either all applicants or none.

To study the equilibrium implications of costly interviews, suppose that all types in the set A apply to the firm, and let the hiring standard $\underline{v}_F$ be given by (4). Then, the firm evaluates applicants iff

$$E[\theta - w_E | v_F \geq \underline{v}_F, v_A \in A]\Pr[v_F \geq \underline{v}_F | v_A \in A] \geq c_F + \max\{0, E[\theta - w_E | v_A \in A]\}. \qquad (33)$$

To understand (33), suppose, first, that the firm would not hire an applicant in the absence of an interview— i.e., $E[\theta - w_E | v_A \in A] < 0$. Then, (33) translates to

$$E[\theta - w_E | v_F \geq \underline{v}_F, v_A \in A]\Pr[v_F \geq \underline{v}_F | v_A \in A] \geq c_F.$$

That is, the firm evaluates an applicant only if the (positive) match surplus from a hired applicant, multiplied by the probability of hiring a random applicant from a pool A, exceeds the interview cost. Now suppose that the firm would hire an applicant in the absence of an interview. In this case, (33) translates to

$$-E[\theta - w_E | v_F < \underline{v}_F, v_A \in A]\Pr[v_F < \underline{v}_F | v_A \in A] \geq c_F. \qquad (34)$$

That is, the expected gain from screening out poor matches, multiplied by the probability that a bad match is detected and denied employment, exceeds the interview cost.

Who does the firm attract if it does not evaluate applicants? If the firm could not interview applicants, then there would always be an equilibrium in which the firm offers $w_E = \underline{w} + c_A$, and only job seekers with type $v_A \geq \underline{w} + c_A$ apply to the firm. When the interview cost is c_F, this

remains an equilibrium as long as

$$-E[\theta - w_E | v_F < \underline{v}_F, v_A \geq w_E] \Pr [v_F < \underline{v}_F | v_A \geq w_E] < c_F.$$

That is, for sufficiently high interview costs, the firm can credibly commit not to interview applicants, and all job seekers with $v_A \geq w_E$ apply and are hired. In fact, this application behavior maximizes the firm's profit, given that it does not interview applicants.

One implication of costly interviews is that the firm will attract fewer applicants and, therefore, will pay a lower wage. To see this, suppose that the firm hires only after an interviews. Then, the optimal wage satisfies the first-order condition

$$\Pr [v_F \geq \underline{v}_F, v_A \geq \underline{v}_A] = (E[\theta - w_E^* | v_F \geq \underline{v}_F, \underline{v}_A] \Pr [v_F \geq \underline{v}_F, \underline{v}_A] - c_F) \left(-\frac{d\underline{v}_A}{dw_E} \right) \Bigg|_{w_E = w_E^*},$$

which leads to a lower wage when compared to (19).

If the firm hires only after an interview and it evaluates all applicants, then the comparative statics of improved selection and recruitmet of Section 5 still hold. Suppose, now, that the firm does not interview workers, so that all types $v_A \geq \underline{w} + c_A$ apply and are hired. In this case, raising h_A is always beneficial for the firm. The key is in the sorting and dispersion effects in (28) of higher h_A. If the firm does not evaluate workers, then $\underline{v}_A = w_E$, and the firm makes a zero profit on the marginal applicant and, thus, the dispersion effect is always zero. As the sorting effect is always positive, and the precision of job seekers type does not affect application decisions (as $\underline{v}_A = w_E$), then the total effect is always positive.

Finally, suppose that the firm does not currently evaluate applicants but has access to a more discriminating interview. Note that even if (34) holds for a higher h_F and the applicant pool $\{v_A : v_A \geq \underline{w} + c_A\}$, the fact that applicants now face a positive rejection probability will lead them to demand a larger wage premium. In fact, if (34) is satisfied with equality, so that the firm has access to an interview technology that makes it indifferent between interviewing applicants or hiring without an interview, then it will never adopt marginal improvements to the interview's precision.

7.2   Slot constraints

In many realistic settings, firms have a fixed number of vacancies that they need to fill. Then, if all applicants prove to be good matches after the interview, only the top performers will be hired. To accommodate this possibility, suppose that the firm can hire, at most, a measure K of workers. To simplify the analysis, I assume that this constraint is binding, so that total employment is always equal to K.

In the base model, applicants exerted an informational externality, as a larger number of applications would lower the firm's assessment of each applicant and, thus, reduce each applicant's hiring probability. In contrast, when the firm faces only slot constraints, then applicants also impose a congestion externality, as a larger number of applications lowers each applicant's hiring probability when they all vie for a limited number of jobs. The following proposition shows that the firm's hiring rule can still be described by a threshold hiring standard and derives the optimal wage.

Proposition 8. (SC-Equilibrium) *If the firm's employment is limited to a mass K of workers, then*

(i) There exist a marginal applicant $\underline{v}_A^S$ and a hiring standard $\underline{v}_F^S$ such that all types $v_A \geq \underline{v}_A^S$ apply to the firm, and the firm hires an applicant iff $v_F \geq \underline{v}_F^S$. The marginal applicant $\underline{v}_A^S$ and the hiring standard $\underline{v}_F^S$ are the unique solution to

$$\Pr\left[v_F \geq \underline{v}_F^S, v_A \geq \underline{v}_A^S\right] = K, \tag{35}$$

$$\underline{v}_F^S = b_A(\underline{v}_A^S, \frac{c_A}{w_E - \underline{w}}). \tag{36}$$

(ii) Letting $\underline{v}_F^S(\underline{v}_A^S)$ be defined implicitly by (35) and $\underline{v}_A^S(w_E)$ be implicitly defined by

$$\underline{v}_F^S(\underline{v}_A^S) = b_A(\underline{v}_A^S, \frac{c_A}{w_E - \underline{w}}), \tag{37}$$

the optimal wage w_E^* solves

$$\left(\frac{\partial \Pi}{\partial \underline{v}_A^S} + \frac{\partial \Pi}{\partial \underline{v}_F^S} \frac{\partial \underline{v}_F^S}{\partial \underline{v}_A^S}\right) \frac{d\underline{v}_A^S}{\partial w_E} = K. \tag{38}$$

Proposition 8-i shows that slot constraints do not change the assortative nature of equilibrium: all types that believe themselves to be a good match apply to the firm, but only the (mass) K top performers are hired. Figure 4 depicts the reaction functions (35) and (36) for the case that $K < 1/2$. Proposition 8-ii describes the firm's optimal wage. Since the firm's reaction function (35) does not change with the wage, it follows that $d\underline{v}_A^S/\partial w_E$ is always negative: higher wages always generate more applications. The binding slot constraint implies that as the firm attracts more applicants, it must also raise its hiring standard. This implies that the value of the marginal applicant as given by (38) is still positive, albeit smaller than in the case with unlimited vacancies (19).

Because the firm's reaction function (35) is now driven by a slot constraint, the equilibrium effects of improved information are different from the ones obtained in Propositions 5 and 6.

Proposition 9. (Recruitment and Selection) *Fix a wage w_E, and let $p = c_A/(w_E - \underline{w})$ and $\underline{v}_A^S$ as defined by (37). Then,*

(i) increasing h_F always dissuades applicants— i.e., $\frac{d\underline{v}_A^S}{\partial h_F} > 0$; and
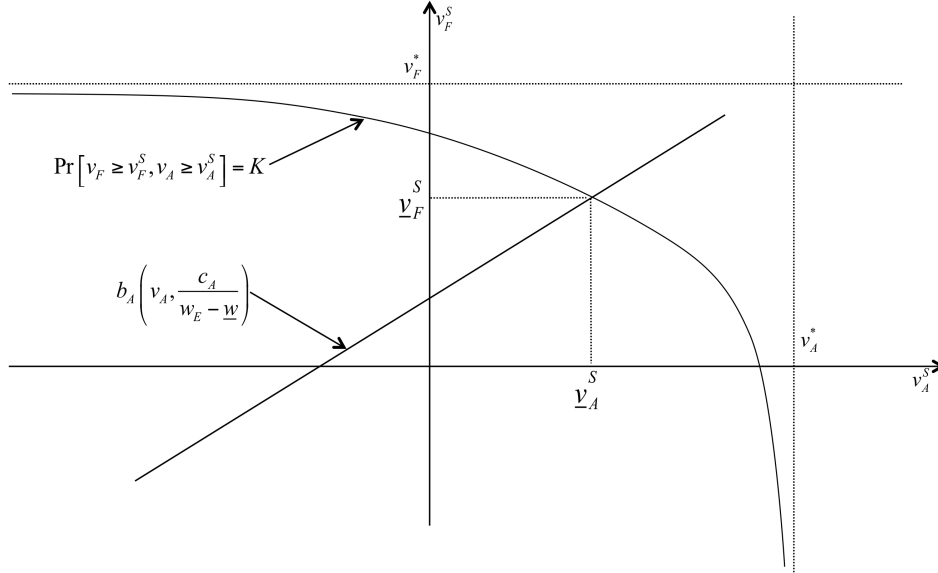
Figure 4: Hiring Standard and Marginal Applicant when the firm hires a mass K of workers.

(ii) if $K < 1/2$ and $h_A > h_0$, then there exist p^S such that increasing h_A dissuades applicants if $p < p^S$.

Unlike Proposition 6, Proposition 9-i shows that improving the interview will always dissuade applicants. The reason is that a higher h_F raises the conditional variance of v_F/v_A and leads to a stochastically-larger order statistics. That is, for a fixed applicant pool, a higher h_F increases the lowest score of the top K performers. This forces the firm to raise the hiring standard to satisfy the slot constraint. Proposition 9-ii shows that better informed job seekers are discouraged from applying if the marginal applicant is a "long-shot"— i.e., when her probability of being hired is low.

While the comparative statics of application decisions do change in the presence of slot constraints, the main insight still holds: because the marginal applicant is still strictly profitable for the firm, discouraging her application also raises hiring costs. In contrast to the setup without slot constraints, this is always the case with a more discriminating interview.

7.3 On-the-job learning about Match Quality

In the model, the firm screens applicants in order to avoid unsuitable matches (if it would otherwise hire all applicants without an interview) or to uncover good matches (if it would otherwise avoid hiring without an interview). Typically, firms also learn progressively about match value once the worker is employed and can limit the impact of adverse matches by terminating the employment

relationship (Jovanovic 1979; see Waldman (2013) for a comparison to alternative learning theories). Indeed, on-the-job learning about match quality, coupled with costless termination, provides firms with an incentive to favor "risky workers" when the uncertainty over match value is higher (Lazear 1995).

The literature has shown that these two informational sources— interviews and on-the-job learning— act as substitutes (Pries and Rogerson 2005), so that firms that find it relatively easy/costless to learn about match value from the workers' performance are less willing to invest in pre-employment screening. The model could accomodate the effect of on-the-job learning on termination of bad matches by imposing a lower bound on the match-specific productivity of a worker. To this end, suppose that during post-hiring employment, the firm can costlessly eliminate matches whose quality does not exceed a given threshold, say $\underline{\theta}$. Then, $f(\theta) = \max[\theta, \underline{\theta}]$ is the productivity of a $\theta-$worker when hired by the firm, and the firm's hiring standard (4) when types in the set A apply now satisfies

$$E\left[f(\theta)|\underline{v}_F^L(A), v_A \in A\right] = w_E.$$

Because f is a non-decreasing transformation of θ, the expectation $E\left[f(\theta)|v_F, v_A \in A\right]$ is monotone in v_F. Therefore, the equilibrium will again be characterized by a threshold hiring rule and a monotone application decision. Furthermore, since $f(\theta) \geq \theta$, the firm will set a lower hiring standard for any application decision; that is, for given w_E, $\underline{v}_F^L(A) \leq \underline{v}_F(A)$, where $\underline{v}_F(A)$ satisfies (4). In summary, as on-the-job learning limits the firm's downside from employing risky workers, the firm rationally sets a lower hiring standard and employs more workers.

While on-the-job learning will affect the firm's incentives to submit applicants to an interview, the main qualitative results regarding the increase of h_F or h_A will still hold in this case, albeit in a different parameter range. For example, when the applicant pool is selective (e.g., when $\underline{w} + c_A >> 0$), a firm may want to advertise to job seekers even in the presence of on-the-job learning. Advertising has little informational effect on match quality in this case, but it reassures the marginal applicant of passing the interview test, thus lowering the hiring costs of the firm.

8   Conclusions

A basic tenet of Human Resource Management is that a limiting factor for pre-employment screening is the costly resources that need to be deployed to investigate each applicant. A firm would surely prefer a selection process that does not require more resources but yet is more informative about a workers's expected productivity. Conversely, it is understood that advertising costs con-

strain firms from providing more information to prospective applicants about the characteristics of the job and the work environment. I show in this paper these views to be incomplete in that firms also incur indirect costs from improved screening or informative advertising. In particular, when the firm cannot contractually cover applicants' costs, improvements that discourage applications raise hiring costs, as the firm must increase the wage premium to attract the same applicant pool.

The driving force in the analysis is job seekers' perceptions of their suitability for the job, which, given the interview process, determine each applicant's likelihood of receiving an offer. For instance, a more discriminating interview can encourage or discourage applications in equilibrium, depending on the quality of the applicant pool. Moreover, informative advertising of firm/job characteristics reduces job seeker's uncertainty about match value, making the interview process less noisy. Whether this leads to more applications depends on the marginal applicant's perceived likelihood of being hired. These effects shape the firm's willingness to improve recruitment or selection. I show that firms will never adopt a perfect interview, even if it is costless, when a more discriminating evaluation makes the marginal applicant less likely to gain employment. Firms may avoid advertising for similar reasons. Interestingly, firms will be unwilling to advertise when applicants are poorly informed and yet apply to the firm.

A maintained assumption in the analysis is that job seekers understand the accuracy with which they will be evaluated and that the firm can commit to such accuracy. Nevertheless, the fact that increasing the interview's precision always benefits the firm when the applicant pool is held constant, raises important concerns about the extent to which firms can, indeed, publicly commit to such practices. These observations regarding common knowlede and commitment to the selection technology are left for future research.

There are two main simplifications of the model. First, only one firm actively evaluates applicants. This obviates the possible effect of competition on the returns to adopting a more discriminating interview or providing information about match quality. Second, the model posits that all uncertainty surrounding the productivity of a worker regards its firm-specific component. In equilibrium, this leads to both positive assortative matching and advantageous selection. This setup can be useful for studying situations in which general human capital can be easily observed, although there is uncertainty over person-job/organization fit. Nevertheless, there are situations in which general human capital is not perfectly known and pre-hiring screening is necessarily imperfect. In this case, a high match value with a given firm may also imply a higher outside option when matching with other firms. This effect can then lead to both adverse and advantageous selection. Alonso (2015a) provides an initial exploration of both scenarios.

9 Appendix A

Proof of Lemma 1: Suppose that all job seekers $v_A \in A$ apply to the firm and are interviewed. As the firm cannot commit ex-ante to arbitrary hiring rules, it will issue an employment offer as long as

$$\mathrm{E}\left[\theta \,|v_F, v_A \in A\right] \geq w_E.$$

We now show that $\mathrm{E}\left[\theta \,|v_F, v_A \in A\right]$ is strictly increasing in v_F with an unbounded range, implying that (i) there is a unique solution to

$$\mathrm{E}\left[\theta \,|\underline{v}_F(A'), v_A \in A'\right] = w_E,$$

and (ii) whenever $v_F \geq \underline{v}_F(A')$, the firm hires the applicant.

We can write

$$
\begin{aligned}
\mathrm{E}\left[\theta \,|v_F, v_A \in A\right] &= \int_A \mathrm{E}\left[\theta \,|v_F, v_A\right] \frac{f\left(v_F, v_A\right)}{\Pr\left[v_F, v_A \in A\right]} dv_A = \int_{A'} \mathrm{E}\left[\theta \,|v_F, v_A\right] \frac{f\left(v_A | v_F\right)}{\Pr\left[v_A \in A | v_F\right]} dv_A \\
&= \int_A \left(\frac{h_0 + h_F}{h_0 + h_F + h_A} v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} v_A\right) \frac{f\left(v_A | v_F\right)}{\Pr\left[v_A \in A | v_F\right]} dv_A \\
&= \frac{h_0 + h_F}{h_0 + h_F + h_A} v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} \int_A v_A \frac{f\left(v_A | v_F\right)}{\Pr\left[v_A \in A | v_F\right]} dv_A.
\end{aligned}
$$

It suffices to show that the second term above is non-decreasing in v_F in order to establish that $\mathrm{E}\left[\theta \,|v_F, v_A \in A'\right]$ increases in v_F without bound, as the first term is strictly increasing and admits neither a lower nor an upper bound.

First, v_F and v_A satisfy the monotone likelihood ratio property (MLRP) as they satisfy it with the random variable θ (Karlin and Rubin 1956); that is, $f\left(v_A | v_F'\right) / f\left(v_A | v_F\right)$ increases in v_A for $v_F' > v_F$. Now, consider, with $v_F' > v_F$ the expression

$$\int_{A'} v_A \left(\frac{f\left(v_A | v_F'\right)}{\Pr\left[v_A \in A' | v_F'\right]} - \frac{f\left(v_A | v_F\right)}{\Pr\left[v_A \in A' | v_F\right]}\right) dv_A. \tag{39}$$

The MLRP of v_F and v_A implies that the function

$$\left(\frac{f\left(v_A | v_F'\right)}{f\left(v_A | v_F\right)} - \frac{\Pr\left[v_A \in A' | v_F'\right]}{\Pr\left[v_A \in A' | v_F\right]}\right) \frac{1}{\Pr\left[v_A \in A' | v_F'\right]}$$

is increasing in v_A, and

$$\int_{A'} \left(\frac{f\left(v_A | v_F'\right)}{f\left(v_A | v_F\right)} - \frac{\Pr\left[v_A \in A' | v_F'\right]}{\Pr\left[v_A \in A' | v_F\right]}\right) \frac{f\left(v_A | v_F\right)}{\Pr\left[v_A \in A' | v_F'\right]} dv_A = 0.$$

Lemma 1 in Persico (2000) then implies that (39) is non-negative. ∎

Proof of Lemma 2: Suppose that the firm hires any applicant that it evaluates if $v_F \geq \underline{v}_F$. Then, the expected gain to an applicant of type v_A from applying is

$$(w_E - \underline{w}) \Pr[v_F \geq \underline{v}_F | v_A] - c_A.$$

The proof of Lemma 1 showed that v_F and v_A satisfy the MLRP. This implies that $v_F | v_A'$ first-order stochastically dominates $v_F | v_A$ when $v_A' > v_A$, meaning that if type v_A is willing to incur the cost c_A and apply, so will any type $v_A' > v_A$. Finally, in our normally distributed example, we have that $v_F | v_A$ is normally distributed with mean $(h_F / (h_F + h_0)) v_A$ and variance independent of v_A. Therefore, for any finite $\underline{v}_F$, $\Pr[v_F \geq \underline{v}_F | v_A]$ is injective and takes any value in $(0, 1)$ for finite v_A. Therefore, for each $\underline{v}_F$, there is a unique $\underline{v}_A(\underline{v}_F)$ such that (7) is satisfied.∎

Proof of Proposition 1: If $w_E - \underline{w} < c_A$, then no job seeker will apply, as the wage premium does not cover the application costs. If $w_E - \underline{w} \geq c_A$, then any sequential equilibrium must satisfy (4) and (7); in particular, Lemma 2 implies that the applicant pool must be of the form $A = \{v_A : v_A \geq \underline{v}_A\}$. Let $b_F(\underline{v}_A, w)$ as defined by (9) and $b_A(v_A, p)$ as defined as by (8). Then, (4) in Lemma 1 can be written as

$$b_F(\underline{v}_A, w_E) = \underline{v}_F,$$

while (7) in Lemma 2 can be written as

$$b_A(\underline{v}_A, \frac{c_A}{w_E - \underline{w}}) = \underline{v}_F.$$

We now show that $b_F(v_A, w_E)$ is strictly decreasing in v_A, while $b_A(v_A, \frac{c_A}{w_E - \underline{w}})$ is strictly increasing in v_A. This implies that given $w_E \geq \underline{w} + c_A$, there is a unique continuation equilibrium: the applicant pool is $A = \{v_A : v_A \geq \underline{v}_A\}$ and the firm's hiring standard is $\underline{v}_F$ which solves (10-11).

First, from joint normality, we can write

$$b_A(v_A, p) = E[v_F | v_A] + \sigma_{v_F | v_A} \Phi^{-1}(p)$$

and, thus,

$$\frac{\partial b_A(v_A, p)}{\partial v_A} = \frac{\partial E[v_F | v_A]}{\partial v_A} = \frac{h_A}{h_0 + h_A} > 0.$$

Second, joint normality of signals also implies

$$E\left[\theta \,\middle|\, v_F, v_A' \geq v_A\right] = v_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} \sigma_{v_A | v_F} \lambda \left(\frac{v_A - E[v_A | \underline{v}_F]}{\sigma_{v_A | v_F}}\right),$$

and, thus,

$$\frac{\partial E\left[\theta \,\middle|\, v_F, v_A' \geq v_A\right]}{\partial v_A} = \frac{h_0 + h_A}{h_0 + h_F + h_A} \lambda' \left(\frac{v_A - E[v_A | \underline{v}_F]}{\sigma_{v_A | v_F}}\right) > 0,$$

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where positivity follows from the positive derivative of the hazard rate of the normal distribution. Lemma 1 already established $\partial E\left[\theta\,|v_F,v'_A \geq v_A\right]/\partial v_F > 0$. Therefore,

$$\frac{\partial b_F(v_A,w_E)}{\partial v_A} = -\frac{\frac{\partial E\left[\theta|v_F,v'_A \geq v_A\right]}{\partial v_A}}{\frac{\partial E\left[\theta|v_F,v'_A \geq v_A\right]}{\partial v_F}} < 0.$$

∎

Proof of Proposition 2: Suppose that the firm can condition payments on whether the applicant incurred the application costs. The firm then offers a contract that pays c, $c_A > c \geq 0$ if the applicant incurred the costs c_A and a wage w_C, $w_C > \underline{w}$ if the applicant is hired. Let $\underline{v}_A$ be the marginal applicant in this case; i.e., $\underline{v}_A$ solves

$$(w_C - \underline{w})\Pr\left[v_F \geq \underline{v}_F|\underline{v}_A\right] = c_A - c, \tag{40}$$
$$E\left[\theta|v_A \geq \underline{v}_A, \underline{v}_F\right] = w_C,$$

and firm profits are

$$\Pi = \int_{\underline{v}_A}^{\infty}\int_{\underline{v}_F}^{\infty}\int_{\Theta}(\theta - w_C)dF(\theta, v_F, v_A) - c\int_{\underline{v}_A}^{\infty}\int_{\Theta}dF(\theta, v_A).$$

Consider, now, a contract that pays $w'_C < w_C$ and $c' > c$ and induces the same marginal applicant. A lower wage induces a lower hiring standard $\underline{v}'_F < \underline{v}_F$ and, thus, $\tilde{\gamma} = \Pr\left[v_F \geq \underline{v}'_F|\underline{v}_A\right] - \Pr\left[v_F \geq \underline{v}_F|\underline{v}_A\right] > 0$. The change in the firm's profits $\Delta\Pi$ from adoptin this new contract is

$$\Delta\Pi = \int_{\underline{v}_A}^{\infty}\int_{\underline{v}'_F}^{\underline{v}_F}\int_{\Theta}(\theta - w'_C)dF(\theta, v_F, v_A) + \Pr\left[v_A \geq \underline{v}_A\right]\left[(w_C - w'_C)\Pr\left[v_F \geq \underline{v}_F|v_A \geq \underline{v}_A\right] - \left(c' - c\right)\right].$$

The first term is non-negative, as $E[\theta|v_A \geq \underline{v}_A, v_F] > w'_C$ for $v_F > \underline{v}'_F$. The term $(w_C - w'_C)\Pr\left[v_F \geq \underline{v}_F|v_A \geq \underline{v}_A\right]$ is the expected reduction in wage payments to each applicant interviewed. We then have that

$$\left(w_C - w'_C\right)\Pr\left[v_F \geq \underline{v}_F|v_A \geq \underline{v}_A\right] \geq \left(w_C - w'_C\right)\Pr\left[v_F \geq \underline{v}_F|\underline{v}_A\right]$$
$$= c' - c + \tilde{\gamma}\left(w'_C - \underline{w}\right)$$
$$> c' - c.$$

where the equality follows from (40). Therefore $\Delta\Pi > 0$, which implies that any contract that does not fully cover the applicant's costs is dominated.

Consider, therefore, the contract $(c, w_C) = (c_A, \underline{w})$. Now, any job seeker is indifferent between incurring the application costs or applying elsewhere. If the firm could optimally choose the

marginal applicant $\underline{v}_A^C$, and given sequentially rational decisions, the optimal choice would satisfy the first-order condition

$$\frac{\partial \Pi}{\partial \underline{v}_A} = -\int_{\underline{v}_F}^{\infty} \int_{\Theta} (\theta - \underline{w}) dF(\theta, v_F, \underline{v}_A^C) + c_A f(\underline{v}_A^C) = 0.$$

which is equivalent to (15). To see that this is the unique equilibrium, consider a potential equilibrium in which job seekers' application is a set $A \neq \{v_A \geq \underline{v}_A^C\}$. Then, the firm, by raising w_C above $\underline{w}$ and lowering c can induce a monotone application $\{v_A \geq \underline{v}_A\}$ with $\underline{v}_A$ arbitrarily close to $\underline{v}_A^C$. As the firm has a profitable deviation, the set A cannot define an equilibrium application decision.■

Proof of Lemma 3: Let $\underline{v}_A (w_E)$ be the unique solution to (10-11). Using the representations (12) and (14) and implicitly differentiating, we have

$$\frac{d\underline{v}_A (w_E)}{dw_E} = \frac{1 - \left[1 - \frac{h_A}{h_0 + h_F + h_A} \lambda'(z)\right] \beta'(w_E)}{\frac{h_F + h_0 \lambda'(z)}{h_0 + h_F}}, \tag{41}$$

where $z = (\underline{v}_A - E[v_A | \underline{v}_F]) / \sigma_{v_A | v_F}$ and $\beta(w_E) = -\sigma_{v_F | v_A} \Phi^{-1} (c_A / (w_E - \underline{w}))$. The standard normal distribution satisfies $0 < \lambda'(z) < 1$; thus, the denominator is positive and bounded. To study the numerator of (41), note that

$$
\begin{aligned}
\beta'(w_E) &= \sigma_{v_F | v_A} \frac{c_A}{\phi \left(\Phi^{-1} (c_A / (w_E - \underline{w}))\right) (w_E - \underline{w})^2} = \\
&= \sigma_{v_F | v_A} \frac{\Phi\left(\frac{\underline{v}_F - E[v_F | \underline{v}_A]}{\sigma_{v_F | v_A}}\right)}{(w_E - \underline{w}) \phi\left(\frac{\underline{v}_F - E[v_F | \underline{v}_A]}{\sigma_{v_F | v_A}}\right)} \\
&= \sigma_{v_F | v_A} \frac{1}{(w_E - \underline{w}) \lambda\left(\frac{\underline{v}_F - E[v_F | \underline{v}_A]}{\sigma_{v_F | v_A}}\right)},
\end{aligned}
$$

where we have exploited the symmetry of $\phi(x)$. First, $\beta'(w_E) > 0$, and it approaches ∞ as $w_E \to \underline{w} + c_A$, and, thus, (41) becomes unbounded from below. In other words, $\underline{v}_A (w_E)$ increases smoothly without bound as $w_E \to \underline{w} + c_A$. Second, the numerator of (41) changes sign at most once. Let $w_{\max}$ be the wage at which (41) is zero, so that for any $w_E > w_{\max}$, increasing the posted wage would discourage applications. As the range of $\underline{v}_A$ has no upper bound, this implies that for each wage above $w_{\max}$ there exists a wage below $w_{\max}$ that induces the same marginal applicant at a lower wage. As firm profits are higher at this lower wage, the higher wage is dominated and would never be offered in equilibrium.■

Proof of Proposition 3: First, we formally derive the expression (17). As $v_i | \theta$ is normally

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distributed we have

$$\int_{\underline{v}_i}^{\infty} dF(v_i|\theta) = 1 - \Phi\left[\frac{v_i - E[v_i|\theta]}{\sigma_{v_i|\theta}}\right] = 1 - \Phi\left[\frac{v_i - \frac{h_i}{h_i+h_0}\theta}{\frac{\sqrt{h_i}}{h_i+h_0}}\right]$$

$$= \Phi\left[\sqrt{h_i}\left(\theta - \frac{h_i + h_0}{h_i}v_i\right)\right] = \Phi\left[z_i(\theta, \underline{v}_i)\right],$$

with $z_i(\theta, \underline{v}_i)$ defined by (18). Then, from independence of $v_F|\theta$ and $v_A|\theta$ we obtain

$$\int_{-\infty}^{\infty} \int_{\underline{v}_F}^{\infty} \int_{\underline{v}_A}^{\infty} (\theta - w_E) \, dF(\theta, v_A, v_F) = \int_{-\infty}^{\infty} (\theta - w_E) \int_{\underline{v}_F}^{\infty} dF(v_F/\theta) \int_{\underline{v}_A}^{\infty} dF(v_A/\theta) dF(\theta)$$

$$= \int_{-\infty}^{\infty} (\theta - w_E) \, \Phi\left[z_A(\theta, \underline{v}_A)\right] \Phi\left[z_F(\theta, \underline{v}_F)\right] dF(\theta).$$

Let $\underline{v}_F(w_E)$ and $\underline{v}_A(w_E)$ be the solutions to (10) and (11). As the firm sets $\underline{v}_F$ optimally given the applicant pool $\{v_A : v_A \geq \underline{v}_A\}$, applying the envelope theorem gives the first-order condition

$$\left(\frac{d\Pi}{dw_E} = \right) \frac{\partial\Pi}{\partial w_E} + \frac{\partial\Pi}{\partial \underline{v}_A} \frac{d\underline{v}_A(w_E)}{dw_E} = 0.$$

Let $\mu = \Pr[v_F \geq \underline{v}_F, v_A \geq \underline{v}_A]$ be the probability that a random job seeker applies and is hired. Given the unit mass of job seekers, μ is also the employment level of the firm. Then, we can write the previous first-order condition as:

$$-\mu - \frac{d\underline{v}_A(w_E)}{dw_E} \int_{-\infty}^{\infty} \int_{\underline{v}_F}^{\infty} (\theta - w_E^*) \, dF(\theta, \underline{v}_A, v_F) = 0,$$

or

$$-\mu - \frac{d\underline{v}_A(w_E)}{dw_E} E[\theta - w_E^* | v_F \geq \underline{v}_F, \underline{v}_A] \Pr[v_F \geq \underline{v}_F, \underline{v}_A] = 0.$$

From this expression, we can readily obtain (19).

To show that $\lim_{c_A \to 0} \underline{v}_A = \underline{v}_A^0$, with $\underline{v}_A^0$ being the solution to (20) and (21), we rewrite the firm's first-order condition as

$$E[\theta - w_E^* | v_F \geq \underline{v}_F, \underline{v}_A] = \left(-\frac{1}{\frac{d\underline{v}_A(w_E)}{dw_E}}\right) \frac{\Pr[v_A \geq \underline{v}_A | F \geq \underline{v}_F]}{\Pr[\underline{v}_A | v_F \geq \underline{v}_F]}. \tag{42}$$

The rhs of (42) is always positive; the marginal applicant cannot be unprofitable to the firm, even as the costs vanish. We now show that this implies that the marginal applicant $\underline{v}_A$ is bounded from below as $c_A \to 0$. Suppose that this is not the case and that $\underline{v}_A \to -\infty$. Then we have

$$E[\theta | v_F \geq \underline{v}_F, \underline{v}_A] = \underline{v}_A + \frac{h_0 + h_F}{h_0 + h_F + h_A} \sigma_{v_F|v_A} \lambda\left(\frac{\underline{v}_F - E[v_F|\underline{v}_A]}{\sigma_{v_F|v_A}}\right)$$

$$\approx \frac{h_0 + h_F}{h_0 + h_F + h_A} \underline{v}_F + \frac{h_0 + h_A}{h_0 + h_F + h_A} \underline{v}_A.$$

Since $\underline{v}_F \leq w_E$, this expectation becomes unbounded from below as $\underline{v}_A \to -\infty$, and thus the rhs of (42) becomes negative, reaching a contradiction. Thus we must have a finite $\lim_{c_A \to 0} \underline{v}_A$.

A finite marginal applicant implies that (i) $\frac{d\underline{v}_A(w_E)}{dw_E}$ becomes unbounded as $c_A \to 0$ for $w_E \geq \underline{w}$; and (ii) the "hazard rate"

$$\frac{\Pr[v_A \geq \underline{v}_A | F \geq \underline{v}_F]}{\Pr[\underline{v}_A | v_F \geq \underline{v}_F]}$$

remains bounded. Therefore, the rhs of (42) approaches zero. As the wage premium vanishes as $c_A \to 0$, this establishes (20). Equation (21) simply reflects the firm's sequentially rational hiring standard and the fact that $\lim_{c_A \to 0} w_E^* = \underline{w}$. ∎

Proof of Lemma 5: From (8), we have

$$b_A(v_A, p) = \frac{h_F}{h_F + h_0} v_A - \sigma_{v_F | v_A} \Phi^{-1}(p).$$

Differentiating b_A wrt h_F shows that $\partial b^A / \partial h_F \geq 0$ iff $v_A > \tilde{v}_A$, where $\tilde{v}_A$ satisfies $\partial b^A(\tilde{v}_A)/\partial h_F = 0$ and is given by

$$\tilde{v}_A = \frac{(h_F + h_0)^2}{h_0} \frac{\partial \sigma_{v_F | v_A}}{\partial h_F} \Phi^{-1}(p).$$

As $\Phi^{-1}(p)$ increases in p and (22) implies that $\partial \sigma_{v_F | v_A}/\partial h_F > 0$ iff $h_0 < \frac{h_A h_F}{h_0 + h_F + h_A}$, then $\tilde{v}_A$ increases in p iff $h_0 < \frac{h_A h_F}{h_0 + h_F + h_A}$. Finally, from (3), we have that $\partial \sigma_{v_F | v_A}/\partial h_A < 0$ so that $sign(\partial b_A/\partial h_A) = sign(\Phi^{-1}(p))$. Therefore, $\partial b_A/\partial h_A > 0$ if and only if $p > 1/2$. ∎

Proof of Lemma 6: The firm's reaction curve $b_F(\underline{v}_A, w)$ is the hiring standard that satisfies (9). Define

$$G(v_A, v_F) = v_F + \sqrt{\frac{h_A}{(h_0 + h_F)(h_0 + h_A + h_F)}} \lambda(\tilde{z}(v_A, v_F)), \text{ with} \tag{43}$$

$$\tilde{z}(v_A, v_F) = \frac{v_A - \frac{h_A}{h_A + h_0} v_F}{\sigma_{v_A/v_F}}, \text{ and} \tag{44}$$

$$r = \Phi^{-1}(p). \tag{45}$$

We first characterize the behavior of $\tilde{z}$ with changes in r. By replacing the applicant's reaction function (8) into (44), we can write

$$\sigma_{v_A/v_F} \tilde{z}(\underline{v}_A, v_F(\underline{v}_A)) = \left(1 - \frac{h_A}{h_A + h_0} \frac{h_F}{h_F + h_0}\right) \underline{v}_A + \frac{h_A}{h_A + h_0} \sigma_{v_F/v_A} r.$$

Because $\underline{v}_A$ increases with p and, thus, increases with r, and r admits neither a lower nor an upper bound, $\tilde{z}$ also increases monotonically with r in an unbounded fashion.

Since $E[\theta | v_F, v_A \geq \underline{v}_A] = G(\underline{v}_A, v_F)$, the function b_F is implicitly defined by $G(\underline{v}_A, b_F) = w$. Applying the implicit function theorem one obtains

$$\frac{\partial b_F}{\partial h_i} = -\frac{\partial G(\underline{v}_A, v_F)/\partial h_i}{\partial G(\underline{v}_A, v_F)/\partial v_F}.$$

The denominator is always positive, since

$$\frac{\partial G(\underline{v}_A, v_F)}{\partial v_F} = 1 - \frac{h_A}{h_A + h_0 + h_F}\lambda'(\tilde{z}) > 1 - \frac{h_A}{h_A + h_0 + h_F} > 0. \tag{46}$$

Thus, $sign\left[\frac{\partial b_F}{\partial h_i}\right] = -sign\left[\partial G(\underline{v}_A, v_F)/\partial h_i\right]$. Consider, first, the change wrt h_F. After some manipulations, we have

$$\frac{\partial G(\underline{v}_A, v_F)}{\partial h_F} = \sqrt{\frac{h_A}{(h_0 + h_F)^3 (h_0 + h_A + h_F)}}P, \text{ with} \tag{47}$$

$$P = -\frac{(2h_0 + h_A + 2h_F)}{2(h_0 + h_A + h_F)}\lambda(\tilde{z}) + \frac{h_A}{(h_0 + h_A + h_F)}\tilde{z}\lambda'(\tilde{z}). \tag{48}$$

The hazard rate of a normal distribution satisfies $z\frac{\lambda'(z)}{\lambda(z)} \leq 1$, so

$$-\left(\frac{2h_0 + h_A + 2h_F}{h_A}\right) + \frac{\tilde{z}\lambda'(\tilde{z})}{\lambda(\tilde{z})} \leq -\left(2\frac{h_0 + h_F}{h_A} + 1\right) + 1 < 0,$$

and, thus, $\partial G(\underline{v}_A, v_F)/\partial h_F < 0$, implying that $\frac{\partial b_F}{\partial h_F} > 0$.

Consider, now, the change wrt h_A. Again, after some manipulations, we have

$$\frac{\partial G(\underline{v}_A, v_F)}{\partial h_A} = \tau\left(\kappa(\tilde{z}) + \frac{2h_A}{h_0 + h_A}r\lambda'(\tilde{z})\right) \tag{49}$$

with

$$\tau = \frac{(h_0 + h_F)(h_0 + h_A)}{2h_A(h_0 + h_A + h_F)^2}\sigma_{v_A/v_F} > 0,$$

$$\kappa(\tilde{z}) = \lambda(\tilde{z}) - \tilde{z}\lambda'(\tilde{z}). \tag{50}$$

The function $\kappa(\tilde{z})$ is positive, quasiconcave, with a maximum at $\tilde{z} = 0$ and $\lim_{|\tilde{z}|\to\infty}\kappa(\tilde{z}) = 0$. Since $\lambda'(\tilde{z}) \geq 0$, we then have that if $r \geq 0$ (i.e., if the probability that the marginal applicant is hired is at least $1/2$), then $\partial G(\underline{v}_A, v_F)/\partial h_A > 0$ and, thus, $\frac{\partial b_F}{\partial h_A} < 0$. To study the case of $r < 0$, differentiate $\kappa(\tilde{z}) + \frac{2h_A}{h_0 + h_A}r\lambda'(\tilde{z})$ with respect to r to obtain

$$\frac{d}{dr}\left(\kappa(\tilde{z}) + \frac{2h_A}{h_0 + h_A}r\lambda'(\tilde{z})\right) = \left[\left(-\tilde{z} + \frac{2h_A}{h_0 + h_A}r\right)\frac{\partial\tilde{z}}{\partial r}\frac{\lambda''(\tilde{z})}{\lambda'(\tilde{z})} + \frac{2h_A}{h_0 + h_A}\right]\lambda'(\tilde{z}).$$

The ratio $\lambda''(\tilde{z})/\lambda'(\tilde{z})$ is positive, decreasing, and becomes unbounded when $r \to -\infty$ (so that $\tilde{z} \to -\infty$). Moreover, as $r \to -\infty$, we have $\underline{v}_F \approx w$ and

$$\underline{v}_A \approx \frac{h_0 + h_F}{h_F}w + \frac{(h_0 + h_F)\sigma_{v_F/v_A}}{h_F}r,$$

$$\tilde{z} \approx h_0\frac{h_0 + h_A + h_F}{h_F(h_0 + h_A)\sigma_{v_A/v_F}}w + \sqrt{\frac{h_0 + h_F}{h_F}\frac{h_0 + h_A}{h_A}}r,$$

$$\frac{\partial\tilde{z}}{\partial r} \approx \sqrt{\frac{h_0 + h_F}{h_F}\frac{h_0 + h_A}{h_A}},$$

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which leads to

$$\lim_{r \to -\infty} \left(-\tilde{z} + \frac{2h_A}{h_0 + h_A} r \right) \frac{\partial \tilde{z}}{\partial r} = \varsigma \sqrt{\frac{h_0 + h_F}{h_F} \frac{h_0 + h_A}{h_A}} \lim_{r \to -\infty} r, \text{ with}$$

$$\varsigma = \left(-\sqrt{\frac{h_0 + h_F}{h_F} \frac{h_0 + h_A}{h_A}} + \frac{2h_A}{h_0 + h_A} \right).$$

Since $\lim_{r \to -\infty} \frac{\lambda''(\tilde{z})}{\lambda'(\tilde{z})} = \infty$, $\lim_{r \to -\infty} \left(-\tilde{z} + \frac{2h_A}{h_0 + h_A} r \right) \frac{\partial \tilde{z}}{\partial r} \frac{\lambda''(\tilde{z})}{\lambda'(\tilde{z})} = -\infty$ iff $\varsigma > 0$; i.e., iff $\frac{h_F}{h_0 + h_F} \left(\frac{h_A}{h_0 + h_A} \right)^3 > \frac{1}{4}$. Therefore,

$$\lim_{r \to -\infty} \frac{\partial b_F}{\partial h_A} > 0 \text{ iff } \frac{h_F}{h_0 + h_F} \left(\frac{h_A}{h_0 + h_A} \right)^3 > \frac{1}{4}.$$

Therefore, there exists $\check{p}^A$ and associated $\check{\underline{v}}_A = \underline{v}_A \left(\check{p}^A \right)$ such that $\frac{\partial b_F}{\partial h_A} > 0$ for $\underline{v}_A < \check{\underline{v}}_A$ iff $\frac{h_F}{h_0 + h_F} \left(\frac{h_A}{h_0 + h_A} \right)^3 > \frac{1}{4}$. ∎

Proof of Proposition 4: With b_A defined by (8), the equilibrium marginal applicant is implicitly defined by

$$G(\underline{v}_A, b_A (\underline{v}_A, p)) = w,$$

with G defined by (43). We can apply the implicit function theorem to obtain

$$\frac{\partial \underline{v}_A}{\partial h_i} = -\frac{\frac{\partial G}{\partial h_i} + \frac{\partial G}{\partial v_F} \frac{\partial b_A}{\partial h_i}}{\frac{\partial G}{\partial v_A} + \frac{\partial G}{\partial v_F} \frac{\partial b_A}{\partial v_A}}.$$

Letting $\tilde{z}$ be defined by (44),

$$\frac{\partial G}{\partial v_A} = \frac{h_0 + h_A}{h_0 + h_A + h_F} \lambda'(\tilde{z}) > 0,$$

$$\frac{\partial G}{\partial v_F} = 1 - \frac{h_A}{(h_0 + h_A + h_F)} \lambda'(\tilde{z}) > 1 - \frac{h_A}{(h_0 + h_A + h_F)} > 0,$$

$$\frac{\partial b_A}{\partial v_A} = \frac{h_F}{h_0 + h_F} > 0,$$

so

$$sign \left[\frac{\partial \underline{v}_A}{\partial h_i} \right] = -sign \left[\frac{\partial G}{\partial h_i} + \frac{\partial G}{\partial v_F} \frac{\partial b_A}{\partial h_i} \right]. \tag{51}$$

Considering the case of changes in h_F, we have that $\partial G / \partial h_F$ and $\partial G / \partial v_F$ are given by (47) and (46), and

$$\frac{\partial b_A}{\partial h_F} = \frac{h_0}{(h_0 + h_F)^2} v_A - \frac{\partial \sigma_{v_A/v_F}}{\partial h_F} r.$$

To shorten expressions, let $d_{v_A} = \frac{h_A}{h_0 + h_A}$ and $d_{v_F} = \frac{h_F}{h_0 + h_F}$. After some manipulations, we can write

$$sign \left[\frac{\partial G}{\partial h_F} + \frac{\partial G}{\partial v_F} \frac{\partial b_A}{\partial h_F} \right] = sign \left[P + \left(1 - \frac{h_A}{(h_0 + h_A + h_F)} \lambda'(\tilde{z}) \right) \left(\tilde{z} - \frac{1}{2\sqrt{d_{v_F} d_{v_A}}} r \right) \right]$$

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with P defined by (48). We now study the limiting behavior of the rhs of the previous expression as r becomes unbounded. First, for $r \to -\infty$, we have $v_F \approx w$ so that

$$v_A \approx \frac{1}{d_{v_F}}w + \frac{\sigma_{v_F/v_A}}{d_{v_F}}r,$$

$$\tilde{z} \approx \frac{(1 - d_{v_A}d_{v_F})}{d_{v_F}}\frac{w}{\sigma_{v_A/v_F}} + \frac{1}{\sqrt{d_{v_F}d_{v_A}}}r,$$

$$\left(\tilde{z} - \frac{1}{2\sqrt{d_{v_F}d_{v_A}}}r\right) \approx \frac{(1 - d_{v_A}d_{v_F})}{d_{v_F}}\frac{w}{\sigma_{v_A/v_F}} + \frac{1}{2\sqrt{d_{v_F}d_{v_A}}}r.$$

Since, $r \to -\infty$ implies that $\tilde{z} \to -\infty$, we have the following limits:

$$\lim_{r\to-\infty} \lambda(\tilde{z}) = \lim_{r\to-\infty} \lambda'(\tilde{z}) = \lim_{r\to-\infty} \tilde{z}\lambda'(\tilde{z}) = 0, \tag{52}$$

so that

$$\lim_{r\to-\infty} \frac{\partial G}{\partial h_F} + \frac{\partial G}{\partial v_F}\frac{\partial t}{\partial h_F} = \lim_{r\to-\infty} \tilde{z} - \frac{1}{2\sqrt{d_{v_F}d_{v_A}}}r = -\infty.$$

Therefore, there exists $\underline{p}^F$ such that $\left(\frac{\partial G}{\partial h_F} + \frac{\partial G}{\partial v_F}\frac{\partial b_A}{\partial h_F}\right) < 0$ for $p < \underline{p}^F$, implying that $\frac{\partial v_A}{\partial h_F} > 0$ for $p < \underline{p}^F$.

Second, as $r \to \infty$ we have

$$\frac{h_0 + h_F}{h_0 + h_A + h_F}v_F + \frac{h_0 + h_A}{h_0 + h_A + h_F}v_A \approx w$$

implying

$$w \approx \frac{h_0 + h_F}{h_0 + h_A + h_F}\left(d_{v_F}v_A - \sigma_{v_F/v_A}r\right) + \frac{h_0 + h_A}{h_0 + h_A + h_F}v_A$$

$$v_A \approx w + \frac{h_0 + h_F}{h_0 + h_A + h_F}\sigma_{v_F/v_A}r$$

Since $r \to \infty$ implies that $\tilde{z} \to \infty$, we obtain the following limits

$$\lim_{r\to\infty} \frac{\lambda(\tilde{z})}{\tilde{z}} = \lim_{r\to-\infty} \lambda'(\tilde{z}) = 1,$$

so

$$\lim_{r\to\infty}\left(\frac{\partial G}{\partial h_F} + \frac{\partial G}{\partial v_F}\frac{\partial b_A}{\partial h_F}\right) = \lim_{r\to\infty}\left(-\frac{h_0 + h_F}{h_0 + h_A + h_F}\tilde{z} + \left(\frac{h_0 + h_F}{h_0 + h_A + h_F}\right)\left(\tilde{z} - \frac{1}{2\sqrt{d_{v_F}d_{v_A}}}r\right)\right)$$

$$= \lim_{r\to\infty} -\left(\frac{h_0 + h_F}{h_0 + h_A + h_F}\right)\left(\frac{1}{2\sqrt{d_{v_F}d_{v_A}}}r\right) = -\infty$$

Therefore, there exists $\bar{p}^F$ such that $\left(\frac{\partial G}{\partial h_F} + \frac{\partial G}{\partial v_F}\frac{\partial b_A}{\partial h_F}\right) < 0$ for $p > \bar{p}^F$, implying that $\frac{\partial v_A}{\partial h_F} > 0$ for $p > \bar{p}^F$. ∎

Proof of Proposition 5: From the proof of Proposition 4, we have that $sign\left[\frac{\partial v_A}{\partial h_i}\right]$ satisfies (51). Through differentiation, we have

$$\frac{\partial G}{\partial h_A} = \frac{(h_0 + h_F)(h_0 + h_A)}{2h_A(h_0 + h_A + h_F)^2}\sigma_{v_A/v_F}\left(\lambda(\tilde{z}) - \tilde{z}\lambda'(\tilde{z})\right) + \frac{h_0 + h_F}{(h_0 + h_A + h_F)^2}\sigma_{v_F/v_A}r\lambda'(\tilde{z}),$$

$$\frac{\partial t}{\partial h_A} = \frac{h_F}{(h_0 + h_A + h_F)(h_0 + h_A)}\sigma_{v_F/v_A}r,$$

with $\tilde{z}$ and r defined in (44) and (45). After some calculations, we obtain

$$\left(\frac{\partial G}{\partial h_A} + \frac{\partial G}{\partial v_F}\frac{\partial t}{\partial h_A}\right) = \varphi\kappa(\tilde{z}) + \chi\left(\frac{h_0}{h_F}\lambda'(\tilde{z}) + 1\right)r, \tag{53}$$

with $\kappa(\tilde{z})$ defined by (50), and

$$\varphi = \frac{(h_0 + h_F)(h_0 + h_A)}{2h_A(h_0 + h_A + h_F)^2}\sigma_{v_A/v_F} > 0,$$

$$\chi = \left(1 + \frac{h_F}{h_0 + h_A}\right)\sqrt{\left(\frac{h_F}{h_0 + h_F}\right)^3\frac{h_A}{h_0 + h_A}} > 0.$$

As noted in the proof of Lemma 6, the function $\kappa(\tilde{z})$ is quasiconcave, and satisfies $0 < \kappa(\tilde{z}) < \lambda(0)$ with $\lim_{|\tilde{z}|\to\infty}\kappa(\tilde{z}) = 0$. Since $\lambda'(\tilde{z}) > 0$, the term in the rhs of (53) is positive if $r > 0$. Therefore, if $r > 0$, we have $\frac{\partial v_A}{\partial h_A} < 0$. In summary, letting $\bar{p}^A = 1/2$, so that $r > 0$ iff $p > \bar{p}^A$, then $\frac{\partial v_A}{\partial h_A} < 0$ if $p > \bar{p}^A$.

To study the case that $r < 0$, we consider the limit behavior of the lhs of (53) as $r \to -\infty$. Making use of the limits (52), we obtain

$$\lim_{r\to-\infty}\left(\frac{\partial G}{\partial h_A} + \frac{\partial G}{\partial v_F}\frac{\partial t}{\partial h_A}\right) = \chi\lim_{r\to-\infty}r = -\infty.$$

That is, there exists $\underline{p}^A$ such that $\left(\frac{\partial G}{\partial h_A} + \frac{\partial G}{\partial v_F}\frac{\partial t}{\partial h_A}\right) < 0$ for $p < \underline{p}^A$, implying that $\frac{\partial v_A}{\partial h_A} > 0$ for $p < \underline{p}^A$.∎

Proof of Lemma 7: Expected equilibrium profits are given by (17). Then, holding constant the marginal applicant $\underline{v}_A$ and hiring standard $\underline{v}_F$, we have

$$\frac{\partial\Pi}{\partial h_i} = \int_\Theta(\theta - w_E)\phi(z_i(\theta,\underline{v}_i))\frac{\partial z_i}{\partial h_i}\Phi(z_j(\theta,\underline{v}_j))dF(\theta)$$

$$= \frac{1}{2\sqrt{h_i}}\int_\Theta(\theta - w_E)(\theta - \frac{h_i - h_0}{h_i}\underline{v}_i)\phi(z_i(\theta,\underline{v}_i))\Phi(z_j(\theta,\underline{v}_j))dF(\theta). \tag{54}$$

Let $\gamma_i = E\left[\theta|\underline{v}_i, v_j \geq \underline{v}_j\right]$ so that

$$\int_\Theta(\theta - \gamma_i)\phi(z_i(\theta,\underline{v}_i))\Phi(z_j(\theta,\underline{v}_j))dF(\theta) = 0.$$

Then, we can write (54) as

$$
\begin{aligned}
\frac{\partial \Pi}{\partial h_i} &= \frac{1}{2\sqrt{h_i}} \int_\Theta (\theta - \gamma_i)^2 \phi(z_i(\theta, \underline{v}_j)) \Phi(z_j(\theta, \underline{v}_j)) dF(\theta) \\
&\quad + \frac{1}{2\sqrt{h_i}} (\gamma_i - w_E)(\gamma_i - \frac{h_i - h_0}{h_i} \underline{v}_i) \int_\Theta \phi(z_i(\theta, \underline{v}_i)) \Phi(z_j(\theta, \underline{v}_j)) dF(\theta).
\end{aligned}
$$

Let μ denote total employment

$$
\mu = \Pr[v_A \geq \underline{v}_A, v_F \geq \underline{v}_F] = \int_\Theta \Phi(z_A(\theta, \underline{v}_A)) \Phi(z_F(\theta, \underline{v}_F)) dF(\theta),
$$

so that

$$
\begin{aligned}
\frac{\partial \mu}{\partial \underline{v}_i} &= -\Pr[\underline{v}_i, v_j \geq \underline{v}_j], \\
\frac{\partial \mu}{\partial h_i} &= \int_\Theta \phi(z_i(\theta, \underline{v}_i)) \frac{\partial z_i}{\partial h_i} \Phi(z_j(\theta, \underline{v}_j)) dF(\theta) \\
&= \frac{1}{2\sqrt{h_i}} \int_\Theta (\theta - \frac{h_i - h_0}{h_i} \underline{v}_i) \phi(z_i(\theta, \underline{v}_i)) \Phi(z_j(\theta, \underline{v}_j)) dF(\theta) \\
&= \frac{1}{2\sqrt{h_i}} (\gamma_i - \frac{h_i - h_0}{h_i} \underline{v}_i) \int_\Theta \phi(z_i(\theta, \underline{v}_i)) \Phi(z_j(\theta, \underline{v}_j)) dF(\theta) \\
&= \frac{1}{2(h_i + h_0)} (\gamma_i - \frac{h_i - h_0}{h_i} \underline{v}_i) \Pr[\underline{v}_i, v_j \geq \underline{v}_j].
\end{aligned}
\tag{55}
$$

We can then write the direct effect as

$$
\begin{aligned}
\frac{\partial \Pi}{\partial h_i} &= \frac{1}{2(h_i + h_0)} Var[\theta | \underline{v}_i, v_j \geq \underline{v}_j] \Pr[\underline{v}_i, v_j \geq \underline{v}_j] + (\gamma_i - w_E) \frac{\partial \mu}{\partial h_i} \\
&= \frac{\Pr[\underline{v}_i, v_j \geq \underline{v}_j]}{2(h_i + h_0)} \left(Var[\theta | \underline{v}_i, v_j \geq \underline{v}_j] + (\gamma_i - w_E)(\gamma_i - \frac{h_i - h_0}{h_i} \underline{v}_i) \right),
\end{aligned}
$$

which gives (54) by noting that $\partial \mu / \partial \underline{v}_i = -\Pr[\underline{v}_i, v_j \geq \underline{v}_j]$. Consider, now, the indirect effect in (27). The term $\frac{\partial \Pi}{\partial \underline{v}_A}$ is simply the (negative of the) marginal profit made on the marginal hire, which is

$$
\begin{aligned}
\frac{\partial \Pi}{\partial \underline{v}_A} &= \int_\Theta (\theta - w_E) \phi(z_A(\theta, \underline{v}_A)) \frac{\partial z_A}{\partial h_A} \Phi(z_F(\theta, \underline{v}_F)) dF(\theta) = \\
&= -\frac{h_0 + h_A}{\sqrt{h_A}} \int_\Theta (\theta - w_E) \phi(z_A(\theta, \underline{v}_A)) \Phi(z_F(\theta, \underline{v}_F)) dF(\theta) \\
&= -\frac{h_0 + h_A}{\sqrt{h_A}} (\gamma_A - w_E) \int_\Theta \phi(z_A(\theta, \underline{v}_A)) \Phi(z_F(\theta, \underline{v}_F)) dF(\theta) \\
&= -(\gamma_A - w_E) \Pr[\underline{v}_A, v_F \geq \underline{v}_F] \\
&= (\gamma_A - w_E) \frac{\partial \mu}{\partial \underline{v}_A}.
\end{aligned}
\tag{56}
$$

Combining (56) and (55), we then obtain (29). ∎

Proof of Proposition 6: We prove the proposition by showing that as h_F tends to ∞, both the direct effect $\partial\Pi/\partial h_F$ and the effect on applicants $d\underline{v}_A/dh_F$ vanish, but their ratio also tends to zero. Therefore, the total effect also converges to zero, but its sign is always given by $sign[-d\underline{v}_A/dh_F]$ for sufficiently high h_F . Therefore, there exists $\bar{h}_F$, which is a function of the parameters of the model, such that the total effect is negative for $h_F > \bar{h}_F$ whenever $d\underline{v}_A/dh_F < 0$, as determined by Proposition 4.

First, we have that $\sqrt{h_F}\phi(z_F(\theta,\underline{v}_F)) \to \delta_{\underline{v}_F}(\theta)$ as $h_F \to \infty$, where $\delta_{\underline{v}_F}(\theta)$ is the Dirac delta concentrated in $\underline{v}_F$. Therefore, we can approximate the direct effect for large h_F by

$$\frac{\partial\Pi}{\partial h_F} \approx \frac{1}{2h_F}\int_\Theta (\theta - w_E)^2 \delta_{\underline{v}_F}(\theta)\,\Phi(z_A(\theta,\underline{v}_A))dF(\theta).$$

If the interview becomes perfectly informative, then $\underline{v}_F \to w_E$. Therefore,

$$\lim_{h_F\to\infty}\frac{\partial\Pi}{\partial h_F} = \lim_{h_F\to\infty}\frac{1}{2h_F}\left[(\theta - \underline{v}_F)^2\Phi(z_A(\theta,\underline{v}_A))\right]|_{\theta=\underline{v}_F} = 0.$$

Next, consider the indirect effect (27), which we can write using (29) as

$$
\begin{aligned}
\frac{\partial\Pi}{\partial\underline{v}_A}\frac{d\underline{v}_A}{dh_F} &= (\gamma_A - w_E)\left(-\frac{\partial\mu}{\partial\underline{v}_A}\right)\frac{d\underline{v}_A}{dh_F}\\
&= -(\gamma_A - w_E)\Pr[\underline{v}_A, v_F \geq \underline{v}_F]\frac{d\underline{v}_A}{dh_F}.
\end{aligned}
$$

Since

$$\lim_{h_F\to\infty}\Pr[\underline{v}_A, v_F \geq \underline{v}_F] = \Pr[\underline{v}_A, \theta \geq w_E] > 0,$$

the limit of the total effect can be written as

$$
\begin{aligned}
\lim_{h_F\to\infty}\frac{d\Pi}{dh_F} &= \lim_{h_F\to\infty}\frac{d\underline{v}_A}{dh_F}\left(\frac{\partial\Pi/\partial h_F}{d\underline{v}_A/dh_F} + \frac{\partial\Pi}{\partial\underline{v}_A}\right)\\
&= \lim_{h_F\to\infty}\frac{d\underline{v}_A}{dh_F}\left(\frac{\partial\Pi/\partial h_F}{d\underline{v}_A/dh_F} - (\gamma_A - w_E)\Pr[\underline{v}_A, v_F \geq \underline{v}_F]\right)
\end{aligned}
$$

For $c_A > 0$, the marginal applicant is strictly valuable to the firm. Then, the second term in the previous expression is bounded away from zero and negative. We next show that the ratio $(\partial\Pi/\partial h_F)/(d\underline{v}_A/dh_F)$ vanishes as $h_F \to \infty$. To do so, we separately approximate $\partial\Pi/\partial h_F$ and $d\underline{v}_A/dh_F$.

To compute $d\underline{v}_A/dh_F$, we use the representation of profits (17). Define $\mathcal{F}$ as

$$\mathcal{F}(\underline{v}_A) = \int_\Theta (\theta - w_E)\Phi(z_A(\theta,\underline{v}_A))\phi(\tilde{z}_F(\theta,\underline{v}_A))dF,$$

with $\tilde{z}_F(\theta,\underline{v}_A) = \sqrt{h_F}\left[\theta - \underline{v}_A + \sqrt{\frac{1}{h_A+h_0} + \frac{1}{h_F}}r\right]$ and r defined in (45). As F is proportional to the marginal profit made on the marginal hire, the equilibrium marginal applicant is implicitly defined

by

$$\mathcal{F}(\underline{v}_A) = 0.$$

Then,

$$\frac{d\underline{v}_A}{dh_F} = -\frac{\partial \mathcal{F}/\partial h_F}{\partial \mathcal{F}/\partial \underline{v}_A},$$

$$\frac{\partial \mathcal{F}}{\partial h_F} = \int_\Theta (\theta - w_E)\left[\frac{\theta - \underline{v}_A}{2\sqrt{h_F}} + \frac{r}{2\sqrt{(h_A + h_0)(h_A + h_0 + h_F)}}\right]\Phi(z_A(\theta, \underline{v}_A))\phi'(\tilde{z}_F(\theta, \underline{v}_A))dF(\theta),$$

$$\frac{\partial \mathcal{F}}{\partial \underline{v}_A} = -\int_\Theta (\theta - w_E)\left[\frac{h_A + h_0}{\sqrt{h_A}}\phi(z_A(\theta, \underline{v}_A))\phi(\tilde{z}_F(\theta, \underline{v}_A)) + \sqrt{h_F}\Phi(z_A(\theta, \underline{v}_A))\phi'(\tilde{z}_F(\theta, \underline{v}_A))\right]dF(\theta).$$

Define $x_A = \underline{v}_A - \sqrt{\frac{1}{h_A + h_0}}r$. Then, $h_F\phi'(\tilde{z}_F(\theta, \underline{v}_A)) \to \delta'_{x_A}(\theta)$ as $h_F \to \infty$, where $\delta'_{x_A}(\theta)$ is the distributional derivative of the Dirac delta concentrated in x_A. We then obtain the following approximations as $h_F \to \infty$:

$$\frac{\partial \mathcal{F}}{\partial h_F} \approx \int_\Theta (\theta - w_E)\left[\frac{(\theta - \underline{v}_A)}{2\sqrt{h_F}} + \frac{r}{2\sqrt{(h_A + h_0)(h_A + h_0 + h_F)}}\right]\Phi(z_A(\theta, \underline{v}_A))\frac{1}{h_F}\delta'_{x_A}(\theta)\,dF(\theta)$$

$$= \frac{1}{h_F\sqrt{h_F}}\int_\Theta (\theta - w_E)\left[\frac{1}{2}(\theta - \underline{v}_A) + \frac{r}{2\sqrt{(h_A + h_0)(\frac{h_A + h_0}{h_F} + 1)}}\right]\Phi(z_A(\theta, \underline{v}_A))\delta'_{x_A}(\theta)\,dF(\theta),$$

$$\frac{\partial \mathcal{F}}{\partial \underline{v}_A} \approx -\frac{1}{\sqrt{h_F}}\int_\Theta (\theta - w_E)\left[\frac{h_A + h_0}{\sqrt{h_A}}\phi(z_A(\theta, \underline{v}_A))\delta_{x_A}(\theta) + \Phi(z_A(\theta, \underline{v}_A))\delta'_{x_A}(\theta)\right]dF(\theta).$$

The distribution $\delta'_{x_A}(\theta)$ satisfies

$$\int_\Theta \Psi(\theta)\delta'_{x_A}(\theta)\,d\theta = -\int_\Theta \Psi'(\theta)\delta_{x_A}(\theta)\,d\theta = -\Psi'(x_A),$$

for any compactly supported smooth test function Ψ. Define

$$R(\theta) = (\theta - w_E)\Phi(z_A(\theta, \underline{v}_A))f(\theta),$$

$$S(\theta) = \left[\theta - \underline{v}_A + \frac{r}{\sqrt{(h_A + h_0)(\frac{h_A + h_0}{h_F} + 1)}}\right]\frac{R(\theta)}{2}.$$

From these approximations, we readily obtain that

$$\lim_{h_F \to \infty} h_F\sqrt{h_F}\frac{\partial F}{\partial h_F} = -S'(x_A) \neq 0,$$

$$\lim_{h_F \to \infty} \sqrt{h_F}\frac{\partial F}{\partial \underline{v}_A} = -(x_A - w_E)\frac{h_A + h_0}{\sqrt{h_A}}\phi(z_A(x_A, \underline{v}_A)) + R'(x_A) \neq 0.$$

We can now compute the limit

$$\lim_{h_F \to \infty} \frac{\frac{\partial \Pi}{\partial h_F}}{\frac{d\underline{v}_A}{dh_F}} = \lim_{h_F \to \infty} -\frac{\frac{\partial \Pi}{\partial h_F}\frac{\partial F}{\partial \underline{v}_A}}{\frac{\partial F}{\partial h_F}} = \frac{-(x_A - w_E)\frac{h_A + h_0}{\sqrt{h_A}}\phi(z_A(x_A, \underline{v}_A)) + R'(x_A)}{S'(x_A)} \lim_{h_F \to \infty} h_F\frac{\partial \Pi}{\partial h_F} = 0,$$

which follows from

$$\lim_{h_F \to \infty} h_F \frac{\partial \Pi}{\partial h_F} = \lim_{h_F \to \infty} \frac{1}{2} \int_{\Theta} (\theta - \underline{v}_F)^2 \delta_{\underline{v}_F}(\theta) \, \Phi(z_A(\theta, \underline{v}_A)) dF(\theta) = 0.$$

∎

Proof of Proposition 7: We show that whenever the job seeker's precision h_A is sufficiently low, both the direct and the indirect effect in (27) are negative. First, let t_A be such that whenever $c_A + \underline{w} < t_A$ then (i) $\underline{v}_A < 0$, and (ii) $d\underline{v}_A/dh_A > 0$. That is, the marginal applicant is below the average job seeker, and improvements in recruitment dissuade the marginal applicant from applying. Proposition 5 and Proposition 3 ensure that t_A exists. By definition of t_A, the indirect effect in (29) is negative. Since $\underline{v}_A < 0$, the right-hand side of (32) becomes unbounded as $h_A \to 0$. Therefore, there exists $\tilde{h}_A$ such that for any $h_A < \tilde{h}_A$, the indirect effect is negative. Therefore, for $c_A + \underline{w} < t_A$ and $h_A < \tilde{h}_A$, the total effect of improved advertising is negative.∎

Proof of Proposition 8: Suppose that all types $v_A \in A$ apply to the firm, where we must have that the total mass of applicants exceeds K to meet the slot constraint; i.e., $\Pr[v_A \in A] \geq K$. Facing a slot constraint, the firm will only hire the applicants with the highest expected match value. As the firm's inference (6) is strictly monotone in the interview score, and $\Pr[v_F, v_A \in A]$ is continuous in v_F, then the firm's hiring rule will again follow a threshold rule: the firm will set a hiring standard $\underline{v}_F^S$ and hire all applicants whose interview score exceeds $\underline{v}_F^S$, with the hiring standard implicitly given by the binding slot constraint

$$\Pr\left[v_F \geq v_F^S, v_A \in A\right] = K.$$

As the firm's hiring rule is monotone, job seekers application decision will also be monotone in type and satisfying (7). Define

$$\mu(\underline{v}_A^S, \underline{v}_F^S) = \Pr\left[v_F \geq v_F^S, v_A \geq v_A^S\right] = \int_{-\infty}^{\infty} \Phi\left[z_A(\theta, \underline{v}_A^S)\right] \Phi\left[z_F(\theta, \underline{v}_F^S)\right] dF(\theta)$$

with $z_i(\theta, \underline{v}_i^S)$ defined by (18). The function $\mu(\underline{v}_A^S, \underline{v}_F^S)$ gives the total employment when all applicants with types higher than $\underline{v}_A^S$, apply but only those with scores exceeding $\underline{v}_F^S$ are hired. Then, for any wage w_E, the continuation equilibrium with slot constraints is given by

$$\mu(\underline{v}_A^S, \underline{v}_F^S) = K, \tag{57}$$

$$\underline{v}_F^S = b_A(\underline{v}_A^S, \frac{c_A}{w_E - \underline{w}}). \tag{58}$$

Since

$$\frac{\partial \mu}{\partial \underline{v}_A^S} = \int_{-\infty}^{\infty} -\frac{h_A + h_0}{\sqrt{h_A}} \phi\left[z_A\right] \Phi\left[z_F\right] dF(\theta) < 0,$$

$$\frac{\partial \mu}{\partial \underline{v}_F^S} = \int_{-\infty}^{\infty} -\frac{h_F + h_0}{\sqrt{h_F}} \phi\left[z_F\right] \Phi\left[z_A\right] dF(\theta) < 0, \tag{59}$$

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the slope of the hiring standard in (57) satisfies $d\underline{v}_F^S/d\underline{v}_A^S = -\frac{\partial \mu}{\partial \underline{v}_A^S} / \frac{\partial \mu}{\partial \underline{v}_F^S} < 0$. This implies that (57-58) has a unique solution. This establishes that (35) and (36) characterize the unique continuation equilibrium for every wage w_E such that the applicant pool has mass in excess of K.

The firm's profits are

$$\Pi = \int_{-\infty}^{\infty} \int_{\underline{v}_F^S(\underline{v}_A^S)}^{\infty} \int_{\underline{v}_A^S}^{\infty} (\theta - w_E)\, dF(\theta, v_A, v_F),$$

and the optimal wage satisfies the FOC

$$\left(\frac{\partial \Pi}{\partial \underline{v}_A^S} + \frac{\partial \Pi}{\partial \underline{v}_F^S} \frac{\partial \underline{v}_F^S}{\partial \underline{v}_A^S} \right) \frac{d\underline{v}_A^S}{dw_E} - \mu(\underline{v}_A^S, \underline{v}_F^S) = 0$$

Noting that total employment must be equal to K, we obtain the FOC (38).∎

Proof of Proposition 9: Fix a posted wage w_E and let $p = \frac{c_A}{w_E - \underline{w}}$ denote the continuation equilibrium probability that the marginal applicant $\underline{v}_A^S$ is hired. We first investigate $\partial \underline{v}_A^S / \partial h_F$. To this end, define

$$\mu_A(\underline{v}_A^S) = \mu(\underline{v}_A^S, b_A(\underline{v}_A^S, p)) = \int_{-\infty}^{\infty} \Phi\left[z_A(\theta, \underline{v}_A^S)\right] \Phi\left[\tilde{z}_F(\theta, \underline{v}_A^S)\right] dF(\theta),$$

with

$$\tilde{z}_F(\theta, \underline{v}_A^S) = \sqrt{h_F}\left[\theta - \underline{v}_A^S + \sqrt{\frac{1}{h_A + h_0} + \frac{1}{h_F}} \Phi^{-1}(p)\right].$$

That is, $\mu_A(\underline{v}_A^S)$ is the employment level when the firm's hiring standard is such that the marginal applicant is $\underline{v}_A^S$. From Proposition 8 we have that, for a wage w_E, the marginal applicant is given in equilibrium by $\mu_A(\underline{v}_A^S) = K$. Since

$$\frac{\partial \mu_A}{\partial \underline{v}_A^S} = -\int_{-\infty}^{\infty} \left(\frac{h_A + h_0}{\sqrt{h_A}} \phi\left[z_A\right] \Phi\left[\tilde{z}_F\right] + \sqrt{h_F} \Phi\left[z_A\right] \phi\left[\tilde{z}_F\right] \right) dF(\theta) < 0,$$

the implicit function theorem implies that the sign of $\partial \underline{v}_A^S / \partial h_F$ will be given by the sign of $\partial \mu_A / \partial h_F$. We have

$$
\begin{aligned}
\frac{\partial \mu_A}{\partial h_F} &= \int_{-\infty}^{\infty} \left(\frac{1}{2\sqrt{h_F}} \left[\theta - \underline{v}_A^S + \sqrt{\frac{h_F}{(h_A + h_0)(h_A + h_F + h_0)}} \Phi^{-1}(p)\right] \Phi\left[z_A\right] \phi\left[\tilde{z}_F\right] \right) dF(\theta) \\
&= \frac{1}{2\sqrt{h_F}} \int_{-\infty}^{\infty} \left(\left[\theta - \frac{h_A + h_0}{h_A + h_F + h_0}\underline{v}_A^S - \frac{h_F + h_0}{h_A + h_F + h_0}\underline{v}_F^S\right] \Phi\left[z_A\right] \phi\left[\tilde{z}_F\right] \right) dF(\theta) \\
&= \frac{1}{2\sqrt{h_F}} \int_{-\infty}^{\infty} \left(\left[\theta - E[\theta | \underline{v}_F^S, \underline{v}_A^S]\right] \Phi\left[z_A\right] \phi\left[\tilde{z}_F\right] \right) dF(\theta) \\
&= \frac{1}{2\sqrt{h_F}} \left(\left[E[\theta | \underline{v}_F^S, v_A \geq \underline{v}_A^S] - E[\theta | \underline{v}_F^S, \underline{v}_A^S]\right] \right) \int_{-\infty}^{\infty} \Phi\left[z_A\right] \phi\left[\tilde{z}_F\right] dF(\theta) > 0
\end{aligned}
$$

Therefore, improving the evaluation of applicants unambiguously discourages applications, and proves Proposition 9-i.

We now investigate $\partial \underline{v}_A^S / \partial h_A$. To this effect let $b_F^S(\underline{v}_A^S)$ be the firm's hiring standard when the marginal applicant is $\underline{v}_A^S$. That is, b_F^S is implicitly defined by

$$\mu(\underline{v}_A^S, b_F^S(\underline{v}_A^S)) = K.$$

Implicitly differentiating we have

$$\frac{\partial b_F^S}{\partial h_A} = -\frac{\partial \mu / \partial h_A}{\partial \mu / \partial \underline{v}_F^S}.$$

From (59), we have that $\partial \mu / \partial \underline{v}_F^S < 0$, so that $sign\left[\partial b_F^S / \partial h_A\right] = sign\left[\partial \mu / \partial h_A\right]$. From (55), we have

$$\frac{\partial \mu}{\partial h_A} = \frac{1}{2\left(h_A + h_0\right)}(E\left[\theta | \underline{v}_A^S, v_F \geq b_F^S(\underline{v}_A^S)\right] - \frac{h_A - h_0}{h_A}\underline{v}_A^S)\Pr\left[\underline{v}_A^S, v_F \geq b_F^S(\underline{v}_A^S)\right].$$

First, if $K < 1/2$ then the marginal applicant, conditional on being hired, cannot have a negative productivity. That is, we must have $E\left[\theta | \underline{v}_A^S, v_F \geq b_F^S(\underline{v}_A^S)\right] > 0$. Since $h_A > h_0$, the following inequalities hold

$$E\left[\theta | \underline{v}_A^S, v_F \geq b_F^S(\underline{v}_A^S)\right] > \frac{h_A - h_0}{h_A}E\left[\theta | \underline{v}_A^S, v_F \geq b_F^S(\underline{v}_A^S)\right] > \frac{h_A - h_0}{h_A}\underline{v}_A^S.$$

Therefore, we have $\partial \mu / \partial h_A > 0$, and, thus, $\partial b_F^S / \partial h_A$. That is, the firm will raise the hiring standard if applicants become better informed, holding the marginal applicant constant. Applicants' best response is still characterized by (8) so that the effect of increasing h_A is given by Lemma 5. That is, $\partial b_A / \partial h_A > 0$ if and only if $p > 1/2$. Since we have that, for $p < 1/2$,

$$\frac{\partial b_F^S}{\partial h_A} > 0 > \frac{\partial b_A}{\partial h_A},$$

this ensures the existence of a $p^S > 1/2$, such that $\partial \underline{v}_A^S / \partial h_A > 0$ if $p < p^S$.∎

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