Endowments, Factor Prices, and Skill-Biased Technology: Importing Development Accounting into HOV *

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ABSTRACT:

International differences in productivity, endowments and factor prices are essential for understanding cross-country patterns of income and growth, yet have proven of limited value for understanding the factor content of international trade. Part of the reason is that international trade empirics have failed to explain why there are such large international differences in unit input requirements of skilled and unskilled labor: Are the differences due to substitution effects associated with international differences in productivity adjusted factor prices or to factor-augmenting international technology differences? To answer these questions we develop a parsimonious general equilibrium Heckscher-Ohlin-Vanek (HOV) model featuring international technology differences and a failure of productivity adjusted factor price equalization. The three core equations of our model fit data for skilled and unskilled labor in 38 countries, and identify the relative contributions of substitution effects vs. factor-augmenting technology differences. We then show that our estimates of technology differences and skill bias are very similar to estimates obtained using approaches in the directed technical change and development accounting literatures.

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International differences in productivity, endowments and factor prices are essential for understanding cross-country patterns of income and growth, yet have proven of limited use for understanding the factor content of international trade. The development accounting literature has shown that richer countries are more productive and that this productivity advantage is more pronounced for skilled than unskilled labor i.e., technology differences are factor-augmenting and skill-biased (Caselli, 2005, Caselli and Coleman, 2006, Caselli and Feyrer, 2007, Caselli, 2016). The directed technical change literature has shown that this skill bias is systematically related to factor endowments: Relatively skill-abundant countries should have relatively low skilled wages, but nevertheless direct innovation towards improving the productivity of skilled labor. (See Acemoglu, 1998, Caselli and Coleman, 2006, and Acemoglu, 2009. For open-economy empirics see Acemoglu and Zilibotti, 2001 and Blum, 2010).

In contrast, when explaining the factor content of trade, the international trade literature has not been able to establish a cohesive empirical *relationship* between endowments, factor prices, and factor-augmenting productivity e.g., Trefler (1995), Gabaix (1997), Davis and Weinstein (2001) and Nishioka (2012). These and other papers look at pieces of the relationship, but not its entirety: Trefler (1995), Davis and Weinstein (2001) and Nishioka (2012) argue that endowments matter for the factor content of trade, Davis and Weinstein (2001) and Fadinger (2011) argue that factor prices matter, and not since Trefler (1993) has anyone seriously argued that factor-augmenting productivity matters.[1] We argue that a parsimonious extension of the Heckscher-Ohlin-Vanek (HOV) model not only lines up well with the data, but also matches a large number of facts documented in the development accounting and directed technical change literatures. We aim to take the relationship between endowments, factor prices, and factor-augmenting productivity documented in our opening paragraph and introduce it into HOV empirics.

Our starting point is a question that comes out of the contrast between Davis and Weinstein (2001) and Trefler (1993): Are international differences in the use of skilled and unskilled labor due to (1) substitution effects associated with international factor price differences as in Davis and Weinstein (2001) or (2) factor-augmenting international technology differences as in Trefler (1993)? To investigate, we introduce factor-augmenting international productivity differences into a multi-sector HOV model and consider an equilibrium *without* factor price equalization. This model explains four phenomena.

First, it fits the entire supply-side content of the HOV model, by which we mean the following: (*a*) it does very well fitting a *Techniques* equation relating international differences in factor demands ('techniques') to international differences in wages and technology; (*b*) it also does very well fitting a *Wage* or factor market clearing equation relating international differences in factor prices to international differences in endowments and technology; and, (*c*) it provides a very good fit of the *Vanek* equation relating the factor content of trade (itself a function of factor prices and technology) to endowments, provided we treat Government Services as nontradable. These three equations describe the *entire* supply side of the model so there are no other equations we can exploit to answer our international trade question about the relative roles of substitution effects

[1] Marshall (2012) is a rare exception. Additionally, Romalis (2004) relates the structure of production to endowments and Burstein and Vogel (forthcoming) relate endowments to relative factor prices.

and factor-augmenting technology differences. We estimate the relevant parameters using the Techniques and Wage equations, and sometimes even using the Vanek equation; thus, we use more overidentifying restrictions than previous contributions to the literature.

Second, our model explains each of these three equations separately by factor (as is the standard since Vanek 1968 and Leamer 1980) and also for the *ratio* of skilled-to-unskilled labor. Thus, we show that *relative* factor abundance matters for *relative* factor prices, *relative* productivity differences, and the *relative* factor content of trade. The latter is particularly important because the Heckscher-Ohlin (HO) component of HOV is a long-neglected prediction about relative endowments.

Third, relative to the development accounting literature, we offer an alternative, multi-sector, multi-country method of estimating factor-augmenting technology parameters. This amounts to simultaneously estimating the Wage and Techniques equations. We find that richer countries are more productive in their use of both skilled and unskilled labor. (In contrast, Caselli and Coleman (2006) find a *negative* relationship between income and unskilled-augmenting technology.) Further, the ratio of our skilled-to-unskilled technology parameters almost exactly equals those that come off of a CES aggregate production function. This intimately connects the HOV and development accounting literatures.

Fourth, our estimates also support a core prediction of the directed technical change literature: When the elasticity of substitution between skilled and unskilled labor exceeds unity ($\sigma > 1$), then on the balanced growth path, skill-abundant countries are relatively more productive in their use of skilled labor i.e., a country's technical change is directed towards its abundant factor. To our knowledge, Blum (2010) is the only other empirical international trade paper to investigate this issue and he finds the opposite result.[2] Our factor-augmenting technology parameters and this core relationship allow us to identify and estimate the elasticity of substitution between skilled and unskilled labor. Our estimate of 1.67 is squarely within the range of existing estimates from the labor literature.

There are three additional features of the model that we have not discussed. First, we require an equilibrium that allows for international productivity comparisons and this in turn requires that countries produce like goods so that productivity estimates can be based on a comparison of apples with apples. Restated, we require a diversified equilibrium. We also require an equilibrium featuring failure of factor price equalization. There is a tension here in that failure of factor price equalization is usually associated with specialization. One solution is to allow for specialization and impose additional assumptions so that productivity analysis can be justified. The required assumptions have not been discussed in the literature.[3] The solution we adopt uses Ricardian productivity differences to break factor price equalization while still maintaining a diversified equilibrium. Since the Ricardian productivities are isomorphic to quality, the reader may be more comfortable thinking about this as a diversified equilibrium with quality differences. See Schott (2004) or Sutton and Trefler (2016) for evidence on quality-based diversification (what Schott calls

[2]Acemoglu, Gancia and Zilibotti (2015) offer an application to offshoring.

[3]Davis and Weinstein (2001) and Feenstra and Hanson (2000) treat observed diversification as a consequence of aggregation in an equilibrium with specialization, but these papers do not provide aggregation theorems of use here.

‘product overlap’). It turns out to be trivial to calibrate these Ricardian or quality differences to the data, which is an exercise similar to those found in the ‘wedges’ literature. Second, we focus almost exclusively on the supply side, to the exclusion of the demand side. The demand side features in Trefler (1995), Davis and Weinstein (2001), and Caron, Fally and Markusen (2014). Third, as in Trefler and Zhu (2010), we define the factor content of trade in a way that is consistent with the Vanek (1968) equation; however, there may be alternative definitions that are more relevant for policy.

This paper is most closely related to Davis and Weinstein (2001), who raised the question of substitution effects versus technology differences in the context of whether the Vanek equation could be made to fit well by allowing for the failure of factor price equalization and Hicks-neutral international technology differences. There are three notable differences between our work and that of Davis and Weinstein. First, being an older study, they had more limited access to data. (*a*) They only had data for 10 OECD countries, which means that they did not have much variation in the development status of the sample countries. (*b*) They did not have data separately for skilled and unskilled labor, which means that they could not investigate skill bias or directed technical change. (*c*) They did not have factor price data, which means that they could not directly investigate their core claim about the failure of the factor price insensitivity theorem (Leamer and Levinsohn, 1995). We use the World Input-Output Database for 2006, which has data for 38 countries, for skilled and unskilled labor, and for factor prices. Second, they modelled substitution effects and the failure of factor price equalization in a reduced-form way; in contrast, we micro-found our model and, in the process, show that Davis and Weinstein were basing their conclusions on parameters that are not identified by their data. Essentially, one cannot separately identify substitution effects from factor-augmenting technology differences unless one has data on factor prices and the elasticity of substitution between factors, neither of which they used. (See Diamond, McFadden and Rodriguez, 1978.) Third, they considered Hicks-neutral technology differences whereas we show that factor-augmenting technology differences between skilled and unskilled labor are a key feature of the data for HOV, development accounting, and directed technical change.

Sections 1–2 present our general equilibrium model, describe our three estimating equations, and discuss identification. Section 3 describes the data. Section 4 presents our baseline results and evaluates the performance of the model. Section 5 draws out the implications for factor price equalization, substitution effects, factor-augmenting technology differences, skill bias, development accounting, and directed technical change. Section 6 links our results to HOV ‘folklore’, and section 7 concludes.

1. The Model

In subsections 1.1–1.3 we set up a *standard* general equilibrium model with CES preferences, identical firms and multiple sectors. The notation is complex and not used elsewhere in the paper so the reader should go quickly through it. In subsection 1.4 we define the Vanek-relevant factor content of trade. Only in section 2 do we get to the heart of the empircally-relevant theory.

1.1. *Preferences, Endowments and Technology*

Let $i,j = 1,\ldots,N$ index countries, let $g = 1,\ldots,G$ index goods (or industries), and let $\omega \in \Omega_{gi}$ index varieties of a horizontally differentiated good g produced in country i as in Krugman (1980). Preferences are internationally identical and homothetic with the nested structure:

$$U = \prod_{g=1}^{G} \left(U_g\right)^{\eta_g} \quad \text{and} \quad U_g = \left(\sum_{i=1}^{N} \int_{\omega \in \Omega_{gi}} x_{gi}(\omega)^{\frac{\rho_g - 1}{\rho_g}} d\omega \right)^{\frac{\rho_g}{\rho_g - 1}},$$

where $\rho_g > 1$ is the elasticitiy of substitution, $\eta_g > 0$, $\Sigma_g \eta_g = 1$, and $x_{gi}(\omega)$ is a quantity consumed. Let $p_{gi}(\omega)$ be the corresponding price. We assume that trade is costless so that all consumers worldwide face the same price $p_{gi}(\omega)$. Then the price index associated with U_g is $P_g = \left(\sum_{i=1}^{N} \int_{\omega \in \Omega_{gi}} p_{gi}(\omega)^{1-\rho_g} d\omega \right)^{\frac{1}{1-\rho_g}}$.

Let $f = 1,\ldots,K$ index primary factors such as labor. V_{fi} is country i's exogenous endowment of factor f and w_{fi} is its price. Let $\mathbf{w}_i = (w_{1i},\ldots,w_{Ki})$. We assume that factors are mobile across industries within a country and immobile across countries.

Turning to technology, a firm uses both primary factors and intermediate inputs of goods $h = 1,\ldots,G$. The production function is Cobb-Douglas in (*a*) an index of primary factors and (*b*) CES indexes of each of the G intermediate goods. This results in a unit cost function for $\omega \in \Omega_{gi}$ of the form

$$\phi_{gi}(\mathbf{w}_i,\mathbf{p}) = \left[c_{gi}(\mathbf{w}_i)\right]^{\gamma_{g0}} \prod_{h=1}^{G} P_{gh}^{\gamma_{gh}} \tag{1}$$

where

$$P_{gh} = \left(\sum_{j=1}^{N} \int_{\nu \in \Omega_{hj}} \alpha_{gh}\, p_{hj}(\nu)^{1-\rho_h}\, d\nu \right)^{\frac{1}{1-\rho_h}},$$

$\mathbf{p} = \{p_{hj}(\nu) : \nu \in \Omega_{hj}, \forall h,j\}$ is the set of all product prices, $\nu \in \Omega_{hj}$ indexes varieties when used as inputs, and the γ_{gh} are positive constants with $\Sigma_{h=0}^{G} \gamma_{gh} = 1$. $c_{gi}(\mathbf{w}_i)$ is a constant-returns-to-scale unit cost function associated with primary factors. P_{gh} is the unit cost function associated with the CES index of intermediate good h in the production of good g. The α_{gh} are constants that allow for empirically relevant factor intensity asymmetries.[4]

Marginal costs are $\phi_{gi}(\mathbf{w}_i,\mathbf{p})$. Per variety variable costs are $\phi_{gi}(\mathbf{w}_i,\mathbf{p})q_{gi}(\omega)$. As is standard in the literature, we assume that fixed costs are proportional to marginal costs and given by $\phi_{gi}(\mathbf{w}_i,\mathbf{p})\overline{\phi}_g$ for some constant $\overline{\phi}_g > 0$.

1.2. *Firm Behavior*

Profits for any variety $\omega \in \Omega_{gi}$ are

$$\left[p_{gi} - \phi_{gi}(\mathbf{w}_i,\mathbf{p})\right] q_{gi} - \phi_{gi}(\mathbf{w}_i,\mathbf{p})\overline{\phi}_g.$$

[4] We assume that a firm does not buy from itself. Since anything it bought from itself would have zero measure, we do not have to keep track of this in the expression for P_{gh}; however, we will have to keep track of this in the discussion of profit maximization below.

There are two sources of demand for $\omega \in \Omega_{gi}$: (1) Consumers in country j demand $p_{gi}{}^{-\rho_g} P_g^{\rho_g - 1} \eta_g Y_j$ where Y_j is national income. (2) Downstream producers of variety $\nu \in \Omega_{hj}$ each demand $b_{ij}(g,h)\left[q_{hj} + \overline{\phi}_h\right]$ where, by Shephard's Lemma,

$$b_{ij}(g,h;\mathbf{w}_j,\mathbf{p}) = \frac{\partial \phi_{hj}(\mathbf{w}_j,\mathbf{p})}{\partial p_{gi}}.$$

$b_{ij}(g,h;\mathbf{w}_j,\mathbf{p})$ is necessarily complicated notation because we need to track the entire global supply chain. Aggregating over both final and intermediate-input demands for a typical variety $\omega \in \Omega_{gi}$ yields the following result that will be useful later:

Lemma 1 *$q_{gi} = p_{gi}^{-\rho_g} \kappa_g$ for some $\kappa_g > 0$ and all i.*

The proof appears in the appendix. The last line of the proof is an expression for κ_g, from which it is apparent that κ_g is a constant from the firm's perspective.

Using lemma 1, profit maximization for $\omega \in \Omega_{gi}$ yields the standard optimal price:

$$p_{gi} = \frac{\rho_g}{\rho_g - 1} \phi_{gi}(\mathbf{w}_i,\mathbf{p}). \tag{2}$$

Zero profits for $\omega \in \Omega_{gi}$, together with this pricing rule, yield:

$$q_{gi} = (\rho_g - 1)\overline{\phi}_g\,. \tag{3}$$

Turning to factor markets, consider the demand for factor f by firm $\omega \in \Omega_{gi}$. By Shephard's Lemma this (direct) demand per unit of output is

$$d_{fgi}(\mathbf{w}_i,\mathbf{p}) = \frac{\partial \phi_{gi}(\mathbf{w}_i,\mathbf{p})}{\partial w_{fi}}.$$

Factor market clearing in country i is thus

$$V_{fi} = \sum_{g=1}^{G} n_{gi} d_{fgi}(\mathbf{w}_i,\mathbf{p}) \left[q_{gi} + \overline{\phi}_g\right] \tag{4}$$

where $n_{gi} = \int_{\omega \in \Omega_{gi}} d\omega$ is the measure of identical firms producing varieties of g in country i.

1.3. *Equilibrium*

Define $\mathbf{n}^* = \left\{n_{gi}^*\right\}_{\forall g,i}$, $\mathbf{w}^* = \left\{w_{fi}^*\right\}_{\forall f,i}$, and $\mathbf{p}^* = \{p_{gi}^*(\omega)\colon \omega \in \Omega_{gi}, \forall g,i\}$. An equilibrium is a triplet $(\mathbf{w}^*,\mathbf{p}^*,\mathbf{n}^*)$ such that when consumers maximize utility and firms maximize profits, product markets clear internationally for each variety, factor markets clear nationally for each factor, and profits are zero. Market clearing and zero profits imply that all income is factor income ($Y_i = \Sigma_f w_{fi} V_{fi}$) and that trade is balanced. It follows from this definition of equilibrium that $(\mathbf{w}^*,\mathbf{p}^*,\mathbf{n}^*)$ is an equilibrium if it satisfies equations (2)–(4). [5]

[5] From equation (3), firm output $q_{gi} = q_g$ is independent of i. Since $q_g = p_{gi}^{-\rho_g}\kappa_g$, it follows that price $p_{gi} = p_g$ is also independent of i. As discussed in Remark 1 of the appendix, this plays no role and is for expositional simplicity.

1.4. The Factor Content of Trade

Most previous HOV research has not adequately defined the factor content of trade for the case of traded intermediate inputs and international technology differences.[6] Here we follow Trefler and Zhu (2010). We begin by moving from the variety level to the national level. Since there are fixed costs ($\overline{\phi}_g$) and these are made with the same inputs as q_{gi} (see the discussion at the end of section 1.1), we consolidate these into $q_{gi} + \overline{\phi}_g$ and let $Q_{gi} = n_{gi}(q_g + \overline{\phi}_g)$ be output inclusive of fixed costs associated with good g in country i.[7] Let $C_{gij} \equiv n_{gj} p_{gj}^{-\rho_g} P_g^{\rho_g - 1} \eta_g Y_i$ be country i's final demand of good g produced in country j. Let M_{gij} be country i's imports of good g from country j, including imports of both final goods and intermediate inputs. Let $X_{gj} = \Sigma_{i \neq j} M_{gij}$ be country j's exports of good g. Let $\mathbf{Q}_i$, $\mathbf{C}_{ij}$, $\mathbf{M}_{ij}$, and $\mathbf{X}_i$ be $G \times 1$ vectors with gth elements of Q_{gi}, C_{gij}, M_{gij}, and X_{gi}, respectively. Let $\mathbf{B}_{ij}(\mathbf{w}_i,\mathbf{p})$ be a $G \times G$ matrix whose (g,h)-th element is $n_{gi} b_{ij}(g,h)$. This is what a typical producer of a variety of good h in country j demands (per unit of output) from all n_{gi} producers of good g in country i. Define the matrices:

$$\mathbf{Q} \equiv \begin{bmatrix} \mathbf{Q}_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{Q}_N \end{bmatrix}, \qquad \mathbf{C} \equiv \begin{bmatrix} \mathbf{C}_{11} & \cdots & \mathbf{C}_{N1} \\ \vdots & \ddots & \vdots \\ \mathbf{C}_{1N} & \cdots & \mathbf{C}_{NN} \end{bmatrix},$$

$$\mathbf{T} \equiv \begin{bmatrix} \mathbf{X}_1 & -\mathbf{M}_{21} & \cdots & -\mathbf{M}_{N1} \\ -\mathbf{M}_{12} & \mathbf{X}_2 & \cdots & -\mathbf{M}_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{M}_{1N} & -\mathbf{M}_{2N} & \cdots & \mathbf{X}_N \end{bmatrix} \text{ and } \mathbf{B} \equiv \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} & \cdots & \mathbf{B}_{1N} \\ \mathbf{B}_{21} & \mathbf{B}_{22} & \cdots & \mathbf{B}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{N1} & \mathbf{B}_{N2} & \cdots & \mathbf{B}_{NN} \end{bmatrix}$$

where $\mathbf{Q}$, $\mathbf{C}$, and $\mathbf{T}$ are $NG \times N$ and $\mathbf{B}$ is $NG \times NG$. The off-diagonal sub-matrices of $\mathbf{B}$ track trade in intermediate inputs. Output is used either for intermediates ($\mathbf{BQ}$) or consumption final demand ($\mathbf{C}$) or trade $\mathbf{T}$ so that $\mathbf{Q} = \mathbf{BQ} + \mathbf{C} + \mathbf{T}$ or

$$\mathbf{T} = (\mathbf{I}_{NG} - \mathbf{B})\mathbf{Q} - \mathbf{C} \tag{5}$$

where $\mathbf{I}_{NG}$ is the $NG \times NG$ identity matrix.

Let $\mathbf{D}_{fi}(\mathbf{w}_i,\mathbf{p})$ be a $1 \times G$ vector with gth column element $d_{fgi}(\mathbf{w}_i,\mathbf{p})$. Define the $1 \times NG$ vectors $\mathbf{D}_f \equiv [\mathbf{D}_{f1} \cdots \mathbf{D}_{fN}]$ and $\mathbf{A}_f \equiv \mathbf{D}_f(\mathbf{I}_{NG} - \mathbf{B})^{-1}$. Let $\mathbf{T}_i$ be the ith column of $\mathbf{T}$. Then, as shown in Trefler and Zhu (2010, Theorem 1),

$$F_{fi} \equiv \mathbf{A}_f \mathbf{T}_i = \mathbf{D}_f(\mathbf{I}_{NG} - \mathbf{B})^{-1}\mathbf{T}_i \tag{6}$$

is the amount of factor f employed worldwide to produce country i's net trade vector $\mathbf{T}_i$. Trefler and Zhu (2010, eq. 11) as well as the proof of theorem 1 below show that the following equation holds given market clearing and the above definition of the factor content of trade:

[6]For example, Davis and Weinstein (2001) assume that there are no traded intermediates and Antweiler and Trefler (2002) use a definition of the factor content of trade that is only meaningful if all the assumptions of the HOV model hold.

[7]We can treat fixed costs and output separately and have worked out all of the math to do so. However, Trefler and Zhu (2000) find that doing so makes little difference to any of the empirical conclusions.

$$F_{fi} = V_{fi} - s_i V_{fw} - \mathbf{A}_f \left(\mathbf{C}_i - s_i \mathbf{C}_w \right) \tag{7}$$

where s_i is country i's share of world consumption (or share of world income after adjusting for trade imbalances). We say that *consumption similarity* holds if $C_{gij} = s_i C_{gwj}$ i.e., if country i's share of world consumption of each good g produced in country j is equal to s_i. This is a common expression; for instance, if there are no intermediate inputs then $C_{gij} = M_{gij}$ and $C_{gwj} = Q_{gj}$ so that consumption similarity is just a restatement of gravity without distance ($M_{gij} = s_i Q_{gj}$).

By equation (7), the HOV-relevant factor content of trade equals the commonly predicted factor content ($V_{fi} - s_i V_{fw}$) plus a portion corresponding to departures from consumption similarity. Consumption similarity is thus a sufficient condition for the familiar Vanek equation:

Theorem 1 *Trefler and Zhu (2010) Let* $F_{fi} \equiv \mathbf{A}_f \mathbf{T}_i$. *If* $C_{gij} = s_i C_{gwj}$ $\forall g,i,j$, *then*

$$F_{fi} = V_{fi} - s_i \sum_{j=1}^{N} V_{fj}.$$

The proof appears in the appendix. The key insight of this theorem is that, if the Vanek equation fails, it is because of departures from consumption similarity. Among other causes, these departures can be due to non-homotheticies, iceberg trade costs, and the presence of nontraded sectors. We return to this point in our empirical work.

2. Empirical Specification

We now add enough structure to the cost functions for primary factors, $c_{gi}(\mathbf{w}_i)$, to achieve three aims: (1) To disentangle factor-augmenting international technology differences from substitution effects associated with the failure of productivity adjusted factor price equalization (PFPE); (2) To deepen insights from the development accounting literature (e.g., Caselli 2005, Caselli and Coleman 2006); and (3) To highlight previously neglected issues of identification. To this end, we assume that the cost function for primary factors is

$$c_{gi}(\mathbf{w}_i) = \left[\sum_f \frac{\alpha_{fg}}{\delta_{gi}} \left(\frac{w_{fi}}{\pi_{fi}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{8}$$

where the α_{fg} control (exogenous) factor intensities, the π_{fi} are factor-augmenting productivity (technology) parameters, σ is the elasticity of substitution, and the δ_{gi} are Ricardian technology parameters. We will assume that cost functions satisfy equation (8) for the remainder of the paper. When the $\delta_{gi} = 1$ for all g and i, equation (8) is a special case of Trefler's (1993) factor-augmenting technology and PFPE is straightforwardly defined as

$$\frac{w_{fi}}{\pi_{fi}} = \frac{w_{f,\mathrm{us}}}{\pi_{f,\mathrm{us}}} \,. \tag{9}$$

We consider a diversified equilibrium in which the Ricardian technology differences δ_{gi} lead to failure of PFPE.[8] It is straightforward to show that there are δ_{gi} which support a diversified

[8] δ_{gi} prevents international goods price equalization from leading to international productivity adjusted factor price equalization.

equilibrium, but we will need to ensure that our empirical counterparts of δ_{gi}, $\widehat{\delta}_{gi}$, are consistent with such an equilibrium. We will return to this point at the end of section 2.1. Here we review several minor points about diversification. First, the δ_{gi} can be interpreted as differences in quality, in which case our diversification has the flavour of Schott (2004).[9] Schott provides abundant evidence of diversification in his analysis of 'product overlap' at the 10-digit HS level. See also Sutton and Trefler (2016). Second, we treat observed diversifcation as trade in varieties rather than as a function of aggregation bias. We do this both because of Schott's evidence and because this allows for comparison of like goods in our productivity analysis and leads to a tight connection between our theory and empirics. Third, country-level productivity can be loaded onto either the π_{fi} or the δ_{gi} so a normalization is needed. We normalize the δ_{gi} using $\delta_{g,\text{us}} = 1$ for all g and $\Sigma_g \theta_{Lgi}\delta_{gi} = 1 \; \forall i$ where θ_{Lgi} is the share of country i's total labor endowment employed in industry g.

If varieties of good g are produced both by country i and by the United States, then Shephard's lemma implies

$$d_{fgi} = \beta_{fi} d_{fg,\text{us}} / \delta_{gi} \tag{10}$$

where

$$\beta_{fi} \equiv \left(\frac{w_{fi}/\pi_{fi}}{w_{f,\text{us}}/\pi_{f,\text{us}}} \right)^{-\sigma} \left(\frac{\pi_{fi}}{\pi_{f,\text{us}}} \right)^{-1} . \tag{11}$$

These β_{fi} are central to what follows. See the appendix for a proof. The terms in the first and second pairs of parentheses of (11) crystallize the substitution and productivity effects at the heart of Davis and Weinstein (2001) and Trefler (1993), respectively. The (direct) amount of a factor used to produce a unit of output can be high relative to the United States either because of low productivity adjusted wages (substitution effects) or because of low productivity. To avoid confusion, we refer to the $\pi_f \equiv (\pi_{fi}, \ldots, \pi_{fN})$ as 'productivity' parameters and the $\beta_f \equiv (\beta_{fi}, \ldots, \beta_{fN})$ as 'reduced-form' parameters since they capture international differences in both productivity and factor prices.[10]

2.1. *The Three Estimating Equations*

With the additional structure that flows from equation (8), we can now develop our three estimating equations. These equations completely describe the supply side of the model — there are no other supply-side equations of interest. Consider first the Vanek equation. Recall that $F_{fi} = \mathbf{D}_f(\mathbf{I}_{NG} - \mathbf{B})^{-1}\mathbf{T}_i$ is the factor content of trade using observed factor usage $\mathbf{D}_f$. Let $\mathbf{D}_f(\beta_f)$ be a $1 \times GN$ matrix with typical element $\beta_{fi} d_{fg,\text{us}}/\delta_{gi}$ (the right-hand side of equation 10) and define

$$F_{fi}(\beta_f) \equiv \mathbf{D}_f\left(\beta_f\right) \left[\mathbf{I}_{NG} - \mathbf{B}\right]^{-1} \mathbf{T}_i \, . \tag{12}$$

[9]This is similar to calibrating 'wedges' e.g., Hottman, Redding and Weinstein (2014) infer quality as the wedge that rationalizes demand for a given set of prices and quantities.

[10]On a trivial identification issue, we only identify the $\pi_{fi}/\pi_{f,\text{us}}$ and not the π_{fi} and $\pi_{f,\text{us}}$ separately. This is a standard feature of international productivity comparisons.

(We suppress the δ_{gi} as arguments.) Under our cost function assumption (equation 8), $\mathbf{D}_f(\beta_f)$ equals the data $\mathbf{D}_f$ and $F_{fi}(\beta_f)$ equals the data F_{fi}. It follows that the Vanek equation becomes

$$F_{fi}(\beta_f) = V_{fi} - s_i \sum_{j=1}^{N} V_{fj} \, . \tag{V}$$

The equation label (V) is for Vanek.[11] Since $F_{fi}(\beta_f)$ is linear in $\mathbf{D}_f(\beta_f)$ and $\mathbf{D}_f(\beta_f)$ is linear in β_f, $F_{fi}(\beta_f)$ is linear in β_f. Hence, equation (V) can be written as a system of linear equations that uniquely solve for the vector β_f. Notice that the unknown parameters (β_{fi}) show up on the left-hand side of the Vanek equation as in Davis and Weinstein (2001).

Turning to the Wage equation, substitute factor demands (equation 10) into the factor-market clearing condition (equation 4) and solve for productivity adjusted wages to obtain

$$\frac{w_{fi}/\pi_{fi}}{w_{f,\mathrm{us}}/\pi_{f,\mathrm{us}}} = \left[\frac{\pi_{f,\mathrm{us}} V_{f,\mathrm{us}}}{\pi_{fi} V_{fi}}\right]^{1/\sigma} \left(\sum_{g=1}^{G} \frac{d_{fg,\mathrm{us}} Q_{gi}}{\delta_{gi} V_{f\mathrm{us}}}\right)^{1/\sigma} \, . \tag{13}$$

See the appendix for a proof. The first term (in square brackets) shows that productivity adjusted factor prices are decreasing in productivity adjusted factor supplies, *ceteris paribus*. The second term shows that the price of factor f is bid up if output Q_{gi} is large in sectors with high per-unit demands for factor f, *ceteris paribus*. These demands are high when the sector is intensive in factor f ($d_{fg,\mathrm{us}}$ is large) or the sector is unproductive (δ_{gi} is small). Rearranging this equation yields our second estimating equation:

$$W_{fi}(\mathbf{D}_f, \mathbf{Q}, V_{fi}, \delta) \equiv \left[\sum_{g=1}^{G} \frac{d_{fg,\mathrm{us}} Q_{gi}}{\delta_{gi} V_{fi}}\right]^{-1} = \beta_{fi} \tag{W}$$

where $\delta \equiv \{\delta_{gi}\}_{\forall g,i}$ and $W_{fi}()$ is a function. The equation label (W) is for Wage, a short form for 'labor-market clearing.'

Turning to the third and last equation, the Techniques equation, we aggregate equation (10) up to the same level as the Vanek and Wage equations, namely, at the factor-country level. Specifically, taking the employment-weighted average of equation (10) yields $\sum_g \theta_{fgi} d_{fgi}/d_{fg,\mathrm{us}} = \sum_g \theta_{fgi}\beta_{fi}/\delta_{gi}$ where θ_{fgi} is the share of V_{fi} that is employed in industry g. The θ_{fgi} are data and satisfy $\sum_g \theta_{fgi} = 1$. Rearranging to isolate β_{fi} yields

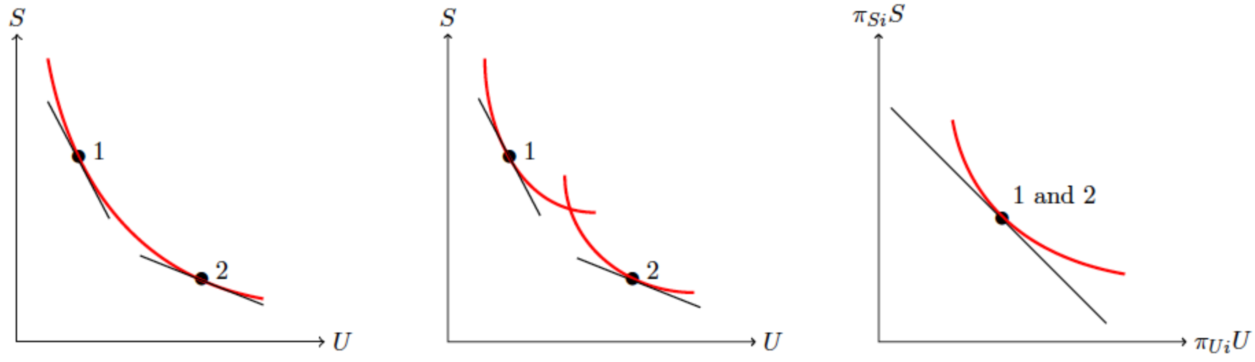
$$T_{fi}(\mathbf{D}_f, \delta) \equiv \frac{\sum_{g=1}^{G} \theta_{fgi}\left(d_{fgi}/d_{fg,\mathrm{us}}\right)}{\sum_{g=1}^{G} \theta_{fgi}/\delta_{gi}} = \beta_{fi} \tag{T}$$

where $T_{fi}()$ is a function. (The dependence of T_{fi} on the θ_{fgi} is suppressed.) Equation (T) also shows that average factor f usage is high when f-intensive industries tend to be unproductive i.e., when θ_{fgi} is large and δ_{gi} small so that $\sum_g \theta_{fgi}/\delta_{gi}$ is large. The equation label (T) is for Techniques. 'Techniques' refers to factor demand choices whereas technology refers to parameters of the cost function.

The only things that are not data in equations (T), (V) and (W) are the δ_{gi} and β_{fi}. We calibrate the δ_{gi} as follows. We have already chosen the normalizations $\delta_{g,\mathrm{us}} = 1$ and

[11] Given that $F_{fi}(\beta_f)$ and F_{fi} are very close, with a slight abuse of language, we refer to $F_{fi}(\beta_f)$ as the 'actual' factor content of trade below and $V_{fi} - s_i \sum_{j=1}^{N} V_{fj}$ as the 'predicted' factor content.

Figure 1: Identification



(*a*): Identical technologies
Choose curvature

(*b*): Identical curvature
Choose tech. diff.

(*c*): Technology differences
with PFPE

$\sum_g \theta_{Lgi}\delta_{gi} = 1$. From equation (10), $\delta_{gi} = (d_{fg,\mathrm{us}}/d_{fgi})\beta_{fi}$. Hence, $\delta_{gi} = \delta_{gi}/\sum_g \theta_{Lgi}\delta_{gi} = (d_{fg,\mathrm{us}}/d_{fgi})/\sum_g \theta_{Lgi}(d_{fg,\mathrm{us}}/d_{fgi})$. This establishes that we can calibrate the δ_{gi} using data on factor usages d_{fgi}.[12] Note that since the calibrated δ_{gi} satisfy equation (10), they are consistent with a diversified equilibrium.

This calibration of δ_{gi} depends on f and so is not unique. In our empirics we have two factors, skilled labor S and unskilled labor U, and so work with the geometric mean of the two: starting with $\delta_{gi} = [(d_{Ug,\mathrm{us}}/d_{Ugi})\beta_{Ui}]^{1/2}[(d_{Sg,\mathrm{us}}/d_{Sgi})\beta_{Si}]^{1/2}$ and following the steps in the previous paragraph yields

$$\widehat{\delta}_{gi} \equiv \frac{(d_{Ug,\mathrm{us}}/d_{Ugi})^{1/2}(d_{Sg,\mathrm{us}}/d_{Sgi})^{1/2}}{\sum_{g'=1}^{G}\theta_{Lg'i}(d_{Ug',\mathrm{us}}/d_{Ug'i})^{1/2}(d_{Sg',\mathrm{us}}/d_{Sg'i})^{1/2}} \,. \tag{14}$$

This is how we calibrate the δ_{gi} for the remainder of the paper.

To conclude, equations (T), (V) and (W) with the δ_{gi} set equal to the $\widehat{\delta}_{gi}$ are our three estimating equations.

2.2. *Identification*

A surprising conclusion emerges from examination of equations (T), (V) and (W): By themselves they cannot identify the factor-augmenting technology parameters π_{fi} nor the substitution effects associated with failure of PFPE i.e., they cannot be used to answer our major question. In these equations the only unknown parameters are the β_{fi}. Further, the only place where w_{fi}, π_{fi} and σ appear are in the β_{fi}. Let $\widehat{\beta}_{fi}$, $\widehat{\pi}_{fi}$ and $\widehat{\sigma}$ be estimates of β_{fi}, π_{fi} and σ, respectively, so that

$$\widehat{\beta}_{fi} = \left(\frac{w_{fi}/\widehat{\pi}_{fi}}{w_{f,\mathrm{us}}/\widehat{\pi}_{f,\mathrm{us}}}\right)^{-\widehat{\sigma}}\left(\frac{\widehat{\pi}_{fi}}{\widehat{\pi}_{f,\mathrm{us}}}\right)^{-1} \,. \tag{15}$$

Hence, given data on factor prices and given estimates $\widehat{\beta}_{fi}$, we cannot uniquely identify $(\widehat{\pi}_{fi}\,,\widehat{\sigma})$: We can only identify combinations of the $\widehat{\pi}_{fi}$ and $\widehat{\sigma}$. This lack of identification is well-known

[12] Intuitively, a Ricardian technology difference $\delta_{gi}/\delta_{g,\mathrm{us}}$ is the average difference in input requirements $d_{fg,\mathrm{us}}/d_{fgi}$ after purging them of their factor-augmenting productivity and wage components β_{fi}.

(Diamond et al., 1978) and intimately connected to the main concerns of this paper. To see this, suppose that we observe data on factor prices and the amounts of U and S per unit of output used in two different countries i.e., suppose we observe (w_{Ui},w_{Si}) and (d_{Ugi},d_{Sgi}) for countries $i = 1,2$. Figure 1(a) plots an isoquant in (U,S) space. Points correspond to (d_{Ugi},d_{Sgi}) and slopes to $-w_{Ui}/w_{Si}$. Now consider the problem of estimating cost or demand functions that are consistent with these data. One approach is to make the identifying assumption that technologies are internationally identical and then fit the data by adjusting the curvature of the isoquant. See panel (a). In our CES context this means adjusting σ. Another approach is to assume that there are international technology differences so that isoquants differ across countries. See panel (b). In our CES context this means adjusting the π_{fi}. In between there are countless other possibilities involving mixtures of curvature and international technology differences. That is, σ and the π_{fi} are not identified.

Trefler (1993) is the special case where PFPE is imposed so that $\widehat{\beta}_{fi} = (\widehat{\pi}_{fi}/\widehat{\pi}_{f,\text{us}})^{-1}$ i.e., the π_{fi} are identified, but not σ. It follows that Trefler could not address the Davis and Weinstein (2001) question about the importance of substitution effects when PFPE fails. The point is further illustrated in panel (c) where the axes are productivity adjusted factor inputs so that international differences in technology and factor prices disappear. Since all data for an industry are on a single point, substitution effects along an isoquant cannot be examined and σ cannot be estimated.

Davis and Weinstein (2001) is the special case in which PFPE fails and there are only Hicks-neutral productivity differences ($\pi_{Si} = \pi_{Ui} = \pi_i$). From equation (15), this implies $\widehat{\beta}_{fi} = (w_{fi}/w_{f,\text{us}})^{-\widehat{\sigma}}(\pi_i/\pi_{us})^{\widehat{\sigma}-1}$ which means that one cannot use the reduced-form $\widehat{\beta}_{fi}$ to infer the w_{fi}, π_{fi} and σ separately i.e., to make claims about whether it is international differences in factor prices or technology that are needed to 'fix' the Vanek equation. Since Davis and Weinstein do not use data on factor prices w_{fi} or external estimates of the elasticity of substitution σ, their reduced-form estimates cannot support their claims. And, given Diamond et al.'s (1978) more general non-identification result, this identification problem holds for any cost function. Surprisingly, then, Davis and Weinstein (2001) ask a great question but do not answer it.[13]

Given these identification issues our strategy is as follows. We estimate the β_{fi} from equations (T) and (W), choose a value of $\widehat{\sigma}$ that comes out of a directed technical change equation and that is also consistent with the labor literature, and use wage data together with the $\widehat{\beta}_{fi}$ and equation (15) to back out the $\widehat{\pi}_{fi}$.[14] We can then examine whether the $\widehat{\pi}_{fi}$ are consistent with PFPE (i.e. $w_{fi}/\widehat{\pi}_{fi} = w_{f,\text{us}}/\widehat{\pi}_{f,\text{us}}$) and answer the Davis and Weinstein question. This in turn is linked to the development accounting exercise of Caselli and Coleman (2006) in which wage and endowment data are used to calculate and characterize the skill bias of technology and the directed technical change of Acemoglu (1998) in which the skill bias is partially explained by endowments.

[13] What makes lack of identification surprising in their context is that their approach is very intuitive. They assume a reduced-form relationship between wages and endowments that is reminiscent of that in Katz and Murphy (1992). In our setting this is $\ln(w_{Si}/w_{Ui}) = \xi \ln(V_{Si}/V_{Ui})$ for some constant ξ. It would thus seem that endowments can be used in place of wages and estimates of $-1/\xi$ can be used in place of σ. We have tried without success to write down a cost function to support this intuition. Part of the problem is that the Katz and Murphy logic is based on the demand for labor in a single sector model. Adding in market clearing (demand *and* supply) and multiple sectors undermines this logic i.e., $\ln(w_{Si}/w_{Ui}) = \xi \ln(V_{Si}/V_{Ui})$ is incompatible with labor-market clearing (equation 4).

[14] We discuss our choice of $\widehat{\sigma}$ in detail below.

3. The Data

Unless otherwise noted, all data come from the World Input-Output Database (WIOD) as assembled by Timmer, Dietzenbacher, Los, Stehrer and Vries (2015). This data set has the advantage of providing information on the full world input-output matrix $\mathbf{B}$ and satisfying all data identities. Our data cover 38 developed and developing countries and 22 industries in the year 2006.[15] Countries and industries are listed in the appendix. WIOD includes trade data ($\mathbf{T}_i$), input-output data ($\mathbf{B}$), output data (Q_{gi}), and data on labor by industry, type, and country (V_{fgi}). Direct input requirements are $d_{fgi} = V_{fgi}/Q_{gi}$. Skilled workers (S) are those possessing some tertiary education. Unskilled workers (U) are the remainder of the labor force. The wage (w_{fi}) for each factor in a given country is given by aggregate compensation to the factor divided by the aggregate number of workers possessing that level of education.[16]

4. Results

4.1. The Wage and Techniques Equations

We begin by estimating the β_{fi} from the (W) and (T) equations. We do this separately by factor. To this end, we stack the T_{Ui} and W_{Ui} and regress the stacked vector on a set of country dummies to estimate the β_{Ui}. We then repeat this for skilled labor to estimate the β_{Si}. Denote these OLS estimates by $\widehat{\beta}_{fi}$. (Throughout this paper a β_{fi} with a 'hat' always refers to these estimates.) To deepen our understanding of these estimates, note that for each (f,i) pair, equation (W) defines a β^W_{fi} that makes the Wage equation fit perfectly. Likewise, equation (T) defines a β^T_{fi} that makes the Techniques equation fit perfectly. Our OLS estimator satisfies $\widehat{\beta}_{fi} = (\beta^W_{fi} + \beta^T_{fi})/2$.[17] [18]

Figure 2 presents the results. The left- and right-hand plots display results for unskilled and skilled labor, respectively. The top row plots the Wage equation (W), meaning, it plots

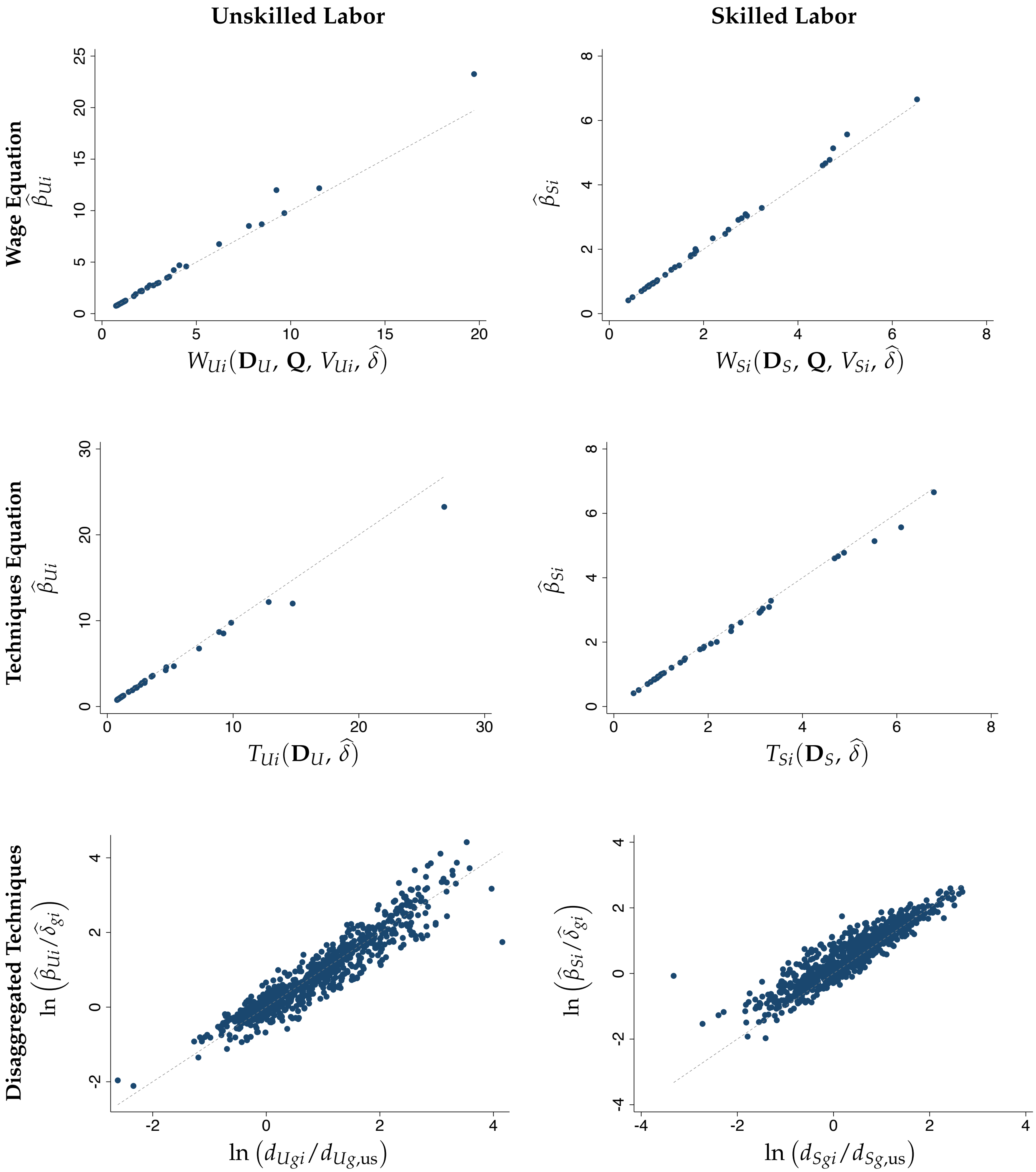
[15] We use 2006 because it is the most recent year before the Great Recession and the subsequent trade collapse.

[16] The Techniques and Wage equations (T) and (W) are defined so as to be unit free and thus naturally scaled. In contrast, the Vanek equation (V) is not unit free. Therefore, throughout this paper, we scale the Vanek equation by σ_f where $\sigma^2_f \equiv \Sigma_i (V_{fi} - s_i V_{fw})^2/N$. (This is a variance because $V_{fi} - s_i V_{fw}$ has a zero mean.) All the important results in this paper are invariant to the choice of scaling: Scaling simply eases visual exposition.

[17] Alternatively, we could have done feasible GLS. That is, for factor f, let h^W_f and h^T_f be estimates of the inverse of the variances of the (W) and (T) equations. Then the feasible GLS estimator is $\widehat{\beta}^{FGLS}_{fi} = \beta^W_{fi} h^W_f + \beta^T_{fi} h^T_f$. $\widehat{\beta}^{FGLS}_{fi}$ is virtually identical to the OLS estimate $\widehat{\beta}_{fi}$. Note that feasible GLS is GMM. Also note that GMM with optimal weighting is both biased in small samples (Altonji and Segal, 1996) and unworkable here because the optimal-weight GMM estimator will fit one equation perfectly and set the weight on the other equation to 0. That is, it will either choose β^W_{fi} and $h^T_f = 0$ or choose β^T_{fi} and $h^W_f = 0$. Finally, since the correlation between β^W_{fi} and β^T_{fi} is 0.99, all these estimators yield very similar estimates.

[18] We have not indicated where the error terms come from. We can be very precise about this. The *only* reason equations (W) and (T) do not fit perfectly is because equation (10) does not fit perfectly. Define the equation (10) error ε_{fgi} implicit via $d_{fgi} = \beta_{fi} d_{fg,\text{us}}/\widehat{\delta}_{gi} + \varepsilon_{fgi}$. Substituting this into factor-market clearing $\sum_g d_{fgi} Q_{gi} = V_{fi}$ and solving for β_{fi} yields $W_{fi} = \beta^W_{fi} + \varepsilon^W_{fi}$ where $\varepsilon^W_{fi} \equiv \sum_g \varepsilon_{fgi} Q_{gi} / \sum_g (d_{fg,\text{us}}/\widehat{\delta}_{gi}) Q_{gi}$. Turning to equation ($T$), start with $(d_{fgi}/d_{fg,\text{us}}) = \beta_{fi}/\widehat{\delta}_{gi} + (\varepsilon_{fgi}/d_{fg,\text{us}})$, average across the g using employment weights θ_{gi}, and solve for β_{fi} to obtain $T_{fi} = \beta^T_{fi} + \varepsilon^T_{fi}$ where $\varepsilon^T_{fi} \equiv \sum_g (\varepsilon_{fgi}\theta_{gi}/d_{fg,\text{us}}) / \sum_g (\theta_{gi}/\widehat{\delta}_{gi})$. In short, the error terms in equations (W) and (T) are data-weighted averages of the errors ε_{fgi} from equation (10).

Figure 2: Performance of the Wage and Techniques Equations: Two-Equation Approach



Notes: The left-hand side plots are for unskilled labor, the right-hand side plots are for skilled labor. The top panels are the Wage equation (W). The middle panels are the Techniques equations (T). The bottom panels are relative factor demands from equation (10), namely, $\ln(d_{fgi}/d_{fg,\mathrm{us}}) = \ln(\beta_{fi}/\delta_{gi})$. All equations are evaluated at the estimated values $\widehat{\beta}_{fi}$ and calibrated values $\widehat{\delta}_{gi}$. In the top and middle panels, each observation is a factor and country (f,i) while in the bottom panels each observation is a factor, industry and country (f,g,i). The 45° line is displayed in each panel.

$W_{fi}(\mathbf{D}_f, \mathbf{Q}, V_{fi}, \widehat{\delta})$ against $\widehat{\beta}_{fi}$. The middle row plots the Techniques equation (T), meaning, it plots $T_{fi}(\mathbf{D}_f, \widehat{\delta})$ against $\widehat{\beta}_{fi}$. It is clear that the fit is very good.

The Techniques and Wage equations are not collinear. However, there is only one source of error that prevents them from fitting perfectly, namely, that equation (10) does not fit perfectly.[19] If equation (10) were to fit perfectly, then the (W) and (T) equations would fit perfectly. To investigate the fit of equation (10), the bottom panels of figure 2 plot $\ln d_{fgi}/d_{fg,\text{us}}$ against $\ln \beta_{fi}/\widehat{\delta}_{gi}$. As is apparent, the fit is very good. The R^2 for unskilled and skilled labor are 0.89 and 0.84, respectively. This explains why the (W) and (T) equations *both* fit so well.

The $\widehat{\beta}_{fi}$ are reported in appendix table A1. Note that we can easily reject the hypothesis that $\widehat{\beta}_{Ui} = \widehat{\beta}_{Si}$ for all i at way less than the 1% level.[20]

4.2. The Vanek Equation

We begin by plotting the factor content of trade against its prediction. Since we predict techniques so well, actual factor demands $\mathbf{D}_f$ are very close to estimated factor demands $\mathbf{D}_f(\widehat{\beta}_f)$ and, consequently, actual factor contents F_{fi} are very close to estimated factor contents $F_{fi}(\widehat{\beta}_f)$. We therefore only report the results for $F_{fi}(\widehat{\beta}_f)$.[21] The top row of figure 3 plots $F_{fi}(\widehat{\beta}_f)$ against $V_{fi} - s_i V_{fw}$. The left- and right-hand panels are for unskilled and skilled labor, respectively. The good news for the Vanek equation is that the correlation is very high: the spearman rank correlation is 0.97 for unskilled labor and 0.98 for skilled labor. There are two outliers, China to the right and the United States to the left. The correlations without these outliers are also very high, 0.96 and 0.98 for unskilled and skilled labor, respectively. Also note that the share of observations for which $F_{fi}(\widehat{\beta}_f)$ and $V_{fi} - s_i V_{fw}$ have the same sign (the 'sign test') is 0.95 for both unskilled and skilled labor. This good fit of the Vanek equation for skilled and unskilled labor is a new result in the literature. [22]

4.2.1. Missing Trade

There is only one problem with the performance of the Vanek equation, namely, Trefler's (1995) 'missing trade.' This is apparent from the displayed 45° line, which is steeper than a line of best fit.[23] It is obvious that if one wants to explain missing trade then one must deal with trade costs and especially nontraded services e.g., Trefler (1995) and Davis and Weinstein (2001). Further, an immediate implication of theorem 1 and equation (7) is that if nontradables lead to departures from the Vanek equation then they do so via departures from consumption similarity.

[19]See foonote 18 above.

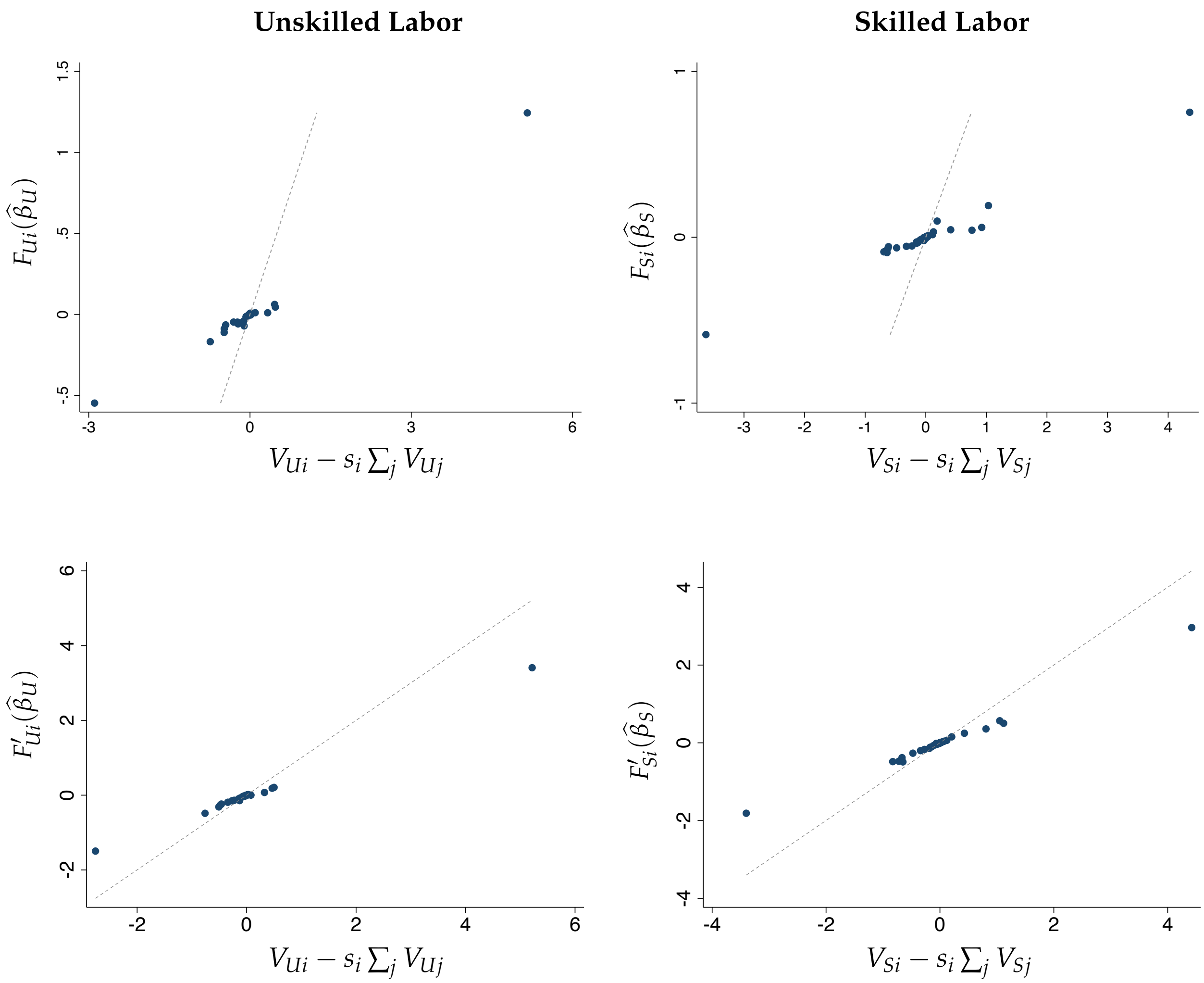
[20]$F^{38}_{76} = 27.56$ where the 1% critical value is $F^{38}_{76} = 1.88$.

[21] There are two reasons for the lack of a perfect fit. The first is the error term ε_{fgi} described in footnote 18. The second is the equation (7) error term $\epsilon_{fi} \equiv \mathbf{A}_f(\mathbf{C}_i - s_i\mathbf{C}_w)$.

[22]The good fit has been documented by Trefler and Zhu (2010) for the case of aggregate labor. We cannot directly compare our results to Davis and Weinstein (2001) because they use aggregate labor and Hicks-neutral technology, but the results are much better than those associated with their most similar specification (T3).

[23]The slope from an OLS regression of $F_{fi}(\widehat{\beta}_f)$ on $V_{fi} - s_i V_{fw}$ is 0.23 for unskilled labor and 0.16 for skilled labor.

Figure 3: Performance of the Vanek Equation



Notes: All panels plot the Vanek equation (V). The left- and right-hand columns are for unskilled and skilled labor, respectively. The top row uses the factor content of trade i.e., $F_{fi}(\widehat{\beta}_f)$. The bottom row uses the Government Services adjusted factor content of trade i.e., $F'_{fi}(\widehat{\beta}_f)$. Each observation is a factor and country (f,i) and the most extreme observations in each panel are the United States (to the left) and China (to the right). All lines are 45° lines.

Since this paper is primarily concerned with the role of factor prices and factor-augmenting technology, and since the handling of missing trade is orthogonal to these issues (i.e., it has no impact on how we estimate the β_{fi}), we focus solely on the most obvious source of nontradables, namely, Government Services, which *by definition* should be nontraded.[24]

We follow Trefler and Zhu (2010) in our treatment of Government Services. For expositional simplicity, assume for the moment that there are no intermediate inputs so that consumption similarity is the same as gravity without distance. Also, let $g = G$ denote Government Services. Had consumption similarity held in Government Services, country i would have consumed a share s_i of the Government Services produced by country j (Q_{Gj}). Hence, i's imports of Government Services from j would have been $s_i Q_{Gj}$ and country j's exports to the world would have been $(1 - s_j)Q_{Gj}$ (production less consumption). Let $\mathbf{T}'_i$ be the vector $\mathbf{T}_i$ in equation (6), but with the elements corresponding to M_{Gij} and X_{Gj} replaced by $s_i Q_{Gj}$ and $(1 - s_j)Q_{Gj}$, respectively. The easy generalization of $\mathbf{T}'_i$ to include intermediate inputs, which is what we use for the remainder of the paper, is described in the appendix.

With $\mathbf{T}'_i$ in hand we can compute the adjusted factor content of trade as $F'_{fi} \equiv \mathbf{D}_f \left[\mathbf{I}_{NG} - \mathbf{B}\right]^{-1} \mathbf{T}'_i$ or, using estimated techniques,

$$F'_{fi}(\widehat{\beta}_f) \equiv \mathbf{D}_f \left(\widehat{\beta}_f\right) \left[\mathbf{I}_{NG} - \mathbf{B}\right]^{-1} \mathbf{T}'_i \, . \tag{16}$$

The bottom row of figure 3 plots $F'_{fi}(\widehat{\beta}_f)$ against $V_{fi} - s_i V_{fw}$. As is apparent, missing trade is much less: The OLS slope is 0.65 for unskilled labor and 0.66 for skilled labor. Online appendix figure B1 drops the outliers China and the United States and shows that the fit inside the 'pack' is very good. Further, we could raise the slope even more if we treated other sectors such as Construction as nontradable.

Our approach is surprisingly similar to that of Davis and Weinstein (2001), who deal with nontradables by netting out the endowments used to produce nontradables i.e., by netting out the factor content of nontradables. The following lemma establishes this.

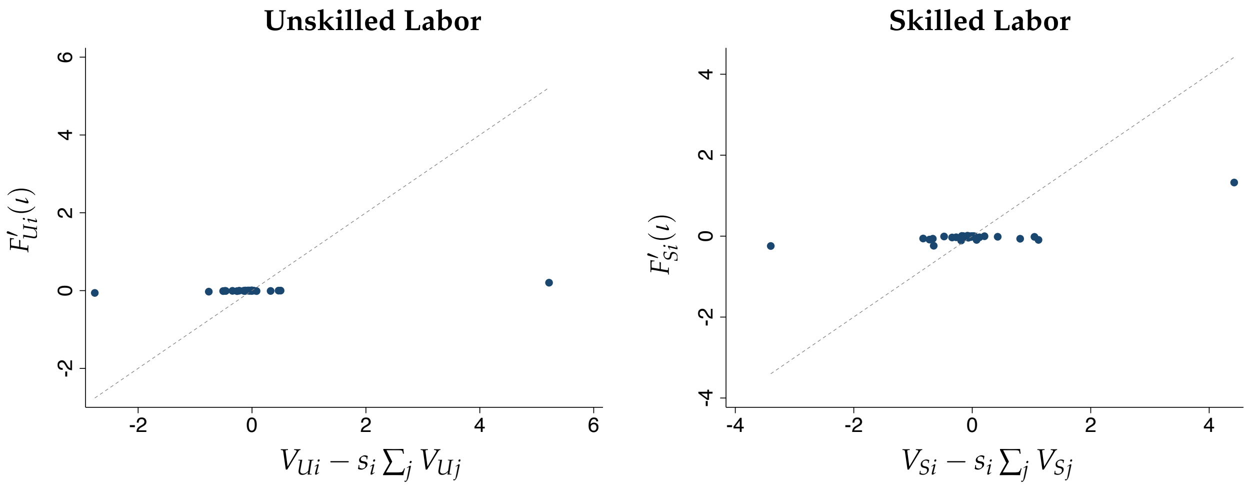
Lemma 2 *Let G' be the set of nontradable goods. Define $V'_{fi} \equiv \sum_{g \in G'} A_{fgi} C_{gii}$ as the factor content of nontradable consumption or, equivalently, the endowments used to produce nontradable consumption. Let $V'_{fw} \equiv \sum_j V'_{fj}$. Assume that consumption similarity holds for tradable goods: $C_{gij} = s_i C_{gwj}$ for all i and j and all $g \notin G'$. Recall from equation (7) that $F_{fi} = V_{fi} - s_i V_{fw} - \mathbf{A}_f \left(\mathbf{C}_i - s_i \mathbf{C}_w\right)$. Then*

1. $\mathbf{A}_f \left(\mathbf{C}_i - s_i \mathbf{C}_w\right) = V'_{fi} - s_i V'_{fw}$ *and* $F_{fi} = V_{fi} - s_i V_{fw} - \left[V'_{fi} - s_i V'_{fw}\right]$
2. $\mathbf{A}_f \left(\mathbf{C}_i - s_i \mathbf{C}_w\right) = F'_{fi} - F_{fi}$ *and* $F'_{fi} = \left(V_{fi} - s_i V_{fw}\right)$

Part 1 states that under the conditions of the lemma, one can derive the type of estimating equation first examined by Davis and Weinstein. The second part derives our estimating equation. Together,

[24]That is, System of National Accounts manuals instruct national statistical agencies to exclude from this sector all government services that are sold via market transactions. By way of example, Canadian postal services are sold to the public but police services are not so that Government Services excludes the post office but not the police. Since by definition Government Services are not sold on markets, they are nontraded — we do not see California state troopers patrolling the streets of São Paolo. We therefore need to treat Government Services differently.

Figure 4: Performance of the Vanek Equations ($\beta_{fi} = 1$)



Notes: Both panels plot the Vanek equation (V). The left- and right-hand columns are for unskilled and skilled labor, respectively. The plots use the Government Services adjusted factor content of trade, but sets the $\beta_{fi} = 1$ i.e., $F'_{fi}(\iota)$. The 45° line is displayed. Each observation is a factor and country (f, i).

the two parts imply that the approaches are equivalent. The term $\mathbf{A}_f(\mathbf{C}_i - s_i\mathbf{C}_w)$ appears on the right side in Davis and Weinstein and on the left side in our work.

The advantage of putting $\mathbf{A}_f(\mathbf{C}_i - s_i\mathbf{C}_w) = V'_{fi} - s_iV'_{fw}$ on the right side is that netting out the factor content of nontradable endowments is intuitive. The disadvantage is that this term is endogenous and so belongs on the left side. Empirically, if we put it on the right side there is not much improvement in missing trade (the slope is only 0.28 for unskilled labor and 0.29 for skilled labor). The reason is simple: $V'_{fi} - s_iV'_{fw}$ is small relative to $V_{fi} - s_iV_{fw}$ so that the latter term dominates and the slope does not rise much.

4.2.2. The Role of the $\widehat{\beta}_f$ for the Vanek Equation

To investigate the role of the $\widehat{\beta}_f$ for the fit of the Vanek equation, in figure 4 we set $\beta_f = (\beta_{f1}, \ldots, \beta_{fN}) = \iota$ where ι is an N-vector of ones. $F'_{fi}(\iota)$ is the factor content of trade if all countries use US techniques save for differences in δ_{gi}. We plot $F'_{fi}(\iota)$ against $V_{fi} - s_iV_{fw}$. The fit is horrible in two dimensions. First, there is an increase in missing trade, which is related to previous findings (Trefler, 1995, Davis and Weinstein, 2001, Trefler and Zhu, 2010) that missing trade is exacerbated when all countries are forced to have the same choice of techniques.[25] Second, the rank correlation deteriorates: 0.44 for unskilled labor and 0.26 for skilled labor. We conclude from this that the β_f play a central role for understanding the Vanek prediction.

[25]Slopes are 0.03 and 0.19 for unskilled and skilled labor, respectively.

4.3. *Spirit of HO*

When we teach the Heckscher-Ohlin model to our students we focus on the role of *relative* abundance and *relative* factor intensities. In contrast, since Vanek (1968) and Leamer (1980), empirical research has examined the Vanek equation one factor at a time. We return to the earlier tradition by examining skilled relative to unskilled labour. This also serves as a 'stress test' of our results.

Figure 5 reports the results in relative terms. The Wage and Techniques terms are in ratios. We put the Vanek equation in differences rather than ratios because — since F'_{fi} can be positive, negative or zero — F'_{Si}/F'_{Ui} is both hard to interpret and becomes extreme when F'_{Ui} is close to 0. Turning to figure 5, each equation does extremely well when examined in terms of skilled relative to unskilled labour! No previous paper has subjected its results to this stress test. Further, these results will be of great interest to those who teach the pre-Vanek/Leamer characterization of the Heckscher-Ohlin model.

4.4. *The Full-Information, Three-Equation Approach*

We want to ensure that we have exploited *all* of the supply-side information about productivity in estimating the β_{fi}. The only source that we have not yet exploited is the comparative advantage information contained in trade flows, which suggests that we should also use the Vanek equation for estimation of the β_{fi}. A reason for being cautious is that trade flows combine information from both the supply and demand sides and so are 'contaminated' by demand. We therefore begin by understanding how to quarantine this contamination. The intuition is simple: If consumption similarity holds then demand patterns are proportional across countries and the Vanek equation is uncontaminated by demand. We formalize this.

Lemma 3 *Suppose $C_{gij} = s_i C_{gwj}$ $\forall g,i,j$. Then equation (V) is equivalent to*

$$F_{fi}(\iota) = \beta_{fi}^{-1} V_{fi} - s_i \textstyle\sum_j \beta_{fj}^{-1} V_{fj} \ . \tag{VT}$$

Further, the β_{fi} that make this equation fit perfectly are given by

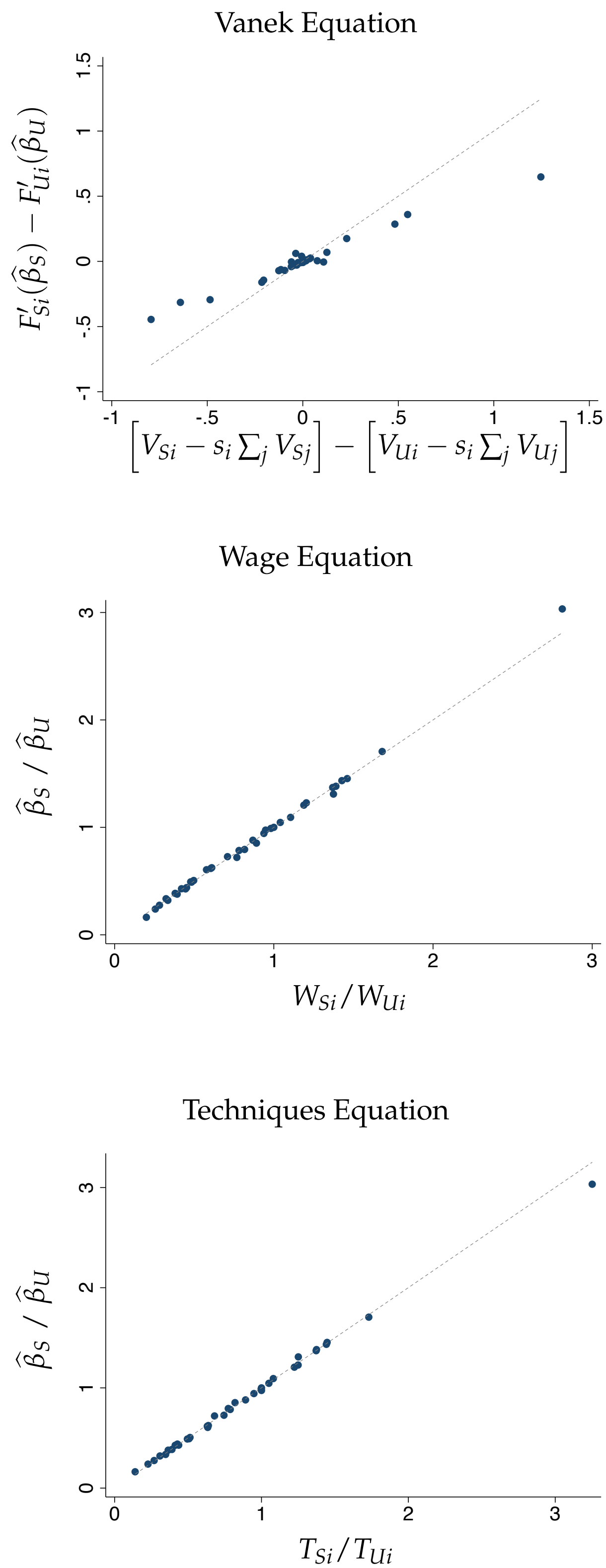
$$(\beta_{fi}^{VT})^{-1} \equiv \frac{s_i / V_{fi}}{s_{us}/V_{f,us}} + \frac{s_i}{V_{fi}} \left(\frac{F_{fi}(\iota)}{s_i} - \frac{F_{f,us}(\iota)}{s_{us}} \right). \tag{17}$$

Equation (VT) states that under consumption similarity we can generalize Trefler's (1993) Vanek equation to the case where there are Ricardian productivity differences and where productivity adjusted factor price equalization fails.[26] Equation (17) states that if we estimated the β_{fi} solely from the modified Vanek equation (VT) then we would end up with $\beta_{fi} = \beta_{fi}^{VT}$.

We next show that the β_{fi}^{VT} capture important aspects of productivity from the development accounting literature. To make sense of the first term on the right-hand side of equation (17),

[26] Under productivity adjusted factor price equalization, $(\beta_{fi})^{-1} = \pi_{fi}/\pi_{f,us}$. Without Ricardian productivity differences ($\delta_{gi}/\delta_{g,us} = 1$), $F_{fi}(\iota)$ is the factor content of trade using US techniques (denoted F_{fi}^{us}). Hence equation (VT) becomes $F_{fi}^{\text{us}} = \pi_{fi} V_{fi} - s_i \sum_j \pi_{fj} V_{fj}$, which is Trefler's equation.

Figure 5: Differencing Across Factors: Two Equation Approach (T and W)



Notes: The top row plots the difference in the Government Services adjusted factor content of trade $F'_{Si}(\widehat{\beta}_S) - F'_{Ui}(\widehat{\beta}_U)$ against the difference in the predicted factor content of trade $\left[V_{Si} - s_i \sum_j V_{Sj}\right] - \left[V_{Ui} - s_i \sum_j V_{Uj}\right]$. The middle row plots W_{Si}/W_{Ui} against $\widehat{\beta}_{Si}/\widehat{\beta}_{Ui}$. The bottom row plots relative unit input requirements (T_{Si}/T_{Ui}) against $\widehat{\beta}_{Si}/\widehat{\beta}_{Ui}$. All lines are 45∘ lines.

consider a single-good economy with an aggregate production function

$$Y_i = \left[\alpha_U(\pi_{Ui}V_{Ui})^{\frac{\sigma-1}{\sigma}} + \alpha_S(\pi_{Si}V_{Si})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{18}$$

where Y_i is both output and income. Equating the marginal product of factor f with its factor price yields

$$w_{fi} = \alpha_f(\pi_{fi})^{\frac{\sigma-1}{\sigma}}\left(Y_i/V_{fi}\right)^{\frac{1}{\sigma}} \ . \tag{19}$$

Dividing by the corresponding equation for the United States and substituting out Y_i using $s_i = Y_i/\sum_j Y_j$ yields

$$\beta_{fi}^{-1} = \frac{s_i/V_{fi}}{s_{\text{us}}/V_{f,\text{us}}}$$

That is, the first term in equation (17) comes straight out of the most basic Development Accounting exercise.[27]

The second term in equation (17) is the information about productivity contained in trade flows. Specifically, after dividing through by s_i to control for country size, if factor f in country i has a larger factor content of trade than in the United States then factor f is revealed by trade to be more productive in i than in the United States.

This discussion demonstrates that if consumption similarity holds — or empirically by equation (7), if the Vanek equation fits well — then the Vanek equation is a third (and final) source of information about productivity. Moving to estimation using the Vanek equation, we note that $F'_{fi}(\beta_f)$ is linear in $\beta_f \equiv (\beta_{f1},\ldots,\beta_{fN})$ so that equation (V) can be rewritten as

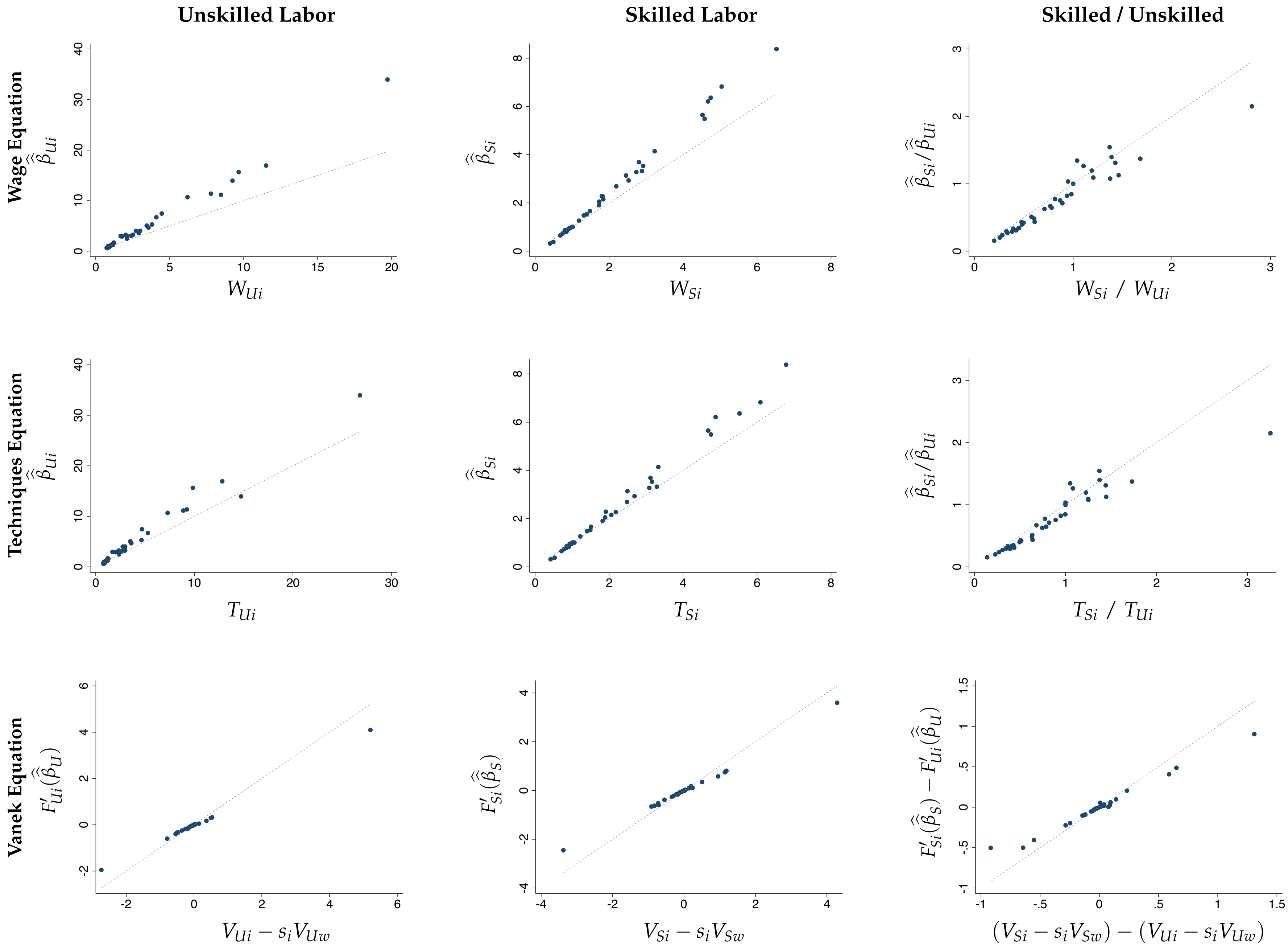
$$H_{fi}(V_{f1},\ldots,V_{fN},F'_{f1}(\iota),\ldots,F'_{fN}(\iota),\widehat{\delta}) = \beta_{fi} \tag{V'}$$

for some function H_{fi} which depends only on data. Doing this allows us to simply stack equations (W), (T) and (V') and estimate the β_{fi} using OLS and country-factor dummy variables. Let $\widehat{\widehat{\beta}}_f$ denote the vector of estimates.

The first thing to note about the estimates is that $\widehat{\widehat{\beta}}_f$ is very close to the two-equation estimate $\widehat{\beta}_f$: the correlation for each factor is 0.99. The second thing to note is that the full-information, three-equation approach produces good results. These are on display in figure 6. The three columns from left to right are for unskilled labor, skilled labor, and the ratio of skilled to unskilled labor. The three rows from top to bottom are for the Wage equation, the Techniques equation, and the Vanek equation. In all nine panels, the fit is very good. Turning specifically to the Vanek equation, notice that there is less missing trade than before. The slope coefficient from a regression of $F'_{fi}(\widehat{\widehat{\beta}}_f)$ on $V_{fi} - s_iV_{fw}$ is now 0.77 for unskilled labor and 0.78 for unskilled labor. The rank correlation statistics for each are high at 0.99. Online appendix figure B1 redoes the figure without the US and Chinese outliers.

[27]Here are the details. $w_{fi}/w_{f,\text{us}} = (\pi_{fi}/\pi_{f,\text{us}})^{\frac{\sigma-1}{\sigma}}\left[(s_i/V_{fi})/(s_{\text{us}}/V_{f,\text{us}})\right]^{\frac{1}{\sigma}}$. Rearranging yields $(w_{fi}/w_{f,\text{us}})^{\sigma}(\pi_{fi}/\pi_{f,\text{us}})^{1-\sigma} = (s_i/V_{fi})/(s_{\text{us}}/V_{f,\text{us}})$. From equation (15), the left-hand side is β_{fi}^{-1}. The π_{fi} that satisfies this equation is exactly the same measure of productivity as in Caselli and Coleman (2006) for the case without capital. See their footnote 7.

Figure 6: Full-Information, Three-Equation Approach



Notes: The left-hand column plots are for unskilled labor, the middle column plots are for skilled labor, and the right-hand column plots are for skilled relative to unskilled labor. The top row is the Wage equation (W), the middle row is the Techniques equation (T), and the bottom row is the Vanek equation (V). All equations are evaluated at the full-information, three-equation estimates of the β_{fi} and calibrated values of the δ_{gi}. Each observation is a factor and country. The 45° line is displayed in each panel.

Table 1: Test Statistics for the Fit of the Vanek Equation

Factor Content of Trade	Unskilled Labor: Rank Corr.	Unskilled Labor: Variance Ratio	Unskilled Labor: Sign Test	Unskilled Labor: Slope Test	Skilled Labor: Rank Corr.	Skilled Labor: Variance Ratio	Skilled Labor: Sign Test	Skilled Labor: Slope Test
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1. F_{fi}	0.953	0.038	0.868	0.193	0.974	0.016	0.947	0.127
2. $F_{fi}(\widehat{\beta}_f)$	0.965	0.052	0.947	0.226	0.982	0.027	0.947	0.163
3. $F'_{fi}(\widehat{\beta}_f)$	0.978	0.424	0.947	0.648	0.990	0.452	0.921	0.664
4. $F_{fi}(\widehat{\beta}_f)$	0.962	0.078	0.895	0.276	0.976	0.089	0.921	0.29
5. $F'_{fi}[\iota]$	0.441	0.001	0.763	0.034	0.256	0.052	0.474	0.185
6. $F'_{fi}(\widehat{\widehat{\beta}}_f)$	0.995	0.588	0.947	0.765	0.995	0.607	0.974	0.776

Notes: This table presents test statistics for the fit of the Vanek equation (V) for different specifications of the factor content of trade. In row 1, the actual factor content of trade is used. In row 2, the factor content of trade is calculated using $\widehat{\beta}_f$ (the two-equation estimate of β_f) and equation (12). In row 3, the factor content of trade is adjusted for nontraded Government Services using equation (16). In row 4, the nontraded Government Services adjustment is put on the right-hand side of the Vanek equation as in part 1 of lemma 2 and as in Davis and Weinstein (2001). In row 5, the factor content of trade is again adjusted for nontraded Government Services using equation (16), but all elements of the vector $\widehat{\beta}_f$ are set to 1. In row 6, the factor content of trade is again adjusted for nontraded Government Services using equation (16), but the estimate of β_f is from the three-equation approach. 'Rank Corr.' is the rank or Spearman correlation between the factor content of trade and $V_{fi} - s_i V_{fw}$. 'Variance Ratio' is the variance of the factor content of trade divided by the variance of $V_{fi} - s_i V_{fw}$. 'Sign Test' is the proportion of observations for which the factor content of trade and $V_{fi} - s_i V_{fw}$ have the same sign. 'Slope Test' is the OLS slope estimate from a regression of the factor content of trade on $V_{fi} - s_i V_{fw}$.

It is conventional in HOV papers to report a large number of test statistics. While we feel that the plots tell the full story, we give a nod to convention in Table 1. The table notes explain the familiar tests.

5. Factor Prices, Substitution Effects, Factor Bias, and Directed Technical Change

Up to this point, much of the paper has been concerned with issues surrounding the specification and fit of the Vanek, Wage and Techniques equations, issues which depend only on the reduced-form β_{fi}. We now turn to a host of questions which depend critically on the underlying structural parameters i.e., on the elasticity of substitution between skilled and unskilled labor (σ) and the factor-augmenting technology parameters (π_{fi}).

The intuition for how we can extract π_{fi} from the β_f comes from Caselli and Coleman (2006) and our discussion of identification. Simplify notation with the normalization $\pi_{f,\text{us}} = 1$ and by normalizing wages using $w_{f,\text{us}} = 1$ for $f = S,U$. Then from equation (15)

$$\pi_{fi} = (\beta_{fi}/w_{fi}^{-\sigma})^{1/(\sigma-1)} . \tag{20}$$

What this says is that for a given value of σ, international factor price differences generate international differences in factor demands according to $w_{fi}^{-\sigma}$, which in turn generates differences in the estimates β_{fi}. Any variation in the β_{fi} not explained by variation in factor prices must be due to international technology differences (the π_{fi}). How so depends on whether or not σ is greater or less than unity. For example, suppose that a country has greater unskilled-intensive demand (relative to the United States) than is predicted based on its wages and the elasticity of substitution. If skilled and unskilled labor are substitutes ($\sigma > 1$), then unskilled-augmenting productivity must be higher in this country. If skilled and unskilled labor are complements ($\sigma < 1$), then unskilled-augmenting productivity must be lower. Thus, we need an estimate of σ.

Typical values for the elasticity of substitution between skilled and unskilled labor range between 1.4 and 2. Although we discuss and justify our choice of σ in detail in section 5.5, we start near the midpoint with σ=1.67.[28] Since our results are completely insensitive to how we estimate the β_{fi}, we use our two-equation estimates ($\widehat{\beta}_{fi}$). Plugging these into equation (20) generates our productivity estimates. These are reported in appendix table A1. We turn now to using those estimates to answer substantive questions.

5.1. *Are Productivity Adjusted Factor Prices Equalized?*

Leamer and Levinsohn's (1995, p. 1360) factor price insensitivity theorem states that in the FPE set, factor prices are insensitive to endowments.[29] In the context of the factor-market clearing condition (equation 13), this means that the impact on wages of differences in productivity adjusted factor endowments

$$S_{fi} \equiv \left[\frac{\pi_{f,\mathrm{us}} V_{f,\mathrm{us}}}{\hat{\pi}_{fi} V_{fi}} \right]^{1/\sigma}$$

are exactly offset by differences in industrial composition

$$R_{fi} \equiv \left(\sum_{g=1}^{G} \frac{d_{fg,\mathrm{us}} Q_{gi}}{\widehat{\delta}_{gi} V_{f\mathrm{us}}} \right)^{1/\sigma} .$$

Restated, Rybczynski effects (R_{fi}) exactly offset supply effects (S_{fi}). Then from equation (13) in logs,

$$\ln \left(\frac{w_{fi} / \widehat{\pi}_{fi}}{w_{f,\mathrm{us}} / \pi_{f,\mathrm{us}}} \right) = \ln S_{fi} + \ln R_{fi} + \ln \epsilon_{fi}^{W} \tag{21}$$

where we have added an error term so that equation (21) is an identity.[30]

Given that equation (21) is an identity we can use a variance decomposition to assess the relative importance of each of the three right-hand side components of (21). We implement this following Bernard, Jensen, Redding and Schott (2007). Consider column 1 of the upper panel of table 2, which deals with unskilled labor. It is a regression of $\ln S_{fi}$ on the left-hand side term of equation

[28] The seminal citation is Katz and Murphy (1992) which provides $\sigma = 1.4$. Caselli and Coleman (2006) use values of σ between 1.1 and 2. Card and Lemieux (2001) find $\sigma = 2.5$. See Ciccone and Peri (2005) for a review of estimate of the elasticity of substitution between skilled and unskilled labor. See Antràs (2004) for estimates of the elasticity of substitution between capital and aggregate labor.

[29] In our context, the theorem is productivity adjusted factor price insensitivity.

[30] The error term is intimately related to ε_{fi}^{W} described in footnote 18.

Table 2: Decomposition of the Wage Equation

Panel A: Unskilled Labor

	(1)	(2)	(3)
	$\ln(S_{Ui})$	$\ln(R_{Ui})$	$\ln(\epsilon^{W}_{Ui})$
$\ln\left(\frac{w_{Ui}}{\pi_{Ui}} / \frac{w_{U\text{us}}}{\pi_{U\text{us}}}\right)$	1.38***	-0.34	-0.040***
	(0.44)	(0.45)	(0.01)
R-squared	0.21	0.02	0.23

Panel B: Skilled Labor

	$\ln(S_{Si})$	$\ln(R_{Si})$	$\ln(\epsilon^{W}_{Si})$
$\ln\left(\frac{w_{Si}}{\pi_{Si}} / \frac{w_{S\text{us}}}{\pi_{S\text{us}}}\right)$	0.89***	0.12	-0.0085***
	(0.22)	(0.22)	(0.00)
R-squared	0.31	0.01	0.22

Panel C: Skilled Relative to Unskilled Labor

	$\ln(S_{SUi})$	$\ln(R_{SUi})$	$\ln(\epsilon^{W}_{SUi})$
$\ln\left(\frac{w_{Si}\pi_{Ui}}{w_{Ui}\pi_{Si}} / \frac{w_{S\text{us}}\pi_{U\text{us}}}{w_{U\text{us}}\pi_{S\text{us}}}\right)$	0.96***	0.027*	0.014**
	(0.01)	(0.01)	(0.01)
R-squared	0.99	0.10	0.15

Notes: Within each panel, each column represents a separate regression. The dependent variable is identified by the column header and the independent variable is identified by the column on the left. All regressions are OLS with 38 observations. Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

(21). The coefficient is large, indicating that the supply component explains most of the variation in wages. Repeating this for $\ln R_{fi}$ in column 2 and $\ln \varepsilon^{W}_{fi}$ in column 3 shows that the supply component is by far the most important. Notice that the coefficients in columns 1–3 must sum to unity by construction. In this sense, the three coefficients provide a variance decomposition of the left-hand side of equation (21) into its components.[31]

We repeat the exercise for skilled labor in panel B of table 2 and skilled relative to unskilled labor in panel C of table 2. As is apparent, the same conclusions emerge, namely, productivity adjusted factor prices are highly sensitive to productivity adjusted factor supplies. We conclude from this that Davis and Weinstein (2001) were correct to emphasize failure of factor price equalization and the role of factor supplies.[32]

[31]It is tempting to interpret the coefficients in table 2 in terms of first moments (slopes); however, the coefficients speak to second moments e.g., a negative coefficient means that the component is associated with a smaller *variance* of factor prices.

[32]Our finding of a lack of 'Rybczynski effects' is consistent with the results of Gandal, Hanson and Slaughter (2004), Lewis (2004) and Blum (2010), though less consistent with the results of Hanson and Slaughter (2002).

5.2. *International Differences in Unit Factor Demands: Substitution Effects or Productivity?*

We can now answer the fundamental question posed by Davis and Weinstein, namely, are international differences in average choice of techniques due more to substitution effects (failure of PFPE) or to international technology differences? We start with the Techniques equation (T), take logs and rewrite it to isolate the object of interest, $\sum_{g=1}^{G} \theta_{fgi} \left(\frac{d_{fgi}}{d_{fg,\mathrm{us}}}\right)$:

$$\ln\left[\sum_{g=1}^{G} \theta_{fgi}\left(\frac{d_{fgi}}{d_{fg,\mathrm{us}}}\right)\right] = \ln\left(\frac{w_{fi}}{\widehat{\pi}_{fi}}\right)^{-\sigma} + \ln\left(\widehat{\pi}_{fi}\right)^{-1} + \ln\left(\sum_{g=1}^{G} \theta_{fgi}/\widehat{\delta}_{gi}\right) + \ln\epsilon_{fi}^{T} \tag{22}$$

where we have added an error term so that equation (22) is an identity.[33] Looking to the right of the equal sign, differences in average techniques are associated with (1) substitution effects due to differences in productivity adjusted factor prices, (2) productivity effects due to differences in factor-augmenting technology, (3) Ricardian productivity differences, and (4) an error term. Similar to our decomposition of the Wage equation, we assess the contribution of each of these four components by separately regressing each on the left-hand side term of equation (22). The coefficients from the four separate regressions again sum to unity.

Table 3 reports the results. Panels A and B present results for unskilled and skilled labor, respectively. They show that most of the variance in techniques relative to the United States is due to international differences in factor-augmenting technology. Factor prices, Ricardian terms, and errors are less important. In contrast, panel C presents results for the ratio of skilled to unskilled labour. (Roughly, panels A and B are about $\ln d_{Ugi}$ and $\ln d_{Sgi}$, respectively, whereas panel C is about $\ln d_{Sgi}/d_{Ugi}$). For variation in relative factor demands it is factor prices that are most important. Recapping, differences in factor-augmenting productivity are most important when analyzing per unit input requirements, but substitution effects caused by departures from PFPE are most important when looking at input requirements of skilled relative to unskilled labor.

5.3. *The Vanek Equation: Technology Differences or Failure of Factor Price Equalization?*

Davis and Weinstein (2001) and Trefler (1993) place substitution effects and factor-augmenting productivity at the forefront of their respective analyses. We are the first to integrate the two within a unified empirical framework and can now examine the relative importance of each. The Vanek equation is not log-linear in the $\widehat{\beta}_{fi}$ so no simple variance decomposition is possible. The

[33] The error term is intimately related to ε_{fi}^{T} described in footnote 18.

Table 3: Decomposition of the Techniques Equation

Panel A: Unskilled Labor

	(1)	(2)	(3)	(4)
	$\ln\left(\frac{w_{Ui}}{\pi_{Ui}}\right)^{-\sigma}$	$\ln\left(\pi_{Ui}\right)^{-1}$	$\ln(\sum_g \theta_{Ugi}/\hat{\delta}_{gi})$	$\ln\left(\epsilon^T_{Ui}\right)$
$\ln\left[\sum_{g=1}^{G}\theta_{Ugi}\left(\frac{d_{Ugi}}{d_{Ug,\text{us}}}\right)\right]$	-0.32***	1.20***	0.087***	0.032***
	(0.09)	(0.09)	(0.02)	(0.01)
R-squared	0.26	0.85	0.36	0.51

Panel B: Skilled Labor

	$\ln\left(\frac{w_{Si}}{\pi_{Si}}\right)^{-\sigma}$	$\ln\left(\pi_{Si}\right)^{-1}$	$\ln(\sum_g \theta_{Sgi}/\hat{\delta}_{gi})$	$\ln\left(\epsilon^T_{Si}\right)$
$\ln\left[\sum_{g=1}^{G}\theta_{Sgi}\left(\frac{d_{Sgi}}{d_{Sg,\text{us}}}\right)\right]$	-0.45*	1.29***	0.15***	0.015***
	(0.27)	(0.22)	(0.05)	(0.00)
R-squared	0.07	0.49	0.19	0.29

Panel C: Skilled Relative to Unskilled Labor

	$\ln\left(\frac{w_{Si}\pi_{Ui}}{\pi_{Si}w_{Ui}}\right)^{-\sigma}$	$\ln\left(\frac{\pi_{Si}}{\pi_{Ui}}\right)^{-1}$	$\ln\left(\frac{\sum_g \theta_{Sgi}/\hat{\delta}_{gi}}{\sum_g \theta_{Ugi}/\hat{\delta}_{gi}}\right)$	$\ln\left(\epsilon^T_{SUi}\right)$
$\ln\left[\frac{\sum_{g=1}^{G}\theta_{Sgi}\left(d_{Sgi}/d_{Sg,\text{us}}\right)}{\sum_{g=1}^{G}\theta_{Ugi}\left(d_{Ugi}/d_{Ug,\text{us}}\right)}\right]$	1.85***	-0.73***	0.16***	-0.29***
	(0.22)	(0.20)	(0.04)	(0.08)
R-squared	0.66	0.28	0.30	0.25

Notes: Within each panel, each column represents a separate regression. The dependent variable is identified by the column header and the independent variable is identified by the column on the left. All regressions are OLS with 38 observations. Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

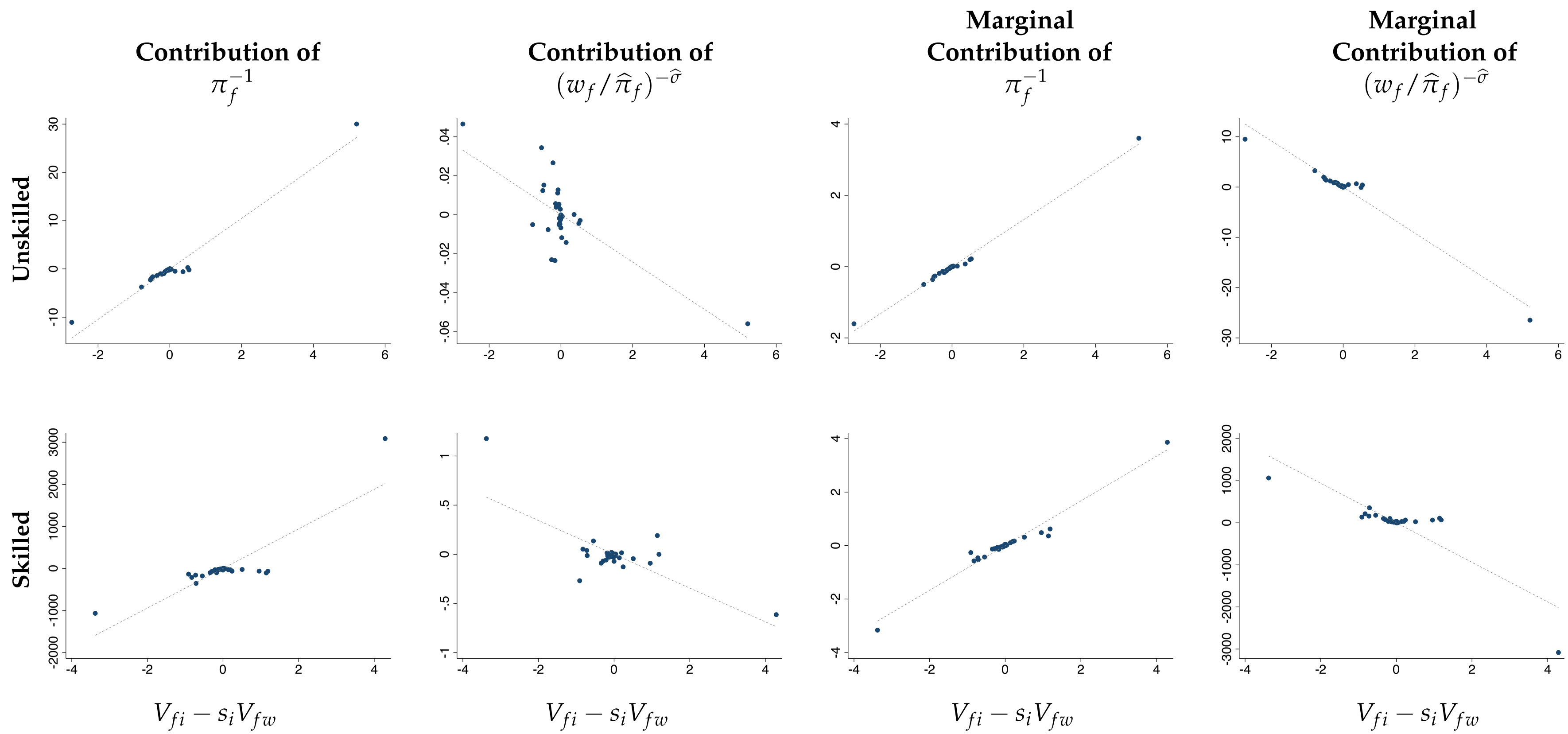
most obvious thing to do is shut down the wage and productivity terms one at a time. Recall from equation (15) together with the normalizations $\pi_{f,\text{us}} = 1$ and $w_{f,\text{us}} = 1$ that

$$\widehat{\beta}_{fi} = (w_{fi}/\widehat{\pi}_{fi})^{-\widehat{\sigma}}\ \widehat{\pi}_{fi}^{-1}$$

can be decomposed into a productivity adjusted factor price term $\left(w_{fi}/\widehat{\pi}_{fi}\right)^{-\widehat{\sigma}}$ and a factor-augmenting technology term $\widehat{\pi}_{fi}^{-1}$.

We begin by shutting down the factor price term and computing what the factor content of trade would look like. In our notation, this is $F'_{fi}(\widehat{\pi}_f^{-1})$ where $\widehat{\pi}_f^{-1} = (\widehat{\pi}_{f1}^{-1}, \ldots, \widehat{\pi}_{fN}^{-1})$. Column 1 of figure 7 plots $F'_{fi}(\widehat{\pi}_f^{-1})$ against $V_{fi} - s_i V_{fw}$ for unskilled labor (top row) and skilled labor (bottom) row. The rank correlations are 0.80 and 0.50, respectively. Thus for unskilled labor and to a lesser extent for skilled labor, factor augmentation is important. We next shut down the factor-augmenting technology term and compute what the factor content of trade would look like. In our notation this is $F'_{fi}([w_f/\widehat{\pi}_f]^{-\widehat{\sigma}})$ where $[w_f/\widehat{\pi}_f]^{-\widehat{\sigma}} = ([w_{f1}/\widehat{\pi}_{f1}]^{-\widehat{\sigma}}, \ldots, [w_{fN}/\widehat{\pi}_{fN}]^{-\widehat{\sigma}})$. Column 2 of figure 7 plots $F'_{fi}([w_f/\widehat{\pi}_f]^{-\widehat{\sigma}})$ against $V_{fi} - s_i V_{fw}$. The fit is horrible and the rank correlations are negative, which means that factor-augmenting technology is exceedingly important.

Figure 7: Vanek Equation: Relative Importance of Substitution Effects vs. Factor Augmentation



Notes: The top row deals with unskilled labor. The bottom row deals with skilled labor. The horizontal axis is always $V_{fi} - s_i V_{fw}$. In the first column, the vertical axis is $F'_{fi}(\widehat{\pi}_f^{-1})$. In the second column the vertical axis is $F'_{fi}([w_f/\widehat{\pi}_f]^{-\widehat{\sigma}})$. In the third column, the vertical axis is $F'_{fi}(\widehat{\beta}_f) - F'_{fi}([w_f/\widehat{\pi}_f]^{-\widehat{\sigma}})$ i.e., the marginal contribution of $\widehat{\pi}_f^{-1}$ after controlling for $(w_f/\widehat{\pi}_f)^{-\widehat{\sigma}}$. In the fourth column the vertical axis is $F'_{fi}(\widehat{\beta}_f) - F'_{fi}(\widehat{\pi}_f^{-1})$ i.e., the marginal contribution of $(w_f/\widehat{\pi}_f)^{-\widehat{\sigma}}$ after controlling for $\widehat{\pi}_f^{-1}$. All lines are OLS best fits.

These results taken together imply that there are important interactions between factor prices and productivity. To examine these we start by defining the 'marginal contribution of productivity'

$$F'_{fi}(\widehat{\beta}_f) - F'_{fi}([w_f/\widehat{\pi}_f]^{-\widehat{\sigma}}) \ .$$

The idea is that if β_f were linear in $[w_f/\widehat{\pi}_f]^{-\widehat{\sigma}}$ and $\widehat{\pi}_f^{-1}$ then this expression would equal $F'_{fi}(\widehat{\pi}_f^{-1})$. So the above term is the marginal contribution of (nonlinearly) adding in factor-augmenting technology. We likewise defined the 'marginal contribution of factor prices' as

$$F'_{fi}(\widehat{\beta}_f) - F'_{fi}(\widehat{\pi}_f^{-1}) \ .$$

These marginal contributions appear in columns 3 and 4 of figure 7, respectively. Two things stand out. First, from column 3, *the marginal contribution of productivity is very important and performs extremely well both in terms of correlations and missing trade.* Second, from column 4, the marginal contribution of factor prices is unimportant. However, because neither productivity nor factor prices performs very well by itself, one must ultimately conclude that both productivity and factor prices are important for understanding the Vanek equation.

5.4. *Development Accounting Revisited: Is There Skill Bias in Cross-Country Technology Differences?*

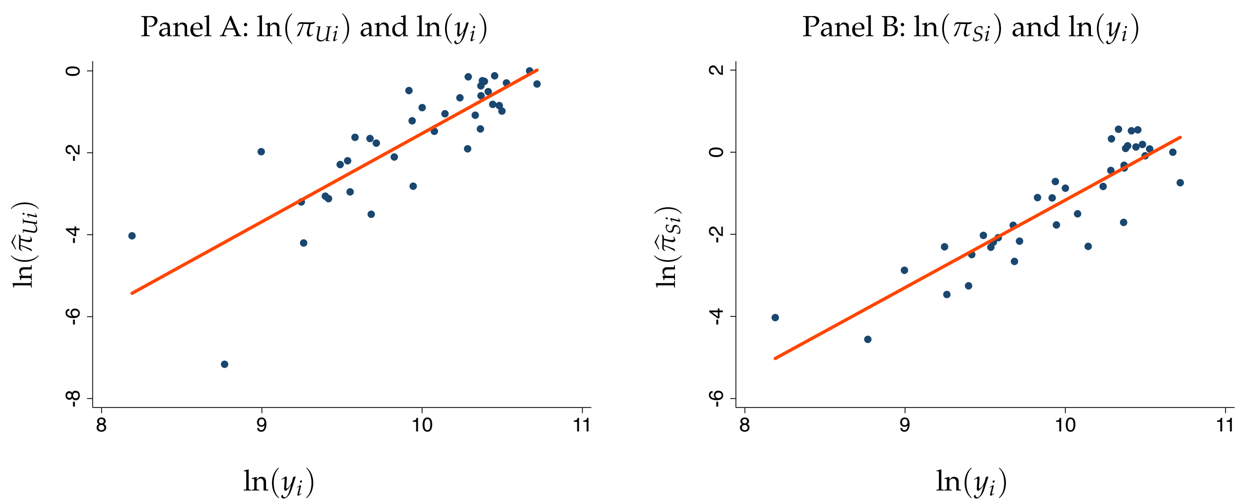
In this subsection we address two questions. First, how similar are our estimates of the π_{fi} to those in the development accounting literature? Second, do our estimates display the skill bias which is so central to Caselli and Coleman (2006)? As discussed in sections 2.2 and 4.4, there are significant methodological similarities between our approach and theirs. Nevertheless, these questions are not trivial because there are also so many differences between the two approaches.[34]

Are our estimates of the π_{fi} close to those of Caselli and Coleman (2006)? The answer is: not quite. Caselli and Coleman (2006, figures 1 and 2) show that while π_{Si} has the expected positive correlation with log real income per worker, π_{Ui} does not. Figure 8 shows that our $\widehat{\pi}_{Si}$ and $\widehat{\pi}_{Ui}$ are *both* positively correlated with log real income per worker. We think that this is a sensible result: As countries grow rich *both* their skilled and unskilled workers become more productive.

More importantly, we confirm Caselli and Coleman's measures of skill bias in cross-country technology differences. To see this, start with an aggregate production function, use equation (19)

[34]By way of just a few examples, Caselli (2016) includes capital and precisely measures endowments of human capital whereas we use highly disaggregated sectoral data and the factor market clearing equation (W).

Figure 8: factor-augmenting International Technology Differences



Notes: Panel A plots $\ln(\widehat{\pi}_{Ui})$ against (log) real income per worker from the Penn World Tables. Each dot is a country. Panel B plots $\ln(\widehat{\pi}_{Si})$ against (log) real income per worker. All lines are OLS best fits.

to equate the ratio of marginal productivities to the ratio of factor prices, and then invert to solve for the Caselli and Coleman technology ratios:

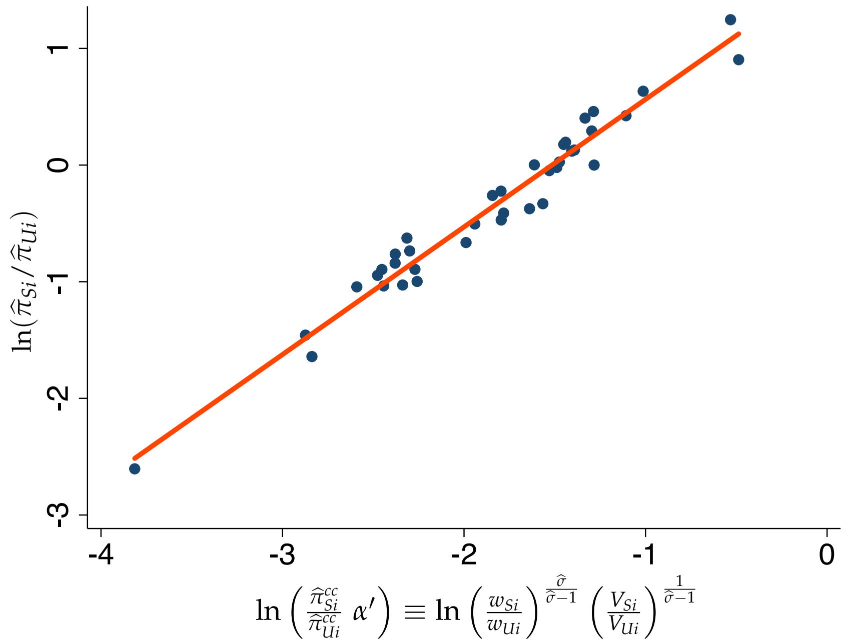
$$\frac{\pi_{Si}^{cc}}{\pi_{Ui}^{cc}} = \alpha' \left(\frac{w_{Si}}{w_{Ui}}\right)^{\frac{\sigma}{\sigma-1}} \left(\frac{V_{Si}}{V_{Ui}}\right)^{\frac{1}{\sigma-1}} . \tag{23}$$

where $\alpha' \equiv (\alpha_S/\alpha_U)^{\frac{\sigma}{1-\sigma}}$. Figure 9 plots the log of these (up to a constant α') against the log of our $\widehat{\pi}_{Si}/\widehat{\pi}_{Ui}$. *The fit is remarkable ($R^2 = 0.95$), which establishes a strong link between the HOV and development accounting literatures. It is also extraordinary that the multi-sector HOV model reproduces results from an aggregate production function.*

5.5. *Directed Technical Change: Are the π_{fi} Biased Towards a Country's Abundant Factor?*

The previous result about skill bias is static i.e., it is derived from a time-invariant production function. In a series of papers summarized in Acemoglu (2009, chapter 15), the author argues that whether innovation is directed towards skilled or unskilled labor will depend on offsetting effects: Innovation will be directed towards the expensive factor (the price effect) and towards the more abundant factor (the market-size effect). Under fairly general assumptions, the market-size effect dominates so that technical change is directed towards a country's abundant factor. The key piece in the proof of Acemoglu's argument deals with the innovation process. Letting η_i be the efficiency

Figure 9: Factor Bias — HOV versus Development Accounting



of R&D in the skill-intensive relative to the unskilled-intensive sectors, Acemoglu (2009, eq. 15.27) derives the following equation, which contains most of the economics of directed technical change:

$$\ln \frac{\pi_{Si}}{\pi_{Ui}} = \gamma_0 + \gamma_1 \ln \eta_i + (\sigma - 1) \ln \left(\frac{V_{Si}}{V_{Ui}} \right) \tag{24}$$

where γ_0 and γ_1 are exogenous parameters of Acemoglu's model.

We investigate this equation and also use it to develop an internally consistent value of σ. Recall that in order to define the $\widehat{\pi}_{fi}$ we needed to choose a value for σ and combine it with $\widehat{\beta}_{fi}$ and w_{fi} where here we treat $\widehat{\beta}_{fi}$ as data. (See equation 20.) We therefore write $\widehat{\pi}_{fi} = \pi_{fi}(\sigma;\ \widehat{\beta}_{fi}, w_{fi})$ and rewrite equation (24) as

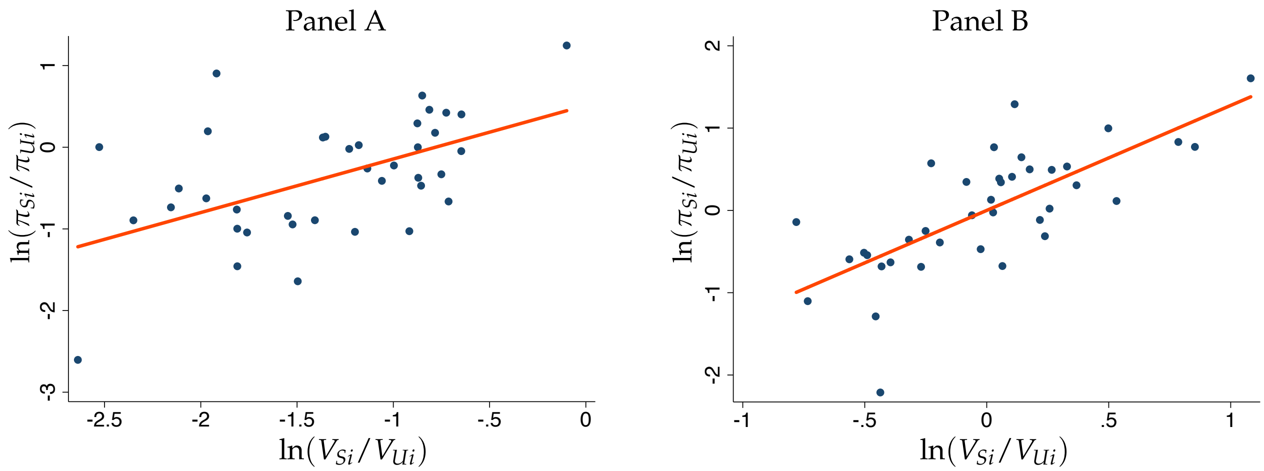
$$\ln \frac{\pi_{Si}(\sigma;\ \widehat{\beta}_{Si}, w_{Si})}{\pi_{Ui}(\sigma;\ \widehat{\beta}_{Ui}, w_{Ui})} = \gamma_0 + \gamma_1 \ln \eta_i + (\sigma - 1) \ln \left(\frac{V_{Si}}{V_{Ui}} \right) . \tag{25}$$

This is a nonlinear equation in σ, which we estimate as follows. Collapse $\gamma_0 + \gamma_1 \ln \eta_i$ into an intercept γ (we relax this below), pick an initial value of σ — call it σ_0 — and run the regression

$$\ln \frac{\pi_{Si}(\sigma_0;\ \widehat{\beta}_{Si}, w_{Si})}{\pi_{Ui}(\sigma_0;\ \widehat{\beta}_{Ui}, w_{Ui}))} = \gamma + (\sigma_1 - 1) \ln \left(\frac{V_{Si}}{V_{Ui}} \right) + \epsilon_i \tag{26}$$

to recover an estimate of σ_1. Iterate until $\sigma_0 = \sigma_1$. For all starting values of σ_0 between 0.5 and 50 we quickly converge to a final value $\widehat{\sigma} = 1.67$, which is the value used throughout this paper. The

Figure 10: Linking HOV with Directed Technical Change



Notes: Panel A plots $\ln(V_{Si}/V_{Ui})$ against $\ln(\pi_{Si}/\pi_{Ui})$ where π_{fi} is evaluated at σ=1.67. Panel B presents a partial regression plot of $\ln(V_{Si}/V_{Ui})$ against $\ln(\pi_{Si}/\pi_{Ui})$ after controlling for a fourth-order polynominal in real income per worker (see equation 27).

t-statistic is 2.93 and the $R^2 = 0.28$. The plot of the data and fitted line appear in the left panel of figure 10.

This establishes three things. First, we have now generated an internally consistent $\widehat{\sigma}$. Second, this $\widehat{\sigma}$ is in the middle of the range of existing estimates of σ. See footnote 28. Third, $\widehat{\sigma} > 1$, which implies that countries indeed direct their innovation towards improving the productivity of their abundant factor.

We can refine the analysis somewhat by recognizing that the intercept γ in equation (26) depends on $\ln \eta_i$ and so is not a constant. We proxy it by income per worker (y_i) and, since we do not know what this function looks like we experiment with polynomials of order 1 through 4 and with semi-parametric estimators. All approaches yield virtually identical results so we only report the 4th-order result. Specifically, we estimate

$$\ln \frac{\pi_{Si}(\sigma;\ \widehat{\beta}_{Si}, w_{Si})}{\pi_{Ui}(\sigma;\ \widehat{\beta}_{Ui}, w_{Ui})} = \sum_{k=0}^{4} \gamma_k (\ln y_i)^k + (\sigma - 1) \ln \left(\frac{V_{Si}}{V_{Ui}} \right) + \epsilon_i \tag{27}$$

using the same iterative procedure as before. This yields $\widehat{\sigma} = 1.89$ ($t = 4.92$, $R^2 = 0.54$). 1.89 is sufficiently close to 1.67 that it has no perceptible impact on our calculated $\pi_{fi}(\sigma;\ \widehat{\beta}_{fi}, w_{fi})$. The right panel of figure 10 displays the partial regression plot. *Once again, the fit is very good and supports the conclusion that countries direct their innovation towards improving the productivity of their* *abundant factors. This firmly ties together the empirics of HOV and directed technical change.*

6. An Observation on Some HOV Folklore

Since Gabaix (1997), there is a sense that something was terribly wrong with the approach in Trefler (1993). Because lemma 3 establishes a close connection between Trefler's approach and our current approach, some comment may be of help. There are two elements of the folklore.

First, depending on how the β_f are estimated, one ends up with very different conclusions about the performance of the Vanek equation. Specifically, return to Trefler's (1993) specification, which is nested by equation (VT) of lemma 3 and recall that β_{fi}^{VT} makes equation (VT) fit perfectly. Using Trefler's data, if one compares $(\beta_{fi}^{VT})^{-1}$ with $w_{fi}/w_{f,\text{us}}$ then one arrives at a positive view of the HOV model with PFPE. On the other hand, if one sets $(\beta_{fi})^{-1} = w_{fi}/w_{f\text{us}}$ and plugs this into equation (VT) one arrives at a negative view. It is important to understand that *this problem does not carry over to our current approach.* In case this is not obvious, recall from the start of section 4.1 that β_{fi}^{W} and β_{fi}^{T} are the values of β_{fi} that respectively make the Wage and Techniques equations fit perfectly. As shown in online appendix figure B2, plugging either β_{fi}^{W} or β_{fi}^{T} into the Vanek equation results in a very good fit. Indeed, the fit is virtually identical to what we saw in the bottom row of figure 3 because empirically, $\widehat{\beta}_{fi} = (\beta_{fi}^{W} + \beta_{fi}^{T})/2 \approx \beta_{fi}^{W} \approx \beta_{fi}^{T}$. Thus the first element of the folklore does not hold in the current setting.

The second element of the folklore is that the estimates of the β_{fi} do not change when trade is set to 0. There are two points of note. (*a*) We do not use the Vanek equation or *any* trade data in estimating $\widehat{\beta}_{fi}$ so this observation is not germane to this paper. (*b*) Setting trade to zero has big impacts elsewhere in the model. In particular, when trade is set to 0 we have $F_{fi}(\widehat{\beta}_f) = 0$. In that case we get a horrible fit of the Vanek equation: A plot of 0 ($= F_{fi}(\widehat{\beta}_f)$) against $V_{fi} - s_i V_{fw}$ is a horizontal line with slope 0 and correlation 0. It obviously matters if trade is set to 0. In summary, the folklore concerns about Trefler's (1993) approach are of considerable interest, but not applicable here.[35]

[35] A deeper explanation of the second element of the folklore appears in online Appendix A.

7. Conclusion

There is a disconnect between the prominent role that international factor-augmenting technology differences play in the development accounting and directed technical change (DTC) literatures versus the uncertain role they currently play in the HOV literature. Part of this uncertainty is due to the unresolved tension between Davis and Weinstein (2001) who emphasized substitution effects (failure of PFPE) versus Trefler (1993) who emphasized factor-augmenting productivity differences. Using a micro-founded but parsimonious extension of the HOV model that allowed for an integrated treatment of factor augmentation and failure of PFPE, we resolved this tension and connected HOV to the development accounting and DTC literatures. Along the way we showed that one cannot identify substitution effects separately from factor augmentation without data on factor prices and the elasticity of substitution between factors. Addressing this identification problem allowed us to make progress towards four goals.

First, we derived three core equations (the Techniques, Wage, and Vanek equations), used the first two to estimate productivity, and showed that the fit of all three equations was very good. (Without a nontradable Government Services correction, the Vanek equation correlation was high, but there was missing trade.) We showed this not only by factor, but also for skilled relative to unskilled labor as in the pre-Vanek Heckscher-Ohlin literature.

Second, we addressed the role of substitution effects versus factor-augmenting productivity differences in three ways. (1) We documented that the factor insensitivity theorem fails i.e., there is substantial international variation in productivity adjusted factor prices, variation that is correlated with endowments. This failure of PFPE provides the potential for large substitution effects. (2) We then showed that international variation in the skilled-to-unskilled ratio of unit inputs $\ln d_{Sgi}/d_{Ugi}$ was largely explained by substitution effects. In contrast, variation in unit input *levels* ($\ln d_{Sgi}$ and $\ln d_{Ugi}$ separately) was largely explained by factor augmentation. (3) For the Vanek equation, both substitution effects and factor augmentation were important, though the latter was relatively more important.

Third, we linked HOV to the development accounting literature first by showing that our estimates of skill- and unskilled-augmenting international technology differences were sensibly increasing in income. More importantly, we showed that the π_{Si}/π_{Ui} displayed exactly the skill bias that is predicted by an aggregate CES production function.

Fourth, we linked HOV to the DTC literature by showing that π_{Si}/π_{Ui} were positively correlated across countries with relative endowments, just as predicted by the DTC literature when the elasticity of substitution σ exceeds unity. Further, the positive correlation implied an estimate of σ that is squarely within the range of existing estimates in the labor literature.

We believe that we have gone a long way towards finally connecting the HOV literature with the development accounting and DTC literatures. We have also provided an identified, micro-founded answer to a question raised by the tension between Davis and Weinstein (2001) and Trefler (1993) about the relative roles of substitution effects (failure of PFPE) and factor-augmenting international technology differences for understanding international variation in unit factor demands and the factor content of trade.

Appendix

Proof of Lemma 1 We start with a preliminary result involving change of indexes. From equation (1), $\phi_{hj}(\mathbf{w}_j,\mathbf{p}) = \left[c_{hj}(\mathbf{w}_j)\right]^{\gamma_{h0}} \prod_{g=1}^{G} P_{hg}^{\gamma_{hg}}$ where $P_{hg} = \left(\sum_{i=1}^{N} \int_{\omega\in\Omega_{gi}} \alpha_{hg}\; p_{gi}(\omega)^{1-\rho_g}\, d\omega\right)^{\frac{1}{1-\rho_g}}$. Also, note that $\partial P_{hg'}^{\gamma_{hg'}}/\partial p_{gi}(\omega) = 0$ for $g' \neq g$ and $\partial P_{hg}^{\gamma_{hg}}/\partial p_{gi}(\omega) = \gamma_{hg} P_{hg}^{\gamma_{hg}-1+\rho_g}\; \alpha_{hg}\; p_{gi}(\omega)^{-\rho_g}$. Hence

$$\begin{aligned} b_{ij}(g,h,\mathbf{w}_j,\mathbf{p}) &= \partial\phi_{hj}(\mathbf{w}_j,\mathbf{p})/\partial p_{gi}(\omega) \\ &= c_{hj}^{\gamma_{g0}} \prod_{g'\neq g} P_{hg'}^{\gamma_{hg'}} \left[\gamma_{hg} P_{hg}^{\gamma_{hg}-1+\rho_g} \alpha_{hg} p_{gi}(\omega)^{-\rho_g}\right] \\ &= \phi_{hj}(\mathbf{w}_j,\mathbf{p})\gamma_{hg} P_{hg}^{-1+\rho_g} \alpha_{hg} p_{gi}(\omega)^{-\rho_g}\;. \end{aligned} \tag{28}$$

As explained in section (1.2), demand for variety $\omega \in \Omega_{gi}$ is the sum of demands for final goods and intermediate inputs: $q_{gi}(\omega) = p_{gi}(\omega)^{-\rho_g} P_g^{\rho_g-1}\eta_g \Sigma_{j=1}^N Y_j + \Sigma_{h=1}^G \Sigma_{j=1}^N \int_{\nu\in\Omega_{hj}} b_{ij}(g,h,\mathbf{w}_j,\mathbf{p})\left[q_{hj}(\nu)+\overline{\phi}_h\right] d\nu$. Substituting in equation (28), the lemma follows with $\kappa_g \equiv P_g^{\rho_g-1}\eta_g\Sigma_{j=1}^N Y_j + \Sigma_{h=1}^G\Sigma_{j=1}^N \int_{\nu\in\Omega_{hj}} \phi_{hj}(\mathbf{w}_j,\mathbf{p})\gamma_{hg} P_{hg}^{-1+\rho_g}\alpha_{hg}\left[q_{hj}(\nu)+\overline{\phi}_h\right] d\nu$. ■

Proof of Theorem 1 Pre-multiplying equation (5) by $\mathbf{A}_f$ yields $\mathbf{A}_f\mathbf{T} = \mathbf{A}_f(\mathbf{I}_{NG}-\mathbf{B})\mathbf{Q} - \mathbf{A}_f\mathbf{C} = \mathbf{D}_f\mathbf{Q} - \mathbf{A}_f\mathbf{C} = \left[\begin{array}{ccc} V_{f1} & \cdots & V_{fN}\end{array}\right] - \mathbf{A}_f\mathbf{C}$. Consider column i of this equation, namely,

$$\mathbf{A}_f\mathbf{T}_i = V_{fi} - \mathbf{A}_f\mathbf{C}_i \tag{29}$$

where $\mathbf{T}_i$ and $\mathbf{C}_i$ are the ith columns of T and C, respectively. Then

$$\mathbf{A}_f\Sigma_j\mathbf{T}_j = \Sigma_j V_{fj} - \mathbf{A}_f\Sigma_j\mathbf{C}_j. \tag{30}$$

Consider each of the three terms in this equation. $V_{fw} \equiv \Sigma_j V_{fj}$ is the world endowment of f. Recall that $\mathbf{T}_j$ is composed of blocks of $G\times 1$ matrices. Let $\mathbf{T}_{ij}$ be the ith block of $\mathbf{T}_j$. Then by inspection of the definition of $\mathbf{T}$ together with balanced trade, $\Sigma_j\mathbf{T}_{ij} = \mathbf{X}_i - \Sigma_j\mathbf{M}_{ji} = \mathbf{0}_G$ where $\mathbf{0}_G$ is the $G\times 1$ vector of zeros. Hence $\Sigma_j\mathbf{T}_j = \mathbf{0}_{NG}$ where $\mathbf{0}_{NG}$ is the $NG\times 1$ vector of zeros. Likewise, $\mathbf{C}_j$ is composed of blocks of $G\times 1$ matrices and $\mathbf{C}_{ji}$ is the ith block of $\mathbf{C}_j$. $\Sigma_j\mathbf{C}_{ji}$ is world consumption of goods produced in country i. Define $\mathbf{C}_{wi} = \Sigma_j\mathbf{C}_{ji}$ and stack the $\mathbf{C}_{wi}$ into an $NG\times 1$ vector denoted by $\mathbf{C}_w$. Thus, equation (30) can be written as $0 = V_{fw} - \mathbf{A}_f\mathbf{C}_w$ or $0 = s_iV_{fw} - \mathbf{A}_f(s_i\mathbf{C}_w)$. Subtracting this from equation (29) yields $F_{fi} = V_{fi} - s_iV_{fw} - \mathbf{A}_f\left(\mathbf{C}_i - s_i\mathbf{C}_w\right)$. But under our assumptions of homothetic demands and costless trade, $C_{gij} = s_iC_{gwj}$ or, in matrix notation, $\mathbf{C}_i = s_i\mathbf{C}_w$. Hence $F_{fi} = V_{fi} - s_iV_{fw}$. ■

Proof of Equation (10)***:*** By Shephard's lemma, $d_{fgi} = \partial\phi_{gi}/\partial w_{fi}$. Hence, $d_{fgi} = \left[\partial\phi_{gi}/\partial c_{gi}\right]\left[\partial c_{gi}/\partial w_{fi}\right] = \left[\gamma_{g0}c_{gi}^{-1}\phi_{gi}\right]\left[\alpha_{fg}\left(w_{fi}\right)^{-\sigma}\left(\pi_{fi}\right)^{\sigma-1}\left(\delta_{gi}\right)^{-1}\left(c_{gi}\right)^{\sigma}\right]$. Recall that at the end of section 1.3 we established that $p_{gi} = p_g$. Hence from equation (2), $1 = p_{gi}/p_{g,\text{us}} = \phi_{gi}(\mathbf{w}_i,\mathbf{p})/\phi_{g,\text{us}}(\mathbf{w}_\text{us},\mathbf{p}) = \left(c_{gi}^{\gamma_{g0}}\prod_{h=1}^G P_{gh}^{\gamma_{gh}}\right) / \left(c_{g,\text{us}}^{\gamma_{g0}}\prod_{h=1}^G P_{gh}^{\gamma_{gh}}\right) = \left(c_{gi}/c_{g,\text{us}}\right)^{\gamma_{g0}}$ or $c_{gi} = c_{g,\text{us}}$ and $\phi_{gi}(\mathbf{w}_i,\mathbf{p}) = \phi_{g,\text{us}}(\mathbf{w}_\text{us},\mathbf{p})$. Hence, $d_{fgi}/d_{fg,\text{us}} = (w_{fi}/w_{f,\text{us}})^{-\sigma}(\pi_{fi}/\pi_{f,\text{us}})^{\sigma-1}(\delta_{gi}/\delta_{g,\text{us}})^{-1}$. Equation (10) follows with the normalization $\delta_{g,\text{us}} = 1$. ■

Remark 1 *If $p_{gi} \neq p_{g,us}$ then we get $d_{fgi} = d_{fg,us}(w_{fi}/w_{f,us})^{-\sigma}(\pi_{fi}/\pi_{f,us})^{\sigma-1}(\delta'_{gi}/\delta'_{g,us})^{-1}$ where $\delta'_{gi} = \delta_{gi}/p_{gi}$. That is, this leads to a reinterpretation of the δ_{gi}, but does not otherwise affect anything in the paper.*

Proof of Equation (W): Recall from the start of section 1.4 that $Q_{gi} = n_{gi}(q_g + \overline{\phi}_g)$. Plugging this into the factor-market clearing equation (4) yields $V_{fi} = \Sigma_g d_{fgi}Q_{gi}$. Substituting in $d_{fgi} = d_{fg,\text{us}}\beta_{fi}/\delta_{gi}$ (equation 10) into this expression delivers $V_{fi} = \beta_{fi}\Sigma_g d_{fg,\text{us}}Q_{gi}/\delta_{gi}$. Equation (W) follows from a simple re-arrangement. ■

Proof of Lemma 2 Part 1. $F_{fi} = V_{fi} - s_i V_{fw} - \mathbf{A}_f(\mathbf{C}_i - s_i\mathbf{C}_w)$: This is proved as part of the proof of theorem 1.

Part 2. By definition $V'_{fi} \equiv \sum_{g\in G'} A_{fgi}C_{gii}$ where G' is the set of nontradable goods. Since $C_{gij} = 0$ for nontradables (i does not buy from j), $V'_{fi} \equiv \sum_j \sum_{g\in G'} A_{fgj}C_{gij}$. Hence $V'_{fw} \equiv \sum_i V'_{fi} = \sum_i \sum_j \sum_{g\in G'} A_{fgj}C_{gij} = \sum_j \sum_{g\in G'} A_{fgj}C_{gwj}$. Hence $V'_{fi} - s_i V'_{fw} = \sum_j \sum_{g\in G'} A_{fgj}(C_{gij} - s_i C_{gwj})$. But consumption similarity holds for tradables i.e., $(C_{gij} - s_i C_{gwj}) = 0$ for $g \notin G'$. Hence $V'_{fi} - s_i V'_{fw} = \sum_j \sum_g A_{fgj}(C_{gij} - s_i C_{gwj}) = \mathbf{A}_f(\mathbf{C}_i - s_i\mathbf{C}_w)$.

Part 3. Let $\mathbf{B}_{gji}$ be a row of the $G \times G$ matrix $\mathbf{B}_{ji}$. Then $\mathbf{B}_{gji}Q_i$ is country i's intermediate demand for good g produced in country j. i's imports of g produced in j are

$$M_{gij} = \mathbf{B}_{gji}Q_i + C_{gij} \; .$$

Adjusted imports impose consumption similarity ($C_{gij} = s_i C_{gwj}$) for $g \in G'$ and are thus

$$M'_{gij} = \mathbf{B}_{gji}Q_i + s_i C_{gwj} \;\; \text{for} \;\; g \in G'.$$

Likewise, exports are

$$X_{gi} = Q_{gi} - B_{gii}Q_i - C_{gii} \; .$$

Adjusted exports impose consumption similarity ($C_{gij} = s_i C_{gwj}$) for $g \in G'$ and are thus

$$X'_{gi} = Q_{gi} - B_{gii}Q_i - s_i C_{gwi} \;\; \text{for} \;\; g \in G'.$$

Consider the section 1.4 expression for the $GN \times 1$ vector $\mathbf{T}_i$. Let $\mathbf{T}'_i$ be a $GN \times 1$ vector whose elements are as follows. When g is tradable ($g \notin G'$), the gi element is X_{gi} and the gj element ($j \neq i$) is $-M_{gij}$. When g is nontradable ($g \in G'$), the gi element is X'_{gi} and the gj element ($j \neq i$) is $-M'_{gij}$. It follows that the nonzero elements of $\mathbf{T}'_i - \mathbf{T}_i$ are either $X'_{gi} - X_{gi} = C_{gii} - s_i C_{gwi}$ or $(-M'_{gij}) - (-M_{gij}) = C_{gij} - s_i C_{gwj}$. That is, the nonzero elements are always of the form $C_{gij} - s_i C_{gwj}$. Since consumption similarity holds for tradables, the zero elements can also be expressed as $C_{gij} - s_i C_{gwj} (= 0)$. Hence $F'_{fi} - F_{fi} = \mathbf{A}_f(\mathbf{T}'_i - \mathbf{T}_i) = \mathbf{A}_f(\mathbf{C}_i - s_i\mathbf{C}_w)$. ■

Consumption Similarity Calculation: From part 3 of the proof of lemma 2, we have already defined M'_{gij}, X'_{gij} and $\mathbf{T}'_i$ in terms of the data $\mathbf{Q}_i$, $\mathbf{B}_{ij}$, and s_i as well as $\mathbf{C}_{wj}$. We define $\mathbf{C}_{wj}$ implicitly using the world goods market clearing condition:

$$\mathbf{Q}_i = \mathbf{C}_{wi} + \sum_{j=1}^{N} \mathbf{B}_{ij}\mathbf{Q}_j. \tag{31}$$

Note that imposing consumption similarity on Government Services affects consumption patterns, but not production patterns $\mathbf{Q}$ or intermediate input usage ratios $\mathbf{B}$.

List of Countries: Australia, Austria, Belgium, Brazil, Bulgaria, Canada, China, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Great Britain, Greece, Hungary, Indonesia, Ireland, Italy, Japan, Korea, Latvia, Lithuania, Malta, Mexico, Netherlands, Poland, Portugal, Romania, Russia, Slovakia, Slovenia, Spain, Sweden, Taiwan, Turkey, USA.

List of Industries and ISIC codes: Agriculture (110), Mining (200), Food, Beverages, Tobacco (311), Textiles and Textile Products (321), Leather and Footwear (323), Wood and Products of Wood (331), Pulp, Paper, Printing, and Publishing (341), Chemicals (351), Rubber and Plastics (355), Non-Metallic Minerals (369), Basic and Fabricated Metals (371), Machinery, nec. (381), Transport Equipment (384), Electrical and Optical Equipment (385), Manufacturing, nec.(390), Electricity, Gas, Water Supply (400), Construction (500), Wholesale and Retail Trade (600), Hotels and Restaurants (630), Transport (700), Finance, Insurance, Real Estate (800), Government Services (900).

Table A1: Values of β_{fi} and π_{fi}

	Unskilled Labor				Skilled Labor			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Country	$\widehat{\beta}_{Ui}$	w_{Ui}	$\widehat{\pi}_{Ui}$	$\frac{w_{Ui}}{\widehat{\pi}_{Ui}}$	$\widehat{\beta}_{Si}$	w_{Si}	$\widehat{\pi}_{S,i}$	$\frac{w_{Si}}{\widehat{\pi}_{Si}}$
Australia	1.13	0.98	1.14	0.86	0.70	0.89	0.44	2.04
Austria	1.22	0.86	0.92	0.93	0.76	0.79	0.37	2.13
Belgium	0.81	1.42	1.76	0.81	0.41	1.10	0.33	3.29
Brazil	12.17	0.07	0.06	1.26	4.60	0.18	0.14	1.31
Bulgaria	8.68	0.07	0.03	2.20	2.91	0.10	0.01	6.63
Canada	1.19	0.97	1.21	0.80	0.93	0.74	0.43	1.74
China	11.99	0.04	0.01	3.50	1.95	0.04	0.00	50.46
Cyprus	1.89	0.44	0.33	1.33	2.48	0.48	0.62	0.77
Czech Republic	2.93	0.26	0.17	1.52	1.44	0.26	0.06	4.39
Denmark	0.79	1.43	1.73	0.83	0.96	0.98	0.89	1.10
Estonia	2.74	0.23	0.11	2.01	4.67	0.20	0.17	1.14
Finland	0.84	0.98	0.72	1.35	1.21	0.77	0.69	1.11
France	0.81	1.30	1.39	0.93	0.84	1.04	0.86	1.21
Germany	1.06	1.03	1.17	0.88	0.93	0.94	0.77	1.22
Great Britain	0.93	1.08	1.10	0.99	1.02	0.90	0.79	1.14
Greece	2.18	0.34	0.22	1.55	1.86	0.38	0.23	1.68
Hunary	3.59	0.23	0.17	1.35	2.61	0.29	0.19	1.52
Indonesia	23.25	0.03	0.02	1.70	5.57	0.07	0.02	4.03
Ireland	0.76	0.87	0.47	1.85	1.04	0.86	0.72	1.18
Italy	1.19	0.75	0.64	1.18	0.51	0.69	0.15	4.73
Japan	1.27	0.74	0.68	1.09	1.36	0.65	0.54	1.20
Latvia	3.48	0.21	0.13	1.60	3.28	0.20	0.10	1.94
Lithuania	4.58	0.17	0.12	1.40	6.66	0.17	0.20	0.85
Malta	2.16	0.40	0.33	1.22	0.83	0.48	0.12	3.97
Mexico	6.75	0.12	0.08	1.42	2.96	0.15	0.04	3.41
Netherlands	0.85	1.14	1.08	1.05	0.84	0.98	0.74	1.33
Poland	4.22	0.17	0.10	1.69	3.04	0.21	0.11	1.92
Portugal	2.75	0.41	0.49	0.84	0.88	0.66	0.29	2.25
Romanis	8.51	0.11	0.10	1.10	2.34	0.17	0.04	4.13
Russia	9.76	0.11	0.11	0.95	4.78	0.12	0.05	2.32
Slovakia	3.00	0.18	0.07	2.57	1.81	0.17	0.03	5.78
Slovenia	2.24	0.43	0.42	1.04	1.78	0.49	0.41	1.22
South Korea	1.69	0.29	0.10	2.90	5.14	0.25	0.35	0.70
Spain	1.08	0.68	0.43	1.58	1.50	0.60	0.52	1.17
Sweden	0.91	1.31	1.69	0.77	0.88	0.88	0.60	1.46
Taiwan	2.52	0.29	0.18	1.61	3.09	0.29	0.24	1.19
Turkey	4.70	0.11	0.04	2.80	2.00	0.19	0.05	4.17
United States	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Notes: Columns 1 and 5 present $\widehat{\beta}_{fi}$. Columns 2 and 6 are data described in section 3. Columns 3, 4, 7, and 8 are calculated using the data in columns 1, 2, 5, and 6, the definition of $\widehat{\beta}_{fi}$, and σ=1.67.

References

Acemoglu, Daron, "Why Do New Technologies Complement Skills? Directed Technical Change And Wage Inequality," *Quarterly Journal of Economics*, November 1998, *113* (4), 1055–1089.

— , *Introduction to Modern Economic Growth*, Princeton: Princeton University Press, 2009.

— **and Fabrizio Zilibotti**, "Productivity Differences," *Quarterly Journal of Economics*, May 2001, *116* (2), 563–606.

— **, Gino Gancia, and Fabrizio Zilibotti**, "Offshoring and Directed Technical Change," *American Economic Journal: Macroeconomics*, July 2015, *7* (3), 84–122.

Altonji, Joseph G and Lewis M Segal, "Small-Sample Bias in GMM Estimation of Covariance Structures," *Journal of Business & Economic Statistics*, July 1996, *14* (3), 353–66.

Antràs, Pol, "Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution," *B.E. Journal of Macroeconomics*, April 2004, *4* (1), 1–36.

Antweiler, Werner and Daniel Trefler, "Increasing Returns and All That: A View From Trade," *American Economic Review*, March 2002, *92* (1), 93–119.

Bernard, Andrew B., J. Bradford Jensen, Stephen J. Redding, and Peter K. Schott, "Firms in International Trade," *Journal of Economic Perspectives*, Summer 2007, *21* (3), 105–130.

Blum, Bernardo S., "Endowments, Output, and the Bias of Directed Innovation," *Review of Economic Studies*, April 2010, *77* (2), 534–559.

Burstein, Ariel and Jonathan Vogel, "International Trade, Technology, and the Skill Premium," *Journal of Political Economy*, forthcoming.

Card, David and Thomas Lemieux, "Can Falling Supply Explain the Rising Return to College for Younger Men? A Cohort-Based Analysis," *Quarterly Journal of Economics*, May 2001, *116* (2), 705–746.

Caron, Justin, Thibault Fally, and James R. Markusen, "International Trade Puzzles: A Solution Linking Production and Preferences," *Quarterly Journal of Economics*, August 2014, *129* (3), 1501–1552.

Caselli, Francesco, "Accounting for Cross-Country Income Differences," in Philippe Aghion and Steven Durlauf, eds., *Handbook of Economic Growth*, Vol. 1 of *Handbook of Economic Growth*, Elsevier, 2005, chapter 9, pp. 679–741.

— , *Technology Differences over Time and Space*, Princeton: Princeton University Press, 2016.

— **and James Feyrer**, "The Marginal Product of Capital," *Quarterly Journal of Economics*, 05 2007, *122* (2), 535–568.

— **and Wilbur John Coleman**, "The World Technology Frontier," *American Economic Review*, June 2006, *96* (3), 499–522.

Ciccone, Antonio and Giovanni Peri, "Long-Run Substitutability Between More and Less Educated Workers: Evidence from U.S. States, 1950-1990," *Review of Economics and Statistics*, November 2005, *87* (4), 652–663.

Davis, Donald R. and David E. Weinstein, "An Account of Global Factor Trade," *American Economic Review*, December 2001, *91* (5), 1423–1453.

Diamond, Peter, Daniel McFadden, and Miguel Rodriguez, "Measurement of the Elasticity of Factor Substitution and Bias of Technical Change," in Melvyn Fuss and Daniel L. McFadden, eds., *Production Economics: A Dual Approach to Theory and Applications Volume II: Applications of the Theory of Production*, Amsterdam: North-Holland, 1978, pp. 125–147.

Fadinger, Harald, "Productivity Differences in an Interdependent World," *Journal of International Economics*, July 2011, *84* (2), 221–232.

Feenstra, Robert C. and Gordon H. Hanson, "Aggregation Bias in the Factor Content of Trade: Evidence from U.S. Manufacturing," *American Economic Review*, May 2000, *90* (2), 155–160.

Gabaix, Xavier, "The Factor Content of Trade: A Rejection of the Heckscher-Ohlin-Leontief Hypothesis," 1997. Mimeo, Harvard University.

Gandal, Neil, Gordon H. Hanson, and M.J.Matthew J. Slaughter, "Technology, Trade, and Adjustment to Immigration in Israel," *European Economic Review*, April 2004, *48* (2), 403–428.

Hanson, Gordon H. and Matthew J. Slaughter, "Labor-Market Adjustment in Open Economies: Evidence from US States," *Journal of International Economics*, June 2002, *57* (1), 3–29.

Hottman, Colin, Stephen J. Redding, and David E. Weinstein, "What is 'Firm Heterogeneity' in Trade Models? The Role of Quality, Scope, Markups, and Cost," Working Paper 20436, National Bureau of Economic Research, August 2014.

Katz, Lawrence F. and Kevin M. Murphy, "Changes in Relative Wages, 1963-1987: Supply and Demand Factors," *Quarterly Journal of Economics*, February 1992, *107* (1), 35–78.

Krugman, Paul R., "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review*, December 1980, *70* (5), 950–959.

Leamer, Edward E., "The Leontief Paradox Reconsidered," *Journal of Political Economy*, 1980, *88*, 495–503.

_ **and James Levinsohn**, "International Trade Theory: The Evidence," in G. M. Grossman and K. Rogoff, eds., *Handbook of International Economics*, Vol. 3 of *Handbook of International Economics*, Elsevier, December 1995, chapter 26, pp. 1339–1394.

Lewis, Ethan, "How Did the Miami Labor Market Absorb the Mariel Immigrants?," Working Papers 04-3, Federal Reserve Bank of Philadelphia 2004.

Marshall, Kathryn G., "International Productivity and Factor Price Comparisons," *Journal of International Economics*, July 2012, *87* (2), 386–390.

Nishioka, Shuichiro, "International Differences in Production Techniques: Implications for the Factor Content of Trade," *Journal of International Economics*, May 2012, *87* (1), 98–104.

Romalis, John, "Factor Proportions and the Structure of Commodity Trade," *American Economic Review*, March 2004, *94* (1), 67–97.

Schott, Peter, "Across-Product versus Within-Product Specialization in International Trade," *Quarterly Journal of Economics*, May 2004, *119* (2), 647–678.

Sutton, John and Daniel Trefler, "Deductions from the Export Basket: Capabilities, Wealth and Trade," *Journal of Political Economy*, June 2016, *124* (3), 826–878.

Timmer, Marcel P., Erik Dietzenbacher, Bart Los, Robert Stehrer, and Gaaitzen J. Vries, "An Illustrated User Guide to the World Input-Output Database: the Case of Global Automotive Production," *Review of International Economics*, August 2015, *23* (3), 575–605.

Trefler, Daniel, "International Factor Price Differences: Leontief was Right!," *Journal of Political Economy*, December 1993, *101* (6), 961–987.

— , "The Case of the Missing Trade and Other Mysteries," *American Economic Review*, December 1995, *85* (5), 1029–1046.

— **and Susan Chun Zhu**, "Beyond the Algebra of Explanation: HOV for the Technology Age," *American Economic Review Papers and Proceedings*, May 2000, *90* (2), 145–149.

— **and** — , "The Structure of Factor Content Predictions," *Journal of International Economics*, November 2010, *82* (2), 195–207.

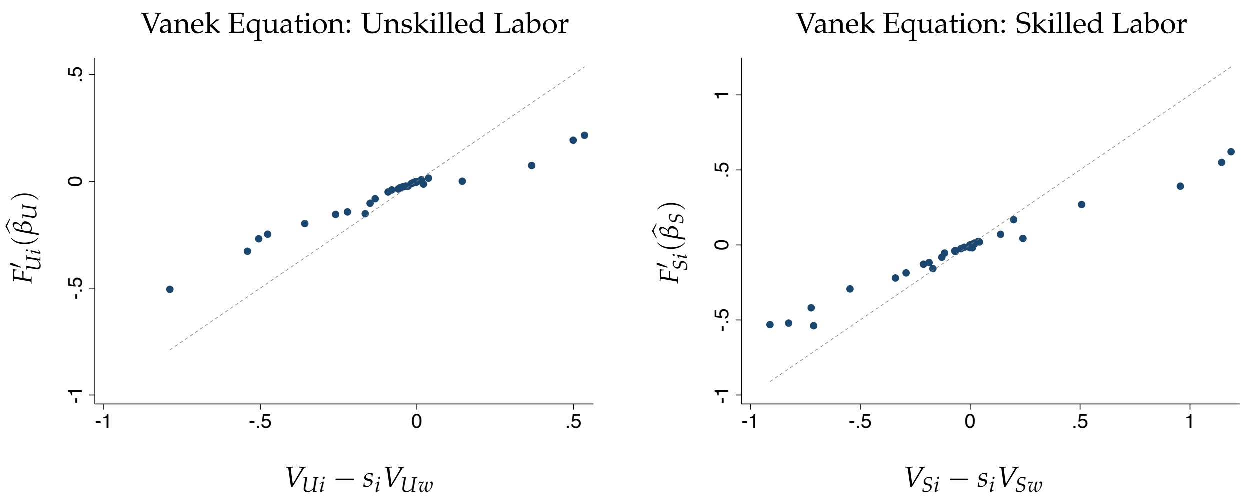
Vanek, Jaroslav, "The Factor Proportions Theory: The N-Factor Case," *Kyklos*, October 1968, *21*, 749–56.

Online Appendix to

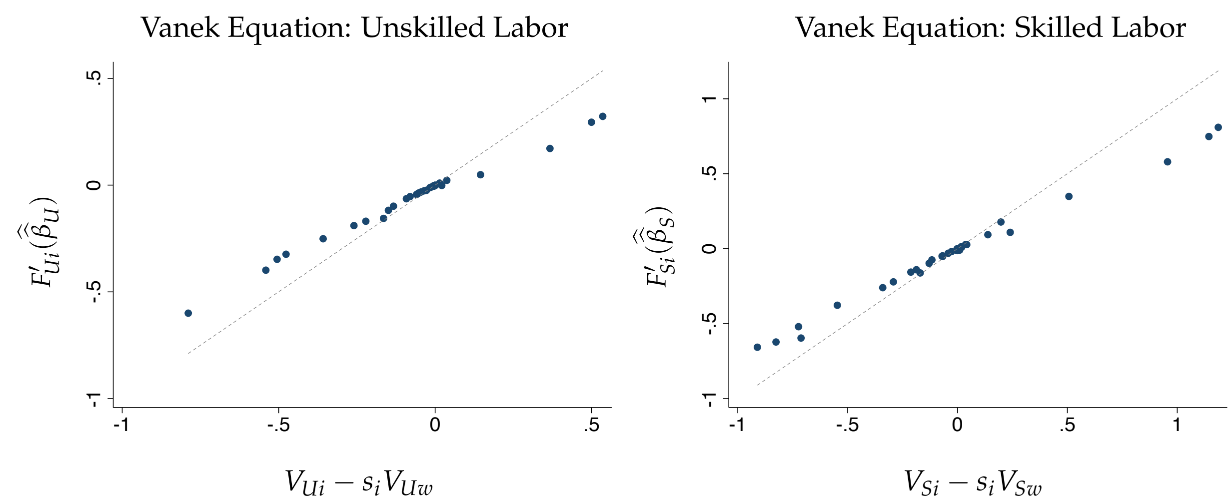
"Endowments, Factor Prices, and Skill-Biased Technology: Importing Development Accounting into HOV"

Figure B1: The Vanek Equation with Outliers (US and China) not Displayed

Panel A. Two-Equation Approach ($\widehat{\beta}_f$, Equations W and T)



Panel B. Three-Equation Approach ($\widehat{\widehat{\beta}}_f$, Equations W, T and V)



Notes: These plots are the same as those appearing in the main text except that the two outliers (China and United States) are not displayed so as to 'unpack' the remaining observations. Panel A corresponds to the bottom row of figure 3. It plots $V_{fi} - s_i V_{fw}$ against the Government Services adjusted factor content of trade (evaluated at the two-equation estimate of β_f, $\widehat{\beta}_f$). Panel B corresponds to the unskilled and skilled figures in the bottom row of figure 6. It plots $V_{fi} - s_i V_{fw}$ against the Government Services adjusted factor content of trade (evaluated at the three-equation estimate of β_f, $\widehat{\widehat{\beta}}_f$). The left panels are for unskilled labor and the right panels are for skilled labor. All lines are 45° lines.

Appendix A. Small Changes in β_f^{VT}

We establish here that when one places the unknown productivity parameters on the 'right-hand side' as in equation (VT), the performance of the Vanek equation is extremely sensitive to small changes in the vector β_f.[36] Start by defining the *predicted* factor content of trade using this approach as $F^*_{fi}(\beta_f) \equiv \beta_{fi}^{-1}V_{fi} - s_i \sum_j \beta_{fj}^{-1}V_{fj}$. Trefler (1993) derives a model in which F^{us}_{fi} is the measured factor content of trade with no Ricardian productivity differences and all techniques set equal to their US values, and $F^*_{fi}(\beta_f)$ is its predicted value (the 'predicted factor content'). See footnote 26. We now show that the relationship between F^{us}_{fi} and $F^*_{fi}(\beta_f)$ is extremely sensitive to differences in β_f even if those differences are very small.

Start by defining two productivity vectors for unskilled labor: β_U^W and β_U^{VT}–the latter of which makes the Vanek equation fit perfectly with productivity terms on the right-hand side such that $F^{us}_{Ui} = F^*_{Ui}(\beta_U^{VT})$–and define the (small) difference between the two as $\varepsilon_U = \left(\beta_U^W\right)^{-1} - \left(\beta_U^{VT}\right)^{-1}$. Because $F^*_{Ui}(\beta_U)$ is linear in its arguments, $F^*_{Ui}(\beta_U^W) - F^*_{Ui}(\beta_U^{VT}) = F^*_{Ui}(\varepsilon_U)$ or

$$F^*_{Ui}(\beta_U^W) - F^*_{Ui}(\beta_U^{VT}) = \varepsilon_{Ui}V_{Ui} - s_i \sum_{j=1}^{N} \varepsilon_{Uj}V_{Uj}$$

where ε_{Ui} is the ith element of ε_U. Now consider the variance of the right-hand side. Suppose that the ε_{Ui} are purely random variables with mean 0 and small variance $\sigma^2_{\varepsilon U} \approx 0.02$.[37] Then the right-hand side is 0 on average. Its variance is $\sigma^2_{\varepsilon U}\text{var}[V_{Ui} - s_i \sum_j V_{Uj}]$. Let σ^2_{FU} and σ^2_{VU} be the variances of F^{us}_{fi} and $V_{Ui} - s_i \sum_j V_{Uj}$, respectively, where the variation is across observations i. Because missing trade is so severe, the variance ratio is $\sigma^2_{FU}/\sigma^2_{VU} = 0.0001$. Hence the variance of the right-hand side is $\sigma^2_{\varepsilon U}\sigma^2_{VU} = \sigma^2_{\varepsilon U}\sigma^2_{FU}/0.0001 = 200\sigma^2_{FU}$! Thus, even though $\sigma^2_{\varepsilon U}$ is small, the right-hand side has a large variance relative to the variance of what is to be explained (σ^2_{FU}). Restated, $F^*_{Ui}(\beta_U^W)$ and $F^*_{Ui}(\beta_U^{VT})$ may be equal on average, but because of missing trade, there is a large variance between the two. Right-hand side approaches are like drunk dart players: Every dart completely misses the dartboard, but if you average them you get a bull's-eye.

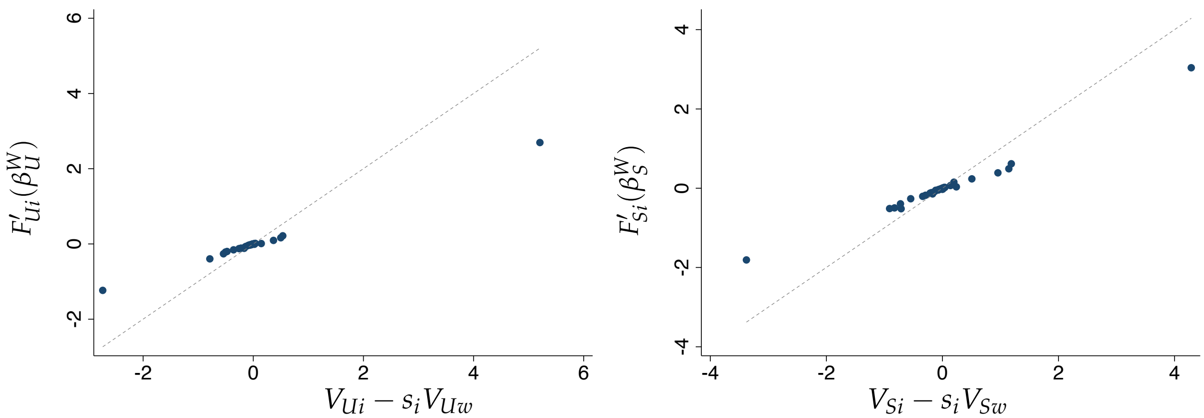
This establishes that the function $F^*_{fi}(\beta_f)$ is very sensitive to the choice of β_f. It thus explains the discrepancy in results between the approaches of Trefler (1993) and Gabaix (1997). Even though they generate similar values of the β_f, they generate very different predictions for the Vanek equation. Similarly, very large differences in the measured factor content of trade can result in very similar differences in β_f. This helps to explain the famous result of Gabaix (1997) in which he shows that setting the measured factor content of trade to zero or setting it equal to its additive inverse affects the resulting values of β_{fi} very little. This reason for the substantial disagreements regarding the performance of RHS approaches (Trefler, 1993, Gabaix, 1997) is new to the literature and serves as a caveat for interpreting results.

[36] This is an additional reason to place the β_f terms on the 'left-hand side' as we do in our main analysis. Note that in figure B2, the performance of the Vanek equation is *not* sensitive to the small differences between β_f^W and β_f^T.

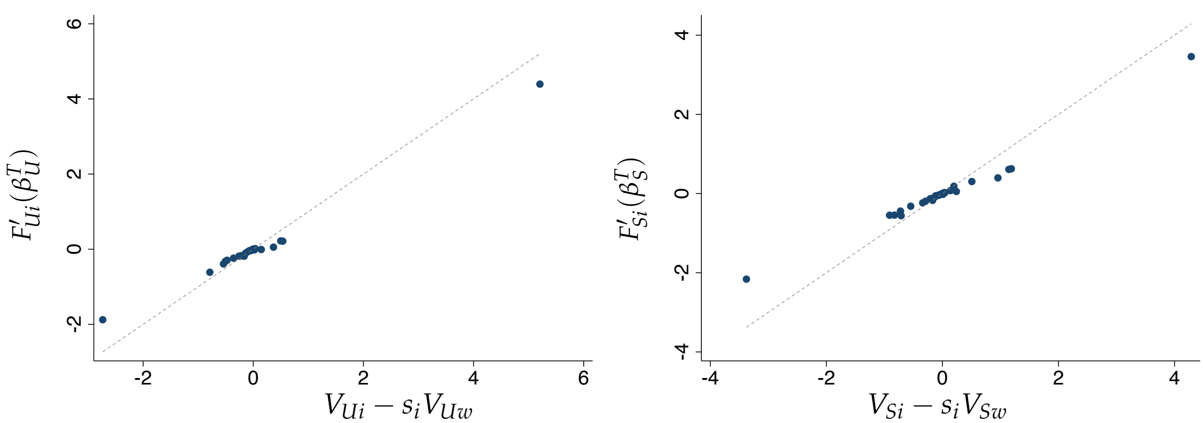
[37] This is the variance of the deviations between $(\beta_{Ui}^W)^{-1}$ and $(\beta_{Ui}^{VT})^{-1}$

Figure B2: Performance of the Vanek Equation Using β_{fi}^W and β_{fi}^T

Productivity Calibrated to the Wage Equation: β_f^W



Productivity Calibrated to the Techniques Equation: β_f^T



Notes: Each panel plots $V_{fi} - s_i V_{fw}$ against the factor content of trade $F'_{fi}(\beta_f)$ from equation (16) i.e., adjusted for nontradable Government Services. In the top row, the β_f that makes the Wage equation fit perfectly (β_f^W) is plugged into the Vanek equation. This yields a very good fit of the Vanek equation for both unskilled labor (left panel) and skilled labor (right panel). In the bottom row, the β_f that makes the Techniques equation fit perfectly (β_f^T) is plugged into the Vanek equation. This again yields a very good fit of the Vanek equation. Each point is a country and all lines are 45° lines.