Business Cycle during Structural Change: Arthur Lewis' Theory from a Neoclassical Perspective.*

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Abstract

We document that the nature of business cycles evolves over the process of development and structural change. In countries with large declining agricultural sectors, aggregate employment is uncorrelated with GDP. During booms, employment in agriculture declines while labor productivity increases in agriculture more than in other sectors. We construct a unified theory of business cycles and structural change consistent with the stylized facts. The focal point of the theory is the simultaneous decline and modernization of agriculture. As capital accumulates, agriculture becomes increasingly capital intensive as modern agriculture crowds out traditional agriculture. Structural change accelerates in booms and slows down in recessions. We estimate the model and show that it accounts well for both the structural transformation and the business cycle fluctuations of China.

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1 Introduction

The nature of economic fluctuations differs systematically at different stages of economic development (Acemoglu and Zilibotti (1997), Aguiar and Gopinath (2007)). China provides a good example. The country has experienced a profound economic transformation with the share of employment in agriculture falling from about 2/3 in 1980 to about 1/3 today. During this time, structural change has systematically accelerated during economic booms and stagnated during recessions, implying procyclical employment in nonagriculture and countercyclical employment in agriculture. Aggregate employment has instead remained smooth and uncorrelated with GDP. The cyclical behavior of labor productivity growth in agriculture is also remarkable: relative labor productivity in agriculture has increased in booms when workers have been leaving the rural sector.

These features are not an anomaly of China. They are shared by the majority of countries at comparable stages of development. Extending the work of Da Rocha and Restuccia (2006) beyond OECD countries, we show that the correlation between agricultural employment and aggregate GDP varies systematically with the relative size of the agricultural sector. While employment in agriculture is procyclical in industrialized countries, it is countercyclical in economies with a large agricultural sector. At the same time, employment in nonagricultural sectors is strongly procyclical in all countries. Finally, downswings in agricultural employment are associated with upswings in the relative productivity and capital intensity of the agricultural sector in developing countries. No such pattern is observed in mature economies where the agricultural sector is typically very small and its behavior shows no significant correlation with aggregate GDP. Moreover, in fully industrialized countries aggregate employment is on average more procyclical and more volatile than in developing and emerging countries.

To explain these observations, we propose a neoclassical theory of growth and structural change where the economy is subject to stochastic productivity shocks. The theory embeds a single exogenous frictions, namely, a time-invarying wedge between agriculture and nonagriculture that prevents the equalization of wages and keep agriculture inefficiently too large.[1] In our theory, the same technological forces drive the structural transformation and business cycle fluctuations at different stages of development. The key drivers of growth and structural change are sector-specific TFP growth (as in Ngai and Pissarides (2007)) and endogenous capital accumulation (as in Acemoglu and Guerrieri (2008)). Investments and capital deepening (and, possibly, differential rate of TFP growth) cause both reallocation from agriculture to nonagriculture and *modernization* of agriculture. To capture modernization, we depart from standard theories of structural change and assume that the rural sector comprises two subsectors, modern and traditional agriculture, producing imperfect substitutes subject to different technologies. In particular, modern agriculture uses capital which is instead not an input in the traditional sector. Under empirically plausible assumptions about technological parameters, agriculture shrinks as a share of total GDP and becomes more productive – not only in an absolute sense but also relative to the rest of the economy. In the long run, the equilibrium converges to an asymptotic balanced growth path where the agricultural sector is small, modernized, and highly productive.

[1] This friction has no bearing on the qualitative predictions but is important for some of the quantitative results as we explain below.

The mechanism in our theory is reminiscent of Lewis (1954), where the existence of a labor-intensive sector makes the supply of labor very elastic. In Lewis' theory, labor supply is infinitely elastic for as long as the traditional sector exists. In such a model, the elasticity would fall discretely as soon as the last worker moves out of the traditional sector. In our model, instead, the elasticity of labor supply declines gradually during the process of modernization of agriculture. A poor economy behaves like a Lewis economy; then, throughout the development process, it becomes more and more similar to a standard neoclassical economy.

Structural change has implications for business cycles. When the agricultural sector is large and predominantly traditional, the economy responds to sector-specific shocks by reallocating workers towards the more productive sectors with limited effects on wages and relative prices. When the agricultural sector is small, this margin of adjustment is muted. Wages and prices respond more to productivity shocks leading to larger swings in labor supply. We show that these model predictions are in line with stylized facts about both structural change and business cycles across countries.

We estimate the growth model using data from China. The key parameters are the elasticities of substitution between the output of the agricultural and nonagricultural sector and, within agriculture, between the two agricultural subsectors. We find both elasticities to be significantly larger than unity. This finding is consistent with the observation that the ratio between the expenditure share in agriculture and nonagriculture is positively correlated with the real GDP ratio between agriculture and nonagriculture. Because a declining agricultural sector is also consistent with nonhomothetic preferences, in an important extension we allow preferences to be nonhomothetic using a generalized Stone-Geary specification as in Herrendorf et al. (2013). The deviation from homothetic preferences is estimated to be very small. While nonhomothetic preferences may be important at the micro level, they do not seem to be an important feature to explain aggregate dynamics. Our findings are in line with the recent evidence in Alvarez-Cuadrado and Poschke (2011).[2]

Having estimated the deterministic model, we introduce stochastic shocks. We show that our model fits well in a quantitative sense salient features of the business of industrializing economies, most notably China. This step is technically challenging. Because, structural change is still ongoing we cannot rely on the standard approach of approximating the model in the neighborhood of a steady state. The moments from the model must then be estimated by simulating a large number of trajectories approaching the steady state, and calculating moments from that.

Among other things, the model explains why positive TFP shocks in nonagriculture cause a temporary acceleration of the process of structural change. Such shocks trigger an increase in total investments and a reallocation of capital and labor out of agriculture. Labor productivity grows, both because of the increase in average TFP and because the resource reallocation reduces misallocation. Interestingly, labor productivity increases more in agriculture than in the rest of the economy, as most

[2]Note that our estimates is different from that obtained by Herrendorf et al. (2013). They estimate a production function with three sectors, agriculture, manufacturing, and services, and find a low elasticity of substitution close to Leontief when using value-added data. Their estimate hinges on an assumption of symmetry, namely that the same elasticity of substitution is imposed across the three sectors. We show that if one relaxes the symmetry assumption the estimated elasticity of substitution between manufacturing and services is indeed close to zero, whereas the elasticity of substitution between agricultural and nonagricultural goods is larger than unity.

of the temporary reallocation of labor away of the agriculture is drawn from the traditional sector. Therefore, agriculture experiences a sharp increase in capital intensity and average labor productivity. The simulations also confirm that as structural change progresses the business cycle properties of the model become increasingly similar to those of advanced economies.

Our research contributes to the existing literature on structural change pioneered by Baumol (1967) which includes, among others, Kongsamut et al. (2001), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), Buera and Kaboski (2009), Alvarez-Cuadrado and Poschke (2011), Herrendorf et al. (2013), Boppart (2014), Comin et al. (2015), Alder et al. (2018). In our model, exogenous TFP growth and capital accumulation induce transition away from agriculture. The properties of the transition are consistent with Acemoglu and Guerrieri (2008), although in their model there is no traditional sector. The closest theoretical contribution in the literature is the recent paper by Alvarez-Cuadrado and Poschke (2011). They study the properties of two-sector models where both sectors use capital and labor as inputs. The elasticity of substitution between capital and labor is assumed to be constant within each sector but can differ across sectors. While in their model there is no explicit distinction between a modern and a traditional agricultural sector, our model nests theirs if we assume that the modern sector uses no labor and that the capital share in the modern sector is 100%.[3] In the general case, our model allows the elasticity of substitution between capital and labor in agriculture to change over the process of development due to the reallocation between traditional and modern agriculture. An advantage of our specification is that, even when the elasticity of substitution between the agricultural and nonagricultural good is larger than unity (which is consistent with our estimate), the labor share in agriculture remains positive, while in the CES technology proposed by Alvarez-Cuadrado and Poschke (2011) it would fall to zero. In the US, the labor share in agriculture is about 30% with no downward trend. Finally, the two models have different quantitative properties at business cycle frequencies (Alvarez-Cuadrado and Poschke (2011) do not consider business cycle fluctuations).

Our work also complements the existing literature on business cycles pioneered by Kydland and Prescott (1982) and Long and Plosser (1983) by adding explicitly the endogenous structural change. The classical multi-sector model focus on the stable economic structures or abstract from growth, for example, Benhabib et al. (1991), Hornstein and Praschnik (1997), Hornstein (2000), Horvath (2000a), Christiano et al. (2001), Kim and Kim (2006), etc. We extends the standard business cycle model to account for the business cycles properties at different stage of development. Related researches on cross-country business cycles differences including Da Rocha and Restuccia (2006) and Aguiar and Gopinath (2007).[4] In an independent study, Yao and Zhu (2018) construct a two-sector model where business cycles is affected, as in our model, by size of the agricultural sector. However, their emphasis

[3] In our model, we assume a Cobb Douglas production function in the nonagricultural sector, while Alvarez-Cuadrado and Poschke (2011) allows for a more general CES technology. Cobb Douglas is for simplicity and can be easily generalized.

[4] In particular, Da Rocha and Restuccia (2006) documents that among OECD countries, economies with larger agricultural sectors have smoother aggregate employment fluctuations and that agricultural employment is less correlated with aggregate GDP. Relative to their paper we show that this pattern holds up when extending the sample of countries to include a large number of countries, including very poor countries that are predominantly agrarian. Moreover, we document several additional salient differences in business cycle properties between poor and rich countries and show that business cycle properties of China are representative of countries with a similar share of agriculture.

is on non-homothetic preferences and they have no role for capital and investment. We view their paper as complementary to ours.

The remainder of the paper is organized as follows. In section 2 we present a set of stylized facts about business cycles and structural change across countries in the world, zooming on China and the US. In Section 3, we present the multisector growth model. In Section 4, we estimate the key structural parameters of the deterministic version of the model using data from China. In Section 5, we compare the quantitative prediction of the model with salient business cycle features. Section 6 concludes. The Appendix includes proofs and other technical material.

2 Facts on Business Cycles and Structural Change

The process of economic development is associated with a significant downsizing of the agricultural sector. In the US, a third of the workforce was employed at farms in 1900. This employment share fell below 2% by 2000. Today, the average employment in agriculture is 4.6% in OECD countries, which compares with 31.6% in non-OECD countries (World Development Indicators 2017). The cross-country correlation between the employment share of agriculture and log GDP per capita is -0.84.

2.1 Modernization of Agriculture

The relative decline of agriculture is accompanied by a number of economic transformations. First, as employment in agriculture declines, the capital intensity of this sector rises faster than in the rest of the economy. Second, labor productivity grows faster in agriculture than in the rest of the economy. Third, as the share of real value added in agriculture falls, so does the relative price of agricultural goods.

Relative Capital Deepening. While capital deepening is pervasive over the development process, it is especially pronounced in agriculture: both the capital-output and the capital-labor ratio grow faster in agriculture than in the rest of the economy. Figures 1-2 show the capital-output (K-Y) ratio in agriculture relative to the aggregate K-Y ratio (henceforth, the *relative K-Y ratio*) over the process of economic development. Figure 1 (panel a) shows that the relative K-Y ratio increased from about 40% in the pre-war period, to about 120% since the 1980s. The cross-country evidence shows a consistent picture: the relative K-Y ratio is significantly lower in developing than in industrialized countries. Figure 2 (panel b) plots the relative K/Y ratio against the employment share of agriculture for the period 1995-2016. For example, that ratio is very low in Sub-Saharan African countries, where the agricultural sector is still very labor intensive and still employs a large proportion of the labor force. The regression coefficient is negative and highly significant. Since data are available over a 22 year panel, we can also study the within-country coevolution of employment shares in agriculture and relative K-Y ratios. To this aim, we split the sample for each country into two observations, the average for the period 1995-2005 and the average for the period 2006-2016, and regress the relative K-Y ratio on the employment share of agriculture and a full set of country dummies. This is identical to running a regression on growth rates. The results (reported in the appendix) show a significant negative relationship between employment in agriculture and relative capital intensive, consistent with the cross-country pattern.

Productivity Gap. Capital deepening and technical change bring about higher labor productivity in all sectors of the economy. However, this growth is larger in agriculture than in the rest of the economy. Consistent with this observation, consider the average labor productivity in nonagriculture relative to agriculture (henceforth, the *productivity gap*). Figure 1 (panel b) shows that the productivity gap declines over time in the US[5]. Figure 2 (panel b) shows a similar pictures across countries: the productivity gap is especially high in poor countries with a large agricultural sector.

The relative sectoral labor income share mirrors the productivity gap.[6] Figure 1 (panel c) shows that in the US the labor-income share in the farm sector relative to that of the nonfarm sector fell over time after World War II. Unfortunately, it is difficult to find comparable data across a large number of countries since the labor income share in agriculture is often poorly measured.

Relative Prices and Quantities. Finally, we are interested in the comovement of relative prices and quantities in agriculture. This comovement is informative about the elasticity of substitution between agriculture and nonagriculture, a key parameter in our they're. We use data for value added at current and constant prices. More formally, let $VA_G \equiv P_G \times Y_G$ and $VA_M \equiv P_M \times Y_M$ denote the value added in agriculture and nonagriculture, respectively, where P_G and P_M are sectoral GDP deflators. Figure 1 (panel d) plots the 5-year growth in the relative value added (at current prices) VA_M/VA_G against the 5-year growth in the relative real output Y_M/Y_G for the US. The figure shows a highly significant positive correlation. The finding is robust to using 1-year and 10-year growth rates instead of 5-year growth rates. Figure 13 in the appendix show that this correlation is strongly positive also in China.

2.2 Business Cycle

Consider, next, economic fluctuations. Panels a and b in Figure 3 compare the business cycle of China with that of the US. While aggregate employment is volatile and highly procyclical in the United States, aggregate employment is acyclical and relatively smooth in China.[7]

Interestingly, aggregate fluctuations in employment are systematically associated with structural change and movements in and out of agriculture. Consider panel c in Figure 3. Until 1960, NBER recessions in the US were associated with a slowdown and reversal of the process of structural change. Namely, the employment share of agriculture fell in booms and increased in downturns. The cyclicality of employment in agriculture is dampened in the later part of the sample and ceases to be visible after 1960. China today looks similar to the US of the early days. Panel d in Figure 3 shows that

[5] Labor productivity is defined as the value added per worker in current prices.

[6] We discuss below the link between the relative labor income shares and the labor productivity gap.

[7] Figure 12 in the appendix shows how the volatility of employment relative to the volatility of output in the US has increased over time. Before 1980, employment was significantly less volatile than output, while after 2000 employment has been more volatile than output.

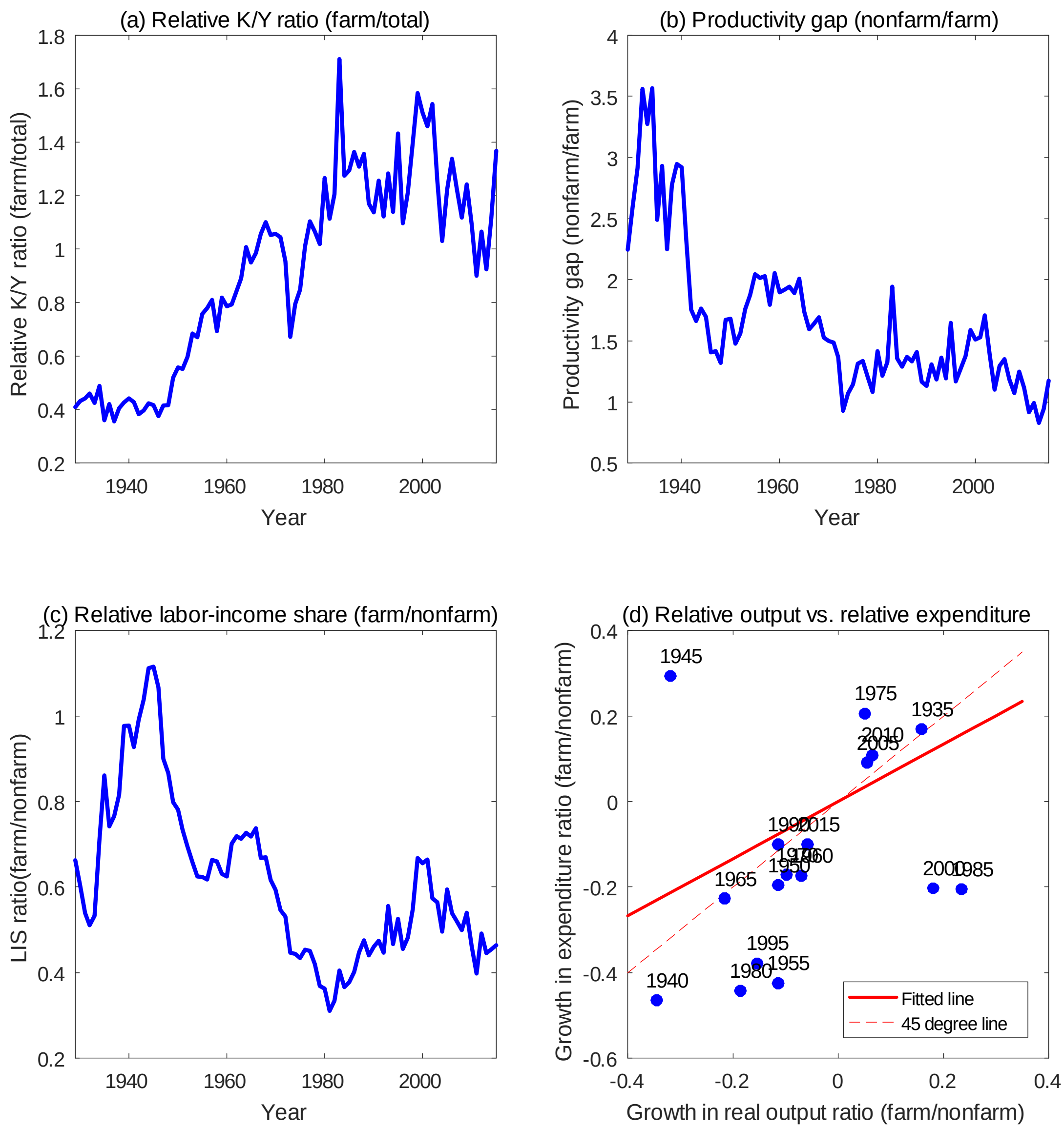

Figure 1: Panel a plots the farm capital-output ratio divided by the total capital-output ratio in the US. Panel b plots the Labor productivity gap over time in the US. The labor productivity gap is measured by the nonfarm value added per worker divided by the farm value added per worker. Panel c plots the labor income share (LIS) ratio in the US. The graph plots labor's income share in the farm sector divided by labor's income share in the nonfarm sector. We compute the labor's income share as the compensation of employees divided by the value-added output excluding the proprietors' income. Panel d plots the growth in relative (current price) value added vs. growth in relative output. The horizontal axis plots the 5-year growth in the relative value added (farm vs. nonfarm) at current prices. The vertical axis plots the annual growth in the relative real output (farm vs. nonfarm). Source: Capital stocks by sectors 1929-2015 are from the U.S. Bureau of Economic Analysis (BEA) Fixed Asset Tables 6.1 "Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods". The value-added output by sectors come from the National Income and Product Accounts (NIPA) Table 1.3.5 "Gross Value Added by Sector". Employment by sectors is from the NIPA Table 6.8A, 6.8B, 6.8C, and 6.8D. Proprietors' income by sectors come from the NIPA Table 1.12. Compensation of employees by sectors come NIPA Table 6.2A, 6.2B, 6.2C, and 6.2D. The real sectoral value added is from NIPA Table 1.3.6. "Real Gross Value Added by Sector, Chained Dollars."

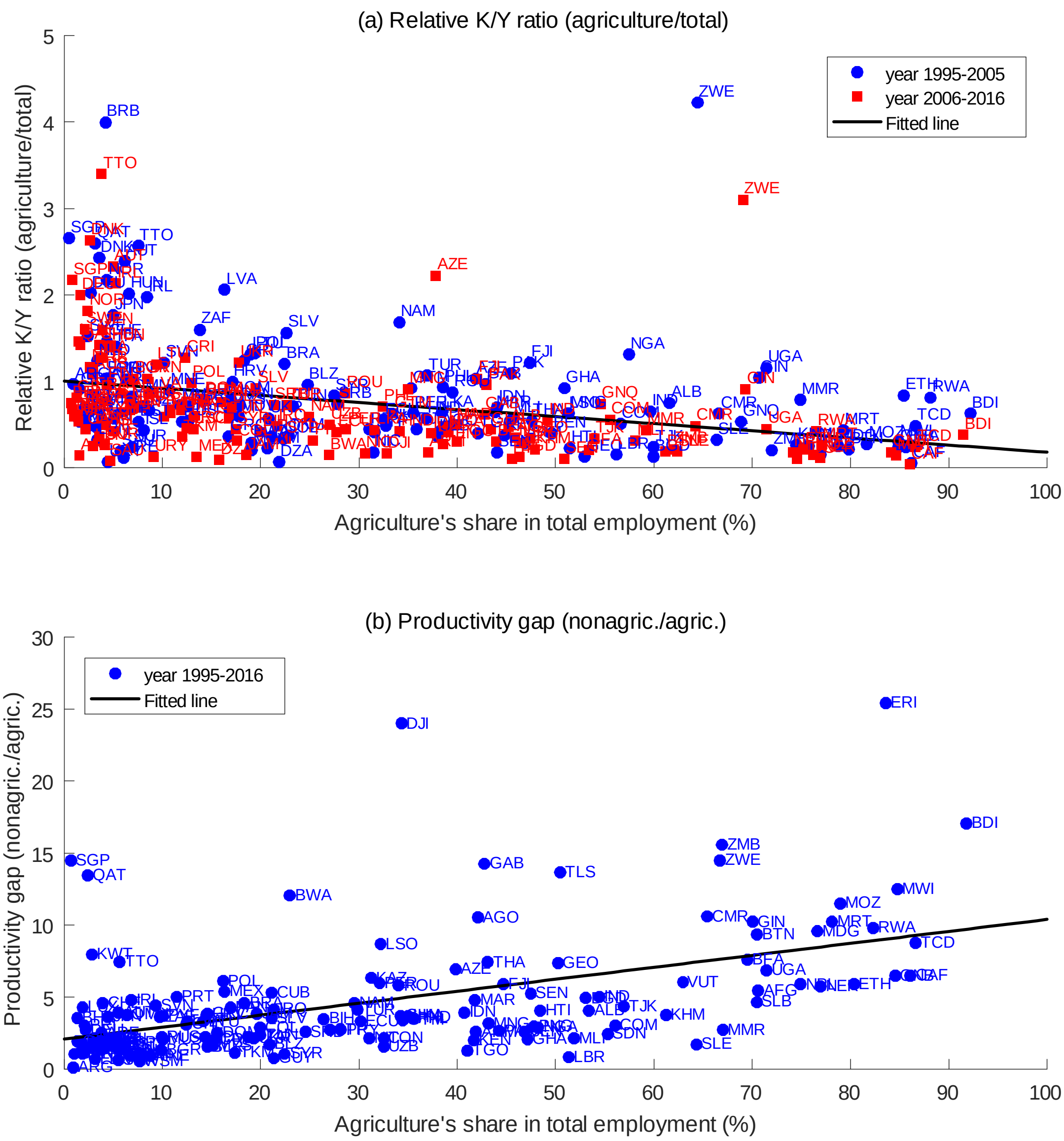

Figure 2: Panel (a) plots the relative capital-output ratio (agriculture vs. total) $\left(K^G/Y^G\right)/\left(K/Y\right)$ vs. the average agricultural employment share across countries. Each country has two observations: the average for the period 1995-2005 and the average for the period 2006-2016. Panel (b) plots labor productivity gap across countries. The labor productivity gap is measured by the nonagriculture value added per worker divided by the agriculture value added per worker. The horizontal axis shows the average employment share of agriculture over the sample period for each country. Source: FAO (K^G is measured by the net capital stock; Y^G is value added, both at current prices) and Penn World Table (capital stock and real GDP at current PPP). Agriculture's employment share comes from ILO modeled estimates. Agriculture's value added output share comes from World Bank database.

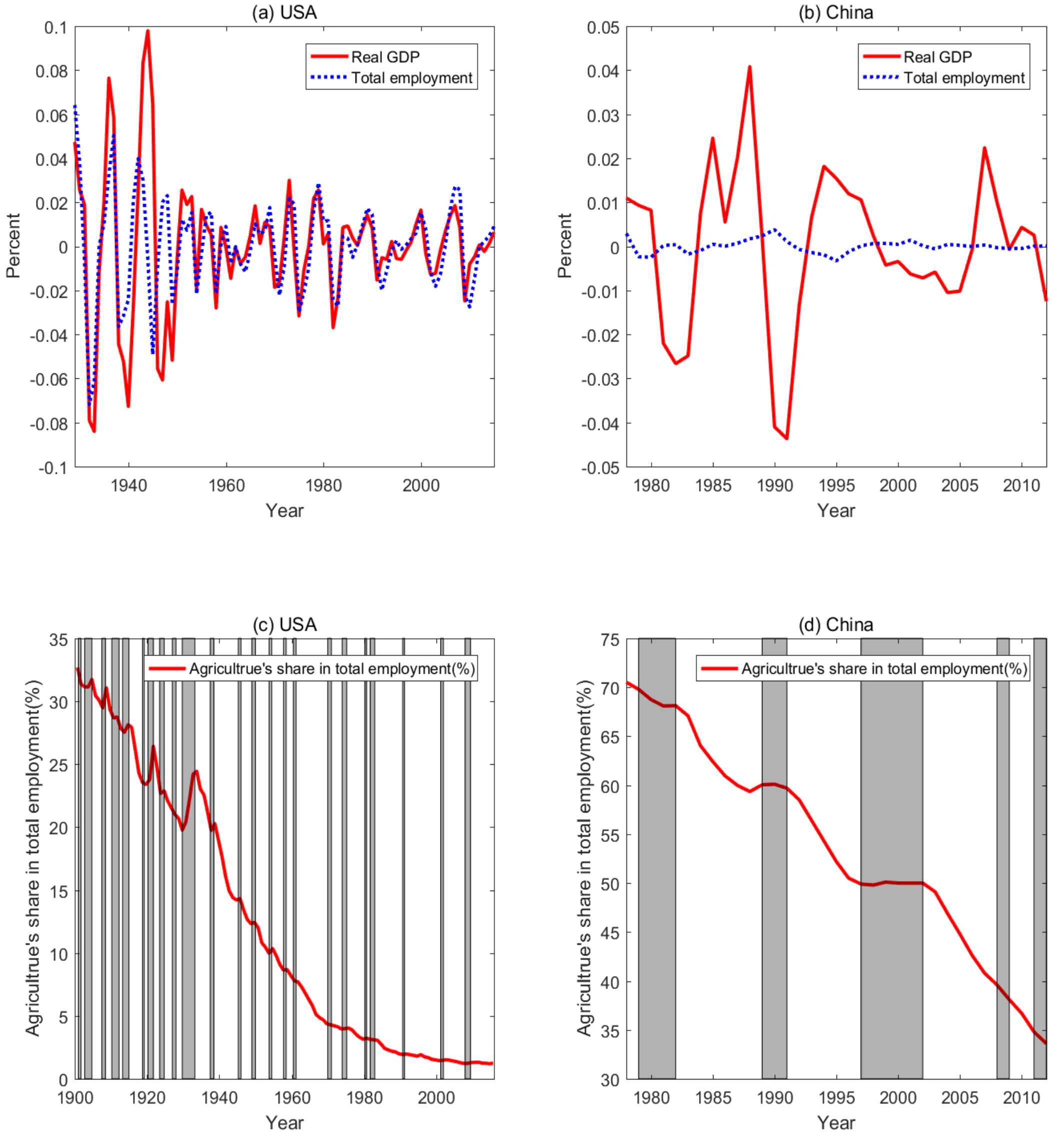

Figure 3: Panel a and b plot the volatility of total employment and real GDP in the US (1929-2015) and China (1978-2012). The figure shows the time evolution in the US (left panel) and China (right panel) of HP-filtered employment and GDP. The HP-filtered use smoothing parameter 6.25 (Ravn and Uhlig 2002). The aggregate employment data in China is from the Statistic Year Book by the China National Bureau of Statistics, Table "Number of employed persons at year-end by three strata of industry". We incorporate a correction proposed by (Holz 2006). The correction takes care of the reclassification of employed workers that was made by the NBS in 1990. Panel c and d plot the agriculture's share in total employment over the business cycles. The left panel plots the farm employment share over the business cycle in the US. Grey ranges denote period classified as recessions by the NBER. The right panel plots the agriculture employment share over the business cycle. Grey ranges denote recessions of the Chinese economy, where the recession is defined as the period of which the real GDP growth rate is below 9.7 percent (the average real GDP growth rate in China during 1978-2012)

structural change – measured by the decline in agricultural employment – accelerates during periods of high growth and slows down or halts during periods of low growth in China.

Panel b of Figure 4 documents that agricultural employment is volatile and countercyclical in China. There is no such pattern in the US in the same period, where the correlation is positive rather than negative. Interestingly, the cyclical pattern of nonagricultural employment is very similar in the US and in China: Panels c and d of Figure 4 shows that nonagricultural employment is highly procyclical and roughly as volatile as GDP both in China and the US. It follows that agricultural and nonagricultural employment are strongly negatively correlated in China.

The stylized facts documented above are consistent with international data.[8] The upper left panel in Figure 5 shows that the volatility of employment relative to GDP is weakly negatively correlated with the employment share of agriculture. This is consistent with the US time series evidence in Figure 58 in the appendix. The cross-country correlation is negative albeit statistically insignificant. Second, the lower left panel in Figure 5 shows that the correlation between aggregate employment and GDP declines strongly with the employment share of agriculture, being large and positive for industrialized countries like the US and negative for countries with a large agricultural sector. This is in line with the US-China contrast discussed above.

The right upper panel in Figure 5 shows that the time-series correlation of the HP-filtered employment in agriculture and nonagriculture is positive (on average) for countries with a small agricultural sector like the US and negative for countries with a large agricultural sector like China.

Finally, we study the dynamics of the productivity gap. Recall that during the process of structural change, the productivity gap falls (see panel b of Figures 1-2). The lower right panel in Figure 5 shows a similar pattern at business-cycle frequencies. Namely, in countries with large agricultural sectors, the productivity gap is negatively correlated with employment in nonagriculture, while this correlation is close to zero in countries with a small agricultural sector. For instance the correlation is -0.54 for China. We conclude that relative productivity and relative employment (in agriculture) move in opposite directions. This happens both during the process of structural change and over the business cycle in countries that are undergoing structural change away from agriculture.

In summary, the characteristics of the business cycles change systematically across different stages of the process of structural change. In countries with a large agricultural sector, we observe:

1. Aggregate employment has a low (and sometimes negative) correlation with GDP;
2. The cyclical employment in agriculture is strongly countercyclical and more pronounced;
3. The labor productivity gap is negatively correlated with employment in nonagriculture.

The third point is important because it suggests that the mechanisms driving structural change is related to those driving the business cycle fluctuations. In economies with a large agriculture, recessions

[8] We use sector-level employment data from the International Labor Organization (ILO). The appendix describes how the data set is constructed.

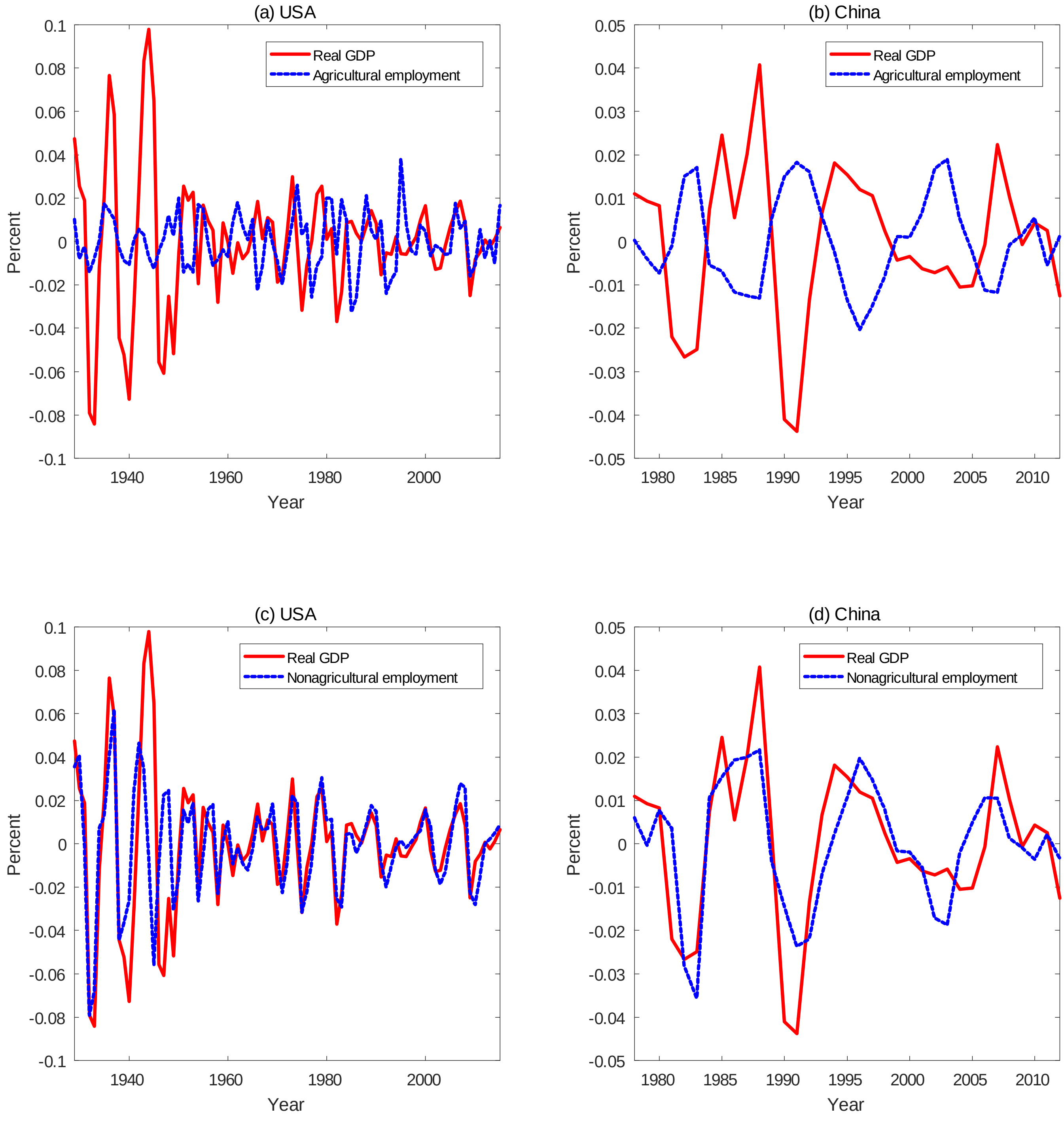

Figure 4: Panel a and b plot the HP-filtered log agricultural employment vs HP-filtered log real GDP in the US 1929-2015 and in China 1978-2012. Panel c and d plot the HP-filtered log nonagricultural employment vs HP-filtered log real GDP in the US and in China. We use a smoothing parameter 6.25 for the HP filter (Ravn and Uhlig 2002). Source: The US employment by sectors is from the NIPA Table 6.8A, 6.8B, 6.8C, and 6.8D. The sectoral employment data in China is from the Statistic Year Book by the China National Bureau of Statistics, Table "Number of employed persons at year-end by three strata of industry". The number is calculated based on the households survey on both urban and rural households in China. The nonagriculture employment is the sum of both employment in the secondary industry and the employment in the tertiary industry. We incorporate a correction proposed by (Holz 2006). The correction takes care of the reclassification of employed workers that was made by the NBS in 1990.

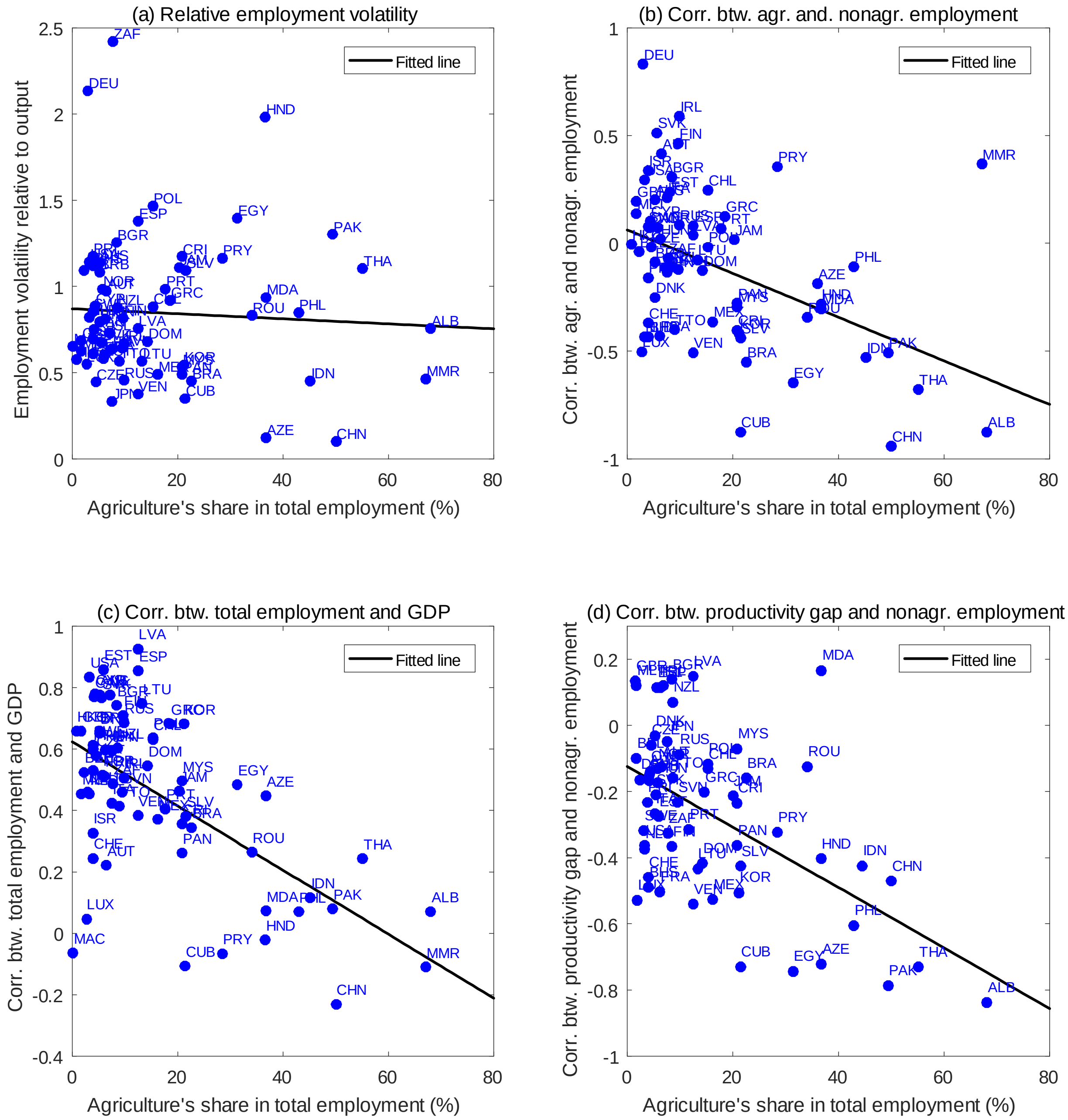

Figure 5: The figure is a cross-country scatter plot of several business cycle statistics. Panel a plots the relative volatility of employment in a sample of 68 countries, where the relative volatility is measured by the standard deviation of HP-filtered log total employment divided by the standard deviation of the HP-filtered log real output. Panel b plots the time series correlation of HP-filtered log nonagricultural employment and HP-filtered log agricultural employment in a sample of 67 countries. Panel c shows the correlation between the HP-filtered log total employment and HP-filtered log real GDP in a sample of 68 countries. Panel d plots the time series correlation of the HP-filtered log productivity gap and the HP-filtered log nonagricultural employment in a sample of 63 countries. We use a smoothing parameter 6.25 for the HP filter (Ravn and Uhlig 2002). The x-axis denotes the mean agriculture's share in total employment over the sample period for each country. Sample period covers 1970-2015 and some countries have fewer observations. We keep the countries that have at least more than 15 years of consecutive observations in order to calculate the business cycle statistics.

are times of slowdown and even reversal of structural change. People stop leaving or even move back to rural areas and recessions have a sullying effect on the productivity of agriculture. In fully modernized economies, farms live an almost an independent life and workers move in and out employment in the urban sector.

3 A Model of Business Cycle with Structural Change

In this section, we present a dynamic general equilibrium model that describes the process of growth and structural change in an economy with a declining agriculture. We show that under appropriate restrictions on the cross-sectoral elasticities and on the sectoral TFP growth rates, the model predicts trajectories of structural change and modernization of agriculture that resemble those observed in the data. We first derive some characterization results. Then, in the following section, we estimate the key structural parameters of the deterministic version of the model using data for China. We show that the model matches accurately the process of structural change of China. Finally, we introduce uncertainty and productivity shocks, and show that the stochastic version of the model with parameters calibrated to the structural change of China is consistent with salient business cycle features of economies undergoing a process of structural change. As documented in the previous section, these are different from those of fully industrialized economies.

3.1 Environment

3.1.1 Production

The consumption good, assumed to be the numeraire, is a CES aggregate of two goods,

$$Y = F\left(Y^G, Y^M\right) = \left[\gamma\left(Y^G\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(Y^M\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{1}$$

We label the sectors producing the two goods *agriculture* (superscript G) and *nonagriculture* (superscript M, as in "manufacturing"), respectively. We denote by $\varepsilon > 0$ the elasticity of substitution between the two goods.[9]

The technology of nonagriculture is described by the following Cobb Douglas production function

$$Y^M = \left(K^M\right)^{1-\alpha} \times \left(Z^M H^M\right)^{\alpha}, \tag{2}$$

where $H^M = hN^M$ is the labor input. N^M denotes the number of workers and h denotes the number of hours worked by each of them. K^M denotes capital and Z^M is a productivity parameter (henceforth, TFP).[10]

[9] The technology parameters γ and ε can alternatively be interpreted as preference parameters, reflecting the relative weight and the elasticity of substitution between goods produced by the agricultural and nonagricultural sector. The same interpretation can be given to the parameters ζ and ω in Equation (3) below.

[10] With Cobb Douglas technology, the distinction between different types of technical progress is immaterial. Thus, referring to Z^M (and later to Z^A) as total factor productivity (TFP) is without loss of generality.

Agriculture is a CES aggregate of two subsectors, modern agriculture (superscript AM) and traditional agriculture (superscript S, as in "subsistence"), which produce imperfect substitutes with an elasticity of substitution $\omega > 0$. More formally,

$$Y^G = \left[\varsigma \left(Y^{AM} \right)^{\frac{\omega-1}{\omega}} + (1-\varsigma) \left(Y^S \right)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \tag{3}$$

where $\varsigma \in (0,1)$. Modern agriculture uses a Cobb-Douglas technology with capital and labor (in the quantitative section we will allow for land in agriculture):

$$Y^{AM} = \left(K^{AM} \right)^{1-\beta} \left(Z^{AM} H^{AM} \right)^{\beta}. \tag{4}$$

Traditional agriculture does not use any capital:

$$Y^S = Z^S H^S. \tag{5}$$

Note that the presence of a traditional sector implies a variable elasticity of substitution between capital and labor in agriculture. When $\omega > 1$, the elasticity of substitution is larger than unity and declines as the economy develops. As K^{AM} grows large, the elasticity of substitution approaches unity.

We assume the TFPs Z^M, Z^{AM}, and Z^S to grow at constant exponential rates g^M, g^{AM}, and g^S, respectively. All goods are produced competitively. Both capital and labor are perfectly mobile across sectors. Capital depreciates at the rate δ.

3.1.2 Households

Agent's preferences are described by a logarithmic utility function.

$$U = \int_0^\infty \left(\theta \log c + (1-\theta) \log (1-h) \right) \times e^{-(\rho-n)t} dt. \tag{6}$$

Here, $c \equiv C/N$ denotes the consumption per capita, $1-h$ is leisure, and ρ is the time discount rate. The population grows at the exogenous rate $n < \rho$. In the analytical section, for simplicity, we assume an inelastic labor supply, set $\theta = 1$ and ignore leisure altogether (i.e., we set $H^i = N^i$). We introduce an endogenous labor-leisure choice in the quantitative analysis below where we estimate the model and study economic fluctuations. When we introduce uncertainty, (6) is replaced by an expected utility function with unit relative risk aversion. We suppress time indexes when it is not a source of confusion.

The representative household maximizes expected utility subject to a set of period budget constraints $Nc + \dot{K} = WN + RK + Tr$, where $K = K^M + K^{AM}$ and $N = N^M + N^{AM} + N^S$ denote the aggregate capital stock and number of workers, respectively. W denotes the after-tax wage that is equalized across sectors in equilibrium; R denotes the equilibrium (gross) interest rate.

Since in the data we see persistent labor wage differences across agriculture and nonagriculture, we introduce an exogenous wedge by assuming that the government taxes wages in nonagriculture at the rate τ.[11] The government runs a balanced budget each period and rebates the tax proceeds to the representative household as lump-sum transfers, denoted by Tr. Thus, $Tr = \tau W^M H^M$, where W^M denotes the nonagriculture pre-tax wage. In equilibrium, $W^{AM} = W^S = (1-\tau) W^M = W$. Note that the wedge prevents the equalization of the marginal product of labor across sectors, increasing employment in agriculture and reducing the equilibrium after-tax wage W. This wedge is a stand-in for a variety of frictions leading to rural overpopulation.

3.2 Competitive Equilibrium

Since the wedge τ is the only distortion, we can characterize the recursive competitive equilibrium as the solution to the constrained maximization of a benevolent social planner problem. More formally, the planner maximizes (6) subject to the resource constraint

$$\dot{K} = F\left(Y^G, Y^M\right) - \delta K - C - \tau \bar{W} N^M + Tr, \tag{7}$$

and to the technological constraints implied by equations (1)–(5). Here, $\bar{W}$ denotes the marginal product of labor in manufacturing, i.e., $\bar{W} = W^M$. The planner takes $\bar{W}$ as parametric when calculating the first-order conditions of the maximization problem. Then, she sets $Tr = \tau \bar{W} H^M$. This makes the solution of the social planner problem identical to the distorted competitive equilibrium.

It is useful to introduce some useful normalizations.

Notation 1 *Define:*

$$\begin{aligned} c &\equiv \frac{C}{N},\ \chi \equiv \frac{K}{N}, \\ \kappa &\equiv \frac{K^M}{K},\ \nu^M \equiv \frac{N^M}{N},\ \nu^{AM} \equiv \frac{N^{AM}}{N},\ \nu^S \equiv \frac{N^S}{N} \\ \upsilon &\equiv \frac{\varsigma\left(Y^{AM}\right)^{\frac{\omega-1}{\omega}}}{\varsigma\left(Y^{AM}\right)^{\frac{\omega-1}{\omega}} + (1-\varsigma)\left(Y^S\right)^{\frac{\omega-1}{\omega}}}. \end{aligned}$$

χ is the aggregate capital-labor ratio, the key endogenous state variable of the economy. κ is the share of the aggregate capital stock used in nonagriculture. Since the traditional sector does not use capital, $1-\kappa$ is the corresponding share in modern agriculture. $\nu^i \equiv N^i/N$ denotes the employment share in sector $i \in \{AM, M, S\}$. υ measures the GDP share of modern agriculture in total agriculture. We focus our discussion on the region of the parameter space that will be consistent with the (unconstrained) estimation in our application.

Assumption 1 *We assume:* $\varepsilon > 1$, $\omega > 1$, $\beta > \alpha$, *and* $g^M \geq g^{AM} \geq g^S$.

We characterize equilibrium in two stages. First, we solve the static problem defined the maximized current output per capita subject to the wedge τ and a given aggregate stock of capital per worker

[11] This wedge has no effect on our analytical result but it is important in the quantitative analysis.

χ and the TFP levels. Then, we solve the dynamic equilibrium involving capital accumulation and technical progress. Let $\boldsymbol{x} \equiv \left(\kappa, \nu^S, \nu^{AM}, \nu^M\right)$. Then, given χ, the competitive equilibrium maximizes the distorted output per worker y

$$y(\chi) = \max_{\boldsymbol{x}} f\left(y^G(\boldsymbol{x},\chi), y^M(\boldsymbol{x},\chi)\right) - \tau \bar{W} \nu^M + Tr. \tag{8}$$

subject to the technology constraint

$$f\left(y^G(\boldsymbol{x},\chi), y^M(\boldsymbol{x},\chi)\right) \tag{9}$$
$$= \left[\begin{array}{c} \gamma \left(\left(\begin{array}{c} \varsigma \left(\left(Z^{AM}\right)^{\alpha} \times ((1-\kappa)\chi)^{1-\alpha} \times \left(\nu^{AM}\right)^{\alpha} \right)^{\frac{\omega-1}{\omega}} \\ + (1-\varsigma)\left(Z^S \nu^S\right)^{\frac{\omega-1}{\omega}} \end{array} \right)^{\frac{\omega}{\omega-1}} \right)^{\frac{\varepsilon-1}{\varepsilon}} \\ + (1-\gamma)\left(\left(Z^M\right)^{\alpha} \times (\kappa\chi)^{1-\alpha} \times \left(\nu^M\right)^{\alpha} \right)^{\frac{\varepsilon-1}{\varepsilon}} \end{array} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

and the resource constraints $\kappa \in [0,1]$ and $\nu^M + \nu^{AM} + \nu^S = 1$.

Conditional on the wedge τ, production efficiency requires the equalization across sectors of the marginal product of labor and of the marginal product capital. The equalization of the marginal product of capital in modern agriculture and nonagriculture yields, after rearranging terms:

$$\frac{1-\kappa}{\kappa} = \frac{1-\beta}{1-\alpha}\frac{\gamma}{1-\gamma}\left(\frac{y^G}{y^M}\right)^{\frac{\varepsilon-1}{\varepsilon}} \upsilon. \tag{10}$$

The equalization of the marginal product of labor in modern agriculture and nonagriculture, and in traditional agriculture and nonagriculture yield, respectively,

$$\nu^{AM} = \frac{1}{1-\tau}\frac{1-\alpha}{1-\beta}\frac{\beta}{\alpha}\frac{1-\kappa}{\kappa}\nu^M, \tag{11}$$
$$\nu^S = \frac{1}{\beta}\frac{1-\upsilon}{\upsilon}\nu^{AM}. \tag{12}$$

Combining (10), (11), and (12) with the resource constraint on labor, and solving for ν^M, yields

$$\nu^M = \left(1 + \frac{1}{1-\tau}\frac{1-\kappa}{\kappa}\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\left(1 + \frac{1}{\beta}\frac{1-\upsilon}{\upsilon}\right)\right)^{-1}. \tag{13}$$

Proposition 1 *Given χ and $\mathbf{Z} = \left(Z^M, Z^{AM}, Z^S\right)$, a static competitive equilibrium is characterized by the functions $\kappa = \kappa(\chi,\mathbf{Z})$, $\upsilon = \upsilon(\chi,\mathbf{Z})$ and by the equilibrium employment shares $\nu^M(\kappa(\chi,\mathbf{Z}), \upsilon(\chi,\mathbf{Z}))$, $\nu^{AM}(\kappa(\chi,\mathbf{Z}), \upsilon(\chi,\mathbf{Z}))$, and $\nu^S(\kappa(\chi,\mathbf{Z}), \upsilon(\chi,\mathbf{Z}))$ implicitly defined by Equations (10), (11), (12), (13), and by the technology (9).*

We can then write the aggregate production function subject to constrained productive efficiency as

$$y(\chi,\mathbf{Z}) = \eta(\kappa(\chi,\mathbf{Z}), \upsilon(\chi,\mathbf{Z})) \times (\chi\kappa(\chi,\mathbf{Z}))^{1-\alpha}\left(\nu^M(\kappa(\chi,\mathbf{Z}), \upsilon(\chi,\mathbf{Z}))\right)^{\alpha},$$

where

$$\eta\left(\kappa\left(\chi,\boldsymbol{Z}\right),\upsilon\left(\chi,\boldsymbol{Z}\right)\right)=\left(1-\gamma\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(1+\frac{1-\alpha}{1-\beta}\frac{1-\kappa\left(\chi,\boldsymbol{Z}\right)}{\kappa\left(\chi,\boldsymbol{Z}\right)}\frac{1}{\upsilon\left(\chi,\boldsymbol{Z}\right)}\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{14}$$

3.2.1 Static Equilibrium

We are interested in the properties of the functions $\kappa\left(\chi,\boldsymbol{Z}\right)$ and $\upsilon\left(\chi,\boldsymbol{Z}\right)$. In general, these are involved. However, in this section we establish sharp results for economies where the two elasticities, ω and ε, are close one to another.

We first consider the comparative statics of capital deepening (χ). The main result is that capital accumulation has monotone effects, namely, a relative decline of agriculture (i.e., increasing κ) and modernization (i.e., increasing υ) at all stages of economic transition. A key intermediate step is to prove (see appendix) that

$$\left.\frac{\partial\ln\kappa\left(\chi,\boldsymbol{Z}\right)}{\partial\ln\chi}\right|_{\omega=\varepsilon}=\frac{\left(\varepsilon-1\right)\left(\beta-\alpha\right)\left(1-\kappa\right)}{1+\left(\varepsilon-1\right)\left(\left(\beta-\alpha\right)\left(\kappa-\nu^{M}\right)+\nu^{S}\left(1-\beta\right)\right)}>0, \tag{15}$$

a generalization of a result in Acemoglu and Guerrieri (2008).[12] The following lemma can be established.

Lemma 1 *Suppose $\beta>\alpha$, and $\varepsilon>1$. Then, there exists $\bar{x}>0$ such that, if $\|\omega-\varepsilon\|<\bar{x}$, then, both $\kappa\left(\chi,\boldsymbol{Z}\right)$ and $\upsilon\left(\chi,\boldsymbol{Z}\right)$ are monotone increasing in χ. Moreover, ν^{M}, ν^{M}/ν^{AM}, and ν^{AM}/ν^{S} are monotone increasing in χ while ν^{S} is monotone decreasing in χ.*

The proof (see appendix) exploits (15) and the FOCs of the planning problem. Since κ increases in χ, then Equation (11) implies that the ratio ν^{M}/ν^{AM} also increases with χ. Moreover, optimality requires that the capital labor ratio in modern agriculture be proportional to that in nonagriculture, i.e.,

$$\frac{\left(1-\kappa\right)\chi}{\nu^{AM}}=\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{\kappa\chi}{\nu^{M}}. \tag{16}$$

Thus, as the economy becomes more capital abundant, both nonagriculture and modern agriculture experience capital deepening. The FOC (12) implies then that both ν^{AM}/ν^{S} and υ must increase with χ.[13] Finally, since ν^{M}/ν^{AM} and ν^{AM}/ν^{S} are increasing in χ, ν^{M} must be increasing and ν^{S} decreasing in χ. This leads to the conclusion that capital accumulation triggers modernization in agriculture.

Next, we study the comparative statics of the TFP vector. First, we note that a proportional increase in all TFP levels does not affect κ and υ, i.e., $\kappa\left(\chi,\boldsymbol{Z}\right)=\kappa\left(\chi,\lambda\boldsymbol{Z}\right)$ and $\upsilon\left(\chi,\boldsymbol{Z}\right)=\upsilon\left(\chi,\lambda\boldsymbol{Z}\right)$ for

[12] To see why this derivative has a positive sign, note that $\kappa-\nu^{M}>0$, since nonagriculture is the capital-intensive sector. Moreover, by assumption, $\varepsilon>1$ and $\beta>\alpha$.

[13] To see this, rewrite (12) as

$$\frac{\nu^{AM}}{\nu^{S}}=\left(\frac{\varsigma}{1-\varsigma}\beta\right)^{\varepsilon}\left(\frac{\left(Z^{AM}\right)^{\beta}}{Z^{S}}\right)^{\varepsilon-1}\left(\frac{\left(1-\kappa\right)\chi}{\nu^{AM}}\right)^{\left(1-\beta\right)\left(\varepsilon-1\right)}.$$

This expression shows that the ratio $\frac{\nu^{AM}}{\nu^{S}}$ will increase in χ if and only if the term $\frac{(1-\kappa)\chi}{\nu^{AM}}$ increases in χ.

any $\lambda > 0$. Thus, what matters are the relative TFPs in the three sector. With some abuse of notation, we write $\kappa(\chi, \boldsymbol{Z}) = \kappa(\chi, z^{AM}, z^S)$ and $\upsilon(\chi, \boldsymbol{Z}) = \upsilon(\chi, z^{AM}, z^S)$ where $z^{AM} \equiv (Z^{AM})^{\beta} / (Z^M)^{\alpha}$ and $z^S \equiv Z^S / (Z^M)^{\alpha}$.

Lemma 2 *Suppose $\beta > \alpha$, and $\varepsilon > 1$. Then, $\kappa(\chi, z^{AM}, z^S)$ is decreasing in z^{AM} and increasing in z^S, whereas $\upsilon(\chi, z^{AM}, z^S)$ is increasing in z^{AM} and decreasing in z^S. Moreover, ν^{AM}, ν^{AM}/ν^S and ν^{AM}/ν^M are increasing in z^{AM} and decreasing in z^S.*

Lemma 2 establishes that higher TFP in nonagriculture relative to modern agriculture (while holding constant Z^S) causes a reallocation of both capital and labor towards nonagriculture. Likewise, higher TFP in modern relative to traditional agriculture (while holding constant Z^M) causes the modernization of agriculture.

3.2.2 Dynamic Equilibrium

In this section, we characterize the dynamic equilibrium. We continue to exploit the equivalence between the distorted planning solution and the competitive equilibrium. We continue to assume an inelastic labor supply, for simplicity. Thus, we can write:

$$\max_{[c_t, \boldsymbol{x}_t, \chi_t]_{t=0}^{\infty}} U = \int_0^{\infty} \log(c) \times e^{-(\rho-n)t} dt$$

subject to the resource constraint

$$\dot{\chi} = f\left(y^G(\boldsymbol{x}, \chi), y^M(\boldsymbol{x}, \chi)\right) - \left(\delta + n + g^M\right)\chi - c,$$

where $f(y^G, y^M)$ is given by (9), to the exogenous law of motion of TFPs, $\dot{Z}_t^M / Z_t^M = g^M$, $\dot{Z}_t^{AM} / Z_t^{AM} = g^{AM}$, and $\dot{Z}_t^S / Z_t^S = g^S$. The problem is subject to a vector of initial conditions $(\chi_0, \boldsymbol{Z}_0) = (\bar{\chi}_0, \bar{\boldsymbol{Z}}_0)$.

It is useful to break down the analysis into two steps. First, we solve the static problem, involving constrained productive efficiency, as discussed in the previous section. This yields a set policy functions $\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t), \nu^S(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t)), \nu^{AM}(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t)), \nu^S(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t))$. Rearranging the set of static equilibrium conditions allows us to eliminate ν^{AM} and ν^S from the maximization problem. The present-value Hamiltonian can then be written as

$$H(c_t, \chi_t, \boldsymbol{Z}_t, \xi_t) = e^{-(\rho-n)t} \log(c_t) + \xi_t \left(\begin{array}{c} \eta(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t)) \times \left(\frac{\chi_t \kappa(\chi_t, \boldsymbol{Z}_t)}{\nu^M(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t))}\right)^{1-\alpha} \\ \times \left(\nu^M(\kappa(\chi_t, \boldsymbol{Z}_t), \upsilon(\chi_t, \boldsymbol{Z}_t))\right) - (\delta + n)\chi_t - c_t \end{array} \right),$$

where ξ_t is a dynamic Lagrange multiplier. The following proposition characterizes the equilibrium:

Proposition 2 *The dynamic competitive equilibrium is characterized by the following system of ordi-*

nary differential equations:

$$\begin{aligned}
\frac{\dot{c}_t}{c_t} &= \left(\eta\left(\kappa\left(\chi_t,\boldsymbol{Z}_t\right),\upsilon\left(\chi_t,\boldsymbol{Z}_t\right)\right)\right)^{\frac{1}{\varepsilon}}(1-\gamma)(1-\alpha)\times \qquad (17)\\
&\quad \left(\frac{\kappa\left(\chi_t,\boldsymbol{Z}_t\right)\chi_t}{Z_t^M\nu^M\left(\kappa\left(\chi_t,\boldsymbol{Z}_t\right),\upsilon\left(\chi_t,\boldsymbol{Z}_t\right)\right)}\right)^{-\alpha}-\delta-\rho\\
\dot{\chi}_t &= \eta\left(\kappa\left(\chi_t,\boldsymbol{Z}_t\right),\upsilon\left(\chi_t,\boldsymbol{Z}_t\right)\right)\times\left(\chi_t\kappa\left(\chi_t,\boldsymbol{Z}_t\right)\right)^{1-\alpha}\times \qquad (18)\\
&\quad \left(Z_t^M\nu^M\left(\kappa\left(\chi_t,\boldsymbol{Z}_t\right),\upsilon\left(\chi_t,\boldsymbol{Z}_t\right)\right)\right)^{\alpha}-(\delta+n)\chi_t-c_t\\
\frac{\dot{Z}_t^M}{Z_t^M} &= g^M,\ \frac{\dot{Z}_t^{AM}}{Z_t^{AM}}=g^{AM},\ \frac{\dot{Z}_t^S}{Z_t^S}=g^S
\end{aligned}$$

where $\eta(\kappa,\upsilon)$ is given by (14), $\nu^M(\kappa,\upsilon)$ satisfies (13), and $\kappa(\chi_t,\boldsymbol{Z}_t)$ and $\upsilon(\chi_t,\boldsymbol{Z}_t)$ are the static equilibrium policy functions. The solution is subject to a vector of initial conditions $(\chi_0,\boldsymbol{Z}_0)=\left(\bar{\chi}_0,\bar{\boldsymbol{Z}}_0\right)$ and a transversality condition (see the appendix).

Equation (17) is a standard Euler equation for consumption. For constant TFPs, the growth rate of consumption is decreasing in χ because the aggregate production function exhibit decreasing returns to capital. Exogenous technical change is the only source of growth in the long run.

It is useful to rewrite the equilibrium conditions in terms of an autonomous system of differential equations. To this aim, we differentiate with respect to time the set of static equilibrium (10), (11), (12), and (13). After rearranging terms, we obtain:

$$\frac{\dot{\kappa}_t}{\kappa_t} = (1-\kappa_t)\frac{\left(\begin{array}{c}\left(\alpha g^M-\beta g^{AM}+(\beta-\alpha)\frac{\dot{\chi}_t}{\chi_t}\right)+\\ \left(\frac{1}{\omega-1}-\frac{(\beta-\alpha)\left(1-\nu^M(\kappa_t,\upsilon_t)\right)}{1-\upsilon_t(1-\beta)}\right)\frac{\dot{\upsilon}_t}{\upsilon_t}\end{array}\right)}{\frac{1}{\varepsilon-1}+(\beta-\alpha)\left(\kappa_t-\nu^M(\kappa_t,\upsilon_t)\right)}, \qquad (19)$$

$$\frac{\dot{\upsilon}_t}{\upsilon_t} = \frac{(1-\upsilon_t)\left(\beta g^{AM}-g^S+(1-\beta)\left(\frac{\dot{\chi}_t}{\chi_t}-\frac{\dot{\kappa}_t}{\kappa_t}\frac{\kappa_t-\nu^M(\kappa_t,\upsilon_t)}{1-\kappa_t}\right)\right)}{\frac{1}{\omega-1}+\frac{(1-\upsilon_t)(1-\beta)(1-\nu^M(\kappa_t,\upsilon_t))}{1-\upsilon_t(1-\beta)}}. \qquad (20)$$

This dynamic system is defined up to a pair of initial conditions: $\kappa_0=\kappa(\chi_0,\boldsymbol{Z}_0)$ and $\upsilon_0=\upsilon(\chi_0,\boldsymbol{Z}_0)$ consistent with the static equilibrium conditions at time zero.

Corollary 1 *The dynamic competitive equilibrium is fully characterized by the solution to the autonomous system of ordinary differential equations (17)-(18)-(19)-(20) and the exogenous law of motion $\dot{Z}_t^M/Z_t^M=g^M$, after setting $\kappa(\chi_t,\boldsymbol{Z}_t)=\kappa_t$ and $\upsilon(\chi_t,\boldsymbol{Z}_t)=\upsilon_t$ for $t>0$, with initial conditions $\kappa(\chi_0,\boldsymbol{Z}_0)=\kappa\left(\bar{\chi}_0,\bar{\boldsymbol{Z}}_0\right)\equiv\kappa_0$ and $(\chi_0,\boldsymbol{Z}_0)=\upsilon\left(\bar{\chi}_0,\bar{\boldsymbol{Z}}_0\right)\equiv\upsilon_0$.*

Equations (19)-(20) allow us to eliminate Z_t^{AM} and Z_t^S from the dynamic system, while only retaining their initial levels and their growth rates. In other words, $\kappa_0=\kappa\left(\bar{\chi}_0,\bar{\boldsymbol{Z}}_0\right)$ and $\upsilon_0=\upsilon\left(\bar{\chi}_0,\bar{\boldsymbol{Z}}_0\right)$ are sufficient statistics. If κ_0 and υ_0 are set at the static equilibrium level at time zero, Equations (19)-(20) guarantee that κ_t and υ_t will also be consistent with the static equilibrium in all future periods.

This characterization allows us to prove conditions under which the economy converges to an asymptotic balanced growth path (ABGP) where the agriculture is fully modernized and the share of nonagriculture in total GDP is unity.

Proposition 3 *Let $k^M = \kappa\chi$ and $k^{AM} = (1-\kappa)\chi$. Then, there exists an Asymptotic Balanced Growth Path (ABGP) such that*

$$\begin{aligned}
\frac{\dot{c}_t}{c_t} &= \frac{\dot{\chi}_t}{\chi_t} = \frac{\dot{k}^M}{k^M} = g_M; \\
\kappa &= \nu^M = 1;\ \frac{\dot{\kappa}_t}{\kappa_t} = \frac{\dot{\upsilon}_t}{\upsilon_t} = 0; \\
\frac{\dot{k}^M}{k^M} &= g^M,\ \frac{\dot{k}^{AM}}{k^{AM}} = g^M - (\varepsilon-1)\beta\left(g^M - g^{AM}\right); \\
\frac{\dot{N}^M}{N^M} &= n,\ \frac{\dot{N}^{AM}}{N^{AM}} = n - (\varepsilon-1)\beta\left(g^M - g^{AM}\right); \\
\frac{\dot{N}^S}{N^S} &= \frac{\dot{N}^{AM}}{N^{AM}} - (\omega-1)\left[\left(g^{AM} - g^S\right) + (1-\beta)\left(g^M - g^{AM}\right)\right].
\end{aligned}$$

Along the ABGP

$$\left(\frac{c}{\chi}\right)^* = \left(\frac{g^M + \delta + \rho}{1-\alpha}\right) - \left(g^M + \delta + n\right), \tag{21}$$

$$\left(\frac{\chi}{Z^M}\right)^* = \left(\frac{(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}(1-\alpha)}{g^M + \delta + \rho}\right)^{\frac{1}{\alpha}}. \tag{22}$$

If $\varepsilon > 1$, $\omega > 1$, $\beta > \alpha$, and $g^M \geq g^{AM} \geq g^S$, then, the ABGP is asymptotically stable, i.e., given a vector of initial conditions $\left(\chi_0/Z_0^M, \kappa_0, \upsilon_0\right)$ close to the ABGP, the economy converges to the ABGP.

The ABGP features a vanishing perfectly modernized agriculture. This result hinges on two assumptions. First, the elasticity of substitution between nonagriculture and agriculture is larger than unity. Second, technical progress is at least as high in nonagriculture as in agriculture. Note that as long as ε is not too large, β is not too large and the TFP growth gap between nonagriculture and modern agriculture is not too high, capital accumulation in modern agriculture remains positive in the ABGP (i.e., k^{AM} grows at a positive rate). In addition, the ABGP features modernization of agriculture: the traditional sector vanishes both as GDP share and employment share of total agriculture. This is due to the combination of a high elasticity of substitution ($\omega > 1$) as long as technical progress is not faster in the traditional sector than in modern agriculture.

Our theory bears predictions about the labor income shares and the productivity gap. To highlight them, we move from the planner's allocation to its decentralized counterpart. Denote by $LIS^j \equiv W^j H^j / P^j Y^j$ for $j \in \{G, M\}$ the labor income share in sector j. The labor share in nonagriculture is constant, owing to the Cobb-Douglas production function. The labor share in agriculture declines as long as the (labor-intensive) traditional sector shrinks, consistent with Figure 1 above. More precisely, $LIS^G = \beta\upsilon + 1 - \upsilon$, implying that the labor share in agriculture declines from unity – when capital is very low and the agriculture is dominated by the traditional sector – to β when agriculture is fully modernized.[14] The theory also predicts a declining productivity gap in line with the data in Figures 1 and 2. The two predictions are two sides of the same coin.[15]

3.2.3 Equilibrium in the Lewis Model

A particular case for which a global characterization of the equilibrium dynamics is available is one where the output of traditional and modern agriculture are perfect substitutes (i.e., where $\omega \to \infty$), implying that labor productivity and the wage are fixed in terms of the agricultural product. This model is interesting for its close relationship with the seminal contribution of Lewis (1954). In addition, it illustrates why the dynamics of κ and υ may be nonmonotone. Intuitively, when ω is large, labor in the traditional sector is a close substitute of capital in modern agriculture. When capital is scarce, it is then efficient to allocate its entire stock to nonagriculture and defer the modernization of agriculture to a later stage in which capital is more abundant.

The technical analysis is available in the online appendix. Here, we zoom on the main qualitative findings. The dynamic equilibrium evolves through three stages. In the first stage, (*Early Lewis* stage), capital is very scarce, all agricultural production takes place in the traditional sector ($\upsilon = 0$) and all capital is allocated to the manufacturing sector ($\kappa = 1$). Capital accumulation brings about a steady increase in the relative price of agricultural goods and a growth in the real wage. The interest rate decreases over time.

At some point, the increase in the capital stock and the growing price of the agricultural good make it efficient to activate modern agriculture allocating part of the capital stock to this subsector. We enter then the *Advanced Lewis* stage. During this stage the share of capital in modern agriculture (κ) increases over time. Employment increases in both nonagriculture and modern agriculture and declines in the traditional sector (thus, υ increases). The sectoral capital-labor ratios are constant over time, and so are factor prices.

Finally, when the labor force reserve in traditional agriculture is exhausted, the economy enters the third stage (*Neoclassical* stage). In this stage, all the agricultural production takes place in modern farms using capital ($\upsilon = 1$). Since $\varepsilon > 1$ and nonagriculture is more capital intensive than agriculture, the output share of nonagriculture keeps growing. There is positive wage growth and a declining interest rate.

Figure 6 summarizes the equilibrium dynamics during economic transition, assuming no technical progress, for simplicity. Each panel has the aggregate capital labor ratio χ on the horizontal axis. Panel a plots the share of labor in each sector. The labor share in the traditional sector (ν^S) starts high

[14] One could obtain a declining labor share by assuming an aggregate CES production function in agriculture.with a high elasticity of substitution between capital and labor, like in Alvarez-Cuadrado et al. (2017). However, such alternative model would feature, counterfactually, an ever declining labor share that would converge to zero in the long run. In our model, like in the data, the labor share in agriculture declines but remains bounded away of zero.

[15] In an undistorted economy, $APL^M/APL^A = \left(LIS^M/LIS^A\right)^{-1}$ by definition. In the distorted economy, the labor tax in nonagriculture opens a wedge between the marginal product of labor in the two sectors that is reflected in the equilibrium prices. Thus, one can show that $APL^M/APL^A = \left((1-\tau) \times LIS^M/LIS^A\right)^{-1}$.

and declines with χ in both the Early and Advanced Lewis stages. The labor share in nonagriculture increases throughout the entire transition. The labor share in modern agriculture is nonmonotone: it is zero in the Early Lewis stage, takes off in the Advanced Lewis stage, and declines again in the neoclassical stage.

Panel b plots the factor price dynamics. During the Early Lewis stage, the interest rate falls and wages increase with capital accumulation. Wages and interest rates are flat during the Advanced Lewis stage. Eventually, the interest rate resumes its fall and wages resume their increase during the Neoclassical stage.

Panel c plots the productivity gap, which tracks the dynamics of υ. The gap is constant during the Early Lewis stage, when $\upsilon = 0$. It falls in χ during the Advanced Lewis stage, when the traditional sector declines as a share of agricultural output. It is constant again in the Neoclassical stage, when $\upsilon = 1$.

Finally, panel d plots the relative capital-output ratio $\frac{K^G}{P^G Y^G} / \frac{K^M}{P^M Y^M}$. The ratio stays at zero during the Early Lewis stage. It increases during the Advanced Lewis stage. Eventually, it becomes constant during the Neoclassical stage.*

In summary, the model with $\omega \to \infty$ has three well-distinct stages.

3.3 Discussion of the Main Assumptions

In this section, we discuss the plausibility of the key assumptions. The assumptions that $g^M \geq g^{AM} \geq g^S$ and that $\beta > \alpha$ are in line with the common wisdom that manufacturing is the most dynamic sector of the economy and that capital-intensive agricultural activities experience more technical progress (e.g., the invention of new machines) than traditional labor-intensive activities. Our estimates confirm this wisdom.

The assumption that $\varepsilon > 1$ deserves further scrutiny. We proceed in two steps. First, we consider a class of two-sector model (that encompasses ours) where agricultural and nonagricultural goods enter a CES aggregate production function, as in Equation 1. Assuming the price of the aggregate consumption good Y to be the numeraire, one obtains the following standard isoelastic demand condition:

$$\frac{P_t^G}{P_t^M} = \frac{\gamma}{1-\gamma} \left(\frac{Y_t^G}{Y_t^M} \right)^{-\frac{1}{\varepsilon}}.$$

Rearranging terms, and taking log on both sides yields

$$\ln \left(\frac{P_t^G Y_t^G}{P_t^M Y_t^M} \right) = \ln \left(\frac{\gamma}{1-\gamma} \right) + \frac{\varepsilon - 1}{\varepsilon} \ln \left(\frac{Y_t^G}{Y_t^M} \right).$$

We take this equilibrium condition to the data. Since, empirically, one cannot reject the hypothesis that the logarithm of the expenditure and output ratio feature unit roots, it is appropriate to take it

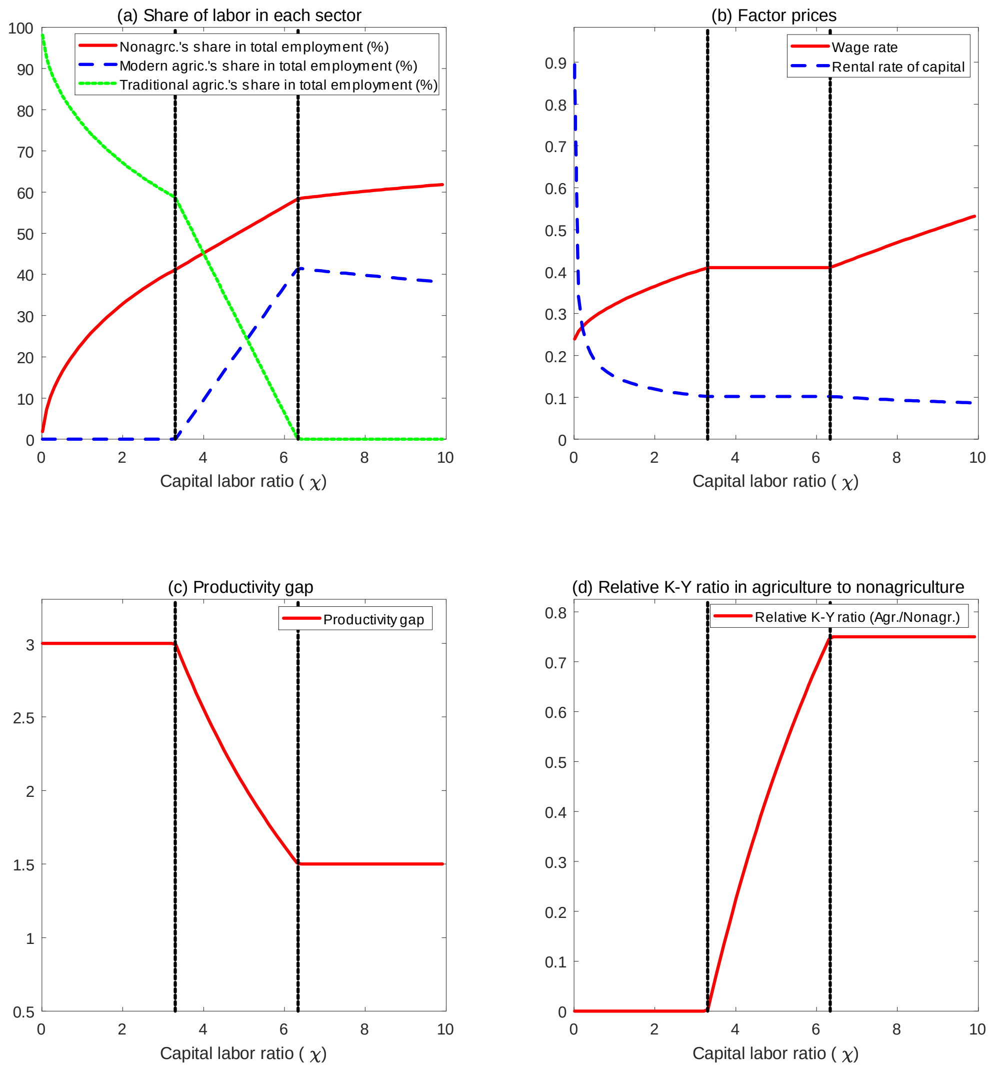

Figure 6: The figure illustrates the allocations and prices as a function of capital per worker, χ, in a version of the Lewis model ($\omega \to \infty$ and no technical change).

in first differences.

$$\Delta \ln \left(\frac{P_t^G Y_t^G}{P_t^M Y_t^M} \right) = \frac{\varepsilon - 1}{\varepsilon} \Delta \ln \left(\frac{Y_t^G}{Y_t^M} \right) \tag{23}$$

The model predicts a positive correlation between the first difference of the expenditure ratio and that of the real output ratio if $\varepsilon > 1$. The correlation is instead negative if $\varepsilon < 1$. Figure 1 (panel d) and Figure 13 in the appendix provide scatter plots of the five-year changes for the US and China, respectively. In both cases, the correlation is strongly positive, indicating that $\varepsilon > 1$.[16]

Second, we consider a class of models where there is also a service sector. We are motivated by the finding of Herrendorf et al. (2013) who estimate a three-sector CES production function where the elasticity of substitution between agricultural goods, manufacturing goods, and services is assumed to be the same. Using a value added approach, they estimate a very low elasticity of substitution between manufacturing, services and agriculture in the US, which might suggest a violation of our assumption. To compare our results with theirs, we extend our model by assuming that nonagriculture comprises two sectors: manufacturing and services. More formally, we assume that

$$Y^M = \left[\hat{\gamma} \left(Y^{Manuf} \right)^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}} + (1 - \hat{\gamma}) \left(Y^{Serv} \right)^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}} \right]^{\frac{\epsilon_{ms}}{\epsilon_{ms}-1}}$$

where the superscripts $Manuf$ and $Serv$ denote manufacturing and services, respectively, and the parameter ϵ_{ms} represents the elasticity of substitution between services and manufacturing in production of the nonagricultural good. Then, we estimate our model using the same sectoral value added data (and the same estimation procedure) used by Herrendorf et al. (2013). More formally, we estimate the following production function:

$$Y = \left[\begin{array}{c} \gamma \left(\left[\hat{\gamma} \left(Y^{Manuf} \right)^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}} + (1 - \hat{\gamma}) \left(Y^{Serv} \right)^{\frac{\epsilon_{ms}-1}{\epsilon_{ms}}} \right]^{\frac{\epsilon_{ms}}{\epsilon_{ms}-1}} \right)^{\frac{\varepsilon-1}{\varepsilon}} \\ + (1 - \gamma) \left(Y^G \right)^{\frac{\varepsilon-1}{\varepsilon}} \end{array} \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

This specification is similar to that used in a different context by (Krusell, Ohanian, Rios-Rull, and Violante 2000). Note that the generalized model nests the specification of Herrendorf et al. (2013) as the particular case in which $\epsilon_{ms} = \varepsilon$. Since their model allows for nonhomothetic preferences with a Stone-Geary demand system, we also generalize preferences in the same direction to make sure that the two models are perfectly nested.

We estimate ϵ_{ms} to be close to zero, indicating a high degree of complementarity between manufacturing and services, consistent with Herrendorf et al. (2013). In contrast, we obtain an estimate of the agriculture-nonagriculture elasticity $\varepsilon = 2.36$, significantly larger than unity. Thus, our model strongly rejects the null hypothesis that $\epsilon_{ms} = \varepsilon$, confirming a high elasticity of substitution between

[16] Simple OLS regressions on US data in line with equation (23), imply estimates of ε of 3.3 and 2.0 based on 5-year and annual changes, respectively. For China the corresponding estimates are $\varepsilon = 4.1$ and $\varepsilon = 6.8$ for 5-year and annual, respectively (sample period: 1985-2012). The total changes over the entire 1985-2012 period imply $\varepsilon = 4.2$. In all cases, the data strongly reject the hypothesis that agricultural and nonagricultural goods are complements.

agricultural and nonagricultural goods. We find a negligible nonhomotheticity preference parameter for the agricultural good, and a significant one for the service sector.

Since the focal point of our theory is the decline and modernization of the agricultural sector, in the rest of the paper we continue to ignore the distinction between manufacturing and services.

4 Estimating the Model

In this section, we estimate our model. We generalize the model in three dimensions. First, we use discrete time. Second, we introduce an endogenous labor supply choice as in Equation (6). Third, we introduce land as a fixed factor in modern agriculture.[17] We could also add land in the traditional sector. However, in the spirit of Lewis (1954), we want to retain the property of a traditional sector working as a labor force reserve at a constant marginal cost. Formally, this linearity is important as it delivers a high elasticity of substitution between capital and labor in agriculture at early stages of development. A complete description of the discrete-time model is found in provided in the technical appendix online.

Next, we go through the following two steps:

1. We estimate the deterministic model with constant productivity growth in each sector. To this end, we start by calibrating some parameters externally. Then, we estimate the structural parameters and initial conditions to match moments of the structural change of China between 1985 and 2012.

2. We introduce productivity shocks. We estimate stochastic processes for the three TFP shocks and simulate the model. Then, we evaluate the ability of the stochastic model to account for the business cycle properties of China. Finally, we use the estimated model to forecast how the Chinese business cycle will evolve as economic development progresses further. The results can be compared with the cross-country stylized facts documented in Section 2.[18]

Parameters calibrated externally: On the preference side, we assume a 4% annual time discount rate. Note that log preferences in Equation (6) ensure that there is no trend in labor supply on the ABGP. θ is chosen so that in the long run agents work one third of their time. We set the annual population growth rate (n) to 1.5% following Acemoglu and Guerrieri (2008). This parameter has no significant impact on the results. Capital is assumed to depreciate at a standard 5% annual rate.

Estimated parameters based on nonagricultural production data: We set $\alpha = 0.5$ to match the labor-income share in the nonagricultural sector in China (see Bai and Qian (2010)). Then, we estimate g^M using standard growth accounting based on a Cobb Douglas production function – as

[17] We assume $Y^{AM} = \left(K^{AM}\right)^{1-\beta-\beta_T} \left(Z^{AM}H^{AM}\right)^{\beta} T^{\beta_T}$, where T is land and β_T is the output elasticity of land.

[18] Alternatively, we could have used time-series data for the US to estimate the model. We prefer to use the data for China because the available time-series for the US cover a period when the employment in agriculture is already quite low (20.3% of the total US employment in 1929) and the subsistence activity captured by our traditional agriculture sector is arguably tiny. In contrast, China has a large and declining share of employment in agriculture ranging from 62.4% in 1985 to 33.6% in 2012.

in the model – to match the trend of real GDP in the nonagricultural sector of China between 1985 to 2012. This yields $g^M = 6.5\%$.

Parameters estimated based on endogenous moments: We estimate the remaining parameters using the Simulated Method of Moments. The parameters we estimate are

$$\left\{\varepsilon, \omega, \tau, \theta\, \gamma, \varsigma, \beta, \beta_T, g^{AM}, g^S, Z^M_{1985}, Z^{AM}_{1985}, Z^S_{1985}\right\}.$$

We normalize $Y_{1985} = 1$ and target (the natural logarithms of) the following empirical annual observations from 1985 to 2012 in China:[19] (i) the share of agricultural employment in total employment; (ii) the share of capital in agriculture relative to the total capital stock; (iii) the ratio of real output in agriculture to total GDP; (iv) the relative value added share of agriculture, evaluated at current prices (i.e., the expenditure share of agricultural goods); (v) the aggregate GDP growth; (vi) the initial and final aggregate capital-output ratio; and (vii) the 1985-2012 change in the productivity gap between agriculture and nonagriculture, adjusted for rural-urban wage differences. We calculate this gap as the ratio of labor productivity in nonagriculture to agriculture times the ratio of wages in agriculture to nonagriculture. Recall that in the model the gap equals $(1 - \upsilon (1 - \beta)) / \alpha$, i.e., the ratio of the labor-income share in agriculture to the labor-income share in nonagriculture.[20]

This procedure yields 143 moment conditions – 28 for each of the annual moments (i)-(v) plus moments (vi)-(vii). The estimation is based on equal weights on the annual moments (i)-(v) and equivalent weights for moments (vi) and (vii) (i.e., 14 times the weight on the annual moments for each of the two K/Y ratios, and 28 times for the relative deviation of the output gap).

We measure real output in agriculture and nonagriculture following the same approach as China's National Bureau of Statistics. The National Bureau of Statistics measures real growth in agricultural and nonagricultural output using prices in a base year. The base year was updated in 1980, 1990, 2000, 2005, and 2010. The levels of real sectoral output are chained when a new base year is adopted. Therefore, in the year of change in base year prices the real levels are by construction invariant to using the new or the old set of prices. This approach is similar to that pursued in the U.S. National Income and Product Accounts before 1996.[21][22]

[19] We choose the year 1985 as our initial period because it was a turning point in internal migration policy. In earlier years restrictions on labor mobility between rural and urban areas were very severe. These restrictions were relaxed in January 1985, following the issuance of the "Ten Policies on Further Active Rural Economy" from the CPC Central Committee and the State Council.

[20] We could alternatively have targeted the empirical labor-income shares directly. However, such data are available only up until 2003. From 2004 onwards, the labor income shares are not comparable to their counterparts in the pre-2004 period. See Bai and Qian (2010) for details. As it turns out, the overall change in the ratio of labor income shares is comparable to the change in the labor productivity gap: over the 1985-2003 period these ratios fall by 13 and 14 log points, respectively.

[21] To be consistent with the empirical data, we start the model in 1980 and base the 1985-1990 growth rates using 1980 as the base year, then update and chain the output levels using 1990 relative prices, etc.

[22] As is well known, this approach introduces a bias when the relative prices change substantially over time. In Figure 14 in the appendix we quantify the magnitude of this bias in China by plotting the series measured in the same way as does the NBS against the exact measures of real sectoral output in the model. As is clear from the figure, the average bias is 0.2 percentage points: the real GDP growth is overstated by 0.5 percentage points over the sample period 1985-1990. Then it decreases over time. In contrast, the bias would be negligible if real growth were calculated using chain weighting.

Nonhomothetic preferences: We also estimate a model with non-homothetic Stone-Geary preferences, where the agricultural good is a necessity. This is for comparability with earlier studies which point at an important role of income effects (see, e.g., Boppart (2014)). To this end, we reinterpret the goods M and G as final goods that are allocated to consumption (denoted C^M and C^G) and investment (denoted X^M and X^G), where $Y^G = C^G + X^G = Nc^G + X^G$ and $Y^M = C^M + X^M = Nc^M + X^M$. More formally, we replace the per-capita utility function in equation (6) by

$$u\left(c^G, c^M, h\right) = \theta \log\left(\left[\gamma\left(c^G - \bar{c}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(c^M\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right) + (1-\theta)\log(1-h),$$

while the investment good continues to be a CES aggregation of the agricultural and nonagricultural goods,

$$\left[\gamma\left(X^G\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(X^M\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} = I.$$

The aggregate resource constraint (7) can be written as

$$\dot{K} = I - \delta K - \tau \bar{W} H^M + Tr. \tag{24}$$

Finally, we denote the total expenditure by $Y = P^G Y^G + P^M Y^M$, the total consumption expenditure as $C = P^G C^G + P^M C^M$, and the total investment expenditure as $I = P^G X^G + P^M X^M$. The constant $\bar{c}$ is added to the list of parameters we estimate. The baseline model is a restricted version with $\bar{c} = 0$.

Table 1 summarizes the results of the estimation.

Estimation results: The estimated elasticities of substitution ε and ω are in all cases significantly larger than unity, consistent with the assumption of Proposition 3 (see also Section 3.3 above). In the homothetic (CRRA) economy ε is slightly lower than that implied by estimating equation (23) on Chinese data (where we obtain $\varepsilon \geq 4$ with the exact estimate depending on the data frequency).

With Stone-Geary preferences (second column), the estimated ε falls. This is not surprising. With homothetic preferences there is only a substitution effect: nonagricultural products turn relatively cheaper due to capital accumulation and faster technical progress causing a decline in the relative demand of agricultural goods. To match the declining expenditure share on agricultural good observed in the data, the price elasticity of the relative demand must be sufficiently high (recall that if $\varepsilon = 1$ the expenditure shares would not change at all). In contrast, with Stone-Geary preferences, the expenditure share in agriculture falls also because of an income effect. Therefore, the Stone Geary model has a potential of lowering the estimate of ε (even, possibly, turning it smaller than unity). However, the estimated subsistence level $\bar{c}$ turns out to be very small, corresponding to 5% of GDP in 1985. Therefore, the estimated ε falls only marginally, and the remaining parameters are also very similar to the case with homothetic preferences.

	Parameters Set Exogenously	CRRA	StGeary	NoPrGap
n	popul. growth rate (15+	1.5%	1.5%	1.5%
δ	capital deprec. rate	5%	5%	5%
$(1+\rho)^{-1}$	discount factor	0.96	0.96	0.96
θ	pref. weight on consumption	0.73	0.73	0.71
	Parameters Estimated Within Model			
	Targeting Data on Nonagr. Prod.	CRRA	StGeary	NoPrGap
α	labor share in nonagr.	0.50	0.50	0.5
g_M	nonagr. TFP growth rate	6.5%	6.5%	6.5%
Z^M_{1985}	initial TFP level in nonagr.	4.33	4.45	3.42
	Targeting Empirical Moments	CRRA	StGeary	NoPrGap
$\bar{c}$	Subsist. level in food cons.	–	0.05	0.05
ε	ES btw agric. and nonagric. cons.	3.60	3.36	4.00
ω	ES btw modern and trad. agr.	9.00	9.00	8.22
γ	weight on agric. goods	0.61	0.60	0.54
ς	weight on modern-agr. output	0.40	0.39	0.50
$1-\beta-\beta^{LAND}$	capital's income share in modern agr.	0.14	0.13	0.11
β	labor's income share in modern-agric.	0.61	0.60	0.68
τ	labor wedge	0.76	0.75	0.73
g^{AM}	TFP growth rate in modern-agr.	6.1%	6.1%	5.9%
g^S	TFP growth rate in trad. sector	0.9%	0.9%	1.0%
Z^S_{1985}	initial TFP level in trad. agr.	1.23	1.18	1.35
Z^{AM}_{1985}	initial TFP level in modern-agr.	2.26	2.25	2.42

Table 1: Calibration Results

In the third column, we show the estimation results when we do not impose the productivity gap as a target. Since the model without traditional economy cannot generate a declining productivity gap by construction, one might worry that the estimation results hinge on the inclusion of the productivity gap as a targeted moment. However, the results are highly robust, being quantitatively very similar to those in the second column. In particular, the data demand a large traditional sector ($\zeta = 0.5$) and estimated elasticities that are very similar to the baseline case. The nonhomotheticity continues to be small. In other words, the estimated model predicts a declining productivity gap over the process of structural change even though this is not a target moment.

The estimated elasticity of substitution between modern and traditional agriculture ω is in all cases very large. The productivity growth rate is very high in both manufacturing and modern agriculture, reflecting the high growth rate of the Chinese economy. To avoid the unrealistic implication of a long run annual growth rate in excess of 6%, we assume that the productivity growth rate in both nonagriculture and modern agriculture gradually (linearly) declines to 1.8% over the 2012-2112 period. Thereafter, the economy grows at an annual 1.8% growth rate. This assumption is for the sake of realism and has negligible effects on the quantitative results. Finally, we note that the estimated productivity growth in modern and traditional agriculture satisfy the (sufficient) conditions for convergence to the ABGP set forth in Proposition 3.

4.1 Accounting for Structural Change In China

In this section, we show that the benchmark model fits well the data along salient dimensions of the process of structural change. Figures 7 and 8 plot the time series for the seven empirical targets against the implications of the estimated model (with and without homothetic preferences). Figure 7shows that, in line with the data, the model predicts that the shares of employment, capital, value added, and expenditure in the agricultural sector relative to total should be falling over time.

Figure 8 displays two aggregate variables, an index of the log GDP per capita (index) and the capital-output ratio, and the productivity gap measured by the output per worker in agriculture relative to total output per worker. It shows in addition the demeaned log of the productivity gap. The estimated model captures well the trend in GDP and capital-output ratio. It also captures the falling productivity gap, although the data feature large swings.

Finally, Figure 9 shows the demise of the traditional agriculture. The employment share of the modern sector in total agriculture increases from about 25% in 1985 to almost one in 2012. The output share (corresponding to the variable v in the model) exhibits a similar behavior. Note that in the data we do not observe a distinction between traditional and modern agriculture, and thus the transition from traditional to modern agriculture is identified from the change in the productivity gap (the bottom panel of Figure 8). The decline of the traditional sector is due to both relative TFP growth and fast capital accumulation.

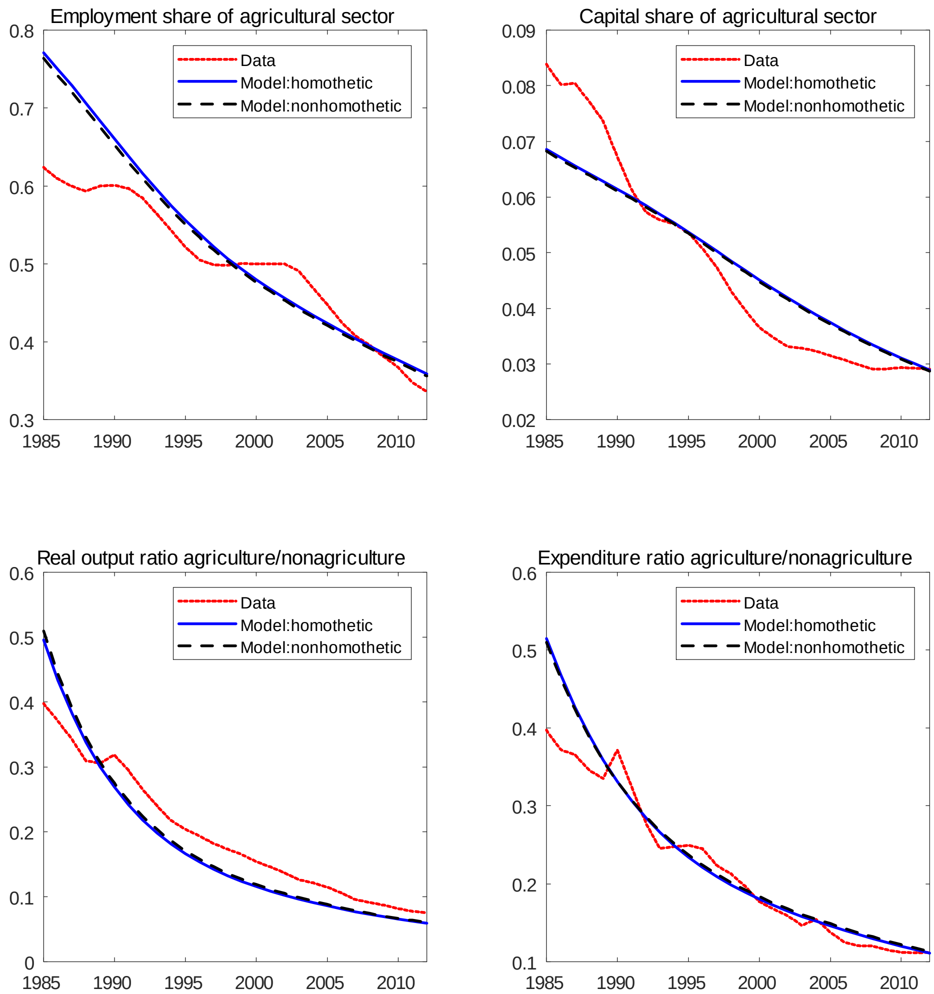

Figure 7: Structural change in model versus targeted empirical moments. The homothetic model is solid lines while the nonhomothetic model is dashed lines. The top left panel displays agricultural employment as a share of total employment. The top right panel displays the share of aggregate capital invested in agriculture. The bottom left panel displays the agricultural value added as a share of aggregate GDP at current prices. The bottom right displays the expenditure on agricultural goods as a share of aggregate GDP.

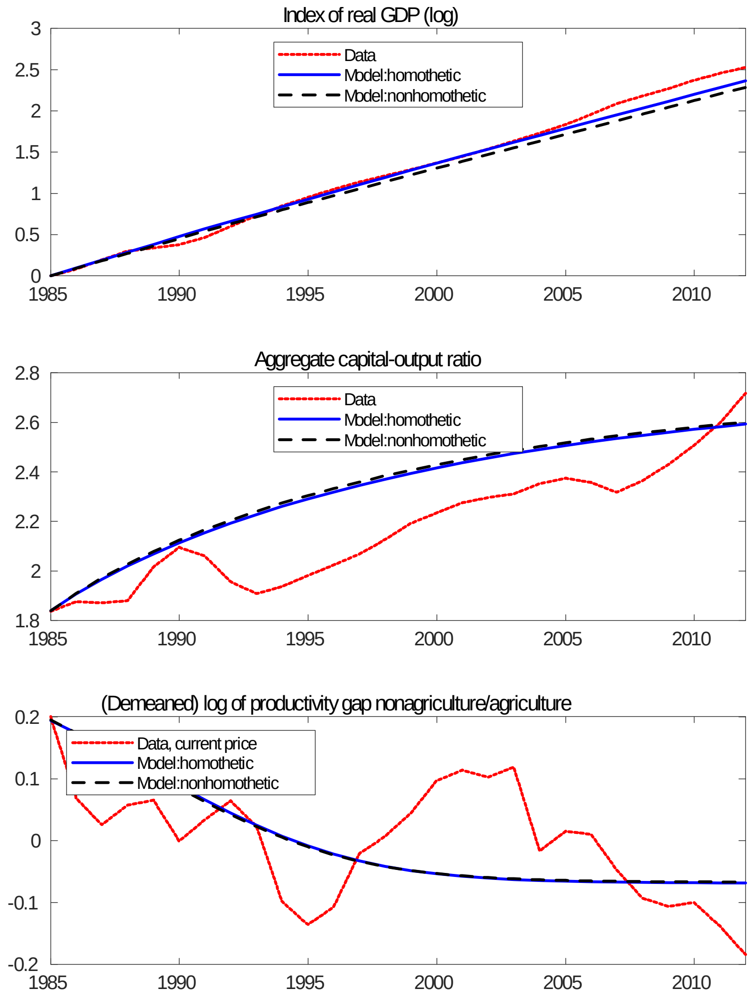

Figure 8: Structural change in model versus targeted empirical moments. Solid blue lines: homothetic model. Dashed blue lines: nonhomothetic model is dahsed lines. The top panel displays an index of real GDP (logarithm). The middle panel displays the capital-output ratio. The bottom panel displays changes the labor productivity gap.

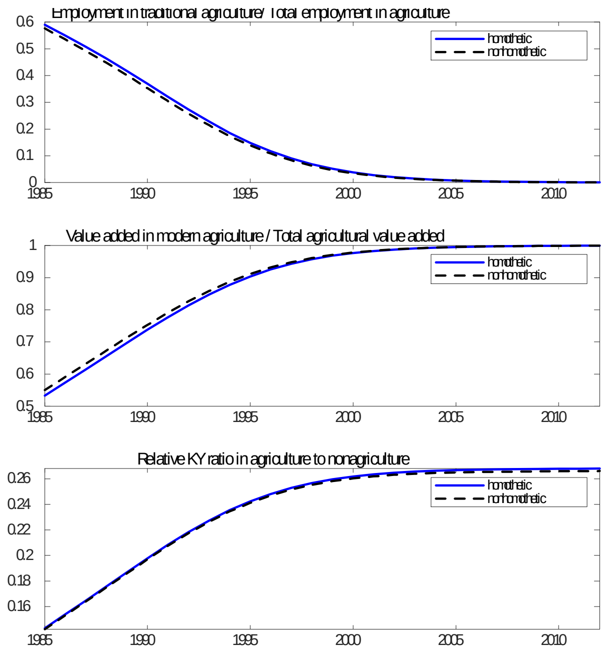

Figure 9: Structural change, according to the models. Dashed blue lines: homothetic model. Dotted black lines: nonhomothetic model. Panel A displays the share of agricultural employment working in traditional sector. Panel B displays the evolution of υ; value added in modern agriculture as a share of total agricultural value added, in current prices.

5 Business Cycle Analysis

In this section, we introduce uncertainty in the form of shocks to the three TFPs.

5.1 Estimating the stochastic process for TFP

The estimation of the process for technology shocks presents a measurement problem because, as mentioned above, we do not have separate measurement of traditional and modern agriculture – we only observe the total. To overcome this problem we exploit two equilibrium conditions in the theory, namely that the marginal product of capital is equated across manufacturing and modern agriculture, and that the marginal product of labor is the same in traditional and modern agricultural. Combining these conditions with direct annual observations of capital, labor, and value added in agriculture and non-agriculture yields a uniquely identified sequence $\{Z_t^M, Z_t^{AM}, Z_t^S\}_{t=1985}^{2012}$.

We decompose the TFP level into a trend and a cyclical part, assuming that the trend is deterministic. to this aim, define $z_t^j \equiv \log\left(Z_t^j\right) - \log\left(\bar{Z}_t^j\right)$, where $\bar{Z}_t^j = \bar{Z}_0^j\left(1+g^j\right)^t$ is the deterministic trend for $j \in \{M, AM, S\}$. We assume that the stochastic process is VAR(1), such that

$$\begin{bmatrix} z_{t+1}^M \\ z_{t+1}^{AM} \\ z_{t+1}^S \end{bmatrix} = \begin{bmatrix} \phi^M & 0 & 0 \\ 0 & \phi^{AM} & 0 \\ 0 & 0 & \phi^S \end{bmatrix} \cdot \begin{bmatrix} z_t^M \\ z_t^{AM} \\ z_t^S \end{bmatrix} + \epsilon_t, \tag{25}$$

where $\epsilon_t = A \cdot \tilde{\epsilon}_t$, A is a 3×3 matrix and $\tilde{\epsilon}_t$ denotes a vector of orthogonal i.i.d. shocks. Off-diagonal elements in the matrix A will capture correlation between the three TFP innovations ϵ_t^j.

The estimated parameters are: $\hat{\phi}^M = 0.63$, $\hat{\phi}^{AM} = 0.9$, and $\hat{\phi}^S = 0.42$, all significant at 1% level.[23] The persistence of the three shocks on a quarterly basis, is 0.89, 0.97, and 0.81, respectively. The estimated correlation matrix is

$$Corr\,(A) = \begin{bmatrix} 1 & 0.42^{**} & -0.48^{**} \\ 0.42^{**} & 1 & -0.15 \\ -0.48^{**} & -0.15 & 1 \end{bmatrix}.$$

Note that innovations to nonagriculture and modern agriculture are positively correlated, whereas innovations to nonagriculture and traditional agriculture are negatively correlated. Finally, the implied standard deviation of the innovations in (25) are given by $\sigma\left(\epsilon_t^M\right) = 0.042$, $\sigma\left(\epsilon_t^{AM}\right) = 0.036$, and $\sigma\left(\epsilon_t^S\right) = 0.053$.

We assume that the realization of the stochastic productivity shock is observed after capital is installed in each sector. Therefore, capital can only adjust in the following period.

5.2 Simulating the stochastic economy

We simulate the model using the estimates in Table 1 that were obtained to match the structural change of China, assuming that the economy starts with the initial condition of 1985. We augment the model with the stochastic process for TFPs discussed above.

5.2.1 Method

Solving the model is a nontrivial task. We cannot approximate the economy around a balanced growth path. Instead, we proceed as follows.[24] We first solve for a stochastic one-sector version of our model without agriculture, using standard methods. We assume that our benchmark three-sector model converges to this one-sector model after 250 periods. Proposition 3 shows that the ABGP of our benchmark economy indeed converges to the balanced growth path of this one-sector model. We then solve the economy recursively for each time period, back to period $t = 0$.

The stochastic process for $z_t \equiv [z_t^M, z_t^{AM}, z_t^S]'$ is approximated by a 27-state Markov chain with three realizations for each shock, using a standard Tauchen method (Tauchen 1986). There are two continuous state variables, κ and χ. We approximate the next-period decision rules for (c_{t+1}, h_{t+1}) with piecewise linear functions over the state variable $\left(\kappa_{t+1}, \chi_{t+1}, z_{t+1}\right)$. We solve for the optimal decisions

[23] Note that the autocorrelation matrix has no off-diagonal elements. If we allow for off-diagonal elements in this matrix, we obtain:

$$\begin{aligned} \log z_{t+1}^M &= 0.72^{***} \log z_t^M - 0.07 \log z_t^A + 0.12 \log z_t^S + \varepsilon_{t+1}^M \\ \log z_{t+1}^A &= -0.03 \log z_t^M + 0.906^{***} \log z_t^A - 0.05 \log z_t^S + \varepsilon_{t+1}^A \\ \log z_{t+1}^S &= 0.10 \log z_t^M + 0.02 \log z_t^A + 0.438^{***} \log z_t^S + \varepsilon_{t+1}^S \end{aligned}$$

Since all off-diagonal coefficients are insignificant, we set them to zero and estimate a restricted VAR.

[24] A more detailed description of the algorithm can be found in the online appendix.

on a grid with 75 grid points for κ and χ. The location of this grid is adjusted over time. In period t the grid for χ_t is distributed from $0.90\bar{\chi}_t$ to $1.1\bar{\chi}_t$ where $\bar{\chi}_t$ denotes the deterministic trend. Similarly, the grid for κ_t distributed from $\bar{\kappa}_t - 0.025$ to $\bar{\kappa}_t + 0.025$. We verify that realized optimal (κ_t, χ_t) never exceeds these brackets. Given decision rules for (c_{t+1}, h_{t+1}), the optimal control variables follow from the state and the optimality conditions. In particular, we solve for current-period optimal choices for $\left(\chi_{t+1}, \kappa_{t+1}, h_t, c_t, \upsilon_t, \nu_t^M, \nu_t^{AM}, \nu_t^S\right)$. The decision rules for χ_{t+1}, κ_{t+1}, h_t, c_t, ν_t^M, υ_t are approximated by piecewise linear functions over (κ_t, χ_t, z_t) and the decision rules for ν_t^{AM} and ν_t^S follow directly from the optimality conditions once the values for κ_t and ν_t^M are determined.

We start each sample economy off with an initial value for $\hat{\chi}_{1980}$ such that the deterministic model reaches the empirical value for $\hat{\chi}_{1985}$ in 1985. Moreover, κ_{1985} is set to the 1985 value on the ABGP. We then simulate 1000 versions of the economy starting in 1980 and calculate statistics from 1985 to 2185.

5.2.2 Results

Both the empirical data and the simulated data are filtered to remove trends. We report results based on HP-filtered data (Table 2 in the main text) and data in First Differences (Tables 4 in the online appendix). The upper panel of each table presents the business cycle statistics for China 1985-2012. The lower panel reports the same statistics for the simulated economies. Tables 2 reports results based on HP-filter (with an HP parameter of 6.25), whereas Tables 4 reports results based on first differences. In the discussion we focus on the HP-filtered data. As is clear from the tables, the empirical and theoretical business-cycle properties are almost invariant to the choice of filter. Figure 10 shows the impulse response functions for employment, value added, and the productivity gap to sectoral TFP shocks. Although the model has correlated shocks, here we illustrate the dynamics to shocks individually, holding the other TFP shocks at the zero level.

Productivity gap. The model predicts correlations between the productivity gap and sectoral labor supply that are qualitatively in line with the empirical observation. The productivity gap is countercyclical; it decreases when the employment in nonagriculture is high and decreases when the employment in modern agriculture is high. However, the volatility of the fluctuations in the productivity gap is only about one third of its empirical counterpart. Intuitively, a positive productivity shock in nonagriculture attracts workers from agriculture and inducing modernization in the agricultural sector. Interestingly, a cyclical boom works like a temporary acceleration of the process of structural change discussed in the previous section (cf. Lemma 2).

The impulse response functions in Figure 10 illustrate the mechanism. An increase in Z^M (upper panels) triggers a shift of labor and capital (the latter is not shown in the figure) to nonagriculture. In the period when the shock occurs, only labor adjusts. Subsequently, capital accumulates in the more productive sector, sourced from both modern agriculture and net capital accumulation. Labor is sourced from traditional agriculture and, to a lesser extent, from modern agriculture. Thus, labor supply in agriculture falls simultaneously with modernization of agriculture: both the average capital intensity and the average labor productivity increase in agriculture. This in turn causes the productivity

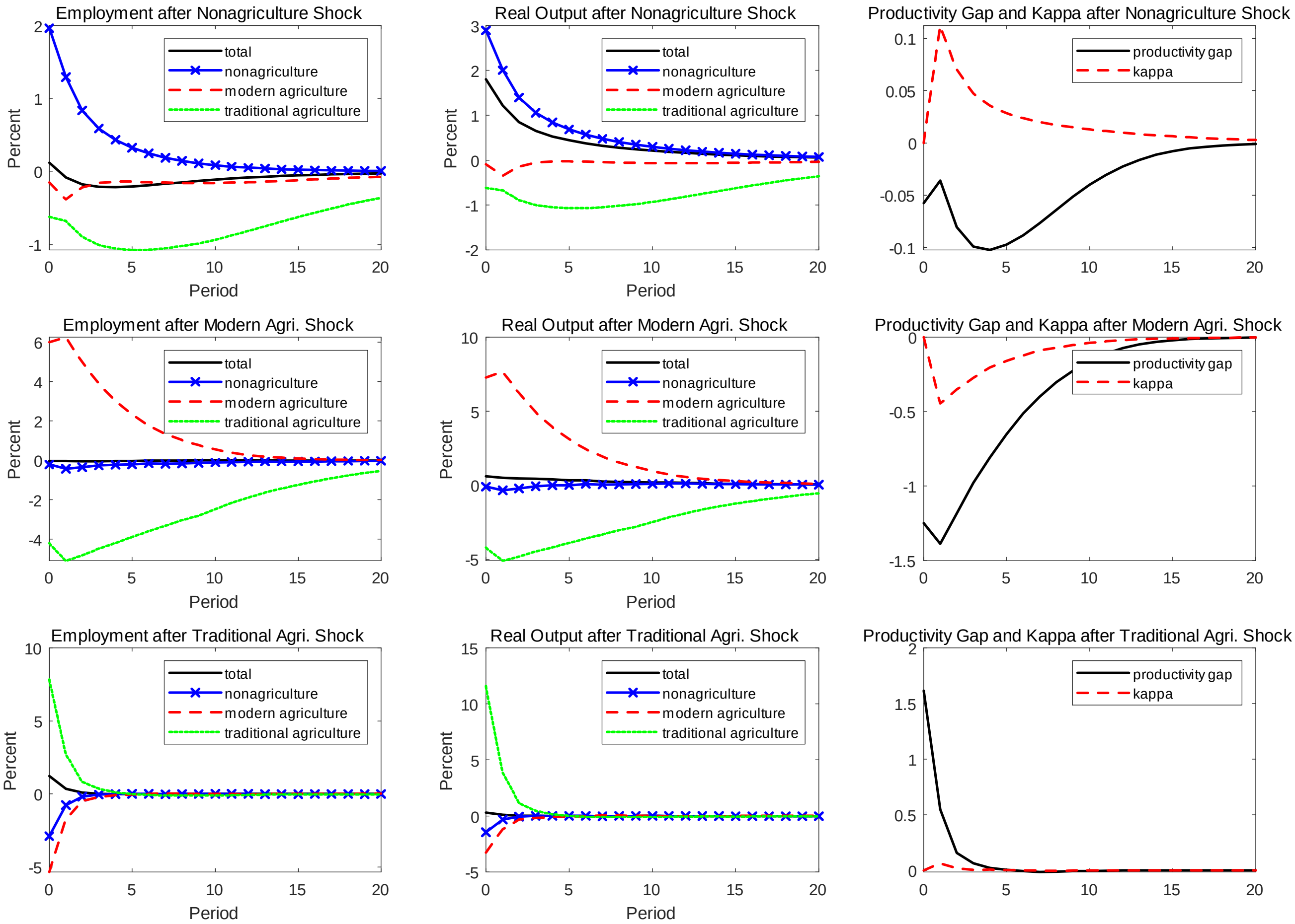

Figure 10: **Impulse Response Functions.** All graphs show impulse response as percentage deviation from the deterministic path. The three top panels show responses to a one-standard deviation change in nonagricultural TFP (Z^M). The three middle panels show responses to modern agricultural TFP (Z^{AM}). The three bottom panels show responses to traditional agricultural TFP (Z^S).

gap to fall (upper right panel of Figure 10). The impulse response of sectoral employment shows that a shock to Z^S (bottom panels) has a similar (but opposite) effect on the employment and output dynamics: a higher Z^S causes labor to be reallocated to traditional agriculture sourced from modern agriculture and nonagriculture. This in turn increases the relative size of traditional agriculture in total agriculture thereby lowering the average labor productivity in agriculture and increasing the labor income share. Thus, shocks to Z^M and Z^S induce a positive comovement in the productivity gap and agricultural employment. In contrast, a positive shock to modern agriculture induces an increase in total employment in agriculture and a decline of employment in the traditional sector and in the productivity gap. Thus, shocks to modern agriculture mitigate the countercyclicality of the productivity gap.

Sectoral labor supply and GDP. The dynamics of the productivity gap is instructive for understanding the comovement between sectoral labor supply and GDP. Both in the model and in the data (see Table 2) employment in nonagriculture ν^M is positively correlated with GDP, consumption, and investment whereas employment in agriculture ν^G is negatively correlated with GDP, consumption, and investment. The reason for the asymmetry between sectors, reflected in opposite signs of the comovements of GDP with ν^M and ν^G, lies in the presence of misallocation and in the role of the traditional agriculture as a labor reserve. Recall that since $\tau = 0.76$, employment in agriculture is suboptimally too large. The presence of a wedge between urban and rural sectors in China is in line with a number existing studies, see e.g., Brandt and Zhu (2010), Cheremukhin et al. (2017). Thus, reallocation to nonagriculture has a positive efficiency effect. When TFP in nonagriculture increases, ν^M increases, GDP goes up both by the direct effect of a productivity increase and because of reduction in misallocation. Both capital and labor move towards nonagriculture, with a fall in the capital labor ratio. While the net effect on employment in modern agriculture is ambiguous, the share of agricultural workers employed in modern agriculture unambiguously increases (see Lemma 2).

The labor reserve amplifies reallocation since many workers can leave agriculture without much effect on the marginal product of labor. This effect is reminiscent of the mechanism in Lewis (1954) at business cycle frequencies. Both the reduction in misallocation and the low opportunity cost of sourcing workers from agriculture result in a strong correlation between employment in nonagriculture and GDP. In contrast, when TFP increases in agriculture the positive direct effect on GDP of a higher average TFP is dampened (and possibly offset) by the increasing misallocation. Thus, employment in agriculture and GDP can move in opposite directions. Although the sign of the net effect hinges on parameters, in our calibration the correlation happens to be negative as it is in the data.

Expenditure and value added.. Expenditure and value added exhibit a cyclical pattern similar to that of sectoral labor: expenditure on nonagricultural good is strongly procyclical, while expenditure on agricultural good is acyclical in the data and only weakly procyclical in the model. The single aspect that the model gets qualitatively wrong is that it is inconsistent with the empirical observation that agricultural value added is positively (negatively) correlated with nonagricultural labor (agricultural labor) in China.

Volatility of labor supply and GDP. The model predicts a low volatility of employment and a

	c	i	$\frac{P^G y^G}{P}$	$\frac{P^M y^M}{P}$	$\frac{APL^G}{APL^M}$	n^G	n^M	n
	A. HP Filtered China Data, 1985-2012					$std\,(y) = 1.7\%$		
$\frac{std(x)}{std(y)}$	0.99	3.53	1.63	1.34	2.04	0.64	0.73	0.10
$corr(x, y)$	0.70	0.65	0.06	0.95	-0.17	-0.69	0.73	-0.23
$corr(x, n^G)$	-0.60	-0.31	-0.37	-0.55	0.48	1.00	-0.94	0.48
$corr\left(x, n^M\right)$	0.60	0.37	0.41	0.57	-0.54	-0.94	1.00	0.04
	B. HP Filtered Model, Homothetic model					$std\,(y) = 1.7\%$		
$\frac{std(x)}{std(y)}$	0.27	2.39	1.09	1.18	0.62	1.03	1.07	0.42
$corr\,(x, y)$	0.81	0.99	0.30	0.97	-0.38	-0.25	0.73	0.43
$corr\left(x, n^G\right)$	-0.08	-0.25	0.78	-0.43	0.73	1	-0.75	0.69
$corr\left(x, n^M\right)$	0.45	0.75	-0.31	0.87	-0.74	-0.75	1	-0.21

Table 2: Summary Statistics for China data and Model: HP-filtered

low correlation with GDP, although both are larger than in the data (note that employment volatility is particularly low in China even compared with countries at similar stage of development, see Figure 4). Moreover, the model is consistent with the observation that aggregate labor supply is highly correlated with employment in agriculture and approximately uncorrelated with employment in nonagriculture. The low volatility and low correlation of employment with total GDP stem from the availability of a larger labor reserve in agriculture that be reallocated across sectors without generating large wage fluctuations. Therefore, labor supply is less volatile (and less correlated with GDP) than in a one-sector economy.

Consumption and investments. Finally, the model shares many of the features of a standard one-sector RBC model. Investment is more volatile and consumption is less volatile than output. Note that consumption volatility is too low in the model relative to the data. Last but not least important, the model generates the same volatility of GDP as in the data.

5.3 Evolution of the Business Cycle during Structural Change

In this section, we simulate the model beyond the stage of structural change reached by China in 2012. This allows us to forecast the future evolution of business cycles and to compare the predictions of the model for an economy at the stage of development of China with that of a fully industrialized economy. We focus on four statistics: (i) the correlation between agricultural employment and productivity gap; (ii) the correlation between agricultural and nonagricultural employment, (iii) the correlation between total employment and GDP, and (iv) the volatility of employment relative to GDP.

The results are shown in Figure 11. Each dot in the figure represents a statistic covering a 28-year rolling window. The first dot on the right of each figure (i.e., that corresponding of the largest agricultural share) corresponds to simulations over the period 1985-2012, the second dot corresponds to 1986-2013, and so on. In each figure, the process of economic development yields a movement from left to right. A fully industrialized economy is an economy with an employment share of agriculture lower than 10%.

The top left panel shows that the correlation between the size of the agricultural sector and the

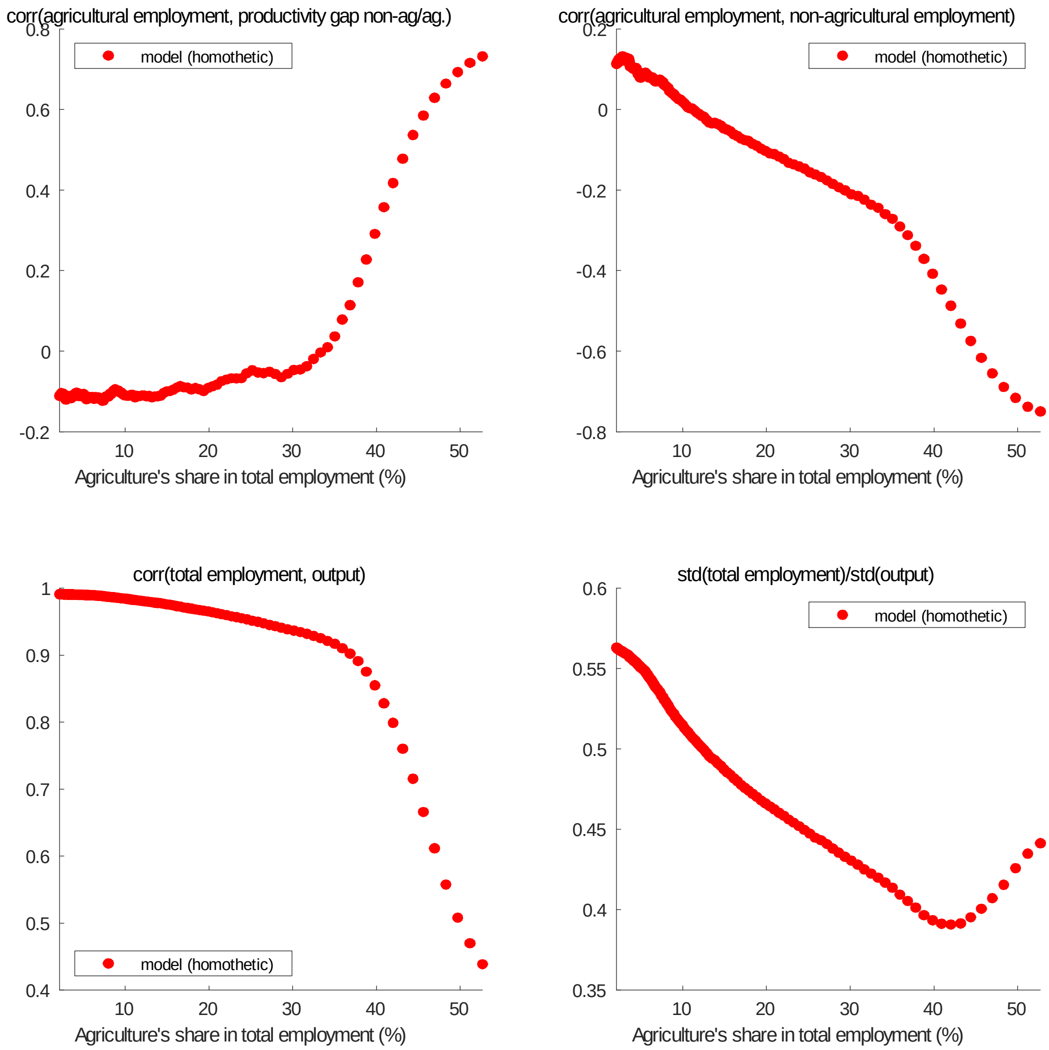

Figure 11: **Business Cycle during Structural Change.** The graphs show the evolution of business cycle statistics as a function of the employment share in agriculture. Each dot shows a statistic covering a 28-year rolling window. Simulated data are HP-filtered. The upper left panel shows the correlation between employment in agriculture and the productivity gap. The upper right panel shows the correlation between employment in agriculture and employment in nonagriculture. The lower left panel shows the correlation between total employment and GDP. The lower right panel shows the volatility of aggregate employment relative to GDP.

productivity gap decreases as structural change proceeds (i.e., as the share of agriculture falls). The reason is that as the agricultural sector modernizes and the Lewis sector shrinks, the economy converges to a Hansen-Prescott economy with a constant productivity gap. The productivity gap is entirely explained by technology parameters and by the exogenous wedge τ.

The top right panel shows that the correlation between employment in agriculture and nonagriculture increases as the share of agriculture falls. In particular, the correlation is negative and large in absolute value as long as the employment share in agriculture is between 40%-50% (which is close to the value for today's China), and is about zero for fully modernized economies. The reason is twofold. First, a large agricultural sector works supplies a buffer: when there are sector-specific shocks, it is possible to move labor in and out of agriculture. Second, the labor reserve in the traditional sector offers a high elasticity of substitution between sectoral employments. As this sector shrinks, the effective elasticity of substitution across sectors falls.

The bottom left panel shows that the correlation between total employment and GDP increases from 40% to 100% as the share of agriculture falls.[25] The reason is manifold. First, in a multisector economy productivity shocks can be absorbed by reallocating of labor across sectors without requiring large swings in the marginal product of labor and wages. In contrast, in the one sector economy wages fluctuate more since the only margin of adjustment is labor-leisure. Second, an economy with a large agricultural production has a low aggregate capital-output ratio. Therefore, movements in aggregate capital cause large swings in the marginal product of capital making consumption more positively correlated with GDP through a standard Euler equation mechanism. In turn, this lowers the fluctuations in labor supply originating from income effects. Finally, in our calibration of initial conditions the aggregate labor supply declines over time causing an increase in the average Frisch elasticity of labor supply.

The bottom right panel shows that the volatility of employment relative to GDP is lower for an economy like China than in a fully industrialized economy. This is a natural implication of the discussion above of the employment-GDP correlation. However, in our calibration, employment volatility decreases with development at earlier stages of the process of structural change (i.e., when agriculture employs more than 45% of the total hours worked). The reason for this nonmonotone behavior is that when traditional agriculture is large, fluctuations are largely driven by TFP shocks to this sector (Z^S). The estimated persistence of the TFP shock in traditional agriculture happen to be low which implies a large response in labor supply. We believe that this result may partly arise from measurement error, which is particularly important in the traditional sector (where, recall, we retrieve TFP shocks indirectly rather than through standard growth accounting, because of lack of data). Measurement error is likely to exaggerate the volatility and underestimate the persistence of shocks to Z^S. We return to this in the robustness section.

[25] Recall that in a one-sector business cycle model temporary TFP shocks to nonagriculture are the only source of fluctuations in both employment and GDP. Therefore, the correlation must be unity.

5.4 Robustness analysis

This section explores three robustness analysis exercises: (1) introducing capital adjustment costs; (2) assuming shocks to Z^S have the same persistence of as shocks to Z^{AM}; and (3) assuming a large food subsistence level and a unit elasticity between agriculture and nonagriculture ($\varepsilon \approx 1$).

5.4.1 Capital adjustment costs

In our benchmark model capital in each sector is set one period in advance, and after one period reallocation of capital between sectors can occur without cost. It is important to investigate the effect of introducing additional costs of reallocation of capital between agriculture and nonagriculture. Indeed, capital adjustment costs are standard in the quantitative DSGE literature (cf. Christiano et al. (2005); Smets and Wouters (2007)), including papers studying business cycles in models with multiple sectors (see e.g. Horvath (2000b); Bouakez et al. (2009); Iacoviello and Neri (2010)).

Following Bouakez et al. (2009) and Iacoviello and Neri (2010), we consider a canonical capital adjustment model where it is costly to change the rate of investment. Recall that for each capital stock j, the law of motion for capital is given by $K_{t+1}^j = (1-\delta) K_t^j + I_t^j$, where $j \in \{M, G\}$ and I_t^j is the effective investment in sector j. The cost of investment is reflected in an aggregate resource constraint for investment goods,

$$\begin{aligned} \left[\gamma \left(X_t^G\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\left(X_t^M\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} &= \Psi_t^G\left(\frac{I_t^G}{K_t^G}, K_t^G\right) \\ &+ \Psi_t^M\left(\frac{I_t^M}{K_t^M}, K_t^M\right) + I_t^G + I_t^M, \end{aligned}$$

where the terms Ψ_t^{AG} and Ψ_t^M reflect the adjustment cost. Recall also that X_t^i is the quantity of good $i \in \{M, G\}$ allocated to investment. We assume that the adjustment cost function Ψ has a standard quadratic form,

$$\Psi_t^j\left(\frac{I_t^j}{K_t^j}, K_t^j\right) = \frac{\xi}{2}\left(\frac{I_t^j}{K_t^j} - \delta - g_t^j\right)^2 K_t^j,$$

where the parameter g_t^j is the growth rate of capital K_t^j in period t in the deterministic structural transition and ξ is a nonnegative adjustment cost parameter. It follows that as long as the capital stock K_t^j grows at exactly the same rate as in the deterministic transition, g_t^j, the adjustment costs are zero. However, when the investment rate deviates from this level then quadratic costs are incurred.

We set the adjustment cost parameter to $\xi = 2.5$, which is slightly lower than the annual equivalent to the value of ξ estimated by Iacoviello and Neri (2010) based on quarterly data for the US ($\xi = 11$). Note that in an annual model it is impossible to change the capital stock more often than annually, so our benchmark economy effectively embeds some investment sluggishness since sectoral capital is set one period in advance. Moreover, note that by assumption the structural change dynamics would not be affected by the capital adjustment cost. We therefore keep all other parameters the same as in the benchmark homothetic economy.

The effects of introducing adjustment costs is that investments become more sluggish. Therefore, consumption must change more in response to a TFP shock, which in turn increases its volatility. Sluggish capital also affects sectoral labor supply because constant capital in the short run implies that moving labor in and out of modern agriculture and nonagriculture becomes less advantageous. This implies, in turn, that aggregate labor move less in response to shocks to z^M and z^{AM} and more to shocks to z^S.

Panel B of Table 3 shows the results. For convenience, the statistics for the benchmark economy are restated as panel A of this table. The main effect of introducing adjustment costs is that the cyclical behavior of aggregate labor supply and consumption is more in line with the data: n becomes negatively correlated with GDP and positively correlated with nonagricultural labor supply, as it is for China (see panel A of Table 2). Moreover, consumption becomes substantially more volatile, more negatively correlated with agricultural labor supply, and get a more reasonable correlation with GDP. However, aggregate investment is too smooth in this adjustment-cost experiment.

As the structural change progresses, the correlation between n and GDP now increase more, while the initial fall in employment volatility becomes more pronounced, see panels C and D of Figure 10.5 in the appendix.

5.4.2 Modifying the TFP process for traditional sector, z^S

In the benchmark model the persistence of shocks to z^S is calibrated to be much lower than that of shocks to z^{AM} ($\hat{\phi}^S = 0.42$ versus $\hat{\phi}^{AM} = 0.90$). We argued above that the transitory nature of shocks to traditional sector z^S was the cause behind the falling volatility of aggregate employment in the initial phase of the transition (see the lower right panel of Figure 11).

Note that since neither traditional nor modern agricultural production are directly observed, measurement error in agricultural capital and employment will show up as movements in the two TFP levels. This will affect the estimated TFP processes for z^S and z^M. To evaluate how the results change in response to modifications of the TFP process in traditional sector, we consider a sensitivity analysis where TFP shocks to traditional agriculture has the same persistence as shocks to modern agriculture ($\hat{\phi}^S = \hat{\phi}^{AM} = 0.9$). Moreover, we adjust the volatility of the innovations to z^S, $\sigma\left(\epsilon_t^S\right)$, so that the stationary variance of z^S is kept constant. The results are shown in panel C of Table 3. The main effect of a higher persistence of shocks to z^S is that the aggregate volatility of labor supply n falls and the correlation between n and GDP increase somewhat. Moreover, the relative volatility of employment is predicted to increase monotonically during structural change (see panel D of Figure 10.5 in the appendix). Lowering the standard deviation of innovations to z^S, $\sigma\left(\epsilon_t^S\right)$, has similar qualitative effects.

5.4.3 A large subsistence level of agriculture

Herrendorf et al. (2013) emphasize the role of a large subsistence component in agriculture, i.e., a large $\bar{c}$, combined with a low elasticity of substitution between goods. They argue that these features are needed to account for structural change in the US after 1950, provided that the elasticity of substitution

	c	i	$\frac{P^G y^G}{P}$	$\frac{P^M y^M}{P}$	$\frac{APL^G}{APL^M}$	n^G	n^M	n
	A. Benchmark Economy						$std\,(y) = 1.7\%$	
$\frac{std(x)}{std(y)}$	0.27	2.39	1.09	1.18	0.62	1.03	1.07	0.42
$corr\,(x, y)$	0.81	0.99	0.30	0.97	-0.38	-0.25	0.73	0.43
$corr\,(x, n^G)$	-0.08	-0.25	0.78	-0.43	0.73	1	-0.75	0.69
$corr\,(x, n^M)$	0.45	0.75	-0.31	0.87	-0.74	-0.75	1	-0.21
	B. Capital Adjustment Cost						$std\,(y) = 1.7\%$	
$\frac{std(x)}{std(y)}$	0.71	1.47	1.22	1.22	0.71	1.35	1.05	0.45
$corr\,(x, y)$	0.99	0.99	0.14	0.96	-0.41	-0.49	0.58	-0.27
$corr\,(x, n^G)$	-0.45	-0.49	0.74	-0.68	0.78	1	-0.92	0.93
$corr\,(x, n^M)$	0.53	0.77	-0.09	0.88	-0.50	-0.56	1	0.14
	C. Same autocorrelation for Z^S and Z^{AM}						$std\,(y) = 1.6\%$	
$\frac{std(x)}{std(y)}$	0.27	2.40	0.97	1.18	0.38	0.82	0.97	0.32
$corr\,(x, y)$	0.80	0.99	0.33	0.98	-0.68	-0.34	0.83	0.54
$corr\,(x, n^G)$	-0.15	-0.35	0.72	-0.48	0.51	1	-0.70	0.50
$corr\,(x, n^M)$	0.49	0.86	-0.15	0.93	-0.77	-0.70	1	0.08
	D. Cobb-Douglas with Subsistence						$std\,(y) = 1.6\%$	
$\frac{std(x)}{std(y)}$	0.42	2.44	1.21	1.16	0	0.86	0.65	0.41
$corr\,(x, y)$	0.80	0.99	0.41	0.99	0	-0.08	0.92	0.73
$corr\,(x, n^G)$	-0.26	-0.06	0.82	-0.09	0	1	-0.32	0.53
$corr\,(x, n^M)$	0.73	0.93	0.13	0.93	0	-0.32	1	0.52
	E. Orthogonal Shocks						$std\,(y) = 1.5\%$	
$\frac{std(x)}{std(y)}$	0.25	2.43	1.25	1.19	0.67	1.20	1.08	0.48
$corr\,(x, y)$	0.79	0.99	0.16	0.96	-0.06	-0.19	0.68	0.49
$corr\,(x, n^G)$	-0.06	-0.18	0.87	-0.40	0.66	1	-0.76	0.69
$corr\,(x, n^M)$	0.40	0.68	-0.48	0.84	-0.53	-0.76	1	-0.22

Table 3: Robustness analysis, benchmark model versus alternative model. All statistics refer to HP-filtered simulated data for 1985-2012.

is restricted to be the same across services, manufacturing, and agriculture (see our discussion in Section 3.3). Recall that when allowing for a subsistence level in agricultural goods, estimated model requires a very small $\bar{c}$. Nevertheless, it is interesting to investigate how a large $\bar{c}$ would influence the structural change and the business-cycle properties of the economy. We therefore reestimate the non-homothetic version of our model when imposing $\varepsilon = 1$ and removing the traditional agricultural sector. This implies a very large subsistence level in agriculture: the estimated $\bar{c}$ amounts to 98% of agricultural consumption in 1985. We label this economy as the *subsistence economy*.

As can be expected from the analysis of Section 4, the model with a low ε and a large $\bar{c}$ provides a poorer fit of China's structural change than does our benchmark model. After 1990 the subsistence economy implies a too large expenditure share of agriculture and too low real output share. Note that a higher ε, which the unrestricted model calls for, will ameliorate the model's predictions on these moments. The model also predicts a too large capital share in agriculture, especially 1985-2005, and a too low employment share of agriculture.

The business-cycle properties of the subsistence economy are presented in panel D of Table 3. Most of the qualitative properties of this economy are similar to those of the benchmark economy. However, the subsistence economy implies a substantially higher correlation between n and GDP. Note also that absent a traditional agricultural sector, the aggregate production function for agricultural goods becomes a Cobb-Douglas. This implies that this economy cannot generate any movements in the productivity gap (see Proposition 3).

Both the subsistence economy and the benchmark economy were estimated using the empirical levels of the expenditure share and output share of agriculture in China, and are therefore tailored to match the overall change between 1985 and 2012. In Section 3.3 we emphasized that the changes in expenditure and output shares of agriculture are consistent with $\varepsilon > 1$ even at higher frequencies, including 1-year and 5-year changes. It is therefore interesting to examine what our business-cycle models would predict about implied elasticities for changes at higher frequencies.

When plotting the 5-year changes in output share against changes in expenditure share of agriculture in our benchmark model, the observations fall on a line with an upward slope of 0.72. This is slightly lower than the empirical regression line for 5-year changes in China (slope of 0.81, see Figure 13 in the appendix). According to equation (23), this model's implied regression line correctly recovers the ε we assumed in the benchmark economy ($\varepsilon = 3.6$). A similar exercise for the subsistence economy yields a regression line with slope 0.5, implying $\varepsilon = 2$. Thus the combination of $\varepsilon = 1$ with a high $\bar{c}$ can to some extent stand in for a larger elasticity of substitution even at high frequencies, although the magnitude of this elasticity is quantitatively too small.

6 Conclusion

Business cycle fluctuations in countries undergoing structural transformation differ systematically from business cycles in industrialized countries. We document that countries with large declining agricultural sectors – including China – have aggregate employment fluctuations that are smooth and acyclical, while these countries experience volatile and procyclical reallocation of labor between agriculture and nonagriculture. We provide an unified theoretical framework for studying business cycle during structural change. The central aspect of the theory is the modernization of agriculture that occurs during the structural change: agriculture is becoming increasingly capital intensive and less labor intensive as a large traditional sector is crowded out. With a large traditional sector the expansion of manufacturing draws workers from traditional agriculture, ensuring smooth aggregate employment and large reallocation of workers between sectors. This process is driven by capital accumulation and differential productivity growth between agriculture and nonagriculture. We calibrate the model to China and show that the model is consistent both with China's structural transformation and with the business cycle properties of China. Moreover, the model is consistent with the changing business cycle properties as the economy goes from a poor economy with a large agricultural sector to a modern industrialized economy with negligible agricultural employment.

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7 Appendix A: Proofs

This appendix contains proofs and analytical derivations.

7.1 Derivation of Equation 15

Proof. We prove here both Lemma 1 and its corollary. We start by deriving the expression in (15). The FOC (10) that yields the equalization of the marginal product of capital can be rewritten as

$$\frac{1-\kappa}{\kappa} = \varsigma \frac{\gamma}{1-\gamma} \frac{1-\beta}{1-\alpha} \left(\frac{Y^G}{Y^M}\right)^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{Y^{AM}}{Y^G}\right)^{\frac{\omega-1}{\omega}}.$$

Taking logarithms, and letting $\omega = \varepsilon$ yields

$$\ln(1-\kappa) - \ln\kappa = \ln\left(\varsigma \frac{\gamma}{1-\gamma} \frac{1-\beta}{1-\alpha}\right) + \frac{\varepsilon-1}{\varepsilon} \ln\left(\frac{y^G}{y^M}\right)$$

Substituting in the expressions for y^G and y^G, and differentiating with respect to $\ln\chi$ yields

$$\left(\frac{\varepsilon-1}{\varepsilon}(\beta-\alpha) - \frac{1}{\varepsilon}\frac{1}{1-\kappa}\right)\frac{\partial\ln\kappa}{\partial\ln\chi} = -\frac{\varepsilon-1}{\varepsilon}(\beta-\alpha)\times\left(1-\frac{\partial\ln\nu^M}{\partial\ln\chi}\right) \tag{26}$$

Next, consider (13). Differentiating with respect to $\ln\chi$ yields

$$\begin{aligned} \frac{\partial\ln\left(\nu^M\right)}{\partial\ln\chi} \quad = \quad & \frac{\nu^{AM} + (1+(1-\kappa)(\varepsilon-1)(1-\beta))\,\nu^S}{(1+\nu^S(1-\beta)(\omega-1))} \frac{1}{1-\kappa}\frac{\partial\ln\kappa}{\partial\ln\chi} \\ & + \frac{\nu^S(\omega-1)(1-\beta)}{1+\nu^S(1-\beta)(\varepsilon-1)} \end{aligned} \tag{27}$$

Plugging-in (27) into (15) and simplifying terms leads to:

$$\frac{\partial\ln\kappa}{\partial\ln K} = \frac{(\varepsilon-1)(\beta-\alpha)(1-\kappa)}{1+(\varepsilon-1)((\beta-\alpha)(\kappa-\nu^M)+\nu^S(1-\beta))} > 0. \tag{28}$$

■

7.2 Proof of Lemma 2

Proof. Having defined $z_S \equiv Z^S / \left(Z^M\right)^\alpha$ and $z_A \equiv \left(Z^{AM}\right)^\beta / \left(Z^M\right)^\alpha$, the four static equilibrium conditions and the definition of Ξ can be expressed as

$$\frac{\nu^S}{\nu^{AM}} = \left(\frac{1}{\beta}\frac{1-\varsigma}{\varsigma}\right)^\varepsilon \left(\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\Xi\right)^{-(1-\beta)(\varepsilon-1)} \left(\frac{z_S}{z_A}\right)^{\varepsilon-1} \quad (29)$$

$$\frac{\nu^{AM}}{\nu^M} = \frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{1-\kappa}{\kappa} \quad (30)$$

$$\left(\frac{\nu^{AM}}{\nu^M}+\frac{\nu^S}{\nu^M}\right)\kappa\chi = \left(\frac{1}{\nu^M}-1\right)\kappa\chi = \Xi - \kappa\chi \quad (31)$$

$$\left(\frac{1-\kappa}{\kappa}\right)(\Xi)^{(\beta-\alpha)(\varepsilon-1)} = \left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^\varepsilon \left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{\beta(\varepsilon-1)} (z_A)^{(\varepsilon-1)} \quad (32)$$

$$\Xi \equiv \frac{\kappa\chi}{\nu^M} = \left(\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\right)^{-1}\frac{(1-\kappa)\chi}{\nu^{AM}} \quad (33)$$

We start with the comparative statics for z_A. Rewrite eq. (32) as

$$\Xi = \left(\left(\frac{1-\kappa}{\kappa}\right)^{-1}\left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^\varepsilon \left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{\beta(\varepsilon-1)}(z_A)^{(\varepsilon-1)}\right)^{\frac{1}{(\beta-\alpha)(\varepsilon-1)}} \quad (34)$$

Substitute (29)-(30) into (31) to get rid of the ratios $\frac{\nu^S}{\nu^{AM}}$ and $\frac{\nu^{AM}}{\nu^M}$ and obtain an equation in κ and Ξ;

$$\frac{\Xi}{\kappa\chi}-1 = \frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\frac{1-\kappa}{\kappa} + \left(\frac{1-\varsigma}{\alpha}\frac{\gamma}{1-\gamma}\right)^\varepsilon\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{(\varepsilon-1)(\beta-\alpha)}(\Xi)^{-(\varepsilon-1)(1-\alpha)}(z_S)^{\varepsilon-1}$$

Substitute in eq. (34), and simplify

$$\frac{1}{\chi}\left(\frac{\kappa}{1-\kappa}\right)^{\frac{1}{(\beta-\alpha)(\varepsilon-1)}}\left(\left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^\varepsilon\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{\beta(\varepsilon-1)}\right)^{\frac{1}{(\beta-\alpha)(\varepsilon-1)}}(z_A)^{\frac{1}{\beta-\alpha}}$$

$$= \frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta} - \frac{\beta-\alpha}{\alpha(1-\beta)}\kappa + \left(\frac{1-\varsigma}{\alpha}\frac{\gamma}{1-\gamma}\right)^\varepsilon\left(\varsigma\frac{\gamma}{1-\gamma}\frac{\beta}{\alpha}\right)^{-\frac{(1-\alpha)}{(\beta-\alpha)}\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{-\frac{(1-\alpha)}{(\beta-\alpha)}(\beta(\varepsilon-1)-\varepsilon)+(\varepsilon-1)(\beta-\alpha)}$$
$$\times(1-\kappa)^{\frac{(1-\alpha)}{(\beta-\alpha)}}(\kappa)^{-\frac{1-\beta}{\beta-\alpha}}(z_A)^{-(\varepsilon-1)\frac{(1-\alpha)}{(\beta-\alpha)}}(z_S)^{\varepsilon-1}$$

The LHS is increasing in z_A and in κ. The RHS is decreasing in z_A and in κ. It follows that an increasing in z_A must be associated with a decline in κ. Equation (30) then implies that ν^{AM}/ν^M increases. Suppose $\partial\ln\Xi/\partial\ln z_A \geq 0$. Then the RHS of eq. (29) unambiguously falls, implying that the ratio $\frac{\nu^S}{\nu^{AM}}$ must also fall. Suppose instead that $\partial\ln\Xi/\partial\ln z_A < 0$. Combine eq. (30) with the identity $1-\nu^S=\nu^M+\nu^{AM}$,

$$\begin{aligned} 1-\nu^S &= \left(1+\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\frac{\kappa}{1-\kappa}\right)\nu^{AM} \\ &= \chi\left(\frac{1}{\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}}-\frac{\beta-\alpha}{\alpha\left(1-\beta\right)}\kappa\right)\frac{1}{\Xi} \end{aligned}$$

Since $\partial\ln\kappa/\partial\ln z_A<0$ and since $\partial\ln\Xi/\partial\ln z_A<0$ by assumption, the RHS must increase. It follows that ν^S must fall, implying that the ratio ν^S/ν^{AM} must also fall. This proves that $\partial\ln\left(\nu^S/\nu^{AM}\right)/\partial\ln z_A<0$. It follows immediately that both υ and ν^{AM} must increase.

Consider now the comparative statics for z_A. Substitute (29)-(30) into (31) to get rid of the ratios $\frac{\nu^S}{\nu^{AM}}$ and $\frac{\nu^{AM}}{\nu^M}$ and obtain an equation in κ and Ξ;

$$\frac{1}{1-\kappa}=\frac{\chi}{\Xi-\chi}\left[\left(1+\left(\frac{1}{\beta}\frac{1-\varsigma}{\varsigma}\right)^{\varepsilon}\left(\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\Xi\right)^{-(1-\beta)(\varepsilon-1)}\left(\frac{z_S}{z_A}\right)^{\varepsilon-1}\right)\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}-1\right]$$

Rewrite (32) as,

$$\frac{1}{1-\kappa}=\left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^{-\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{-\beta(\varepsilon-1)}\left(z_A\right)^{-(\varepsilon-1)}\left(\Xi\right)^{(\beta-\alpha)(\varepsilon-1)}+1.$$

Equate these expressions and rearrange to get one equation in Ξ;

$$\begin{aligned} &\ln\left(\left(\frac{1}{\beta}\frac{1-\varsigma}{\varsigma}\right)^{\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{(1-\beta)(\varepsilon-1)+1}\left(\Xi\right)^{-(1-\beta)(\varepsilon-1)}\left(z_S\right)^{\varepsilon-1}+\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}-\frac{\Xi}{\chi}\right)\left(z_A\right)^{\varepsilon-1}\right) \\ = &\ln\left(\frac{1}{\chi}\left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^{-\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{-\beta(\varepsilon-1)}\right)+\left(\beta-\alpha\right)\left(\varepsilon-1\right)\ln\left(\Xi\right)+\ln\left(\Xi-\chi\right) \qquad (35) \end{aligned}$$

Differentiate equation (35) w.r.t. z_S, and rearranging terms yields

$$\begin{aligned} &\frac{\left(\varepsilon-1\right)\left(\frac{1}{\beta}\frac{1-\varsigma}{\varsigma}\right)^{\varepsilon}\left(\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\right)^{-(1-\beta)(\varepsilon-1)-1}\left(\Xi\right)^{-(1-\beta)(\varepsilon-1)}\left(\frac{z_S}{z_A}\right)^{\varepsilon-1}}{\left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^{-\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{-\beta(\varepsilon-1)}\left(z_A\right)^{-(\varepsilon-1)}\left(\Xi\right)^{(\beta-\alpha)(\varepsilon-1)}\left(\frac{\Xi-\chi}{\chi}\right)} \\ = &\left[\begin{array}{c} \left(\beta-\alpha\right)\left(\varepsilon-1\right)+\frac{\Xi}{\Xi-\chi} \\ +\frac{(1-\beta)(\varepsilon-1)\left(\frac{1}{\beta}\frac{1-\varsigma}{\varsigma}\right)^{\varepsilon}\left(\frac{z_S}{z_A}\right)^{\varepsilon-1}\left(\frac{\alpha}{\beta}\frac{1-\beta}{1-\alpha}\right)^{-(1-\beta)(\varepsilon-1)-1}(\Xi)^{-(1-\beta)(\varepsilon-1)}+\frac{\Xi}{\chi}}{\left(\varsigma\frac{\gamma}{1-\gamma}\frac{1-\beta}{1-\alpha}\right)^{-\varepsilon}\left(\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}\right)^{-\beta(\varepsilon-1)}(z_A)^{-(\varepsilon-1)}(\Xi)^{(\beta-\alpha)(\varepsilon-1)}\left(\frac{\Xi-\chi}{\chi}\right)} \end{array}\right]\frac{\partial\ln\Xi}{\partial\ln z_S} \end{aligned}$$

Recall that $\Xi=\frac{\kappa\chi}{\nu^M}>\chi$ due to $\kappa>\nu^M$. Therefore both coefficient on $\partial\ln\Xi/\partial\ln z_S$ is positive. It follows that $\partial\ln\Xi/\partial\ln z_S>0$. Now take log on both sides of equation (32) and differentiate w.r.t. $\ln z_S$; $0=\partial/\partial\ln z_S\ln\left[\left(1-\kappa\right)/\kappa\right]+\left(\beta-\alpha\right)\left(\varepsilon-1\right)\partial\ln\Xi/\partial\ln z_S$. Since $\left(\beta-\alpha\right)\left(\varepsilon-1\right)\partial\ln\Xi/\partial\ln z_S>0$, it must be that $\partial/\partial\ln z_S\ln\left[\left(1-\kappa\right)/\kappa\right]<0$, which in turn implies that $\partial/\partial\ln z_S\ln\kappa>0$. Recall that $\nu^{AM}=\chi\left(1-\kappa\right)/\Xi$. Since both Ξ and κ are increasing in z_S, ν^{AM} must be falling in z_S. Now substitute

(30) into (31) to obtain an equation in $\frac{\nu^S}{\nu^{AM}}$, κ and Ξ;

$$\left(\frac{\Xi}{\chi}-1\right)\frac{1}{1-\kappa}=\left(1+\frac{\nu^S}{\nu^{AM}}\right)\frac{\beta}{\alpha}\frac{1-\alpha}{1-\beta}-1$$

Since both κ and Ξ are increasing in z_S, it follows that the ratio $\frac{\nu^S}{\nu^{AM}}$ must also be increasing in z_S. Since $\nu^S/\nu^{AM}=\beta^{-1}(1-\upsilon)/\upsilon$, it follows immediately that υ must fall in z_S. ∎

7.3 Proof of Proposition 3

Proof. We start by evaluating Equations (17)-(18) under the ABGP conditions. Note that (14) implies that $\eta(1,1)=(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}$. Thus,

$$\begin{aligned}\frac{\dot{c}_t}{c_t} &= g_M=(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}(1-\alpha)\left(\frac{\chi}{Z^M}\right)^{-\alpha}-\delta-\rho,\\ \frac{\dot{\chi}_t}{\chi_t} &= g_M=(1-\gamma)^{\frac{\varepsilon}{\varepsilon-1}}\times\left(\frac{\chi}{Z^M}\right)^{-\alpha}-(\delta+n)-\frac{c}{\chi}.\end{aligned}$$

Solving for c/χ and χ/Z^M yields the expressions in (21) and (22). Therefore, (17)-(18) hold true under the ABGP conditions. It is straightforward to see that under the ABGP conditions (in particular, when $\kappa=\upsilon=1$) (19)-(20) yields $\frac{\dot{\kappa}}{\kappa}=\frac{\dot{\upsilon}}{\upsilon}=0$. Likewise, (10) holds true when $\kappa=\upsilon=1$.

Next, consider the asymptotic growth rates of the sectoral capital. Taking logarithms and differentiating with respect to time the definitions of k^M and k^{AM} yields $\dot{k}^M/k^M=\dot{\kappa}/\kappa+\dot{\chi}/\chi=g^M$ and $\dot{k}^{AM}/k^{AM}=-(1-\kappa)^{-1}\times\dot{\kappa}/\kappa+\dot{\chi}/\chi=g^M-(\varepsilon-1)\beta\left(g^M-g^{AM}\right)$.

Next, consider the asymptotic growth rates of the sectoral employments of labor. First, observe that Equation (13) yields $\nu^M=1$ at the ABGP conditions $\kappa=\upsilon=1$. Second, note that $\nu^M=1$ implies that $\dot{N}^M/N^M=\dot{N}/N=n$. In order to establish the growth rate of N^{AM}, observe that taking logarithms on both side of Equation (11), differentiating with respect to time, and using the ABGP conditions and Equation (19) yields

$$\frac{\dot{N}^{AM}}{N^{AM}}=-\frac{1}{1-\kappa}\frac{\dot{\kappa}}{\kappa}+\frac{\dot{N}^{AM}}{N^{AM}}=n-(\varepsilon-1)\beta\left(g^M-g^{AM}\right).$$

Finally, to establish the growth rate of N^S, observe that taking logarithms on both side of Equation (12), differentiating with respect to time, and using the ABGP conditions and Equation (20) yields

$$\frac{\dot{N}^S}{N^S}=-\frac{1}{1-\upsilon}\frac{\dot{\upsilon}}{\upsilon}+\frac{\dot{N}^{AM}}{N^{AM}}=\frac{\dot{N}^{AM}}{N^{AM}}-(\omega-1)\left[\left(g^{AM}-g^S\right)+(1-\beta)\left(g^M-g^{AM}\right)\right].$$

To establish convergence, we linearize the dynamic system in a neighborhood of the ABGP. The system has three predetermined variables (χ,κ,υ) and one jump variable (c). Therefore, we must prove that the linear system has three negative eigenvalues and one positive eigenvalue. The rest of the proof is devoted to establish that this is the case.

Let $\tilde{\chi}=\frac{\chi}{Z^M}$ and $\tilde{c}=\frac{c}{Z^M}$, implying that $\frac{d\tilde{\chi}/dt}{\tilde{\chi}}=\frac{\dot{\chi}}{\chi}-g^M$. Then, we can write the dynamic system

(17)-(18)-(19)-(20). We can rewrite the system as

$$\begin{aligned}
\frac{d\tilde{c}/dt}{\tilde{c}} &= \eta\left(\kappa_t,\upsilon_t\right)^{\frac{1}{\varepsilon}}\left(1-\gamma\right)\left(1-\alpha\right)\left(\frac{\kappa_t\tilde{\chi}_t}{\nu^M\left(\kappa_t,\upsilon_t\right)}\right)^{-\alpha}-\delta-\rho-g^M \\
\frac{d\tilde{\chi}/dt}{\tilde{\chi}} &= \eta\left(\kappa_t,\upsilon_t\right)^{1-\alpha}\left(\frac{\kappa_t\tilde{\chi}_t}{\nu^M\left(\kappa_t,\upsilon_t\right)}\right)^{-\alpha}\kappa_t-\delta-\tilde{c}_t/\tilde{\chi}_t-n-g^M \\
\frac{\dot{\kappa}}{\kappa} &= \left(1-\kappa\right)\frac{\left(\begin{array}{c}\left(\alpha g^M-\beta g^{AM}+\left(\beta-\alpha\right)\left(\frac{d\hat{\chi}/dt}{\hat{\chi}}+g^M\right)\right)\\+\left(\frac{1}{\omega-1}-\frac{\left(\beta-\alpha\right)\left(1-\nu^M\right)}{1-\upsilon\left(1-\beta\right)}\right)\frac{\dot{\upsilon}}{\upsilon}\end{array}\right)}{\frac{1}{\varepsilon-1}+\left(\beta-\alpha\right)\left(\kappa-\nu^M\right)} \\
\frac{\dot{\upsilon}}{\upsilon} &= \left(1-\upsilon\right)\frac{\left(\begin{array}{c}\beta g^{AM}-g^S+\\\left(1-\beta\right)\left(\frac{\dot{\chi}_t}{\chi_t}-\frac{\dot{\kappa}_t}{\kappa_t}\frac{\kappa_t-\nu^M\left(\kappa_t,\upsilon_t\right)}{1-\kappa_t}\right)\end{array}\right)}{\frac{1}{\omega-1}+\frac{\left(1-\upsilon\right)\left(1-\beta\right)\left(1-\nu^M\right)}{1-\upsilon\left(1-\beta\right)}}
\end{aligned}$$

where we use Equation (13) implying that

$$\frac{1-\nu^M}{\nu^M}=\frac{1-\kappa}{\kappa}\frac{1-\alpha}{1-\beta}\left(\frac{\beta}{\alpha}+\frac{1}{\alpha}\frac{1-\upsilon}{\upsilon}\right).$$

The transversality condition (TVC) becomes

$$\lim_{t\to\infty}\xi e^{-(\rho-n)t}K=0$$

Substitute the condition that

$$\lim_{t\to\infty}\frac{\dot{K}}{K}=n+g^M$$

The TVC becomes

$$\lim_{t\to\infty}\frac{\dot{\xi}}{\xi}+g^M+n<\rho-n$$

Because

$$-\frac{\dot{c}}{c}=\left(1-\theta\right)\left(1-\sigma\right)\frac{\dot{h}}{h}\frac{h}{1-h}+\frac{\dot{\xi}}{\xi}+n$$

Then we have

$$\lim_{t\to\infty}\frac{\dot{c}}{c}=-\lim_{t\to\infty}\frac{\dot{\xi}}{\xi}-n$$

where we use $\lim_{t\to\infty}\frac{\dot{h}}{h}=0$. Plug $\lim_{t\to\infty}\frac{\dot{\xi}}{\xi}=-g^M-n$ into the TVC to get

$$-g^M-n+g^M+n<\rho-n$$

In the end, the TVC can be rewritten as

$$\rho-n>0$$

Letting $\Psi = (\tilde{c}, \tilde{\chi}, \kappa, \upsilon)'$, we can write the system of differential equations as

$$A\dot{\Psi} = f(\Psi)$$

where

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & a & 1 & b \\ 0 & c & d & 1 \end{bmatrix}$$

and, after

$$\begin{aligned}
a &= -\frac{(1-\kappa)(\beta-\alpha)}{\frac{1}{\varepsilon-1} + (\beta-\alpha)(\kappa-\nu^M)} \\
b &= -(1-\kappa)\frac{\left(\frac{1}{\omega-1} - \frac{(\beta-\alpha)(1-\nu^M)}{1-\upsilon(1-\beta)}\right)}{\frac{1}{\varepsilon-1} + (\beta-\alpha)(\kappa-\nu^M)} \\
c &= -\frac{(1-\upsilon)(\omega-1)(1-\beta)}{\left(1 + \frac{1-\upsilon}{\upsilon}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{\alpha}(\omega-1)\nu^M\right)} \\
d &= -\frac{(1-\upsilon)(\omega-1)(1-\beta)\left(1 - \frac{1-\alpha}{(1-\beta)\alpha}\frac{\nu^M}{\kappa}\left(\beta + \frac{1-\upsilon}{\upsilon}\right)\right)}{1 + (\omega-1)\frac{1-\upsilon}{\upsilon}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{\alpha}\nu^M}
\end{aligned}$$

$$f = \begin{bmatrix} \eta(\kappa_t, \upsilon_t)^{\frac{1}{\varepsilon}}(1-\gamma)(1-\alpha)\left(\frac{\kappa_t\tilde{\chi}_t}{\nu^M(\kappa_t,\upsilon_t)}\right)^{-\alpha} - \delta - \rho - g^M \\ \eta(\kappa_t, \upsilon_t)\left(\frac{\kappa_t\tilde{\chi}_t}{\nu^M(\kappa_t,\upsilon_t)}\right)^{-\alpha}\kappa_t - \delta - \tilde{c}_t/\tilde{\chi}_t - n - g^M \\ (1-\kappa)\frac{\beta\left(g^M - g^{AM}\right)}{\frac{1}{\varepsilon-1}+(\beta-\alpha)(\kappa-\nu^M)} \\ \frac{(1-\upsilon)(\omega-1)\left(g^{AM}-g^S+(1-\beta)\left(g^M-g^{AM}\right)\right)}{1+\frac{1-\upsilon}{\upsilon}\frac{1-\kappa}{\kappa}\frac{1-\alpha}{\alpha}\nu^M(\omega-1)} \end{bmatrix}$$

where we use the

$$\begin{aligned}
\frac{1-\nu^M}{\nu^M} &= \frac{1-\kappa}{\kappa}\frac{1-\alpha}{(1-\beta)}\left(\frac{\beta}{\alpha} + \frac{1}{\alpha}\frac{1-\upsilon}{\upsilon}\right) \\
\frac{1-\nu^M}{1-\upsilon(1-\beta)} &= \nu^M\frac{1-\kappa}{\kappa}\frac{1-\alpha}{(1-\beta)}\frac{1}{\alpha\upsilon} \\
\frac{\kappa-\nu^M}{1-\kappa} &= -1 + \frac{\nu^M}{\kappa}\frac{1-\alpha}{(1-\beta)\alpha}\left(\beta + \frac{1-\upsilon}{\upsilon}\right)
\end{aligned}$$

The inverse of matrix A is given by

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{1}{bd-1}(a-bc) & -\frac{1}{bd-1} & \frac{b}{bd-1} \\ 0 & \frac{1}{bd-1}(c-ad) & \frac{d}{bd-1} & -\frac{1}{bd-1} \end{bmatrix}$$

Along the approximate balanced growth path,

$$a^* = 0, b^* = 0, c^* = 0, d^* = 0$$

Now we compute

$$J(\Psi) = \begin{pmatrix} J_1(\Psi) \\ J_2(\Psi) \\ J_3(\Psi) \\ J_4(\Psi) \end{pmatrix} = A^{-1}(\Psi) f(\Psi)$$

where

$$\begin{aligned}
J_1(\Psi) &= \eta^{\frac{1}{\varepsilon}} (1-\gamma)(1-\alpha) \left(\frac{\kappa \tilde{\chi}}{\nu^M(\kappa, \upsilon)} \right)^{-\alpha} - \delta - \rho - g^M \\
J_2(\Psi) &= \eta \left(\frac{\kappa \tilde{\chi}}{\nu^M(\kappa, \upsilon)} \right)^{-\alpha} \kappa - \delta - \tilde{c}(\tilde{\chi})^{-1} - n - g^M \\
J_3(\Psi) &= \frac{1}{bd-1}(a-bc) \left(\left(\frac{\kappa \tilde{\chi}}{\nu^M(\kappa, \upsilon)} \right)^{-\alpha} \kappa - \delta - \tilde{c}/\tilde{\chi} - n - g^M \right) \\
&\quad - \frac{1}{bd-1}(1-\kappa) \frac{\beta \left(g^M - g^{AM} \right)}{\frac{1}{\varepsilon - 1} + (\beta - \alpha)(\kappa - \nu^M)} \\
&\quad + \frac{b}{bd-1} \frac{(1-\upsilon)(\omega - 1)\left(g^{AM} - g^S + \left(g^M - g^{AM} \right)(1-\beta) \right)}{1 + \frac{1-\upsilon}{\upsilon} \frac{1-\kappa}{\kappa} \frac{1-\alpha}{\alpha} (\omega - 1) \nu^M} \\
J_4(\Psi) &= \frac{1}{bd-1}(c-ad) \left(\left(\frac{\kappa \tilde{\chi}}{\nu^M(\kappa, \upsilon)} \right)^{-\alpha} \kappa - \delta - \tilde{c}/\tilde{\chi} - n - g^M \right) \\
&\quad + \frac{d}{bd-1}(1-\kappa) \frac{\beta \left(g^M - g^{AM} \right)}{\frac{1}{\varepsilon - 1} + (\beta - \alpha)(\kappa - \nu^M)} \\
&\quad - \frac{1}{bd-1} \frac{(1-\upsilon)(\omega - 1)\left(g^{AM} - g^S + \left(g^M - g^{AM} \right)(1-\beta) \right)}{1 + \frac{1-\upsilon}{\upsilon} \frac{1-\kappa}{\kappa} \frac{1-\alpha}{\alpha} (\omega - 1) \nu^M}
\end{aligned}$$

From (13) it follows that

$$\begin{aligned}
\dot{\nu}^M &= \nu^M \left(\frac{\dot{\kappa}}{\kappa} \frac{1-\nu^M}{1-\kappa} + \frac{\dot{\upsilon}}{\upsilon} \frac{1-\nu^M}{1-\upsilon(1-\beta)} \right) \\
&= \nu^M \left(\frac{\dot{\kappa}}{\kappa} \frac{\nu^M}{\kappa} \frac{1-\alpha}{(1-\beta)} \left(\frac{\beta}{\alpha} + \frac{1}{\alpha} \frac{1-\upsilon}{\upsilon} \right) + \frac{\upsilon}{\upsilon} \frac{1-\nu^M}{1-\upsilon(1-\beta)} \right)
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{\partial \nu^M}{\partial \kappa} &= \left(\frac{\nu^M}{\kappa} \right)^2 \frac{1-\alpha}{(1-\beta)} \left(\frac{\beta}{\alpha} + \frac{1}{\alpha} \frac{1-\upsilon}{\upsilon} \right) \\
\frac{\partial \nu^M}{\partial \upsilon} &= \frac{\nu^M}{\upsilon} \frac{1-\nu^M}{1-\upsilon(1-\beta)}.
\end{aligned}$$

Computing the Jacobian evaluated at the balanced growth path $(\tilde{c}, \tilde{\chi}, \kappa, \upsilon)'$, we obtain

$$J = \begin{bmatrix} 0 & J_{12}^* & J_{13}^* & 0 \\ J_{21}^* & J_{22}^* & J_{23}^* & 0 \\ 0 & 0 & J_{33}^* & 0 \\ 0 & 0 & J_{43}^* & J_{44}^* \end{bmatrix}$$

with determinant given by $-J_{12}^* J_{21}^* J_{33}^* J_{44}^*$ and four eigenvalues given by

$$\begin{bmatrix} \frac{1}{2}J_{22}^* + \frac{1}{2}\sqrt{\left(J_{22}^*\right)^2 + 4J_{12}^* J_{21}^*} \\ \frac{1}{2}J_{22}^* - \frac{1}{2}\sqrt{\left(J_{22}^*\right)^2 + 4J_{12}^* J_{21}^*} \\ J_{33}^* \\ J_{44}^* \end{bmatrix},$$

where

$$\begin{aligned} J_{12}^* &= -\alpha\left(1-\gamma\right)^{\frac{\varepsilon}{\varepsilon-1}}\left(1-\alpha\right)\left(\tilde{\chi}^*\right)^{\alpha-1} < 0 \\ J_{21}^* &= -\left(\tilde{\chi}^*\right)^{-1} < 0 \\ J_{22}^* &= \left(\tilde{\chi}^*\right)^{-1}\left(\rho - n\right) > 0 \\ J_{33}^* &= -\left(\varepsilon - 1\right)\beta\left(g^M - g^{AM}\right) < 0 \\ J_{44}^* &= -\left(\omega - 1\right)\left(g^{AM} - g^S + \left(g^M - g^{AM}\right)\left(1-\beta\right)\right) < 0 \end{aligned}$$

Thus, three eigenvalues are negative, while one is positive ($\frac{1}{2}J_{22}^* + \frac{1}{2}\sqrt{\left(J_{22}^*\right)^2 + 4J_{12}^* J_{21}^*} > 0$), establishing the result. ■

8 Appendix B: Data Construction

In this appendix, we describe how we have constructed the data on business-cycle facts across countries.

The data for aggregate GDP, capital stocks, investment are from the World Development Indicators. The data for value added in agriculture and capital stocks in agriculture is from the FAO. The data for sectoral employment comes from the International Labor Organization (ILO).

The data set is constructed as follows. First, we use data from the labor force surveys, households surveys, official statistics, and population censuses. We exclude data from firm surveys. Second, we exclude data that are not representative of the whole country. In particular, we exclude data from some countries which report data that only cover the urban population. Third, if multiple sources exist for the same country and these data cover overlapping time periods, we merge (by chaining) the different sources provided that data in the overlapping time periods are small. However, if the differences are large across different sources, we only retain the most recent data source, provided that the sample period is at least 15 years. If the most recent data cover less than 15 years, we retain the less recent data series (provided the sample covers at least 15 years).

Fourth, if multiple sources exist for the same country and these data do not cover overlapping time periods, then we do not merge the data. Instead we retain only the most recent data, provided that the sample period is at least 15 years. Again, if the most recent data covers less than 15 years, we use less recent data, provided that the data cover at least 15 years. The country is dropped if there are no data series longer than 15 years.

The resulting data set covers the following countries and time periods. For panels a and c in Figure 3 (aggregate employment and GDP), the sample comprises 68 countries. The sample time periods for each country are the following

Country	Start	End	Country	Start	End
ALB	1983	2015	ITA	1977	2015
AUS	1983	2015	JAM	1992	2015
AUT	1983	2015	JPN	1970	2015
AZE	1990	2015	KOR	1970	2015
BEL	1983	2015	LTU	1998	2015
BGR	2000	2015	LUX	1983	2015
BHS	1989	2015	LVA	1996	2015
BRA	1981	2014	MAC	1989	2012
BRB	1981	2015	MDA	1999	2015
CAN	1970	2012	MEX	1995	2015
CHE	1991	2015	MLT	2000	2015
CHL	1975	2015	MMR	1978	1994
CHN	1985	2012	MYS	1980	2015
CRI	1980	2013	NLD	1987	2015
CUB	1995	2014	NOR	1972	2015
CYP	1999	2015	NZL	1986	2015
CZE	1993	2015	PAK	1973	2008
DEU	1983	2015	PAN	1982	2015
DNK	1972	2015	PHL	1971	2015
DOM	1996	2015	POL	1999	2015
EGY	1989	2015	PRI	1970	2011
ESP	1970	2015	PRT	1974	2015
EST	1995	2015	PRY	1997	2015
FIN	1970	2015	ROU	1997	2015
FRA	1970	2015	RUS	1997	2015
GBR	1983	2015	SLV	1994	2015
GRC	1983	2015	SVK	1994	2015
HKG	1978	2008	SVN	1995	2015
HND	1990	2015	SWE	1970	2015
HUN	1992	2015	THA	1971	2015
IDN	1985	2015	TTO	1977	2015
IRL	1983	2015	USA	1970	2015
ISL	1991	2015	VEN	1975	2013
ISR	1970	2015	ZAF	2000	2015

For panel b in Figure 3 (agricultural versus nonagricultural employment), the sample comprises 67 countries. The sample time periods for each country are the following,

Country	Start	End	Country	Start	End
ALB	1994	2014	ITA	1977	2015
AUS	1970	2015	JAM	1992	2015
AUT	1983	2015	JPN	1970	2015
AZE	1983	2015	KOR	1970	2015
BEL	1983	2015	LTU	1998	2015
BGR	2000	2015	LUX	1983	2015
BHS	1989	2011	LVA	1996	2015
BRA	1983	2014	MDA	1999	2015
BRB	1981	2015	MEX	1995	2015
CAN	1970	2012	MLT	2000	2015
CHE	1991	2015	MMR	1978	1994
CHL	1975	2015	MYS	1980	2015
CHN	1985	2012	NLD	1987	2015
CRI	1980	2013	NOR	1972	2015
CUB	1995	2014	NZL	1986	2015
CYP	1999	2015	PAK	1973	2008
CZE	1993	2015	PAN	1982	2015
DEU	1983	2015	PHL	1971	2015
DNK	1972	2015	POL	1999	2015
DOM	1996	2015	PRI	1970	2011
EGY	1989	2015	PRT	1974	2015
ESP	1970	2015	PRY	1997	2015
EST	1989	2015	ROU	1997	2015
FIN	1970	2015	RUS	1997	2015
FRA	1970	2015	SLV	1994	2015
GBR	1983	2015	SVK	1994	2015
GRC	1983	2015	SVN	1993	2015
HKG	1978	2008	SWE	1970	2015
HND	1990	2015	THA	1971	2015
HUN	1992	2015	TTO	1977	2010
IDN	1985	2015	USA	1970	2015
IRL	1983	2015	VEN	1975	2013
ISL	1991	2015	ZAF	2000	2015
ISR	1970	2015			

For panel d in Figure 3 (productivity gap versus nonagricultural employment), the sample comprises 63 countries. The sample time periods for each country are the following,

Country	Start	End	Country	Start	End
ALB	1994	2014	JAM	1993	2015
AUS	1990	2015	JPN	1970	2015
AUT	1983	2015	KOR	1970	2015
AZE	1990	2015	LTU	1998	2015
BEL	1995	2015	LUX	1995	2015
BGR	2000	2015	LVA	1996	2015
BHS	1989	2011	MDA	1999	2015
BRA	1981	2014	MEX	1995	2015
BRB	1990	2014	MLT	2000	2015
CHE	1991	2015	MYS	1980	2015
CHL	1975	2015	NLD	1987	2015
CHN	1985	2012	NOR	1972	2015
CRI	1980	2013	NZL	1986	2015
CUB	1995	2014	PAK	1973	2008
CYP	1999	2015	PAN	1982	2015
CZE	1993	2015	PHL	1971	2015
DEU	1991	2015	POL	1999	2015
DNK	1972	2015	PRI	1971	2011
DOM	1996	2015	PRT	1995	2015
EGY	1989	2015	PRY	1997	2015
ESP	1995	2015	ROU	1997	2015
EST	1995	2015	RUS	1997	2015
FIN	1975	2015	SLV	1994	2015
FRA	1970	2015	SVK	1995	2015
GBR	1990	2015	SVN	1995	2015
GRC	1995	2015	SWE	1980	2015
HND	1990	2015	THA	1971	2015
HUN	1995	2015	TTO	1984	2010
IDN	1987	2015	USA	1970	2015
IRL	1995	2015	VEN	1975	2013
ISL	1997	2015	ZAF	2000	2015
ITA	1990	2015			

9 Appendix E: Appendix Tables and Figures

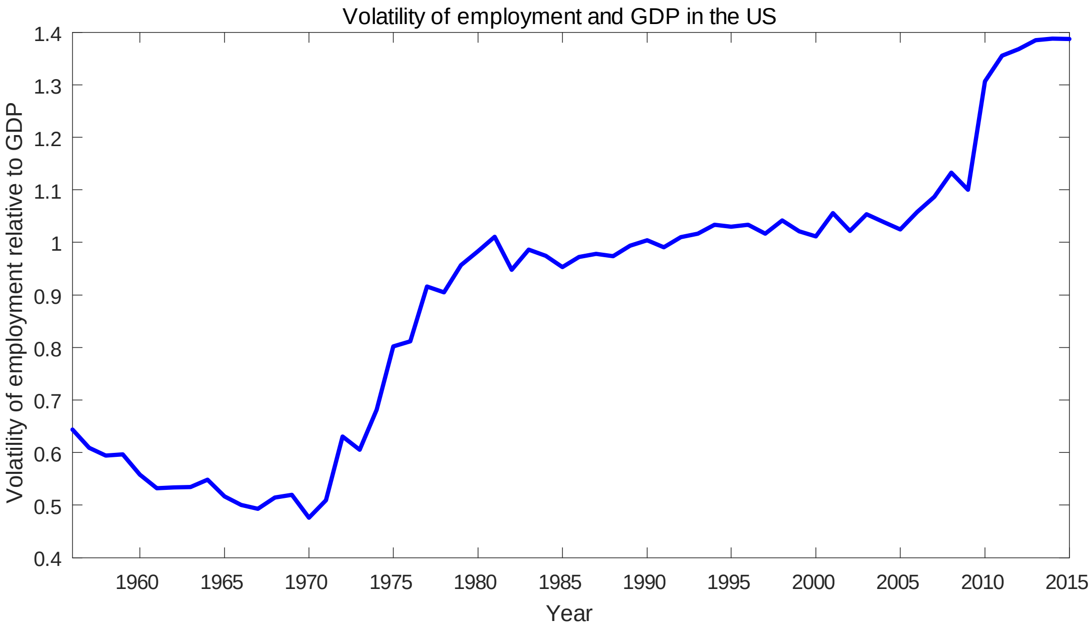

Figure 12: The figure shows the time evolution of the volatility of total employment in private sector (excluding government) relative to the GDP in the US from 1955 to 2015. The relative volatility is measured by the standard deviation of the HP-filtered log total employment divided by the HP-filtered log real output, both of which are computed on a 28-year rolling window. The x-axis denotes the end year of the sample window. The HP-filter use the smoothing parameter 6.25 (Ravn and Uhlig 2002). Source: Employment in private sectors is from the NIPA Table 6.8A, 6.8B, 6.8C, and 6.8D. The GDP in current price is deflated by the implicit price deflators from NIPA Table 1.1.9.

	c	i	$\frac{P^G y^G}{P}$	$\frac{P^M y^M}{P}$	$\frac{APL^G}{APL^M}$	n^G	n^M	n
A. FD- Filtered China Data, 1985-2012: $std\,(y) = 2.4\%$								
$\frac{std(x)}{std(y)}$	1.27	3.34	1.82	1.31	2.39	1.00	0.76	0.33
$corr(x, y)$	0.57	0.63	0.12	0.93	-0.09	-0.57	0.66	-0.25
$corr(x, n^G)$	-0.74	-0.34	-0.38	-0.38	0.35	1.00	-0.50	0.71
$corr\left(x, n^M\right)$	0.32	0.37	0.40	0.53	-0.52	-0.50	1	0.19
B. FD- Filtered Model, Homothetic model $std\,(y) = 2.7\%$								
$\frac{std(x)}{std(y)}$	0.30	2.36	1.11	1.25	0.72	1.10	1.27	0.49
$corr\,(x, y)$	0.80	0.99	0.24	0.95	-0.42	-0.30	0.69	0.18
$corr\left(x, n^G\right)$	-0.22	-0.27	0.80	-0.51	0.79	1	-0.78	0.75
$corr\left(x, n^M\right)$	0.55	0.66	-0.40	0.88	-0.81	-0.78	1	-0.52

Table 4: Summary Statistics for China data and Model: First-differenced

10 Online Appendix

10.1 The Lewis Model

In this section we provide the details of the analysis in Section 3.2.3. For simplicity, we abstract from technical progress and set $Z^M = Z^{AM} = Z^S = 1$. Moreover, we set $\tau = 0$. Endogenous capital accumulation is then the only source of transition. We continue to assume that $\varepsilon > 1$ and $\beta > \alpha$.

Stage 1 (Early Lewis). Consider an economy in which capital is very scarce. When $\chi < \underline{\chi}$, then, $\nu^{AM} = 0$, $\nu^M > 0$, $\nu^S > 0$, and $\kappa = 1$.[26] Intuitively, because capital is scarce, it is optimal to use it only in nonagriculture, where it is an essential factor, to take advantage of the high relative price of the nonagricultural good. Over time, employment grows in nonagriculture and falls in agriculture.[27]

The average labor productivity is higher in nonagriculture than in agriculture, reflecting the nonagriculture uses capital. More formally, the productivity gap is given by the inverse ratio of the labor-income shares in the two sectors, which equals $1/\alpha$.

Consider, next, the evolution of the aggregate capital-output ratio and of the factor prices in the Early Lewis stage. If the agricultural and nonagricultural goods were perfect substitutes, both the wage and the interest rate would stay constant as capital accumulates. However, for $\varepsilon < \infty$ capital accumulation triggers an increase in the relative price of agricultural goods and a growth in the real wage. Wage growth in turn causes capital deepening in the nonagricultural sector and a declining interest rate.

As capital accumulation progresses, the relative price of the agricultural good increases. Once capital is sufficiently abundant, (i.e., as $\chi \geq \underline{\chi}$), the relative price of agriculture is so high that it becomes optimal to put some capital in the modern agricultural sector. At this point the modernization process of agriculture starts and the economy enters the Advanced Lewis stage.

Stage 2 (Advanced Lewis). In this stage the equalization of factor returns across the two sectors implies that they have a constant capital-labor ratios. These are given by

$$k^{AM} \equiv \frac{(1-\kappa)\chi}{\nu^{AM}} = \left(\frac{1-\zeta}{\beta\zeta}\right)^{\frac{1}{1-\beta}}, \; k^M \equiv \frac{\kappa\chi}{\nu} = \frac{\beta(1-\alpha)}{\alpha(1-\beta)} k^{AM}. \tag{37}$$

[26] In particular, $\underline{\chi} = \frac{\beta(1-\alpha)}{\alpha(1-\beta)} \left(\frac{(1-\zeta)}{\beta\zeta}\right)^{\frac{1}{1-\beta}} \frac{\Xi}{1+\Xi}$, where $\Xi \equiv \left(\frac{\alpha(1-\gamma)}{\gamma(1-\zeta)^{\frac{\varepsilon-1}{\varepsilon}}}\right)^{\varepsilon} \left(\frac{\beta(1-\alpha)}{\alpha(1-\beta)} \left(\frac{(1-\zeta)}{\beta\zeta}\right)^{\frac{1}{1-\beta}}\right)^{(\varepsilon-1)(1-\alpha)}$.

[27] The key equilibrium condition is the equalization of the marginal product of labor in nonagriculture and traditional agriculture. Using the implicit function theorem, we can show that ν^M is an increasing function of χ. More formally,

$$\frac{\nu^M}{1-\nu^M} = \left(\frac{\alpha(1-\gamma)}{\gamma(1-\zeta)^{\frac{\varepsilon-1}{\varepsilon}}}\right)^{\varepsilon} \left(\frac{\chi}{\nu^M}\right)^{(\varepsilon-1)(1-\alpha)}, \tag{36}$$

where the LHS is increasing in ν^M and the RHS is increasing in χ and decreasing in ν^M. Thus, standard differentiation implies that $\partial\nu^M/\partial\chi > 0$.

The share of capital that goes to nonagriculture declines over the process of development:

$$\kappa = \frac{K^M}{L^M}\frac{L^M}{L}\frac{1}{\chi} = \frac{1+\left(\frac{(1-\zeta)}{\beta\zeta}\right)^{\frac{1}{1-\beta}}\frac{\beta}{1-\beta}\frac{1}{\chi}}{1+\frac{\alpha}{1-\alpha}\left(\frac{1+\Xi}{\Xi}\right)} \tag{38}$$

The optimal allocation of labor in manufacturing and modern agriculture yields

$$\nu^M = \frac{\chi + k^{AM}\frac{\beta}{1-\beta}}{k^{AM}\frac{\beta}{1-\beta}\left(\frac{1+\Xi}{\Xi}\right)+k^M},\ \nu^{AM} = \frac{\beta}{1-\beta}\frac{1+\Xi}{\Xi}\nu^M - \frac{\beta}{1-\beta}. \tag{39}$$

These expressions shows that employment in both manufacturing and modern agriculture increase as χ grows. Since the sectoral capital-labor ratios are constant, this also implies that capital and output in these sectors are increasing at the expense of a falling production of the traditional agriculture. Since factor prices are constant while the aggregate capital intensity in the economy is increasing, then the aggregate share of GDP accruing to capital grows while the labor share falls.

An interesting observation is that throughout this stage the expenditure share on agriculture and nonagriculture remain constant, even though $\varepsilon \neq 1$. To understand why, consider an economy without a Lewis sector. In this case, when $\varepsilon > 1$ and $\beta > \alpha$, capital accumulation would imply that the capital-intensive sector (in our case, nonagriculture) would grow faster over time. Although this implies an increase in the relative price of the agricultural product, the expenditure share on nonagricultural goods would increase over time. However, reallocation within agriculture with the decline of the Lewis sector offsets this force by increasing labor productivity in agriculture.

More formally, we can show that $\frac{P^MY^M}{P^GY^G} = \frac{1-\gamma}{\gamma}\left(\frac{Y^M}{Y^G}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \Psi$, where $\Psi \equiv \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{\alpha}{1-\zeta}\right)^{\varepsilon-1}\left(k^M\right)^{(1-\alpha)(\varepsilon-1)}$. This immediately implies that the productivity gap between agriculture and nonagriculture is shrinking, since

$$\frac{\frac{P^MY^M}{L^M}}{\frac{P^GY^G}{L^G}} = \Psi\frac{1-\nu^M}{\nu^M},$$

and, recall, ν^M is increasing in the Advanced Lewis stage. In particular, the productivity gap (which is the inverse of the ration between the labor income share in the two sectors) declines from $1/\alpha$ to β/α in this stage, where, recall, β is the labor income share in modern agriculture. Finally, in the Advanced Lewis stage, the capital-output ratio in agriculture increases relative to the capital-output ratio in nonagriculture. More formally,

$$\frac{\frac{K^G}{P^GY^G}}{\frac{K^M}{P^MY^M}} = \Psi\frac{\alpha}{1-\alpha}\left(\frac{1+\Xi}{\Xi} - \frac{\frac{\alpha}{1-\alpha}\left(\frac{1+\Xi}{\Xi}\right)+1}{\frac{1}{k^M}\chi + \frac{\alpha}{1-\alpha}}\right),$$

which is increasing in χ.

Stage 3 (Neoclassical). As the process of capital accumulation proceeds, the labor reserve in traditional agriculture becomes eventually exhausted. This happens when

$$\bar{\chi} = \frac{\beta}{\beta + \Xi} \left(\frac{(1-\zeta)}{\beta\zeta} \right)^{\frac{1}{1-\beta}} \left(1 + \frac{(1-\alpha)\,\Xi}{\alpha\,(1-\beta)} \right) > \underline{\chi}.$$

For any $\chi > \bar{\chi}$, $\nu^S = 0$ and $\upsilon = 1$. Henceforth, the economy exhibit standard properties of neoclassical models. In particular, if $\varepsilon > 1$ and $\beta > \alpha$, the nonagriculture sector grows in relative size, capital share (i.e., κ increases) and expenditure share. The productivity gap remains constant at β/α and the relative (agriculture vs. nonagriculture) capital-output ratio is also constant. During this stage, the interest rate falls and the real wage increases as capital accumulates.

Proposition 4 *Suppose $\varepsilon > 1$, $\beta > \alpha$ and $\omega \to \infty$. Then, as χ grows, economic development progresses through three stages:*

1. ***Early Lewis:*** *If $\chi \leq \underline{\chi}$, then, $\nu^{AM} = \upsilon = 0$, $\kappa = 1$. Moreover, ν^M is increasing and ν^S is decreasing in χ. The interest rate is decreasing and the wage rate is increasing in χ. The (average labor) productivity gap is constant and equal to $1/\alpha$.*

2. ***Advanced Lewis:*** *If $\chi \in [\underline{\chi}, \bar{\chi}]$ then, ν^M and ν^{AM} are increasing linearly in χ while ν^S is falling linearly in χ (cf. equation (39)). Therefore, υ is increasing in χ. Moreover, κ is decreasing in χ (cf. equation (38)). The capital-labor ratio in nonagriculture and modern agriculture is constant (cf. equation (37)), but the relative nonagriculture-to-agriculture capital-output ratio is falling in χ. The interest rate and the wage rate are constant, implying that the aggregate labor income share is falling. The (average labor) productivity gap is monotonically decreasing.*

3. ***Neoclassical:*** *If $\chi \geq \bar{\chi}$, then, $\nu^S = 0$ and $\upsilon = 1$. ν^M is increasing and ν^{AM} is decreasing in χ. Moreover, κ is increasing in χ. The capital-labor ratio is increasing in χ in both nonagriculture and modern agriculture, but the relative nonagriculture-to-agriculture capital-output ratio is constant. The interest rate is decreasing in χ and the wage rate is increasing in χ, while the aggregate labor income share is falling. The (average labor) productivity gap is constant. As χ becomes arbitrarily large, $\kappa \to 1$, $\nu^M \to 1$ and the expenditure share of agriculture tends to zero.*

10.2 Discrete Time Model

In this section we provide a complete description of the discrete time model with endogenous labor supply estimated in Section 4. Our baseline discrete time model adds the following model features to the continuous-time model: (1) endogenous labor supply (2) land as a factor of production in modern-agriculture sector (3) TFP shocks (4) capital stocks in each sector are predetermined.

Time t is discrete, indexed by $0, 1, 2, \ldots$. Given the initial capital stock in each sector, i.e., $\bar{K}_0\kappa_0$ and $\bar{K}_0(1-\kappa_0)$, and initial TFP levels, $Z_0^i, i = AM, M, S$, the representative household maximizes expected utility:

$$\max E_0 \sum_{t=0}^{\infty} \mu^t \left(\theta \log c_t + (1-\theta) \log (1-h_t)\right)$$

subject to the budget constraint

$$N_t c_t + K_{t+1} = W_t N_t + (R_t - \delta) K_t + Tr_t$$

where $K_t = K_t^M + K_t^{AM}$, $N_t = N_t^M + N_t^{AM} + N_t^S$. W_t denotes the after-tax equilibrium wage. $Tr_t = \tau W_t^M h_t N_t^M$ denotes the lump-sum transfer from the government to the representative household. Note that μ denotes the discount factor.

The production side is identical to the model in the text, except that the production of modern agriculture has been modified to include land

$$Y_t^{AM} = \left(K_t^{AM}\right)^{1-\beta-\beta^L} \left(Z_t^{AM} H_t^{AM}\right)^{\beta}$$

where the land income share is denoted by $\beta^L \geq 0$. We assume $\beta + \beta^L < 1$.

As explained in Section 3.2, we can exploit the equivalence between the competitive equilibrium and the distorted social planner problem, and write the Lagrangean as:

$$L = E_0 \sum_{t=0}^{\infty} \mu^t \left\{ \begin{array}{c} \theta \log c_t + (1-\theta) \log (1-h_t) + \\ \xi_t \left[\begin{array}{c} Y_t + (1-\delta) K_t - c_t N_t - K_{t+1} \\ -\tau W_t^M H_t^M + Tr_t \end{array} \right] \end{array} \right\}$$

where we use the notation $\chi_t, \kappa_t, \nu_t^M, \nu_t^{AM}, \nu_t^S, \upsilon_t$ introduced in the main text, but we modify the notations of η_t and introduce $\tilde{\eta}_t$

$$\begin{aligned} \eta_t &\equiv \left[\gamma \left(\frac{Y_t^G}{Y_t^M}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ \tilde{\eta}_t &\equiv \left[\gamma + (1-\gamma) \left(\frac{Y_t^M}{Y_t^G}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

Therefore, by definition $Y_t = Y_t^M \eta_t$ and $Y_t = Y_t^G \tilde{\eta}_t$. Recall that $H_t^i \equiv h_t N_t^i$.

The FOC with respect to ν_t^{AM} and ν_t^S are as in the continuous time problem.

$$\begin{aligned} \gamma \left(Y_t^G\right)^{1-\frac{1}{\varepsilon}} \upsilon_t \beta \frac{1}{\nu_t^A} &= (1-\tau)(1-\gamma) \left(Y_t^M\right)^{1-\frac{1}{\varepsilon}} \alpha \frac{1}{\nu_t^M} \\ \gamma \left(Y_t^G\right)^{1-\frac{1}{\varepsilon}} (1-\upsilon_t) \frac{1}{\nu_t^S} &= (1-\tau)(1-\gamma) \left(Y_t^M\right)^{1-\frac{1}{\varepsilon}} \alpha \frac{1}{\nu_t^M} \end{aligned}$$

Therefore, we have

$$\begin{aligned} \frac{\nu_t^{AM}}{\nu_t^M} &= \frac{\gamma \left(Y_t^G\right)^{1-\frac{1}{\varepsilon}} \upsilon_t \beta}{(1-\tau)(1-\gamma) \left(Y_t^M\right)^{1-\frac{1}{\varepsilon}} \alpha} \\ \frac{\nu_t^S}{\nu_t^M} &= \frac{\gamma \left(Y_t^G\right)^{1-\frac{1}{\varepsilon}} (1-\upsilon_t)}{(1-\tau)(1-\gamma) \left(Y_t^M\right)^{1-\frac{1}{\varepsilon}} \alpha} \end{aligned}$$

sum up together to have the expenditure ratio agr./non-agr. as

$$\frac{\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}}{(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}}=\frac{1-\nu_t^M}{\nu_t^M}\frac{(1-\tau)\,\alpha}{(1-\upsilon_t)+\upsilon_t\beta}$$

and express ν_t^M as

$$\nu_t^M=\left(1+\frac{\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}}{(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}}\frac{(1-\upsilon_t)+\upsilon_t\beta}{(1-\tau)\,\alpha}\right)^{-1}$$

Those with respect to c_t and h_t yield, respectively:

$$\theta\frac{1}{c_t}=\xi_t N_t, \tag{40}$$

$$\frac{1-\theta}{1-h_t}=\xi_t Y_t^{\frac{1}{\varepsilon}}\left[\begin{array}{c}\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}\left(\upsilon_t\frac{\beta}{h_t}+(1-\upsilon_t)\frac{1}{h_t}\right)\\ +(1-\tau)(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\frac{\alpha}{h_t}\end{array}\right]. \tag{41}$$

Substituting the FOCs with respect to ν_t^{AM} and ν_t^S into (41) yields

$$\frac{1-\theta}{1-h_t}=\xi_t Y_t^{\frac{1}{\varepsilon}}(1-\tau)\,\alpha\,(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\frac{1}{h_t\nu_t^M}. \tag{42}$$

Combining (42) with (40) yields

$$\frac{1-\theta}{\theta}\frac{c_t}{1-h_t}=(1-\tau)\,Y_t^{\frac{1}{\varepsilon}}\alpha\,(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}\frac{1}{\nu_t^M h_t N_t}.$$

The FOC w.r.t. κ_{t+1} and K_{t+1} yield, respectively (after combining the two equations and rearranging terms):

$$\xi_t = E_t\left[\mu\xi_{t+1}\left(Y_{t+1}^{\frac{1}{\varepsilon}}\gamma\left(Y_{t+1}^G\right)^{-\frac{1}{\varepsilon}}\varsigma\left(\frac{\upsilon_{t+1}}{\varsigma}\right)^{-\frac{1}{\omega-1}}\left(1-\beta-\beta^L\right)\frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}}+1-\delta\right)\right] \tag{43}$$

$$\xi_t = \mu E_t\left[\xi_{t+1}\left(Y_{t+1}^{\frac{1}{\varepsilon}}\frac{1}{K_{t+1}}\frac{1}{\kappa_{t+1}}(1-\gamma)\left(Y_{t+1}^M\right)^{1-\frac{1}{\varepsilon}}(1-\alpha)+1-\delta\right)\right]. \tag{44}$$

Substitute $Y_t=\eta_t Y_t^M$ and $Y_t=Y_t^G\tilde{\eta}_t$ Substituting in this into (43) and (44) respectively to eliminate Y yields

$$\xi_t = \mu E_t\left[\xi_{t+1}\left(\gamma\tilde{\eta}_{t+1}^{\frac{1}{\varepsilon}}\varsigma\left(\frac{\upsilon_{t+1}}{\varsigma}\right)^{-\frac{1}{\omega-1}}\left(1-\beta-\beta^L\right)\frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}}+1-\delta\right)\right] \tag{45}$$

$$\xi_t = \mu E_t\left[\xi_{t+1}\left(\eta_{t+1}^{\frac{1}{\varepsilon}}\frac{Y_{t+1}^M}{K_{t+1}^M}(1-\gamma)(1-\alpha)+1-\delta\right)\right]. \tag{46}$$

Equation (40), 45), and (46) yields the standard Euler equations for consumption:

$$
\begin{aligned}
1 &= \frac{\mu}{1+n} E_t \left\{ \frac{c_t}{c_{t+1}} \left[\begin{array}{c} \gamma \tilde{\eta}_{t+1}^{\frac{1}{\varepsilon}} \varsigma \left(\frac{\upsilon_{t+1}}{\varsigma}\right)^{-\frac{1}{\omega-1}} \left(1-\beta-\beta^L\right) \left(\left(1-\kappa_{t+1}\right)\chi_{t+1}\right)^{-\beta-\beta^L} \\ \times \left(Z_{t+1}^{AM} h_{t+1} \nu_{t+1}^{AM}\right)^{\beta} N_{t+1}^{-\beta^L} + 1 - \delta \end{array} \right] \right\} \\
1 &= \frac{\mu}{1+n} E_t \left\{ \frac{c_t}{c_{t+1}} \left[1 + \eta_{t+1}^{\frac{1}{\varepsilon}} \left(1-\gamma\right)\left(1-\alpha\right) \left(\frac{Z_{t+1}^{M} h_{t+1} \nu_{t+1}^{M}}{\kappa_{t+1}\chi_{t+1}} \right)^{\alpha} - \delta \right] \right\}
\end{aligned}
$$

We can simplify the intratemporal condition:

$$
\frac{1-\theta}{\theta} \frac{c_t}{1-h_t} = \left(1-\tau\right) \eta_t^{\frac{1}{\varepsilon}} \alpha \left(1-\gamma\right) \left(\frac{Z_t^M h_t \nu_t^M}{\kappa_t \chi_t} \right)^{\alpha-1}
$$

The resource constraint becomes

$$
\chi_{t+1}\left(1+n\right) = \eta_t \left(\kappa_t\right)^{1-\alpha} \left(Z_t^M \nu_t^M h_t\right)^{\alpha} \chi_t^{1-\alpha} + \left(1-\delta\right)\chi_t - c_t
$$

10.3 Algorithm to Solve the Rational Expectation Equilibrium

We can rewrite the model in discrete time recursively. Denote the state space as $\Theta_t \equiv \left(\hat{\chi}_t, \kappa_t, z_t^M, z_t^{AM}, z_t^S, t\right)$. The Bellman equation (during the structural change transition) is given by

$$
V\left(\Theta_t\right) = \max_{\hat{c}_t, h_t, \kappa_{t+1}, \hat{\chi}_{t+1}, \upsilon_t, \nu_t^M} \left\{ u\left(\hat{c}_t, h_t\right) + \mu E_t V'\left(\Theta_{t+1}\right) \right\}
$$

where use the following notations to detrend the variables

$$
\hat{\chi}_t = \frac{\chi_t}{\Lambda_t^M}, \hat{c}_t = \frac{c_t}{\Lambda_t^M}, z_t^i = \frac{Z_t^i}{\Lambda_t^i}, i = AM, M, S
$$

and

$$
\Lambda_t^i = Z_0^i \Pi_{k=1}^t \left(1+g_k^i\right), i = AM, M, S
$$

We solve the Bellman equation using its detrended first order conditions and budget constraint.

In each period, we have 6 unknown policy function to solve

$$
\hat{c}_t\left(\Theta_t\right), h_t\left(\Theta_t\right), \kappa_{t+1}\left(\Theta_t\right), \hat{\chi}_{t+1}\left(\Theta_t\right), \upsilon_t\left(\Theta_t\right), \nu_t^M\left(\Theta_t\right).
$$

We can reduce the number of unknown policy function further by expressing $\hat{c}_t$ and $\hat{\chi}_{t+1}$ in terms of other state variables and decision variables.

$$
\begin{aligned}
\hat{c}_t &= \left(1-\tau\right)\eta_t^{\frac{1}{\varepsilon}} \left(1-\gamma\right) \alpha \left(\kappa_t \hat{\chi}_t\right)^{1-\alpha} \left(z_t^M h_t \nu_t^M\right)^{\alpha-1} \left(1-h_t\right) \frac{\theta}{1-\theta} \\
\hat{\chi}_{t+1} &= \frac{\eta_t \left(\kappa_t\right)^{1-\alpha} \left(z_t^M \nu_t^M h_t\right)^{\alpha} \left(\hat{\chi}_t\right)^{1-\alpha} + \left(1-\delta\right)\hat{\chi}_t - \hat{c}_t}{\left(1+n\right)\left(1+g_{t+1}^M\right)}
\end{aligned}
$$

where

$$
\begin{aligned}
\eta_t &= \left[\gamma\left(\frac{Y_t^G}{Y_t^M}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
\frac{Y_t^G}{Y_t^M} &= \frac{\left(z_t^{AM}\Lambda_t^{AM}h_t\frac{\nu_t^{AM}}{\nu_t^M}\nu_t^M N_t\right)^{\beta}\left(\left(1-\kappa_t\right)\hat{\chi}_t\Lambda_t^M N_t\right)^{1-\beta-\beta^L}}{\left(z_t^M\Lambda_t^{AM}h_t\nu_t^M N_t\right)^{\alpha}\left(\kappa_t\hat{\chi}_t\Lambda_t^M N_t\right)^{1-\alpha}}\left(\frac{\upsilon_t}{\varsigma}\right)^{-\frac{\omega}{\omega-1}} \\
\frac{\nu_t^{AM}}{\nu_t^M} &= \frac{1-\nu_t^M}{\nu_t^M}\frac{(1-\tau)\alpha}{(1-\upsilon_t)+\upsilon_t\beta}\frac{\upsilon_t\beta}{(1-\tau)\alpha}
\end{aligned}
$$

Therefore, we are left with four unknowns $h_t(\Theta_t), \kappa_{t+1}(\Theta_t), \nu_t^M(\Theta_t), \upsilon_t(\Theta_t)$. We need to solve four nonlinear equations as well.

The first two are the detrended the Euler equations

$$
\hat{c}_t^{-1} = \frac{\mu}{(1+n)\left(1+g_{t+1}^M\right)}E_t\left\{\hat{c}_{t+1}^{-1}\times\left[\eta_{t+1}^{\frac{1}{\varepsilon}}\left(\frac{z_{t+1}^M\nu_{t+1}^M h_{t+1}}{\kappa_{t+1}\hat{\chi}_{t+1}}\right)^{\alpha}(1-\gamma)(1-\alpha)+(1-\delta)\right]\right\} \tag{47}
$$

$$
\hat{c}_t^{-1} = \frac{\mu}{(1+n)\left(1+g_{t+1}^M\right)}E_t\left[\hat{c}_{t+1}^{-1}\left(\gamma\tilde{\eta}_{t+1}^{\frac{1}{\varepsilon}}\varsigma\left(\frac{\upsilon_{t+1}}{\varsigma}\right)^{-\frac{1}{\omega-1}}\left(1-\beta-\beta^L\right)\frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}}+1-\delta\right)\right] \tag{48}
$$

where

$$
\frac{Y_{t+1}^{AM}}{K_{t+1}^{AM}} = \left(\left(1-\kappa_{t+1}\right)\hat{\chi}_{t+1}\Lambda_{t+1}^M\right)^{-\beta-\beta^L}\left(z_{t+1}^{AM}\Lambda_{t+1}^{AM}\lambda_{t+1}^{AM}h_{t+1}\right)^{\alpha_A}N_{t+1}^{-\beta^L}
$$

and

$$
\frac{Y_{t+1}^G}{Y_{t+1}^M} = \frac{\left(z_{t+1}^{AM}\Lambda_{t+1}^{AM}h_{t+1}\nu_{t+1}^{AM}N_{t+1}\right)^{\beta}\left(\left(1-\kappa_{t+1}\right)\hat{\chi}_{t+1}\Lambda_{t+1}^M N_{t+1}\right)^{1-\beta-\beta^L}}{\left(z_{t+1}^M\Lambda_{t+1}^{AM}h_{t+1}\nu_{t+1}^M N_{t+1}\right)^{\alpha}\left(\kappa_{t+1}\hat{\chi}_{t+1}\Lambda_{t+1}^M N_{t+1}\right)^{1-\alpha}}\left(\frac{\upsilon_{t+1}}{\varsigma}\right)^{-\frac{\omega}{\omega-1}}
$$

The other two are the equations about υ_t and υ_t^M.

$$
\upsilon_t = \frac{\varsigma\left(Y_t^{AM}\right)^{\frac{\omega-1}{\omega}}}{\varsigma\left(Y_t^{AM}\right)^{\frac{\omega-1}{\omega}}+(1-\varsigma)\left(Y_t^S\right)^{\frac{\omega-1}{\omega}}} \tag{49}
$$

$$
\nu_t^M = \left(1+\frac{\gamma\left(Y_t^G\right)^{1-\frac{1}{\varepsilon}}}{(1-\gamma)\left(Y_t^M\right)^{1-\frac{1}{\varepsilon}}}\frac{(1-\upsilon_t)+\upsilon_t\beta}{(1-\tau)\alpha}\right)^{-1} \tag{50}
$$

where

$$
\begin{aligned}
Y_t^{AM} &= (1-\kappa_t)^{1-\beta-\beta^L}\left(\hat{\chi}_t\Lambda_t^M\right)^{1-\beta-\beta^L}\left(\frac{\nu_t^{AM}}{\nu_t^M}\right)^{\beta}\left(z_t^{AM}\Lambda_t^{AM}\nu_t^M\right)^{\beta}h_t^{\beta}N_t^{-\beta^L}\\
Y_t^S &= z_t^S\nu_t^M\frac{\nu_t^S}{\nu_t^M}h_t\Lambda_t^S N_t\\
\frac{\nu_t^{AM}}{\nu_t^M} &= \frac{1-\nu_t^M}{\nu_t^M}\frac{(1-\tau)\,\alpha}{(1-\upsilon_t)+\upsilon_t\beta}\frac{\upsilon_t\beta}{(1-\tau)\,\alpha}\\
\frac{\nu_t^S}{\nu_t^M} &= \frac{1-\nu_t^M}{\nu_t^M}\frac{(1-\tau)\,\alpha}{(1-\upsilon_t)+\upsilon_t\beta}\frac{(1-\upsilon_t)}{(1-\tau)\,\alpha}
\end{aligned}
$$

Summary 1 *We solve the Bellman equation backwards from the last period, which corresponds to the long-run approximate balanced growth path with (approximately) only one sector, that is, the non-agr. sector.*

1. *We choose to number of transition period to be a large number ($T = 250$). In practice, we can increases the number until the beginning transition period we are interested in are no longer affected by the choice of T. We can also check whether the economy will converge to the long-run ABGP within the period T.*
2. *We discretize the state space. In the deterministic case, we choose the state space for $\hat{\chi}$ as $[0.5\hat{\chi}_0, 1.5\hat{\chi}^*]$ and choose the state space for κ as $[0.5, 1]$. We discretize both $\hat{\chi}$ and κ using 250 equally spaced grid points; In the stochastic case, we choose the state space for $\hat{\chi}$ as $[0.9\hat{\chi}_t^*, 1.1\hat{\chi}_t^*]$ and choose state space for κ as $[\kappa_t^* - 0.025, \kappa_t^* + 0.025]$. We discretize both $\hat{\chi}$ and κ using 75 equally spaced grid points, where $\hat{\chi}_t^*$ and κ_t^* are the realized path in the deterministic model. We further discretize the joint process for the three types of shocks using 27 grid points using Tauchen's method (Tauchen 1986).*
3. *We solve the transitional path backwards.*
   (a) *In the last period, the economy is almost identical to a one-sector RBC model. Therefore, we set*

   $$
   \begin{aligned}
   \kappa_{T+1}(\Theta_T) &= 1, \upsilon_T(\Theta_T) = 1\\
   \nu_T^M(\Theta_T) &= 1, \nu_T^{AM}(\Theta_T) = 0, \nu_T^S(\Theta_T) = 0
   \end{aligned}
   $$

   We solve the Bellman equation using value function iteration, with linear interpolation between grid points. We can solve for the rest of the policy functions

   $$c_T(\Theta_T), h_T(\Theta_T), \hat{\chi}_{T+1}(\Theta_T)$$

   (b) *From period $t = T-1$ to 1, we solve the nonlinear system of equations (47), (48), (49), and (50) for the policy functions $\kappa_{t+1}(\Theta_t), \upsilon_t(\Theta_t), \nu_t^M(\Theta_t), h_t(\Theta_t)$. In each period, we first express $\hat{c}_t(\Theta_t), \hat{\chi}_{t+1}(\Theta_t)$ in terms of other state variables and decision variables*

10.4 Measuring sector-specific TFP levels: theory

We observe $\{Y, Y^M, K^M, N^M, Y^G, K^G, N^G\}$ but we do not have direct observations of allocations of labor and output across the two agricultural technologies (we also presume that we have already estimated all relevant parameters). We describe the procedure to estimate the three-shock process

To measure the sector-specific TFP levels we impose three equilibrium conditions: (i) marginal return to capital are equated across manufacturing and modern agriculture, and (ii) marginal return to labor is equated across the two agricultural technologies; and (iii) hours per worker h is equalized across sectors. In addition we assume that assume that h is constant over time so that employment is a sufficient statistic for measuring labor input.

Recall the following definitions of sectoral outputs;

$$\begin{aligned}
Y^G &= \left[\varsigma \left(Y^{AM}\right)^{\frac{\omega-1}{\omega}} + (1-\varsigma)\left(Y^S\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \\
Y^M &= \left(K^M\right)^{1-\alpha_M} \times \left(Z^M H^M\right)^{\alpha_M} \\
Y^{AM} &= \left(K^{AM}\right)^{\beta_{AM}} \times \left(Z^{AM} H^{AM}\right)^{\alpha_{AM}} \\
Y^S &= Z^S H^S
\end{aligned}$$

We express aggregate output using current prices,[28] $Y = P^G Y^G + P^M Y^M$.

The marginal products of sector-specific capital and labor are

$$\begin{aligned}
\frac{\partial Y}{\partial K_M} &= P^M \frac{\partial Y_M}{\partial K_M} = P^M (1-\alpha_M) \left(\frac{K^M}{H^M}\right)^{-\alpha_M} \times \left(Z^M\right)^{\alpha_M} \\
\frac{\partial Y}{\partial K_G} &= P^G \frac{\partial Y_G}{\partial Y_{AM}} \frac{\partial Y_{AM}}{\partial K_G} = P^G \varsigma \left(\frac{Y^{AM}}{Y}\right)^{-\frac{1}{\omega}} \beta_{AM} \left(K^G\right)^{\beta_{AM}-1} \times \left(Z^{AM} H^{AM}\right)^{\alpha_{AM}} \\
\frac{\partial Y}{\partial N_S} &= P^G \frac{\partial Y_G}{\partial Y_S} \frac{\partial Y_S}{\partial N_S} = P^G (1-\varsigma) \left(\frac{Y^S}{Y^G}\right)^{-\frac{1}{\omega}} Z^S h \\
\frac{\partial Y}{\partial N_{AM}} &= P^G \frac{\partial Y_G}{\partial Y_{AM}} \frac{\partial Y_{AM}}{\partial N_{AM}} = P^G \varsigma \left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} \alpha_{AM} \left(K^G\right)^{\beta_{AM}} \times \left(H^{AM}\right)^{\alpha_{AM}-1} \left(Z^{AM}\right)^{\alpha_{AM}} h
\end{aligned}$$

We now proceed to measuring the TFP levels in five steps:

1. The marginal product of capital is the same in manufacturing and modern agriculture,

$$P^M (1-\alpha_M) \left(K^M\right)^{-\alpha_M} \times \left(Z^M H^M\right)^{\alpha_M} = \varsigma \left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} P^G \beta_{AM} \left(K^G\right)^{\beta_{AM}-1} \times \left(Z^{AM} H^{AM}\right)^{\alpha_{AM}}$$
$$\Rightarrow$$

[28] The advantage of focusing on aggregate output in terms of current prices instead of specifying the production function is that while a subsistence level in agricultural consumption will be subsumed in the relative prices, it will not affect the equations determining how TFP levels are measured.

$$\left(\frac{Y^{AM}}{Y^G}\right)^{1-\frac{1}{\omega}} = \frac{K^G}{K^M}\frac{P^M Y^M}{P^G Y^G}\frac{1}{\varsigma}\frac{1-\alpha_M}{\beta_{AM}} \tag{51}$$

The variables on the right-hand side of equation (51) are observable. This identifies the ratio Y^{AM}/Y^G.

2. Use the agricultural production function (3) to derive a relationship between the ratios Y^S/Y^G and Y^{AM}/Y^G,

$$1 = \varsigma\left(\frac{Y^{AM}}{Y^G}\right)^{\frac{\omega-1}{\omega}} + (1-\varsigma)\left(\frac{Y^S}{Y^G}\right)^{\frac{\omega-1}{\omega}} \tag{52}$$

Given the imputed ratio Y^{AM}/Y^G, this equation identifies the ratio Y^S/Y^G.

3. The marginal product of labor is equated across the two agricultural sectors. This implies

$$P^G(1-\varsigma)\left(\frac{Y^S}{Y^G}\right)^{-\frac{1}{\omega}} Z^S h = P^G\varsigma\left(\frac{Y^{AM}}{Y^G}\right)^{-\frac{1}{\omega}} \alpha_{AM}\left(K^G\right)^{\beta_{AM}} \times \left(H^{AM}\right)^{\alpha_{AM}-1}\left(Z^{AM}\right)^{\alpha_{AM}} h$$
$$\Rightarrow$$
$$\frac{H^{AM}}{H^S} = \alpha_{AM}\frac{\varsigma}{1-\varsigma}\left(\frac{Y^{AM}}{Y^G}\frac{Y^G}{Y^S}\right)^{1-\frac{1}{\omega}} \tag{53}$$

4. Use the accounting identity $N^G = N^{AM} + N^S$ to identify N^{AM} and N^S;

$$\frac{H^{AM}}{H^S} = \frac{N^{AM}}{N^S} = \frac{N^{AM}}{N^G - N^{AM}} \Rightarrow$$

$$N^{AM} = \frac{N^G}{1 + \left(\frac{H^{AM}}{H^S}\right)^{-1}} \tag{54}$$
$$N^S = N^G - N^{AM} \tag{55}$$

5. Normalize hours per worker to $h = 1$ for all periods (so aggregate hours equals employment). Equations (51)-(55) then allows us to identify Z^M, Z^S, and Z^{AM};

$$\ln\left(Z^S\right) = \ln\left(Y^S\right) - \ln\left(N^S\right) \tag{56}$$
$$\ln\left(Z^{AM}\right) = \frac{1}{\alpha_{AM}}\ln\left(Y^{AM}\right) - \frac{\beta_{AM}}{\alpha_{AM}}\ln\left(K^{AM}\right) - \ln\left(N^{AM}\right) \tag{57}$$

and finally TFP in manufacturing;

$$\ln\left(Z^M\right) = \frac{1}{\alpha_M}\ln\left(Y^M\right) - \frac{1-\alpha_M}{\alpha_M}\ln\left(K^M\right) - \ln\left(N^M\right) \tag{58}$$

10.5 Additional Figures

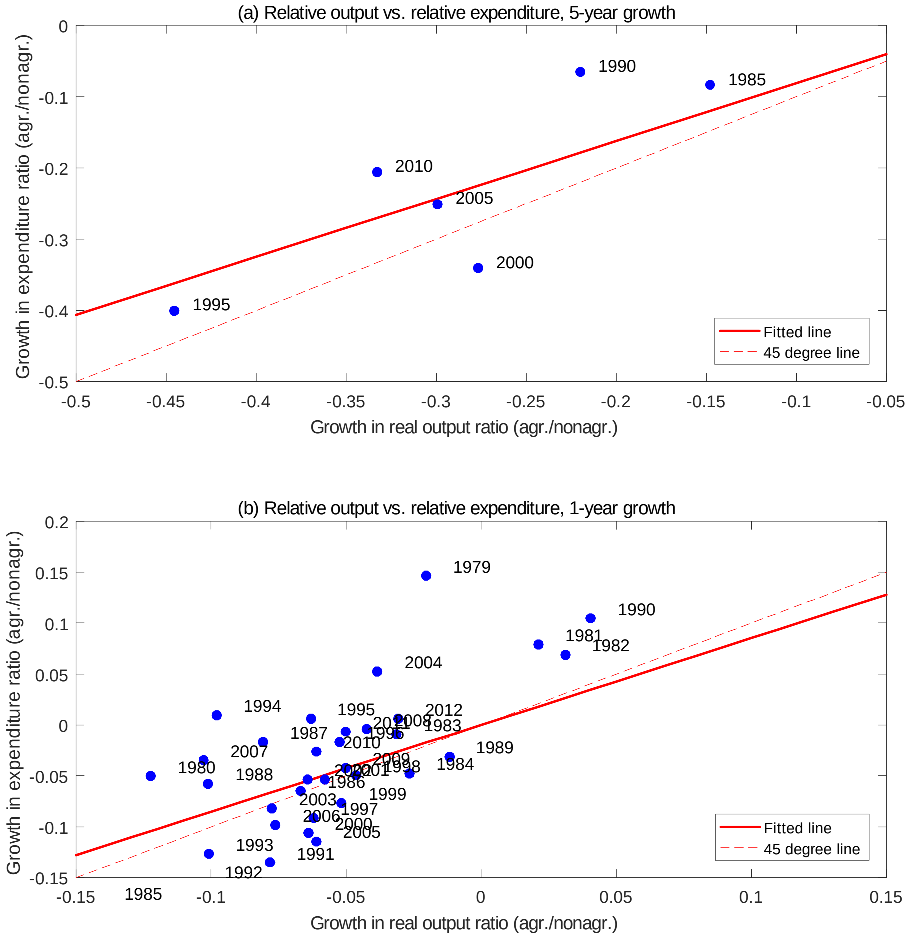

Figure 13: Panel a in the graph shows a scatter plot of 5-year changes in the log of the ratio of real agricultural to nonagricultural output against the log of the corresponding value added share (in current prices). The sample is 1978-2012 for China. An OLS regression without a constant term – in line with equation (23) – yields a slope of 0.8121 (std. err. 0.11, t=7.36, p=0.001), implying $\varepsilon \approx 5.2$. Similarly, Panel b in the graph shows a scatter plot of 1-year changes in the log of the ratio of real agricultural to nonagricultural output against the log of the corresponding value added share (in current prices). An OLS regression without a constant term yields a slope of 0.852 (std. err. 0.132, t=6.41, p=0.000), implying $\varepsilon \approx 6.75$.

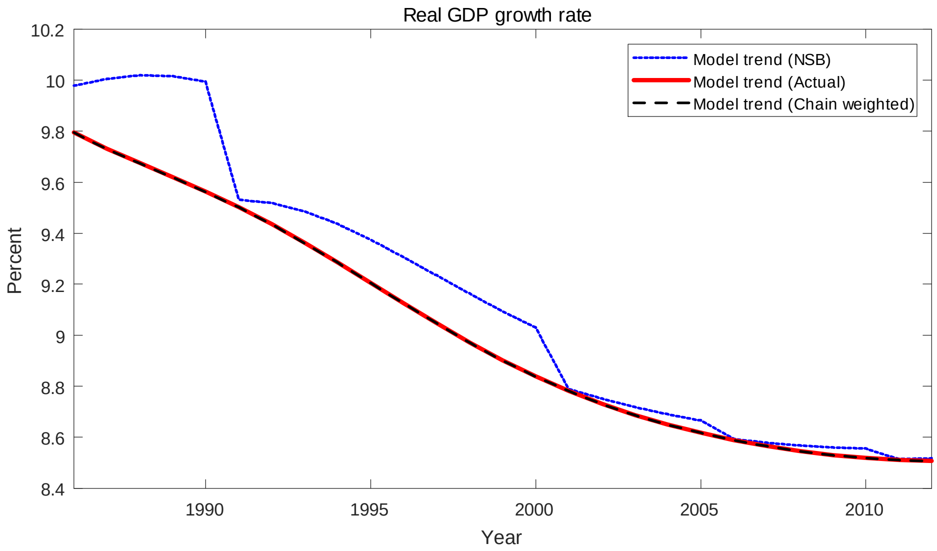

Figure 14: This figure compare different ways of calculating the real GDP growth rate with data.