# Social Insurance, Information Revelation, and Lack of Commitment[*]

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## Abstract

We consider optimal public provision of unemployment insurance when government's ability to commit is imperfect. Unemployed persons privately observe arrivals of job opportunities and choose probabilities of communicating this information to the government. Imperfect commitment implies that full information revelation is generally suboptimal. We define a notion of the social value of information and show that, due to the incentive constraints, it is a convex function of the information revealed. In the optimum each person is provided with an incentive to either reveal his private information fully or not reveal any of it, but the allocation of these incentives may be stochastic. In dynamic economies unemployed persons who enter a period with higher continuation utilities reveal their private information with lower probabilities. The optimal contract can be decentralized by a joint system of unemployment and disability benefits in a way that resembles how these systems are used in practice in developed countries.

# 1  Introduction

The major insight of the normative literature in public finance is that there are substantial benefits from using past and present information to provide individuals with insurance against risk as well as incentives to work. A common assumption of this literature is that the government is a benevolent social planner that can perfectly commit to its policies. An important implication of this assumption is that a more informed government can allocate resources more efficiently.[1] As a result, it is optimal for the government to acquire all the private information that agents have.

The political economy literature has long emphasized that such commitment may be difficult to attain in practice.[2] Over time, self-interested politicians and voters – whom we will broadly refer to as "the government" – are tempted to re-optimize and choose new policies. When the government's ability to commit is imperfect, revealing more information has costs as better informed governments have higher temptation to depart from ex-ante desirable policies. Therefore, relaxing the perfect commitment assumption produces a non-trivial trade-off on how much private information should be revealed to the government.

In this paper we study optimal information revelation and resource allocation when government's ability to commit is imperfect. We consider a simple model of unemployment insurance. Our economy is populated by a continuum of agents who receive privately-observed job opportunities. An agent who has a job opportunity can convert his labor hours into output; an agent without a job opportunity cannot produce any output. The government collects information from the agents and allocates labor supply and consumption. We impose few assumptions on the duration of job opportunities or preferences other than that leisure is a normal good.

The government is benevolent but it cannot perfectly commit to the allocations that it will choose after information is revealed. As a result, it is in general suboptimal to reveal full information to the government. We study static and dynamic economies. In the static economy imperfect commitment is modelled exogenously. The government can pre-commit to some allocations initially, but it can break its promises after receiving information by paying

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[1] The seminal work of Mirrlees (1971) started a large literature in public finance on taxation, redistribution, and social insurance in the presence of private information about individuals' types. Akerlof (1978) work on "tagging" is another early example of how a benevolent government can use information about individuals to improve efficiency. For surveys of the recent literature on social insurance and private information see Golosov, Tsyvinski, and Werning (2006) and Kocherlakota (2010).

[2] There is a vast literature in political economy that studies the frictions that policymakers face. For our purposes, the work of Acemoglu (2003) and Besley and Coate (1998) is particularly relevant. They argue that inefficiencies in a large class of political economy models can be traced back to the lack of commitment. Kydland and Prescott (1977) is the seminal contribution that first analyzed policy choices when the policymaker cannot commit.

an exogenous cost. In the dynamic, infinite-horizon economy there are no external commitment costs and the government is constrained only by the structure of the subgame perfect equilibrium.

Our economy is a simple prototype of a Mirrleesian economy that is used extensively in public finance and other fields.[3] It provides a laboratory to study optimal information revelation in a transparent way. The amount of information that the government receives from an agent coincides with the probability with which that agent reports that he received a job offer. This feature of the model provides us with a natural and simple way of ordering information, which is harder to do in more general environments.

Information revelation has benefits and costs. More information allows the government to provide better insurance on the equilibrium path, but it also increases the temptation for the government to break its promises and deviate from equilibrium actions. In our economy the difference between these benefits and costs is a natural definition of the "value of information".

We show that the value of information is a convex function of the amount of information communicated by an agent. This convexity arises because of the need to provide agents with incentives to reveal information. Incentive constraints induce distortions. The same distorted allocation provides an equal incentive to reveal any amount of information but its costs depends on the mass of agents that receive the distortion. When an agent reveals his information with higher probability, it is possible to adjust distortions so as to further increase the marginal value of the additional information. As a result, the marginal value of information is increasing in the amount of information, establishing convexity.

One implication of this result is that in efficient equilibria any given agent either reveals his information fully or does not reveal it at all. When partial information revelation is necessary, this is achieved through a form of rationing. Ex-ante identical agents are randomly allocated into one of two groups: the group where agents are provided with incentives and reveal their information fully, and the group where agents receive no incentives and reveal no information. This arrangement maximizes the amount of information revealed per unit of distortion. Agents in the second group are better off as their consumption is not distorted by the incentive constraints.

In the dynamic economy, past histories of idiosyncratic shocks introduce heterogeneity across agents. These idiosyncratic histories can be summarized recursively by continuation

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[3]In public finance, some examples of such papers would be Golosov, Kocherlakota, and Tsyvinski (2003), Albanesi and Sleet (2006), Farhi and Werning (2013), Stantcheva (2014), Golosov, Troshkin, and Tsyvinski (2016). Similar settings have been used in international economics (Green (1987), Dovis (2009)), political economy (Yared (2010)), and firm dynamics (Clementi and Hopenhayn (2006)).

utilities. We show that optimal information revelation is decreasing in those utilities, so that an agent who enters a period with higher continuation utility reveals his information with lower probability. We identify normality of leisure as the key force behind this result. When leisure is a normal good, agents with a higher continuation utility work fewer hours and produce less output when employed. It is thus less socially valuable to know whether such agents have job opportunities.

We conclude our paper by comparing the dynamics of the optimal contract under perfect and imperfect commitment, and discuss the implications for decentralization. We focus on the special case of our environment where jobs offer permanent employment. This case has been studied extensively under the assumption that the policymaker can perfectly commit, and those studies provide us with a natural benchmark. We show that under imperfect commitment the optimal contract exhibits mean reversion and has less history dependence. These features help to relax the government's sustainability constraints. The optimal allocations can be decentralized by a two-tier system. In one tier, individuals receive a generous constant benefit and face low incentives to search for jobs; this tier is an absorbing state. In the other tier, individuals receive less generous benefits, that are declining over time and that provide strong incentives to search for jobs.

The two-tier system bears some similarities to the way public unemployment and disability benefits are provided in developed countries. While initially designed as a support system for most sick and severely incapacitated, modern public disability insurance systems often work as an alternative form of insurance against adverse economic shocks orthogonal to health. An extensive literature has documented that such events as recessions or plant closures spur an increase in applications and awards of disability benefits.[4] A common interpretation of these findings is that disability benefits are poorly designed and are misused by those individuals who are able to work. From the lens of our model, the observed substitution between the two systems is instead an efficient outcome when government commitment is imperfect. Since providing incentives to search for jobs is costly ex-post, it might be desirable to let some of the unemployed exit the labor force by taking disability benefits.

**Related literature**. Our paper is related to a relatively small literature that studies social insurance with imperfect commitment. Roberts (1984) was one of the first to explore the implications of imperfect commitment for social insurance. He studied a dynamic economy in which types are private information, but do not change over time. Bisin and Rampini

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[4]To give just one example, Maestas, Mullen, and Strand (2018) estimate that 8.9% of all awards of Social Security Disability Insurance (SSDI) benefits during 2008-2012 was directly induced by the Great Recession. We provide an extensive review of the literature on the utilization of disability insurance in Section 2.1.4.

(2006) pointed out that, in general, it might be desirable to hide information from a benevolent government in a two period economy. They did not characterize how and who should reveal information, which is the focus of our paper. Sleet and Yeltekin (2006), Sleet and Yeltekin (2008), Acemoglu, Golosov, and Tsyvinski (2010), Farhi, Sleet, Werning, and Yeltekin (2012) all studied versions of dynamic private-information economies with imperfect commitment by the policymaker. However, they all made various assumptions on commitment technology and shock processes to ensure that any information revealed by agents becomes obsolete once the government deviates. Thus, they bypassed the core issues we are interested in and they focused, instead, on the case where the standard revelation principle holds and the government learns all private information. To the best of our knowledge, the only paper that has explored the value of information for the provision of social insurance is Aiyagari and Alvarez (1995), however, their focus is on optimal monitoring.

In a broader context, our work is also related to Skreta (2006), Skreta (2015), Gerardi and Maestri (2018), Beccuti and Moller (2018), Doval and Skreta (2018), who build on earlier work of Bester and Strausz (2001), Freixas, Guesnerie, and Tirole (1985) and Laffont and Tirole (1988), to study mechanism design problems in settings where the principal cannot commit. To gain tractability, these papers assume quasi-linear payoffs, which makes their results inapplicable to our insurance setting.

Our unemployment insurance framework is closely related to a large literature that studied the design of unemployment insurance under commitment, such as Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), Atkeson and Lucas (1995), Chetty (2006), Pavoni and Violante (2007). We relax the perfect commitment assumption in such settings. We use Hopenhayn and Nicolini (1997) as the benchmark for decentralization to study the implications of imperfect commitment. Our two-tier decentralization bears some resemblance to the decentralization obtained by Pavoni and Violante (2007) but it is driven by a very different mechanism.

Our result that benefits are convex in the amount of information bears some superficial resemblance to the well-known result in Radner and Stigliz (1984) (see also Chade and Schlee (2002) for generalizations). Despite similar conclusions about convexity, the mechanisms in the two papers are very different. In the Radner-Stigliz model, informational costs and signals are exogenous and the convexity result is local. It is driven by the assumption that the marginal benefit from a small amount of information is zero when the principal is uninformed. In our model, costs and benefits of information are endogenous and determined by incentive constraints. The marginal benefit of information is always strictly positive, but the convexity result is global and originates from optimal incentive provision. The broad conclusion that

lack of concavity in the value of information makes the analysis of such problems more delicate is certainly valid in both settings.

Finally, our recursive characterization builds on the Lagrangian techniques developed by Farhi and Werning (2007) to study a class of principal-agent models under perfect commitment. Those techniques cannot be applied directly to dynamic games such as ours because the value of best deviation, which depends on the strategies of all agents in the economy, makes their Lagrangian non-separable in individual histories. To obtain separability, which is a key step in deriving a recursive formulation, we first need to find an appropriate bound on the value of the best deviation. Finding this bound is an important intermediate result in our analysis.

The paper is organized as follows. In Section 2, we present a static economy that we use to illustrate our main insights under the assumption that the commitment cost is exogenous. In Section 3, we turn to the dynamic economy where commitment costs and agent heterogeneity emerge endogenously as outcomes of the efficient provision of incentives. To make analysis transparent, we focus in this section on the case where shocks are i.i.d. so that private information is valuable only for one period. In Section 4, we generalize our results to persistent shocks, compare contract dynamics under perfect and imperfect commitment, and discuss decentralization.

## 2 Optimal information revelation in a simple model

In this section we use a simple model to illustrate insights about optimal information revelation when the principal's ability to commit is imperfect. Consider a static economy populated by a continuum of ex-ante identical agents and a large player, which we refer to as the "government". Each agent finds a job opportunity with exogenous probability $p$ and does not find one with probability $1 - p$. Whether an agent has found a job or not is private information to that agent. An agent with a job can transform $l$ units of his labor into $l$ units of output; a jobless agent cannot produce any output.

All consumption and labor supply are allocated by the government. Each agent communicates with the government by sending a report about whether he found a job ($H$) or not ($L$). The government then allocates bundles ($c_H, l$) of consumption and output requirement to each agent reporting $H$, and bundles ($c_L$) of consumption to each agent reporting $L$. These bundles must be chosen so that they satisfy the feasibility constraint that we specify below. We assume that an agent who cannot deliver the output requested by the government suffers an arbitrarily high utility cost, so that jobless agents never report $H$ in equilibrium.

Agents are risk-averse and the government is utilitarian. Agents' preferences are given

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by utility function $U(c, l)$ where $c, l \geq 0$. We denote derivatives of this function by using subscripts: $U_c$, $U_l$, etc.

**Assumption 1** *$U$ is strictly increasing and strictly concave in $(c, -l)$, and twice differentiable. Leisure $-l$ is a strictly normal good, in other words, the income effect on labor supply is strictly negative. $U_l(c, 0) = 0$ for all $c$.*

Our economy is a special case of Mirrleesian models that are used extensively to study the design of taxes and social insurance programs under the assumption that the government has perfect commitment. The usual approach is to invoke the revelation principle according to which it is without loss of generality to focus on direct communication between agents and the government whereby the government offers incentive-compatible allocations and agents reveal their private information fully. The optimal allocations maximize welfare subject to those incentive constraints.

The assumption that the government can perfectly commit is crucial in this line of reasoning as the government has an incentive to re-optimize after agents' private information is revealed. Thus, this simple version of the revelation principle fails when commitment is imperfect.

Our goal is to understand properties of the constrained optimal allocations when government's ability to commit is imperfect. For now, we simply assume that there is an exogenous cost $\Upsilon$ that the government must pay when breaking promises. This assumption is a reduced-form way to capture the cost of deviation that emerges endogenously in the infinitely repeated game. It also offers a simple way to parameterize the strength of commitment, with $\Upsilon = \infty$ corresponding to perfect commitment. In Section 2.1.2 we show that our insights hold under much more general types of commitment costs.

Formally, we consider the following three-stage game:

0. The government announces functions $\{c_H(z), c_L(z), l(z)\}$, for all $z \in [0, 1]$.

1. Each agent learns about his job opportunity and draws a random publicly-observed signal $z$. Signals $z$ are drawn from a uniform distribution on $[0, 1]$, independently for each agent. An agent who has a job opportunity chooses the probability $\sigma(z)$ with which he reveals this information by reporting $H$; he reports $L$ otherwise. A jobless agent reports $L$ with probability 1.

2. The government chooses allocations of consumption and labor $\{\tilde{c}_H(z), \tilde{l}(z)\}$ and consumption $\{\tilde{c}_L(z)\}$ for agents who report $H$ and $L$, respectively. Bundles must be feasible, that is, total consumption cannot exceed total output. The government incurs a utility

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cost $\Upsilon > 0$ if allocations differ from those announced in stage 0 for any positive measure of agents.

We make several comments about this environment. First, agents choose from only two messages, $H$ and $L$. This is without loss of generality and additional messages are redundant as it can be easily seen from the proofs below. Second, we allow agent's strategies to depend not only on the agent's job status and his report about it, but also on the realization of the idiosyncratic sunspot variables $z$. This is a technical device that allows the government to randomly assign ex-ante identical agents to different groups and provide them with different incentives. As it will become clear shortly, there are two ways to limit the amount of information revealed to the government. One way is to provide all agents with the same incentives but have them reveal their job status with some interior probability. The other way is to offer different incentives to ex-ante identical agents. One of the key results of this section is that the latter way is preferable to the former.

It will be more convenient to work in the space of utils and labor supply $(u, l)$ instead of consumption and labor $(c, l)$. Let $C(u, l)$ be the cost of providing $u$ utils to an agent who supplies $l$ units of labor. This function is implicitly defined via $U(C(u, l), l) = u$. We denote derivatives with respect to its first and second arguments by $C_1$, $C_2$, $C_{12}$, etc.

**Lemma 1** *$C$ is non-negative, strictly increasing, strictly convex and twice differentiable. Utils and labor are complements, $C_{12} > 0$. $C_2(u, 0) = 0$ for all $u$.*

The proof of this result comes from a straightforward application of the implicit function theorem and is left for the appendix. There we show a stronger statement that (strict) normality of leisure is equivalent to (strict) complementarity of utils and labor hours in the cost function $C$. This property is intuitive: normality implies that the marginal value of leisure is increasing in the agent's utility, hence, the marginal cost of compensating an agent for an extra unit of effort must also be increasing in agent's utility. As we shall see, this property has strong implications for optimal information revelation.

We now consider best responses of agents and the government. Suppose that an agent who has a job expects to receive a bundle $(u_H, l)$ if he reports $H$, and a bundle $(u_L)$ if he reports $L$. This agent reveals some information, i.e. chooses $\sigma > 0$, only if $(u_H, u_L, \sigma)$ satisfies

$$u_H \geq u_L, \ (1 - \sigma)[u_H - u_L] = 0. \tag{1}$$

The first equation is the usual incentive constraint familiar from perfect commitment problems. The second constraint says that an agent is willing to randomize between $H$ and $L$ only if he receives the same utility from the two reports.

Let $x \equiv (u_H, u_L, l, \sigma)$ be a bundle of allocations and reporting probability and let $X$ be the space of all such bundles where (i) $(u_H, l)$ and $(u_L, 0)$ lie in the domain of $C$, (ii) $\sigma \in [0, 1]$, and (iii) $(u_H, u_L, \sigma)$ satisfies (1). We endow $X$ with the Borel $\sigma$-algebra. As a shortcut for the requirement (i), we define the set $\mathcal{C} \subset \mathbb{R}^3$ consisting of all triples $(u_H, u_L, l)$ such that both $(u_H, l)$ and $(u_L, 0)$ lie in the domain of $C$. Convexity of $C$ implies that $\mathcal{C}$ is a convex set. Equilibrium outcomes can then be represented as a probability measure on $X$.

Let $\psi$ be any measure on $X$ that represents stage-0 announcements by the government and stage-1 reporting probabilities. Let $\sigma(x)$ be the reporting probability $\sigma$ that corresponds to point $x \in X$. If the government decides to break its promises in stage 2, it can achieve a payoff $\tilde{W}(\psi)$ that is given by

$$\tilde{W}(\psi) \equiv \max_{\{\tilde{u}_H(x), \tilde{u}_L(x), \tilde{l}(x)\}_{x \in X}} \int \left[ p\sigma(x)\tilde{u}_H(x) + (1 - p\sigma(x))\tilde{u}_L(x) \right] \psi(dx) - \Upsilon, \qquad (2)$$

subject to

$$\int \left[ p\sigma(x)\left( C\left(\tilde{u}_H(x), \tilde{l}(x)\right) - \tilde{l}(x) \right) + (1 - p\sigma(x))C\left(\tilde{u}_L(x), 0\right) \right] \psi(dx) \leq 0,$$

and $\left(\tilde{u}_H(x), \tilde{u}_L(x), \tilde{l}(x)\right) \in \mathcal{C}$ a.s.

If the government breaks promises in stage 2, its allocation choices $\left(\tilde{u}_H(x), \tilde{u}_L(x), \tilde{l}(x)\right)$ do not need to be incentive compatible and satisfy (1). Although the government can in principle choose different bundles $\left(\tilde{u}_H(x), \tilde{u}_L(x), \tilde{l}(x)\right)$ for different $x$, it is immediate to see that it is not optimal to do so, and the solution $\left(\tilde{u}_H^*(x), \tilde{u}_L^*(x), \tilde{l}^*(x)\right)$ to this problem does not depend on $x$. We thus drop $x$ from the notation. Supermodularity of $C$ implies $\tilde{u}_H^* < \tilde{u}_L^*$, so that agents who revealed information are strictly worse off if stage-0 promises are broken. It then follows that the government never breaks stage-0 promises in equilibrium.[5]

We can now define our notion of equilibrium. To make notation compact, for any $x = (u_H, u_L, l, \sigma)$ define function $g(x)$ and $f(x)$ as

$$g(x) \equiv p\sigma u_H + (1 - p\sigma)u_L$$

and

$$f(x) \equiv p\sigma\left\{C(u_H, l) - l\right\} + (1 - p\sigma)C(u_L, 0).$$

Function $g$ captures expected utility that an agent receives from bundle $x$ and $f$ is the net resource cost of that bundle. A *perfect Bayesian equilibrium (PBE)* is a measure $\psi$ on $X$ such

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[5] To see this, suppose agents expected the government to break its promises. Then they would all choose the reporting strategy $\sigma = 0$ in stage 1, leaving the government with the only option of allocating zero consumption and labor to everyone. However, the government can always achieve the same outcome and, in addition, save on the cost $\Upsilon$ by simply offering zero consumption and labor in stage 0. Therefore, in equilibrium stage-0 promises are never broken.

that allocations are feasible, $\int f d\psi \leq 0$, and sustainable, $\int g d\psi \geq \tilde{W}(\psi)$. A measure $\psi^*$ on $X$ is a *best PBE* if it is a PBE and there is no other PBE that gives a weakly higher utility to all agents and a strictly higher utility to a positive mass of agents. Since in our baseline economy all agents are ex-ante identical, any best PBE is a PBE that delivers the highest expected utility to the representative agent, i.e. that is a solution to

$$\max_{\psi:\int f d\psi \leq 0, \int g d\psi \geq \tilde{W}(\psi)} \int g d\psi. \tag{3}$$

Our focus is on characterizing properties of best equilibria. One considerable difficulty arises from the fact that $\tilde{W}(\psi)$ is, in general, a complicated function of $\psi$. A useful intermediate step is to show that the maximization problem (3) can be replaced with an auxiliary problem that is more amenable to analytical analysis.

**Lemma 2** *Let $\psi^*$ be some best PBE. Then there exists a scalar $\lambda^* \geq 0$ that defines a function $W(x)$ given by*

$$W(x) \equiv \max_{(\tilde{u}_H, \tilde{u}_L, \tilde{l}) \in \mathcal{C}} \quad [p\sigma(x)\tilde{u}_H + (1 - p\sigma(x))\tilde{u}_L]$$

$$- \lambda^* \left[ p\sigma(x)\left(C\left(\tilde{u}_H, \tilde{l}\right) - \tilde{l}\right) + (1 - p\sigma(x))C(\tilde{u}_L, 0)\right] - \Upsilon,$$

*such that $\psi^*$ is a solution to*

$$\max_{\psi:\int f d\psi \leq 0, \int(g-W) d\psi \geq 0} \int g d\psi. \tag{4}$$

*Conversely, any $\psi^{**}$ that solves (4) is a best PBE.*

**Proof.** Problem (2) defines a maximization of a concave function over a convex set, thus, its solution is characterized by a saddle point Lagrangian (Luenberger (1969), Theorem 1, p. 224):

$$\tilde{W}(\psi) = \min_{\lambda \geq 0} \max_{\substack{\{\tilde{u}_H(x), \tilde{u}_L(x), \tilde{l}(x)\}_{x \in X} \\ (\tilde{u}_H(x), \tilde{u}_L(x), \tilde{l}(x)) \in \mathcal{C} \text{ a.s.}}} \int \left\{ \begin{array}{c} [p\sigma(x)\tilde{u}_H(x) + (1 - p\sigma(x))\tilde{u}_L(x)] - \Upsilon \\ -\lambda \left[ \begin{array}{c} p\sigma(x)\left(C\left(\tilde{u}_H(x), \tilde{l}(x)\right) - \tilde{l}(x)\right) \\ + (1 - p\sigma(x))C(\tilde{u}_L(x), 0) \end{array} \right] \end{array} \right\} \psi(dx)$$

$$\leq \int \max_{(\tilde{u}_H, \tilde{u}_L, \tilde{l}) \in \mathcal{C}} \left\{ \begin{array}{c} [p\sigma(x)\tilde{u}_H + (1 - p\sigma(x))\tilde{u}_L] \\ -\lambda^* \left[ p\sigma(x)\left(C\left(\tilde{u}_H, \tilde{l}\right) - \tilde{l}\right) + (1 - p\sigma(x))C(\tilde{u}_L, 0)\right] - \Upsilon \end{array} \right\} \psi(dx),$$

where $\lambda^*$ is the Lagrange multiplier corresponding to $\psi^*$. Moreover, this inequality holds with equality at $\psi = \psi^*$. Using this multiplier $\lambda^*$ in the definition of $W(x)$, we obtain $\tilde{W}(\psi) \leq \int W d\psi$ for all $\psi$, with equality at $\psi = \psi^*$. In turn, the latter implies that the

constraint set in the maximization problem (4) is contained in the constraint set of problem (3). Since $\psi^*$ maximizes (3) and lies in the smaller constraint set of problem (4), it must also maximize (4).

Conversely, any $\psi^{**}$ that maximizes (4) is feasible in problem (3) and attains the same payoff as $\psi^*$, therefore, it must be a best PBE. ∎

It is immediate to verify the following useful property of $W$.

**Corollary 1** $W(x)$ *depends on* $\sigma(x)$ *but not on other elements of* $x$.

The maximization problem (4) can then be written in the Lagrangian form as

$$\max_{\psi} \int [g - \zeta f - \chi W] \, d\psi \tag{5}$$

for some multipliers $\zeta > 0$ and $\chi \geq 0$.[6] Since this is a linear problem, $\psi^*$ must assign a positive measure only to those $x^*$ that maximize $g(x) - \zeta f(x) - \chi W(x)$. Such $x^*$ can be found using a two-step procedure. First, we can find optimal allocations $(u_{H,\sigma}, u_{L,\sigma}, l_\sigma)$, for any given reporting strategy $\sigma$. These allocations are solutions to

$$\kappa(\sigma) \equiv \max_{u_H, u_L, l} \; p\sigma u_H + (1 - p\sigma) u_L - \zeta \left[ p\sigma \left( C(u_H, l) - l \right) + (1 - p\sigma) C(u_L, 0) \right],$$

subject to $(u_H, u_L, l) \in \mathcal{C}$ and (1). Second, any optimal reporting strategy $\sigma^*$ can be found by solving

$$\max_{\sigma \in [0,1]} \kappa(\sigma) - \chi W(\sigma). \tag{6}$$

Equation (6) has a natural interpretation. Function $\kappa$ captures the benefits of information revelation in providing insurance, while function $\chi W$ captures the costs arising from imperfect commitment. The difference between these benefits and costs is the value of information; such value is maximized at $\sigma^*$. The next proposition provides insights about these benefits and costs, and derives implications about optimal information revelation.[7] We will encounter these insights throughout our paper.

**Proposition 1** $\kappa$ *is strictly increasing, convex and not linear.* $W$ *is strictly increasing and linear. Therefore, the optimal* $\sigma^*$ *satisfies* $\sigma^* \in \{0, 1\}$. *When both* $\sigma^* = 0$ *and* $\sigma^* = 1$ *are optimal, corresponding allocations are unique and satisfy* $u_{L,0} > u_{L,1} = u_{H,1}$.

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[6] Linearity of (4) and monotonicity of utility in consumption imply that there exist Lagrange mulipliers $\zeta^* > 0$ and $\chi^* \geq 0$ such that $\psi^*$ solves $\max_\psi \int [(1 + \chi^*) g - \zeta^* f - \chi^* W] \, d\psi$ with $\zeta^* \int f d\psi^* = 0$ and $\chi^* \int (g - W) \, d\psi^* = 0$. The equation in the text is obtained by dividing the objective function under the max operator by $1 + \chi^*$ and defining $\zeta \equiv \zeta^* / (1 + \chi^*)$, $\chi \equiv \chi^* / (1 + \chi^*)$.

[7] We thank an anonymous referee for suggesting the simple proof of the proposition.

**Proof.** The incentive constraint (1) must bind for $\sigma = 1$, therefore, we can write $\kappa$ as

$$\kappa(\sigma) = \max_{u,l:(u,u,l)\in\mathcal{C}} u - \zeta\left[p\sigma\left(C\left(u,l\right) - l\right) + \left(1 - p\sigma\right)C\left(u,0\right)\right]. \tag{7}$$

Constraint $(u, u, l) \in \mathcal{C}$ can be written as $U(0,0) \leq u$, $u \leq U(\infty, l)$, $l \geq 0$, and, since $C(U(\infty, l), l) = \infty$ and $C_2(u, 0) = 0$, the last two inequalities must be slack.

Let $(u_\sigma, l_\sigma)$ be the solution to this problem for a given $\sigma$, which must be unique for each $\sigma$ due to strict convexity of $C$. The optimality condition for $l$ is $C_2(u_\sigma, l_\sigma) = 1$, which implies that $l_\sigma > 0$. $\kappa$ is differentiable and by the envelope theorem its derivative satisfies

$$\kappa'(\sigma) = \zeta p\left[-\left(C\left(u_\sigma, l_\sigma\right) - l_\sigma\right) + C\left(u_\sigma, 0\right)\right].$$

This expression implies that $\kappa'(\sigma) > 0$ because $l_\sigma$ is the maximizer of $-(C(u_\sigma, l) - l)$, and therefore $\kappa$ is strictly increasing.

We now rule out several corner solutions. Note that best PBEs must involve some production. Otherwise, all agents would receive the allocation $(u, l) = (U(0,0), 0)$, the sustainability constraint $\int g d\psi^* \geq \tilde{W}(\psi^*)$ would be slack (due to $\Upsilon > 0$), and there would exist some other $\psi^{**}$ giving higher welfare. This implies that there exist at least one $\bar{\sigma} > 0$ that solves (6), with corresponding $u_{\bar{\sigma}} > 0$. Uniqueness of the solution for all $\sigma$ and the theorem of maximum implies $u_\sigma > 0$ for all $\sigma$ in the neighborhood of $\bar{\sigma}$.

The optimality condition for $u_\sigma$ is

$$1 \leq \zeta\left[p\sigma C_1\left(u_\sigma, l_\sigma\right) + \left(1 - p\sigma\right)C_1\left(u_\sigma, 0\right)\right], \tag{8}$$

with equality if $u_\sigma > U(0,0)$. Equation (8) can hold with equality for two distinct $\sigma_1, \sigma_2$ only if $(u_{\sigma_1}, l_{\sigma_1}) \neq (u_{\sigma_2}, l_{\sigma_2})$ since supermodularity of $C$ implies that $C_1(u_\sigma, l_\sigma) > C_1(u_\sigma, 0)$ for all $\sigma$. Therefore $(u_\sigma, l_\sigma)$ must all be distinct in the neighborhood of $\bar{\sigma}$.

For any $\sigma_1$ and $\sigma_2$, we must have

$$\begin{aligned}\kappa(\sigma_2) &\geq u_{\sigma_1} - \zeta\left[p\sigma_2\left(C\left(u_{\sigma_1}, l_{\sigma_1}\right) - l_{\sigma_1}\right) + \left(1 - p\sigma_2\right)C\left(u_{\sigma_1}, 0\right)\right] &(9)\\ &= \kappa(\sigma_1) + \kappa'(\sigma_1)(\sigma_2 - \sigma_1) \text{ for all } \sigma_1, \sigma_2,\end{aligned}$$

with strict inequality if $(u_{\sigma_1}, l_{\sigma_1}) \neq (u_{\sigma_2}, l_{\sigma_2})$. This implies that $\kappa$ is convex, and strictly convex in the neighborhood of $\bar{\sigma}$ (see Section 3.1.3 in Boyd and Vandenberghe (2004)).

Direct extension of these arguments establishes that $W$ is strictly increasing and linear in $\sigma$. Therefore equation (6) either has a unique solution $\sigma^* = 1$, or has two solutions, $\sigma^* = 0$ and $\sigma^* = 1$. We now show that in the latter case $u_0 > u_1$. Suppose $u_0 \leq u_1$, which also implies that

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$C_1(u_0, 0) < C_1(u_1, l_1)$. Since $u_0$ satisfies $1 \leq \zeta C_1(u_0, 0)$, this implies that inequality in (8) for $\sigma = 1$ is strict, and therefore $U(0,0) = u_1 = u_0$, which is impossible. Therefore $u_0 > u_1$. ∎

This proposition offers several insights about costs and benefits of information revelation. Both are increasing in $\sigma$, so better information allows the government to achieve higher welfare on the equilibrium path but also makes it more tempting to deviate. The crucial distinction is that costs increase linearly in $\sigma$ while the benefits are, for the interior solution, strictly convex. Strict convexity emerges due to the incentive constraints. In order to incentivize agents to reveal information, the government needs to distort their allocations. Welfare losses from those distortions depend on the mass of agents who receive distorted allocations. As more information is being revealed, the government can adjust distortions to increase the marginal value of an extra unit of information.

Our result about convexity of the value of information is reminiscent of a well-known result in Radner and Stigliz (1984). These authors study an abstract economy with an exogenous information structure and exogenous costs of information acquisition. They observe that if one assumes that the marginal informativeness of signals is zero when the principal is completely uninformed then the net benefits from information acquisition are locally U-shaped in the amount of information acquired (see also Chade and Schlee (2002) for a discussion). Our value of information function $\kappa - \chi W$ is similarly U-shaped but for completely different reasons. The marginal informativeness of signals is strictly positive in our economy (i.e. $\kappa'(\sigma) > 0$ for all $\sigma$) and strict convexity emerges endogenously due to the need to provide incentives to reveal information. As a consequence, we have a global rather than a local result and we fully characterize the optimal $\sigma^*$.

Proposition 1 provides characterization of the optimal information revelation. It shows that some information revelation is always optimal. When full information revelation is infeasible, then both $\sigma^* = 1$ and $\sigma^* = 0$ must be optimal. Therefore, it is efficient to randomly separate agents into two groups. Agents in one of these groups reveal all their information, while agents in the other group reveal no information. Moreover, agents assigned to the second group are better off than those in the first.

We now describe conditions under which full information revelation is infeasible and provide comparative statics with respect to the commitment cost $\Upsilon$. Let $\varsigma^*(\Upsilon)$ be the fraction of agents with job $a$ opportunity that reveal their information to the government in the best PBE, for a given level of $\Upsilon$. Let $U^{fb}$ and $U^{sb}$ be first- and second-best welfare levels, i.e. the welfare levels that can be achieved under perfect commitment, when the planner can observe the arrival of job offers ("first-best") and when it cannot ("second-best"). Let $\bar{\Upsilon} \equiv U^{fb} - U^{sb}$. It is easy to

show that $\bar{\Upsilon} > 0$.

**Corollary 2** *Sustainability constraint $\int g d\psi \geq \tilde{W}(\psi)$ binds, both $\sigma^* = 0$ and $\sigma^* = 1$ are optimal, and some agents do not reveal their information if and only if $\Upsilon < \bar{\Upsilon}$. $\varsigma^*(\Upsilon)$ is strictly increasing in $\Upsilon$ when $\Upsilon \leq \bar{\Upsilon}$, and equal to 1 for $\Upsilon \geq \bar{\Upsilon}$.*

The formal proof of this corollary requires additional notation and we relegate it to the Online appendix. The intuition for this result is quite simple. If all agents reveal their information fully, the government can achieve the second best welfare level $U^{sb}$. This is sustainable if and only if $\Upsilon \geq \bar{\Upsilon}$. Therefore, when $\Upsilon < \bar{\Upsilon}$, some agents should not reveal their information. Proposition 1 implies that in this case both $\sigma^* = 0$ and $\sigma^* = 1$ are optimal and $\varsigma^*(\Upsilon) < 1$. Since on-the-equilibrium path welfare is strictly increasing in the fraction of agents revealing their information and the sustainability constraint binds for all $\Upsilon \in (0, \bar{\Upsilon})$, function $\varsigma^*(\Upsilon)$ must be strictly increasing on $(0, \bar{\Upsilon})$.

## 2.1 Discussions

### 2.1.1 Heterogeneity and the need for randomization

We chose our baseline environment to illustrate the key mechanism in the most direct way: all agents were ex-ante identical and the Pareto frontier of the set of the PBEs was just one point. Agents' homogeneity implied that the only way to provide them with different incentives was to condition allocations on the realization of sunspot variables $z$.

Sunspot variables can be re-interpreted as any idiosyncratic characteristic of the agent that is not directly related to the insurance provision. Consider our baseline environment but suppose that agents are heterogenous with respect to some characteristic $\xi$, drawn from a continuous probability distribution. To abstract from any additional effect that such characteristic may have on incentive provision, we assume that it is payoff irrelevant. Consider welfare in any PBE that maximizers the sum of agents' expected utilities. It is easy to see that this welfare level coincides with welfare in best PBEs of our baseline economy; moreover, it can be attained by conditioning government strategies on characteristic $\xi$ alone, without using the public randomization device. We show this formally in the Online appendix, where we also extend our analysis to any point on the Pareto frontier of the set of PBEs of this heterogeneous-agent economy.

### 2.1.2 Commitment costs

In our analysis we have assumed that the commitment cost $\Upsilon$ is constant and does not depend on the deviation. Our results hold under much weaker assumptions. Suppose that commitment costs are given by an arbitrary function $\Upsilon(u - \tilde{u})$ that depends on the difference in the utility that an agent was promised in stage 0, $u$, and the utility he eventually receives in stage 2, $\tilde{u}$. We assume that $\Upsilon$ is non-negative, convex and differentiable with $\Upsilon(0) = 0$. Let $(u_H(x), u_L(x), l(x), \sigma(x))$ be the tuple associated with any given $x$. The maximization problem (2) now becomes

$$\tilde{W}(\psi) = \max_{\{\tilde{u}_H(x), \tilde{u}_L(x), \tilde{l}(x)\}_{x \in X}} \int \left[ \begin{array}{c} p\sigma(x)\{\tilde{u}_H(x) - \Upsilon(u_H(x) - \tilde{u}_H(x))\} \\ + (1 - p\sigma(x))\{\tilde{u}_L(x) - \Upsilon(u_L(x) - \tilde{u}_L(x))\} \end{array} \right] \psi(dx),$$

subject to $(\tilde{u}_H(x), \tilde{u}_L(x), \tilde{l}(x)) \in \mathcal{C}$ a.s. and the feasibility constraint. We construct the bound $\int W d\psi$ on this best deviation as in Lemma 2, and verify that $W(x)$ is linear in $\sigma(x)$ (although $W(x)$ may now also depend on $u_H(x), u_L(x)$). Incentive constraints imply that the value of information $\kappa - \chi W$ is still a convex function of $\sigma$, and therefore that the insights about optimal information revelation in Proposition 1 continue to hold. See the Online appendix for details.

### 2.1.3 Moral hazard

In our baseline economy we also assumed that job opportunities arrive with an exogenous probability $p$. It is easy to introduce moral hazard and extend our analysis to the case when this probability is endogenous. Suppose that probability of job arrival is a function of unobservable effort that can take values 0 or 1. Exerting effort carries an additive utility cost $e$. A job offer arrives with probability $p$ if effort is exerted and with probability 0 otherwise. The rest of the set up is exactly as in our baseline economy.

Since effort is costly, conditional on exerting effort, it is always optimal to reveal the arrival of a job opportunity to the government. The key friction is then about the probability with which effort is exerted. The incentive constraint (1) now reads

$$u_H \geq u_L + e/p, \ (1 - \sigma)[u_H - u_L - e/p] = 0,$$

where $\sigma$ denotes the probability with which effort is exerted. It is easy to verify (see the Online appendix) that our arguments continue to apply in these settings with only minor modifications.

### 2.1.4 Decentralization

The optimal allocations described in Proposition 1 can be achieved by a two-tier insurance system, where one tier provides relatively poor benefits and possibly contains additional incentives to search for a job, while the other tier has more generous benefits but access to them is limited. One specific implementation in a competitive equilibrium can be as follows.

Proposition 1 and Corollary 2 imply that best equilibrium allocations consists of a tuple $(u^*_{L,0}, u^*_{L,1}, u^*_{H,1}, l^*_1, \varsigma^*)$, where $\varsigma^*$ is the fraction of agents with job opportunities who reveal their information in equilibrium and work. Define benefits and taxes as $b^{DI} \equiv C(u^*_{L,0}, 0)$, $b^{UI} \equiv C(u^*_{L,1}, 0)$, and $\tau \equiv l^*_1 - C(u^*_{H,1}, l^*_1)$. Note that $b^{DI} > b^{UI}$ since $u^*_{L,0} > u^*_{L,1}$. To implement these allocations as a competitive equilibrium, we follow the interpretation of Section 2.1.1 and assume that agents are heterogeneous in some characteristic $\xi$, which we call "severity of back pain" for concreteness. We let $\bar{\xi}$ be the threshold of back pain given by $p \Pr\left(\xi \leq \bar{\xi}\right) = \varsigma^*$.

The two-tier system is described as follows. Any person who works pays a lump sum tax $\tau$. Any person who does not work qualifies for the unemployment benefit (UI) $b^{UI}$. Moreover, any agent with back pain above the threshold $\bar{\xi}$ is eligible for the disability benefit (DI) $b^{DI}$, which replace the UI benefit, provided that such agent does not work. In the Online appendix, we provide a formal definition of competitive equilibrium and prove that equilibrium allocations coincide with those of the best PBE. The intuition for this result is straightforward. Since the DI benefit provides the highest utility level, all eligible agents choose to take $b^{DI}$. The remaining agents with job opportunities choose to work since unemployment benefits are incentive compatible. Corollary 2 implies that the threshold $\bar{\xi}$ increases in $\Upsilon$, and the second, DI, tier becomes redundant for $\Upsilon \geq \bar{\Upsilon}$.

This two-tier system is similar to the way unemployment and disability insurance operate in practice. In developed countries a person who experiences an involuntary job loss is usually eligible for unemployed benefits. UI benefits are generally temporary and often come with additional stipulations, such as the requirement that recipients be actively engaged in job search. DI benefits, instead, are long-lasting and offer weak incentives for returning to employment.[8] Eligibility for DI is limited to those with certain medical conditions.

Although initially envisioned as a support system for those who are severely incapacitated to be able to perform any work, modern DI has much weaker eligibility requirements. In the U.S., for example, 59% of DI applicants fall into one of two categories: musculoskeletal conditions and mental disorders (Maestas, Mullen, and Strand (2013)). Typical ailments in

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[8]In the U.S. less than 1 percent of those who start receiving disability benefits ever return to the labor force (Bound and Burkhauser (1999)).

those categories are back pain and mental stress which are difficult to verify medically, let alone ascertain whether they prevent a person from being able to work.

As a result, in practice there is a significant degree of substitution between DI and UI programs among many applicants. Autor and Duggan (2003) show that the reduction in stringency of medical screening in the U.S. between 1984 and 2001 caused a decrease in the measured unemployment rate by diverting some workers into disability programs. Numerous papers have documented how exogenous economic shocks – that are orthogonal to health conditions – lead to higher applications and awards of DI benefits. Maestas, Mullen, and Strand (2018) estimate that 8.9% of all awards of Social Security Disability Insurance (SSDI) benefits during 2008-2012 was directly induced by the Great Recession. Black, Daniel, and Sanders (2002) show that the collapse in coal prices in the 1980s led to a significant increase in the number of DI beneficiaries in those counties in the U.S. that were more exposed to the coal industry. Bratsberg, Fevang, and Roed (2013) use Norwegian administrative data and records on mass layoffs in bankruptcy procedures to show that such layoffs more than double the probability of claiming permanent disability. Other studies that document a close substitution between DI and UI utilization include Rege, Telle, and Votruba (2009), Maestas, Mullen, and Strand (2015), Charles, Li, and Stephens (2018), Andersen, Markussen, and Roed (2019).

This evidence is often interpreted as showing that disability benefits are poorly designed as they allow people who are not "truly disabled" into the DI system. Our model suggests a different perspective. It is efficient ex-ante, but costly ex-post, to provide unemployed agents with incentives to search for jobs, thus, UI systems require policymakers to have strong commitment power. If the ability of the political system to commit to such policies is imperfect, it may be more desirable to let some of the unemployed exit the labor force, for example by receiving disability benefits.

## 3   An infinitely repeated game

In this section we extend our analysis to dynamic interactions between agents and the government in an infinite horizon economy. Past idiosyncratic histories of shocks induce heterogeneity across agents and we study which agents should reveal information. Infinitely repeated games are also a natural device for modelling commitment costs.

Consider a simple versions of a dynamic Mirrleesian economy in the spirit of Golosov, Kocherlakota, and Tsyvinski (2003) and Atkeson and Lucas (1995). The economy is populated by a continuum of infinitely-lived ex-ante identical agents and a government. In every

period, each agent receives a privately-observed job opportunity with probability $p$. The job opportunity allows an agent to convert labor into output. The government controls the production process as in Section 2. Agents' period utility $U$ satisfies Assumption 1 and, in addition, is bounded. We normalize $U(0,0) = 0$. The agent's discount factor is $\beta \in (0,1)$ and the government is utilitarian.

The agents and the government play an infinitely repeated game. Each period $t$ is divided in two stages:

1. Each agent learns his job status and draws a random publicly-observed signal $z$ from a uniform distribution. An agent who finds a job chooses the probability $\sigma$ with which he reveals this information by reporting $H$; he reports $L$ otherwise. A jobless agent reports $L$ with probability 1.

2. The government chooses allocations $(c, l)$ for all agents, with $l = 0$ if the agent reported to be jobless, subject to the constraint that total consumption cannot exceed total output.

When submitting a report in stage 1, each agent observes his past reports, his current and past sunspots and job offers, as well as past government actions and past distributions of other agents' reports. The allocation that an agent receives in stage 2 is a function of his current and past reports and sunspots, as well as past government actions and current and past distributions of other agents' report.

The structure of this game is standard and has been used extensively in the macroeconomic literature to study interactions between agents and policymakers in models with limited commitment, at least since the seminal work of Chari and Kehoe (1990). Since such models are well understood, we only sketch the equilibrium structure.[9] All agents are infinitesimal, thus, actions of any given agent do not affect aggregate equilibrium outcomes. The government is a large player, its actions are publicly observed, and agents' strategies are conditioned on them. There is no aggregate uncertainty and the distribution of reports and allocations is deterministic along the equilibrium path.

A PBE consists of strategies of the agents and the government and posterior beliefs such that each player chooses his best response given his posterior beliefs formulated using Bayes' rule. We focus on the PBEs that give the highest ex-ante utility to the agents, i.e. best PBEs.

Standard arguments imply that best equilibria are supported by reverting to a worst PBE – i.e. a PBE that gives the lowest utility to the government – following any deviation by the government from its prescribed equilibrium behavior. It is easy to see that no information is

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[9]The Online appendix contains all the formal definitions and the details of the derivations.

revealed and all players receive a payoff of 0 in any worst equilibrium. This equilibrium is supported by the government's strategy of choosing the best static allocations for any distribution of reports it receives, and by the agents' strategy of reporting $L$ with probability 1. No other PBE can give a lower payoff to the government since the government can always obtain a payoff of 0 for any strategy of the agents by simply ignoring their reports.

We now turn to the characterization of best equilibria. Standard arguments show that agents' past history of shocks and reports can be summarized by the continuation utility $w$, which represents their expected utility from playing equilibrium reporting strategies going forward. It is without loss of generality to restrict attention to equilibria in which agents' strategies depend on their past histories only through $w$. To characterize agents' best responses it is sufficient to restrict attention to one-shot deviations, that is, to deviations from equilibrium strategies that occur only once. These properties are standard (e.g. Atkeson and Lucas (1992)) and we leave their proofs to the Online appendix.

Given these observations, an agent will find it optimal to report $H$ with probability $\sigma$ if his current and continuation utilities from reporting $H$ and $L$ satisfy

$$u_H + \beta w_H \geq u_L + \beta w_L, \ (1 - \sigma)\left[(u_H + \beta w_H) - (u_L + \beta w_L)\right] = 0. \tag{10}$$

We extend the definitions of point $x$ and space $X$ in Section 2 to include $(w_H, w_L)$, where $w_H, w_L \in [0, \bar{U}/(1-\beta))$, with $\bar{U}$ denoting the least upper bound of $U$. Notice that continuation utilities are bounded below by 0 since an agent can always claim to be jobless and receive a period utility which is at least 0. Similarly to the static model, equilibrium strategies are represented by sequences of probability distributions $\left\{\psi_{t,w}\right\}_{t,w}$ over the space $X$. To deliver continuation $w$ they must satisfy

$$w = \int \left[p\sigma\left(u_H + \beta w_H\right) + (1 - p\sigma)\left(u_L + \beta w_L\right)\right] d\psi_{t,w} \text{ for all } t, w. \tag{11}$$

The distribution of continuation utilities at time $t$, denoted by $\pi_t$, is defined recursively through

$$\pi_t\left(w\right) = \int \left[p\sigma \mathcal{I}_{w_H \leq w} + (1 - p\sigma)\mathcal{I}_{w_L \leq w}\right] d\psi_{t,\tilde{w}} d\pi_{t-1} \text{ for } t \geq 0, \tag{12}$$

starting from some initial distribution $\pi_{-1}$, where $\mathcal{I}_{\hat{w} \leq w}$ is an indicator variable that equals 1 when $\hat{w} \leq w$. Allocations are feasible if

$$\int \int f d\psi_{t,w} d\pi_{t-1} \leq 0, \text{ for all } t, \tag{13}$$

where $f(x)$ is the net resource cost of $x$ defined in Section 2.

To characterize the government's best response one needs to verify that the payoff from deviating in any period $t$, which is then followed by a reversion to the worst PBE, is not higher

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than the payoff obtained on the equilibrium path. In any given period $t$, the government faces a distribution of continuation utilities $\pi_{t-1}$, and receives information from the agents captured by $\{\psi_{t,w}\}_w$. The highest payoff that the government can obtain in period $t$ is given by

$$\tilde{W}\left(\{\psi_{t,w}\}_w, \pi_{t-1}\right) = \max_{\tilde{u}_H, \tilde{u}_L, \tilde{l}} \int \int \left[p\sigma(x)\tilde{u}_H + (1 - p\sigma(x))\tilde{u}_L\right]d\psi_{t,w}d\pi_{t-1},$$

subject to $(\tilde{u}_H, \tilde{u}_L, \tilde{l}) \in \mathcal{C}$ and

$$\int \left[p\sigma(x)\left(C\left(\tilde{u}_H, \hat{l}\right) - \hat{l}\right) + (1 - p\sigma(x))C(\tilde{u}_L, 0)\right]d\psi_{t,w}d\pi_{t-1} \leq 0.$$

Strictly speaking, $(\tilde{u}_H, \tilde{u}_L, \tilde{l})$ can vary with $x$, however, it is easy to see that the government will assign the same $(\tilde{u}_H, \tilde{l})$ and $\tilde{u}_L$ to all agents reporting $H$ and $L$, respectively.

The government's best response constraint is

$$\sum_{s=t}^{\infty}\beta^{s-t}\int\int g d\psi_{s,w}d\pi_{s-1} \geq \tilde{W}\left(\{\psi_{t,w}\}_w, \pi_{t-1}\right) \text{ for all } t, \tag{14}$$

where $g$ is the function defined in Section 2. Therefore, $\{\psi_{t,w}^*\}_{t,w}$ is a *best PBE* if it solves

$$\max_{\{\psi_{t,w}\}_{t,w}}\sum_{t=0}^{\infty}\beta^t\int\int g d\psi_{t,w}d\pi_{t-1}, \tag{15}$$

subject to (11), (12), (13) and (14).

## 3.1 Characterization

The sustainability constraint (14) presents a considerable difficulty because the value of the best deviation $\tilde{W}$ is, in general, a complicated function of $(\{\psi_{t,w}\}_w, \pi_{t-1})$. This difficulty can be overcome following the same arguments as in the static model of Section 2. More specifically, a direct extension of Lemma 2 shows that there exists a sequence of functions $\{W_t(\cdot)\}_t$ such that any $\{\psi_{t,w}^*\}_{t,w}$ that maximizes (15), subject to (11), (12), (13) and

$$\sum_{s=t}^{\infty}\beta^{s-t}\int\int g d\psi_{s,w}d\pi_{s-1} \geq \int\int W_t d\psi_{t,w}d\pi_{t-1} \text{ for all } t, \tag{16}$$

is a best PBE. Moreover, $W_t$ can be constructed so that it is linear in $\sigma(x)$ and independent of the other elements of $x$.

The modified maximization problem is amenable to analytical characterization using techniques developed by Farhi and Werning (2007). In particular, we first set up the following Lagrangian:

$$\mathcal{L} = \max_{\{\psi_{t,w}, \pi_{t-1}\}_{t,w}}\sum_{t=0}^{\infty}\bar{\beta}_t\left[g - \zeta_t f - \chi_t W_t\right]d\psi_{t,w}d\pi_{t-1},$$

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subject to (11) and (12). Here, $\bar{\beta}_t$, $\zeta_t$, $\chi_t$ are all functions of the multipliers on (13) and (16), re-organized in a more compact way.[10] We must have $\chi_t \geq 0$, $\zeta_t > 0$ and $\hat{\beta}_t \equiv \bar{\beta}_t / \bar{\beta}_{t-1} \geq \beta$, with equality if and only if $\chi_t = 0$.

Any best PBE must solve this Lagrangian. The converse does not need to be true, as there might be some solutions to the Lagrangian that do not satisfy constraints (13) and (16). However, any solution to the Lagrangian that also satisfies (13) and (16) is a best PBE.

The Lagrangian has a nice linear structure, which greatly simplifies our analysis. We can solve our problem by breaking it into simpler component problems, where each component problem studies the optimal way to deliver any continuation $w$ independently of all other continuation values. Formally, the efficient allocations and reporting probabilities that deliver a continuation $\tilde{w}$ in period $t$ are a solution to

$$K_t(\tilde{w}) \equiv \frac{1}{\bar{\beta}_t} \max_{\{\psi_{s,w}, \pi_{s-1}\}_{s,w}} \sum_{s=t}^{\infty} \bar{\beta}_s \left[g - \zeta_s f - \chi_s W_s\right] d\psi_{s,w} d\pi_{s-1}, \tag{17}$$

subject to (11) and (12), with $\pi_{t-1}(w) = 0$ if $w < \tilde{w}$ and $\pi_{t-1}(w) = 1$ if $w \geq \tilde{w}$. Using this notation, $\mathcal{L} = \max_w K_0(w)$. Moreover, the sequence of value functions $\{K_t\}_t$ has a useful recursive structure:

$$K_t(w) = \max_{\psi: \ (11) \ \text{holds}} \int \left[g - \zeta_t f - \chi_t W_t + \hat{\beta}_{t+1} K_{t+1}\right] d\psi, \tag{18}$$

where $K_{t+1}(x)$ on the right hand side is a shorthand for $p\sigma K_{t+1}(w_H) + (1 - p\sigma) K_{t+1}(w_L)$.

## 3.2 Optimal information revelation

Equation (18) is the dynamic analogue of (5). The main difference is the presence of the continuation value $w$, which was absent in the static setting where all agents were ex-ante identical. In this section we characterize dependence of optimal information revelation on $w$.

Let $\kappa_t(v, \sigma)$ be defined as

$$\kappa_t(v, \sigma) \equiv \max_{u_H, u_L, l, w_H, w_L} \quad p\sigma \left[u_H - \zeta_t(C(u_H, l) - l) + \hat{\beta}_{t+1} K_{t+1}(w_H)\right] \tag{19}$$

$$+ (1 - p\sigma) \left[u_L - \zeta_t C(u_L, 0) + \hat{\beta}_{t+1} K_{t+1}(w_L)\right],$$

subject to $(u_H, u_L, l) \in \mathcal{C}$, $w_H, w_L \in dom(K_{t+1})$, the incentive constraint (10) and

$$v = p\sigma(u_H + \beta w_H) + (1 - p\sigma)(u_L + \beta w_L).$$

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[10] If $\beta^t \zeta_t^*$ and $\beta^t \chi_t^*$ are the multipliers on (13) and (16), then $\bar{\beta}_t = \beta^t \left(1 + \sum_{s=0}^t \chi_s^*\right)$, $\zeta_t = \beta^t \zeta_t^* / \bar{\beta}_t$, and $\chi_t = \beta^t \chi_t^* / \bar{\beta}_t$.

Function $\kappa_t(v, \sigma)$ captures insurance benefits from $\sigma$ "units of information" for an agent who receives utility $v$. Commitment costs are captured by $\chi_t W_t(\sigma)$ and the net benefits are maximized by the value $\sigma_{t,v}$ that solves

$$k_t(v) \equiv \max_{\sigma \in [0,1]} \kappa_t(v, \sigma) - \chi_t W_t(\sigma).$$

This equation extends the expression for value of information (6) to the case when agents receive different expected utilities $v$.

It is clear from equation (18) that function $K_t$ is a concave envelope of $k_t$. Any best PBE $\psi_{t,w}^*$ assigns positive measure to some collection of $v'$s that deliver $w$ in expectation. The equilibrium reporting strategy for any given $v$ is given by $\sigma_{t,v}$. One of the main insights about optimal information revelation in the dynamic setting comes from studying the properties of function $\kappa_t$.

**Lemma 3** $\kappa_t$ is concave in $v$, increasing and convex in $\sigma$, and submodular in $(v, \sigma)$, for all $t$.

**Proof.** We sketch here the main arguments, all the details are left for the Online appendix. Observe, using equation (17), that $K_{t+1}$ is concave. This implies that $\kappa_t(\cdot, \sigma)$ is also concave and, furthermore, that at least one of its solutions, when $\sigma = 1$, satisfies the incentive constraint (10) with equality. Therefore, we can write $\kappa_t$ as

$$\kappa_t(v, \sigma) = \max_{u_H, u_L, l} \quad p\sigma \left[ u_H - \zeta_t \left( C(u_H, l) - l \right) + \hat{\beta}_{t+1} K_{t+1} \left( \frac{v - u_H}{\beta} \right) \right] \qquad (20)$$
$$+ (1 - p\sigma) \left[ u_L - \zeta_t C(u_L, 0) + \hat{\beta}_{t+1} K_{t+1} \left( \frac{v - u_L}{\beta} \right) \right],$$

subject to $(u_H, u_L, l) \in \mathcal{C}$ and $(v - u_H)/\beta, (v - u_L)/\beta \in dom(K_{t+1})$. Thus, $\kappa_t$ is convex (in fact, linear) in $\sigma$.

Let $f_t(u, l; v, \Delta) \equiv u - \zeta_t(C(u, l) - l) + \hat{\beta}_{t+1} K_{t+1}((v - u)/\beta)$ and

$$g_t(v, \Delta) \equiv \max_{u, l} \quad f_t(u, l; v, \Delta),$$

subject to $l \in [0, \Delta]$, $(u, l) \in dom(C)$ and $(v - u)/\beta \in dom(K_{t+1})$. Function $g_t$ is supermodular in $(v, -\Delta)$ by the results in Topkis (2011): *(i)* the constraint set is a sublattice in $s = (u, -l, v, -\Delta)$ (where the points $s$ have the usual ordering on $\mathbb{R}^4$) by Example 2.2.7(a); *(ii)* $f_t$ is supermodular in $(u, -l, v, -\Delta)$ on that sublattice by Theorem 2.6.3 because $C_{12} > 0$ and $K_{t+1}$ is concave; and, therefore, *(iii)* $g_t$ is supermodular in $(v, -\Delta)$ by Theorem 2.7.6. Using the envelope theorem, $\frac{\partial}{\partial \sigma} \kappa_t(v, \sigma) = p[g_t(v, \infty) - g_t(v, 0)] > 0$. Supermodularity of $g_t$ then implies that $\frac{\partial}{\partial \sigma} \kappa_t(v, \sigma)$ is decreasing in $v$, i.e. that $\kappa_t$ is submodular. ■
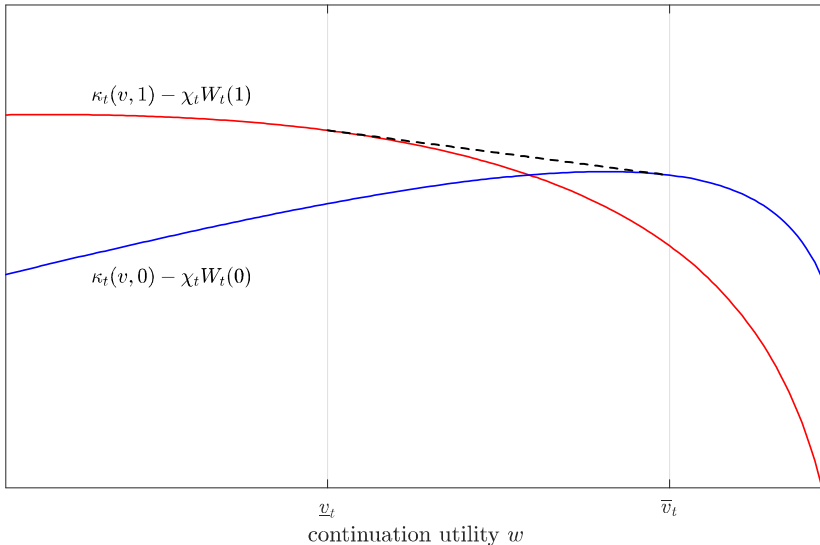
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Figure 1: Solid lines show the functions $\kappa_t(v,1) - \chi_t W_t(1)$ and $\kappa_t(v,0) - \chi_t W_t(0)$. The function $K_t$ is their concave evelope. It coincides with $\kappa_t(v,1) - \chi_t W_t(1)$ for $v \leq \underline{v}_t$, with the dashed line for $v \in (\underline{v}_t, \bar{v}_t)$, and with $\kappa_t(v,0) - \chi_t W_t(0)$ for $v \geq \bar{v}_t$.

A novel insight that emerges in the dynamic economy is that the marginal benefit of an addition unit of information is decreasing in $v$. When employed, agents with higher continuation utility work less due to normality of leisure. This means that they produce less output making it less valuable for the government to know whether they found work. Formally, this result manifests itself as submodularity of $\kappa_t$.

From here we can deduce optimal information revelation in equilibrium. Let $\sigma_{t,w}^*(z)$ be the optimal reporting strategy of an agent who enters period $t$ with continuation $w$ and observes sunspot value $z$. Unlike $\sigma_{t,v}$, which is a scalar, $\sigma_{t,w}^*(z)$ is a random variable since the agent might be randomly assigned to groups with different incentives to reveal information.

**Proposition 2** *The best equilibrium payoff can be achieved with reporting strategies $\sigma_{t,w}^*$ such that $\sigma_{t,w}^*(z) \in \{0,1\}$ for all $z$, and information revelation is decreasing in $w$, i.e. $\Pr(\sigma_{t,w}^* = 1)$ is decreasing in $w$.*

The proof is in the appendix but the main argument can be summarized graphically in Figure 1. This figure shows how the value of full information ($\sigma = 1$) and no information ($\sigma = 0$) vary with $w$. The value function $K_t(w)$ is the concave envelope of these two functions. Submodularity of $\kappa_t$ implies that

$$\{\kappa_t(\cdot,1) - \chi_t W_t(1)\} - \{\kappa_t(\cdot,0) - \chi_t W_t(0)\}$$

is a decreasing function, so that the relative value of full information falls with $w$.

Figure 1 also provides additional insights into how information is revealed. The space of continuation utilities can be partitioned into three regions, defined by cutoffs $\underline{v}_t \leq \bar{v}_t$. Any person with continuation utility $w < \underline{v}_t$ reveals his private information fully, any person with continuation $w > \bar{v}_t$ reveals no information at all. Agents whose continuation lies in the intermediate region $w \in [\underline{v}_t, \bar{v}_t]$, as long as it is non-degenerate, reveal full information and receive utility $\underline{v}_t$ with probability $\frac{\bar{v}_t - w}{\bar{v}_t - \underline{v}_t}$, and reveal no information and receive $\bar{v}_t$ with probability $\frac{w - \underline{v}_t}{\bar{v}_t - \underline{v}_t}$.

Dynamics of information revelation can also be understood by studying properties of (19). The optimality condition for continuation values is $K'_{t+1}(w_{H,t,v}) \leq K'_{t+1}(w_{L,t,v})$, which implies that $w_{H,t,v} \geq w_{L,t,v}$. In order to provide agents with incentives to reveal information the government needs to offer lower continuation values to those agents who report that they did not find a job. Thus, agents with a history of $L$ reports are more likely to be incentivized to reveal information than agents with a history of $H$ reports.

# 4    Extensions

So far we assumed that jobs last for one period. This assumption implies that private information about a job offer has value only for the period in which the offer was received. We now extend our analysis to a more general environment in which jobs are long-lasting. This environment encompasses the assumptions on persistence used in most studies of optimal unemployment insurance under perfect commitment. We then show how relaxing the perfect commitment assumption affects the design of the optimal UI system.

## 4.1    Persistent employment

We consider the following modification of our baseline economy. There is a continuum of agents who can be either employed or unemployed. Each period each unemployed receives a privately observed job offer with probability $p$. If the offer is accepted, he becomes employed and can transform one unit of labor into one unit of consumption; otherwise, he remains unemployed at the beginning of the next period. The worker who was employed in period $t$ either loses his job and becomes unemployed in period $t+1$ with probability $q \geq 0$, or retains it with probability $1 - q$. Job separations are publicly observable. In addition, every person exits the economy, or "dies", with probability $\delta \geq 0$ and a measure $\delta$ of newly born agents enter the economy as unemployed.

All allocations are provided by the government after it receives information from the unemployed about their job offers. Motivated by the literature on probabilistic voting we assume that preferences of the government in period $t$ are given by the sum of utilities of all agents alive in that period.[11] As in Section 3 we study properties of Pareto efficient subgame perfect equilibria. We focus on the point on the Pareto frontier in which the distribution of allocations is stationary and all newly born agents receive the same continuation utility irrespective of the period in which they were born. As in Farhi and Werning (2007), stationarity is achieved by assuming that initial continuation utilities of the agents alive in period $t = 0$ are drawn from an appropriately chosen distribution.

The assumptions about duration of private information made in a variety of models of unemployment insurance fit into this framework. The case $q = 1$, $\delta = 0$ corresponds to the economy considered by Atkeson and Lucas (1995). The case $q = 0$ is a common benchmark that was studied by Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), Chetty (2006), Pavoni and Violante (2007). The UI literature usually focuses on partial equilibrium settings in which the shadow cost of government funds is fixed. To be consistent with that assumption, in our fully specified economy we introduced the exit probability $\delta$ and the distribution of continuation utilities for the initial old.

Formal analysis of this problem is notationally intensive but its key steps closely follow those in Section 2. We relegate all derivations to the Online appendix and only describe the main insights. The best equilibrium is supported by trigger strategies: if the government deviates in period $t$ from the prescribed equilibrium allocation then, starting from period $t + 1$, agents coordinate on the subgame perfect equilibrium that gives the lowest payoff to the period-$t$ government. When information is persistent, i.e. $q < 1$, this worst equilibrium is no longer a no-production economy. Nonetheless, one can show that both the worst equilibrium and the best deviation depend only on the total measure of agents who are either already employed or have revealed to have found employment in period $t$. This feature allows us to write the optimal contracting problem in a recursive way similarly to the one in Section 3. Since information is persistent, past histories are summarized by a two dimensional variable $(w, \iota)$, where $w$ is the continuation utility and $\iota$ is an indicator variable taking value 1 if an agent is employed and 0 otherwise.

Only the unemployed have private information and we denote their reporting strategies by $\sigma_w(z)$. The following proposition shows that the results about optimal information revelation is unchanged when information is persistent.

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**Proposition 3** *The best equilibrium payoff in the steady state can be achieved with reporting strategies $\sigma_w^*$ such that $\sigma_w^*(z) \in \{0,1\}$ for all $z$, and information revelation is decreasing in $w$, i.e. $\Pr(\sigma_w^* = 1)$ is decreasing in $w$.*

The proof of this Proposition is given in the Online appendix. Its two key ingredients are the same as in previous proofs: the incentive constraint implies that the value of information is convex in $\sigma$; and normality of leisure implies that the cost of delivering $w$ is submodular in $(w, \sigma)$. The value function $K$ now depends on a two dimensional variable $(w, \iota)$ and an additional intermediate step is required to show that $K_1(w, 1) \leq K_1(w, 0)$, where $K_1$ denotes the derivative with respect to the first argument.

## 4.2  Contract dynamics and decentralization when jobs are permanent

The case of permanent job offers, $q = 0$, is the standard benchmark in the unemployment insurance literature. Hopenhayn and Nicolini (1997) (HN) fully characterize the optimal contract under perfect commitment by the policymaker. In this section we discuss the implications of relaxing this commitment assumption for the design of unemployment insurance in these settings.

We start with an overview of HN's results in the context of our environment. Commitment constraints are captured by a version of the sustainability constraint (14). Dropping it from the maximization problem recovers the perfect commitment case. The optimal contract is characterized by a pair of Bellman equations. The first one is the Bellman equation for agents who were employed last period. It takes the form

$$K(v, 1) = \max_{u,l,w} \ u - \zeta \{C(u, l) - l\} + \tilde{\beta} K(w, 1),$$

subject to $v = u + \tilde{\beta}w$, $(u, l) \in dom(C)$ and $w \in dom(K)$, where $\tilde{\beta} = \beta(1-\delta)$ is a mortality-adjusted discount factor. An optimality condition for this optimization is $K_1(v, 1) = K_1(w_v, 1)$, where $w_v$ denotes the optimal choice of $w$ given a continuation $v$ at the beginning of the period. One can show that $K(\cdot, 1)$ is strictly concave, which implies that continuation utility, consumption and labor supply of an employed person are the same in all periods, and that they are pinned down by the continuation utility $v$ assigned in the period in which the worker accepts the job.

The value of that continuation utility is determined by the second Bellman equation, which

describes the optimal allocations for the unemployed. It takes the form

$$K\left(v,0\right) = \max_{u_H,u_L,l,w_H,w_L} \quad p\left[u_H - \zeta\left\{C\left(u_H,l\right)-l\right\} + \tilde{\beta}K\left(w_H,1\right)\right] \tag{21}$$

$$+ \left(1-p\right)\left[u_L - \zeta C\left(u_L,0\right) + \tilde{\beta}K\left(w_L,0\right)\right],$$

subject to $(u_H, u_L, l) \in \mathcal{C}$, $w_H \in dom\left(K(\cdot,1)\right)$, $w_L \in dom\left(K(\cdot,0)\right)$, the incentive constraint

$$u_H + \tilde{\beta}w_H \geq u_L + \tilde{\beta}w_L \tag{22}$$

and the promise-keeping constraint

$$v = p(u_H + \tilde{\beta}w_H) + \left(1-p\right)\left(u_L + \tilde{\beta}w_L\right). \tag{23}$$

The initial continuation utility for the newly unemployed, $v_0$, is given by $K_1\left(v_0,0\right) = 0$. Subsequent continuations are determined recursively via

$$K_1\left(w_{L,v},0\right) = K_1\left(v,0\right) + \frac{\mu_v}{1-p},$$

$$K_1\left(w_{H,v},1\right) = K_1\left(v,0\right) - \frac{\mu_v}{p},$$

where $\mu_v > 0$ is the Lagrange multiplier on the incentive constraint (22). The first of these equations implies that the continuation utility, and therefore consumption, of the unemployed falls monotonically with the duration of unemployment. The second shows that continuation utility for the employed also depends on the unemployment duration. HN provide sufficient conditions under which the continuation utility of the employed is decreasing in the length of the unemployment spell.

HN show that optimal allocations can be decentralized by a system of unemployment benefit $b^{UI}$ and a tax on employed workers $\tau$. The nature of the optimal contract implies that the unemployment benefits must be monotonically decreasing in the duration of unemployment. Upon finding a job, a worker is assigned with a tax $\tau$ which is constant in all future periods. The value of the tax depends on the duration on the unemployment. Therefore, the past history of unemployment has a permanent effect on the taxes paid by the employed.

We now contrast this dynamics with the one that emerges in the economy where the government cannot commit and the sustainability constraint binds. The Bellman equation for the employed agent is given by

$$K\left(v,1\right) = \max_{u,l,w} \quad u - \zeta\left\{C\left(u,l\right)-l\right\} + \hat{\beta}K\left(w,1\right) - const,$$

subject to $v = u + \tilde{\beta}w$, $(u,l) \in dom\left(C\right)$ and $w \in dom\left(K(\cdot,1)\right)$, where $\hat{\beta} > \tilde{\beta}$. The law of motion for the continuation utility satisfies $K_1\left(w_v,1\right) = \frac{\tilde{\beta}}{\hat{\beta}}K_1\left(v,1\right)$, which implies that it must

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converge to the point $v_{SS}$ given by $K_1(v_{SS}, 1) = 0$. This mean-reversion property is reminiscent of the results in Sleet and Yeltekin (2006) who showed that commitment constraints induce the policymaker to use a higher effective discount factor than the one of the agents, which reduces inequality and relaxes future sustainability constraints.

The initial continuation utility of the unemployed satisfies $K_1(v_0, 0) = 0$. Since interior reporting strategies are suboptimal, the newly unemployed are divided into two groups: those who are provided with incentives to reveal information and those who are not. Submodularity implies that the continuation utilities of the agents in the two groups, $v_0^{UI}$ and $v_0^{DI}$ respectively, satisfy $v_0^{UI} \leq v_0 \leq v_0^{DI}$.

The allocations for agents in the first group are given by the solution to the problem

$$\max_{\substack{u_H, u_L, l, \\ w_H, w_L}} p\left[u_H - \zeta\{C(u_H, l) - l\} + \hat{\beta}K(w_H, 1)\right] + (1-p)\left[u_L - \zeta C(u_L, 0) + \hat{\beta}K(w_L, 0)\right],$$

subject to (22), (23), for $v = v_0^{UI}$, $(u_H, u_L, l) \in \mathcal{C}$, $w_H \in dom(K(\cdot, 1))$ and $w_L \in dom(K(\cdot, 0))$. It is easy to verify the following property of this maximization problem, which we prove in the Online appendix.

**Lemma 4** *If $v \leq v_0^{UI}$ then $w_{L,v} \leq v_0^{UI}$, the latter inequality is strict if $v_0^{UI}$ is interior, $v_0^{UI} > 0$.*

Together with the monotonicity result in Proposition 3, this lemma implies that any worker who was ever assigned to the group that reveals information must always remain in that group. Moreover, unlike the perfect commitment case, continuation $w_{L,v}$ does not need to be strictly lower than $v$ because the desire to provide incentives may be offset by the desire to avoid excessive inequality induced by the sustainability constraint.

The allocations of the agents who are assigned to the second group, the one that reveals no information, are determined by

$$\max_{u,w} u - \zeta C(u, 0) + \hat{\beta}K(w, 0),$$

subject to $v_0^{DI} = u + \tilde{\beta}w$, $(u, 0) \in dom(C)$ and $w \in dom(K)$. An optimality condition for this problem is $K_1(w_{v_0^{DI}}, 0) = 0$. In the Online appendix, we show that this result implies that it is without loss of generality to set $w_{v_0^{DI}} = v_0^{DI}$, so that agents in the second group receive continuation utility $v_0^{DI}$ in all periods and never reveal their information.[12]

The optimal allocation can be decentralized by a system of taxes on the employed $\tau$, unemployment insurance benefit $b^{UI}$ and disability benefit $b^{DI}$. As in the perfect commitment

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[12] This result does not follow immediately from the discussion above since the value function $K(v, 0)$ is linear in the region $[v_0^{UI}, v_0^{DI}]$ and satisfies $K_1(v, 0) = 0$ for all such $v$, and the optimality conditions are necessary but not sufficient.

case, both $\tau$ and $b^{UI}$ depend on the duration of the unemployment spell, but history dependence is now more limited. In particular, $\tau$ is only temporarily affected by the length of the unemployment spell (which can be interpreted as a temporary tax credit for those who find a job quickly) and eventually reaches some long-run level $\tau_{SS}$. The disability benefit $b^{DI}$ is constant and more generous than the unemployment benefit, but access to the disability system is rationed. This rationing can be achieved in same way as it was done in Section 2, that is, by imposing eligibility requirements that restrict the pool of potential applicant.

# 5 Final remarks

In this paper we took a step towards developing a theory of social insurance in a setting in which the principal cannot commit. We focused on a simple version of a Mirrleesian economy and showed how it can be incorporated into the standard recursive contracting framework with relatively few modifications. The natural extension of this approach is to incorporate it into the richer models of social insurance cited in the introduction. Our methods should also be applicable to other principal-agent environments in which the principal interacts with a large number of agents and cannot commit, such as models of regulation, employer-employee relationships, bargaining and trading with private information.

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# Appendix

## A.1    Proofs for Section 2

**Proof of Lemma 1.**    Monotonicity and concavity of $U$ implies that $U_c > 0$, $U_l < 0$, $U_{cc} < 0$, $U_{ll} < 0$, $U_{cc}U_{ll} - U_{cl}^2 > 0$, for all $c, l > 0$. Direct calculation shows that the marginal effect of lump sum income on labor supply, i.e. the income effect, is given by (see, alternatively, equation (24) in Saez (2001))

$$\text{income effect} = \frac{(U_l/U_c)\,U_{cc} - U_{cl}}{U_{ll} + (U_l/U_c)^2\,U_{cc} - 2\,(U_l/U_c)\,U_{cl}}.$$

The denominator of this expression is the second order optimality condition for labor and is negative. The strict normality of leisure is equivalent to the income effect being strictly negative, i.e.

$$U_{ll} + (U_l/U_c)^2\,U_{cc} - 2\,(U_l/U_c)\,U_{cl} < 0, \;\; (U_l/U_c)\,U_{cc} - U_{cl} > 0. \tag{24}$$

By the implicit function theorem, $U_c C_1 = 1$ and $U_c C_2 + U_l = 0$, thus, $C_1, C_2 > 0$. From the first expression, we get $U_{cc} C_1^2 + U_c C_{11} = 0$, which implies $C_{11} > 0$. From the second expression, we get

$$
\begin{aligned}
0 &= [U_{cc}C_2 + U_{cl}]\,C_2 + U_c C_{22} + U_{cl}C_2 + U_{ll} \\
&= \left[ U_{ll} + U_{cc}\,(U_l/U_c)^2 - 2U_{cl}\,(U_l/U_c) \right] + U_c C_{22}.
\end{aligned}
$$

The expression in square brackets is negative by (24) and, therefore, $C_{22} > 0$.

The cross-partial derivative satisfies $[U_{cc}C_2 + U_{cl}]\,C_1 + U_c C_{12} = 0$. The latter gives $(U_c)^2\,C_{12} = -[U_{cl} - U_{cc}\,(U_l/U_c)]$, which implies $C_{12} > 0$ by (24). The determinant of the Hessian is

$$
\begin{aligned}
C_{11}C_{22} - C_{12}^2 &= \frac{U_{cc}}{U_c^3}\frac{U_{ll} + U_{cc}\,(U_l/U_c)^2 - 2U_{cl}\,(U_l/U_c)}{U_c} - \left( \frac{U_{cl} - (U_l/U_c)\,U_{cc}}{U_c^2} \right)^2 \\
&= \frac{U_{cc}U_{ll} - U_{cl}^2}{U_c^4} > 0,
\end{aligned}
$$

thus, $C$ is strictly convex. Finally, $U_l = 0$ implies $U_c C_2 = 0$ and, since $U_c > 0$, we have $C_2 = 0$. ∎

## A.2    Proofs for Section 3

We first show the following corollary to Lemma 3

**Corollary 3** $\kappa_t\left(\cdot,0\right)-\chi_t W_t\left(0\right)$ *crosses* $\kappa_t\left(\cdot,1\right)-\chi_t W_t\left(1\right)$ *at most once, from below, and*

$$\max\left\{\kappa_t\left(v,1\right)-\chi_t W_t\left(1\right),\kappa_t\left(v,0\right)-\chi_t W_t\left(0\right)\right\}\geq\kappa_t\left(w,\sigma\right)-\chi_t W_t\left(\sigma\right),\ \textit{for all }\sigma.$$

*Moreover,* $K_t\left(v\right)$ *is the concave envelope of the expression on the left hand side.*

**Proof.** $\kappa_t\left(v,\cdot\right)-\chi_t W_t\left(\cdot\right)$ is a linear function for all $v$. Since $\kappa_t$ is submodular in $\left(v,\sigma\right)$ by Lemma 3, we can partition the space of continuation utilities in three regions:

$$\arg\max_{\sigma\in[0,1]}\kappa_t\left(v,\sigma\right)-\chi_t W_t\left(\sigma\right)=\left\{\begin{array}{cc} 1 & \text{for low }v \\ {}[0,1] & \text{for intermediate }v \\ 0 & \text{for high }v \end{array}\right..$$

All three regions exist if $\frac{\partial}{\partial\sigma}\left[\kappa_t\left(v,\sigma\right)-\chi_t W_t\left(\sigma\right)\right]$ changes sign for different values of $v$. $\blacksquare$

**Lemma 5** $K_t$ *is differentiable.*

**Proof.** Observe that, if $x=\left(u_H,u_L,w_H,w_L,l,\sigma\right)$ satisfies (10) and (11) for some $w$, then $x^\varepsilon\equiv\left(\varepsilon u_H,\varepsilon u_L,\varepsilon w_H,\varepsilon w_L,l,\sigma\right)$ satisfies (10) and (11) for some $\varepsilon w$, where $\varepsilon$ is a scalar.

Take any point $v_0$ and let $\left\{\psi_{t,w}^*\right\}_t$ be the corresponding solution to (17). For any $\varepsilon$ in the neighborhood of 1, construct $\{\tilde{\psi}_{t,w}\}_t$ that assigns $\left\{\psi_{t,w}^*\right\}_t$ to points $x^\varepsilon$, rather than $x$. The payoff from this strategy, $\tilde{K}_t\left(v\right)$, with $v=\varepsilon v_0$, is

$$\tilde{K}_t\left(v\right)\equiv\frac{1}{\bar{\beta}_t}\sum_{s=t}^\infty\bar{\beta}_s\left[g\left(x^\varepsilon\right)-\zeta_s f\left(x^\varepsilon\right)-\chi_s W_s\right]d\psi_{s,w}^* d\pi_{s-1}.$$

Direct observation shows that the right hand side is differentiable in $\varepsilon$ and, therefore, $\tilde{K}_t\left(v\right)$ is differentiable at $v_0$. We also have $K_t\left(v\right)\geq\tilde{K}_t\left(v\right)$ with equality at $v_0$. Differentiability of $K_t\left(v\right)$ at $v_0$ (and, since $v_0$ is arbitrary, at any $v$) then follows from Benveniste and Scheinkman (1979). $\blacksquare$

**Lemma 6** *The incentive constraint (10) holds with equality.*

**Proof.** Let's consider a constrained problem (19) where we add an additional constraint

$$u_H+\beta w_H\leq u_L+\beta w_L.$$

We want to show that this constraint is always slack. Suppose it is not slack, which can be only for $\sigma=1$. The problem is convex, thus, we can set up the Lagrangian

$$
\begin{aligned}
\mathcal{L}=\max_{u_H,u_L,l,w_H,w_L}\ & p\left[u_H-\zeta_t\left(C\left(u_H,l\right)-l\right)+\beta K_{t+1}\left(w_H\right)\right]+\left(1-p\right)\left[u_L-\zeta_t C\left(u_L,0\right)+\beta K_{t+1}\left(w_L\right)\right]\\
& +\mu_{t,v}\left(\left\{u_H+\beta w_H\right\}-\left\{u_L+\beta w_L\right\}\right)\\
& -\gamma_{t,v}\left(p\left\{u_H+\beta w_H\right\}+\left(1-p\right)\left\{u_L+\beta w_L\right\}-v\right),
\end{aligned}
$$

subject to $(u_H, u_L, l) \in \mathcal{C}$ and $w_H, w_L \in dom(K_{t+1})$, for some multipliers $\mu_{t,v} < 0$ and $\gamma_{t,v}$. Let $(u_{H,v}, u_{L,v}, w_{H,v}, w_{L,v}, l_v)$ denote an optimal bundle (for convenience, throughout the proof we omit explicit dependence on time from optimal bundles). The first-order conditions w.r.t. $u_H$, $u_L$, $w_H$ and $w_L$ are, respectively,

$$
\begin{aligned}
p\left[1 - \zeta_t C_1\left(u_{H,v}, l_v\right)\right] + \mu_{t,v} &\leq \gamma_{t,v} p, \\
(1-p)\left[1 - \zeta_t C_1\left(u_{L,v}, 0\right)\right] - \mu_{t,v} &\leq \gamma_{t,v}(1-p), \\
p\beta K'_{t+1}\left(w_{H,v}\right) + \beta\mu_{t,v} &\leq \gamma_{t,v}\beta p, \\
(1-p)\beta K'_{t+1}\left(w_{L,v}\right) - \beta\mu_{t,v} &\leq \gamma_{t,v}\beta(1-p),
\end{aligned}
$$

with equality if the solution is interior. They can be re-arranged as

$$
\begin{aligned}
1 - \zeta_t C_1\left(u_{H,v}, l_v\right) + \frac{\mu_{t,v}}{p} &\leq \gamma_{t,v}, \\
1 - \zeta_t C_1\left(u_{L,v}, 0\right) - \frac{\mu_{t,v}}{1-p} &\leq \gamma_{t,v}, \\
K'_{t+1}\left(w_{H,v}\right) + \frac{\mu_{t,v}}{p} &\leq \gamma_{t,v}, \\
K'_{t+1}\left(w_{L,v}\right) - \frac{\mu_{t,v}}{1-p} &\leq \gamma_{t,v}.
\end{aligned}
$$

We assume that $u_{H,v}$ is interior, the other case is immediate. If $\mu_{t,v} < 0$, then,

$$
C_1\left(u_{H,v}, l_v\right) < C_1\left(u_{L,v}, 0\right),
$$

thus $u_{H,v} < u_{L,v}$ by $l_v > 0$ and $C_{12} > 0$, and the incentive constraint implies $w_{H,v} > w_{L,v}$. Thus, $w_{H,v}$ is interior and its first order condition holds with equality. Also, by concavity,

$$
K'_{t+1}\left(w_{L,v}\right) \geq K'_{t+1}\left(w_{H,v}\right)
$$

or

$$
K'_{t+1}\left(w_{L,v}\right) - \frac{\mu_{t,v}}{1-p} > K'_{t+1}\left(w_{H,v}\right) + \frac{\mu_{t,v}}{p},
$$

which violates the first order conditions. Therefore, we must have $\mu_{t,v} = 0$. ∎

**Proof of Proposition 2.** We only need to prove this proposition for the case when $\frac{\partial}{\partial\sigma}\left[\kappa_t\left(v, \sigma\right) - \chi_t W_t\left(\sigma\right)\right]$ changes sign for some $v$, otherwise, the result is trivial. Our solution is continuous in $v$, thus, there must be some $\hat{v}$ such that $\frac{\partial}{\partial\sigma}\left[\kappa_t\left(\hat{v}, \sigma\right) - \chi_t W_t\left(\sigma\right)\right] = 0$. If there is a region for which this holds, we take $\hat{v}$ to the the smallest point at which this equation is satisfied.

By Lemma 6, the incentive constraint (10) holds with equality. Let $(u_{H,v}, u_{L,v}, w_{H,v}, w_{L,v}, l_v)$ denote an optimal bundle that solves (20) for a given $v$, such optimal bundles are independent

of $\sigma$ (for convenience, throughout the proof we omit explicit dependence on time from optimal bundles). If $v = 0$ the solution is trivial, we thus consider the case with $v > 0$. Optimality conditions are as follows. Since $v > 0$, we must either have $u_{H,v} > 0$ or $w_{H,v} > 0$ (or both). In the former case the relevant first order condition is

$$1 - \zeta_t C_1 \left( u_{H,v}, l_v \right) \geq \frac{\hat{\beta}_{t+1}}{\beta} K'_{t+1} \left( \frac{v - u_{H,v}}{\beta} \right).$$

In the latter,

$$1 - \zeta_t C_1 \left( u_{H,v}, l_v \right) \leq \frac{\hat{\beta}_{t+1}}{\beta} K'_{t+1} \left( \frac{v - u_{H,v}}{\beta} \right).$$

Similarly, we must either have $u_{L,v} > 0$ or $w_{L,v} > 0$ (or both), with first order conditions given by, respectively,

$$1 - \zeta_t C_1 \left( u_{L,v}, 0 \right) \geq \frac{\hat{\beta}_{t+1}}{\beta} K'_{t+1} \left( \frac{v - u_{L,v}}{\beta} \right)$$

and

$$1 - \zeta_t C_1 \left( u_{L,v}, 0 \right) \leq \frac{\hat{\beta}_{t+1}}{\beta} K'_{t+1} \left( \frac{v - u_{L,v}}{\beta} \right).$$

We consider all possible cases.

1) Suppose that $u_{H,v}$ is at its lower bound. Then, we immediately have $u_{H,v} < u_{L,v}$ and, using (10), $w_{H,v} > w_{L,v}$. Therefore, by concavity $K'_{t+1} \left( w_{H,v} \right) \leq K'_{t+1} \left( w_{L,v} \right)$, with strict equality if $K_{t+1}$ is not linear on $[w_{L,v}, w_{H,v}]$.

2) Suppose that $u_{H,v}$ is interior and that the constraint $w_L \geq 0$ is slack. Then the relevant first order conditions are

$$
\begin{aligned}
1 - \zeta_t C_1 \left( u_{H,v}, l_v \right) &\geq \frac{\hat{\beta}_{t+1}}{\beta} K'_{t+1} \left( \frac{v - u_{H,v}}{\beta} \right), \\
1 - \zeta_t C_1 \left( u_{L,v}, 0 \right) &\leq \frac{\hat{\beta}_{t+1}}{\beta} K'_{t+1} \left( \frac{v - u_{L,v}}{\beta} \right).
\end{aligned}
$$

If $u_{H,v} \geq u_{L,v}$, then, using $l_v > 0$ and strict convexity of $C \left( \cdot, l \right)$,

$$C_1 \left( u_{H,v}, l_v \right) \geq C_1 \left( u_{L,v}, l_v \right) > C_1 \left( u_{L,v}, 0 \right).$$

The conditions above imply

$$\frac{\hat{\beta}_{t+1}}{\beta} K'_{t+1} \left( \frac{v - u_{L,v}}{\beta} \right) \geq 1 - \zeta_t C_1 \left( u_{L,v}, 0 \right) > 1 - \zeta_t C_1 \left( u_{H,v}, l_v \right) \geq \frac{\hat{\beta}_{t+1}}{\beta} K'_{t+1} \left( \frac{v - u_{H,v}}{\beta} \right).$$

Thus, $w_{H,v} > w_{L,v}$, which violates (10). If instead $u_{H,v} < u_{L,v}$, then (10) implies $w_{H,v} > w_{L,v}$ and by concavity $K'_{t+1} \left( w_{H,v} \right) \leq K'_{t+1} \left( w_{L,v} \right)$, with strict equality if $K_{t+1}$ is not linear on $[w_{L,v}, w_{H,v}]$.

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3) Finally, suppose that $u_{H,v}$ is interior and that the constraint $w_L \geq 0$ binds. Then $w_{H,v} \geq w_{L,v} = 0$ and by concavity $K'_{t+1}(w_{H,v}) \leq K'_{t+1}(w_{L,v})$, with strict equality if $K_{t+1}$ is not linear on $[w_{L,v}, w_{H,v}]$.

Notice that in all cases above $w_{H,v} \geq w_{L,v}$. Consider the derivatives of $\kappa_t(\cdot, 0)$ and $\kappa_t(\cdot, 1)$. If $w_{L,v} > 0$, then $w_{H,v} > 0$ and standard Benveniste-Sheikman arguments imply

$$\frac{\partial}{\partial v}\kappa_t(v, 0) = \frac{\hat{\beta}_{t+1}}{\beta}K'_{t+1}(w_{L,v}),$$

$$\frac{\partial}{\partial v}\kappa_t(v, 1) = \frac{\hat{\beta}_{t+1}}{\beta}\left[pK'_{t+1}(w_{H,v}) + (1-p)K'_{t+1}(w_{L,v})\right].$$

Therefore, $\frac{\partial}{\partial v}\kappa_t(v, 0) \geq \frac{\partial}{\partial v}\kappa_t(v, 1)$. Similar steps prove that $\frac{\partial}{\partial v}\kappa_t(v, 0) \geq \frac{\partial}{\partial v}\kappa_t(v, 1)$ also when $w_{L,v} = 0$. Thus, if $K_{t+1}$ is not linear on $[w_{L,\hat{v}}, w_{H,\hat{v}}]$, we immediately have that $\frac{\partial}{\partial v}\kappa_t(v, 0)\big|_{v=\hat{v}} > \frac{\partial}{\partial v}\kappa_t(v, 1)\big|_{v=\hat{v}}$. Since at $v$ the left and the right derivatives of $\kappa$ are different, the function $k_t(v) \equiv \max\{\kappa_t(v, 1) - \chi_t W_t(1), \kappa_t(v, 0) - \chi_t W_t(0)\}$ is not concave at $\hat{v}$. Thus, there is an interval $(\underline{v}_t, \bar{v}_t)$, $\underline{v}_t < \bar{v}_t$, such that any $w \in (\underline{v}_t, \bar{v}_t)$ is delivered with non-trivial randomization between $\underline{v}_t$ and $\bar{v}_t$.

Suppose, instead, that $K_{t+1}$ is linear on $[w_{L,\hat{v}}, w_{H,\hat{v}}]$ and there is an interval $[\hat{v}'_t, \hat{v}''_t]$ such that $\frac{\partial}{\partial \sigma}[\kappa_t(\hat{v}, \sigma) - \chi_t W_t(\sigma)] = 0$, for all $\hat{v} \in [\hat{v}'_t, \hat{v}''_t]$. Note that $k_t(v)$ is concave in this case, thus, $K_t = k_t$. Also, any $\sigma$ satisfies the first-order condition and, hence, it will be a solution to (18). Since (18) gives necessary, but not sufficient conditions for a best PBE, we need to verify that the strategies described in the text of this proposition satisfy equations (11), (13) and (14). We will do this in two claims.

*Claim 1*: A best PBE can be supported with reporting strategies $\sigma$ that are in $\{0, 1\}$.

Suppose we have a best equilibrium such that, in some history, $\sigma_{\hat{v}} \in (0, 1)$, which would be possible only if $\hat{v} \in [\hat{v}'_t, \hat{v}''_t]$. Construct a different equilibrium where all strategies are unchanged except in $\hat{v}$. In particular, all agents with continuation $\hat{v}$ are assigned randomly to one of two groups as follows. With probability $\sigma_{\hat{v}}$, they are assigned to a group that chooses $\sigma = 1$ and receives $(u_{H,\hat{v}}, u_{L,\hat{v}}, w_{H,\hat{v}}, w_{L,\hat{v}}, l_{\hat{v}})$, and, with probability $1 - \sigma_{\hat{v}}$, they are assigned to a group that chooses $\sigma = 0$ and receives $(u_{L,\hat{v}}, w_{L,\hat{v}})$. With these newly constructed strategies both the amount of information revealed and the distribution of all continuation histories remain unchanged. Since the original equilibrium strategies satisfied (11), (13) and (14), the new strategies must also satisfy these constraints and, therefore, must also be an equilibrium. $\square$

*Claim 2*: If there is a PBE such that, at some $t$ and for some $v' < v''$, we have $\sigma^*_{v'} = 0$ and $\sigma^*_{v''} = 1$, then there is another PBE such that, at the same $t$ we have $\sigma^*_{v'} = 1$ and $\sigma^*_{v''} = 0$.

Since this case is possible only if $K_{t+1}$ is linear in the appropriate region, from the first order conditions above we must have $u_{L,v'} = u_{L,v''}$. To simplify notation we use $w''_H$ to refer

to $w_{H,v''}$, etc. Let $\delta \equiv w_H'' - w_L'' > 0$. We have that a measure $p$ of agents receives $w_L'' + \delta$, a measure $1 - p$ receives $w_L''$, and a measure $1$ receives $w_L'$.

Now adjust continuation utilities and reporting strategies as follows. Suppose $v''$ now does not reveal information (i.e. plays $\sigma = 0$) and receives $w_L'' + \delta$ with probability $\varepsilon''$ and $w_L'$ with probability $1 - \varepsilon''$, where

$$\varepsilon'' \left( w_L'' + \delta \right) + \left( 1 - \varepsilon'' \right) w_L' = w_L''.$$

Similarly, suppose $v'$ plays $\sigma = 1$. This agent receives the same allocations as before, if he reports $L$. Instead, if he reports $H$, he receives $\left( u_{H,v''}, l_{v''} \right)$ with continuation utility $w_L' + \delta$, which is delivered by randomizing between $w_L'' + \delta$ and $w_L'$ with probabilities $\varepsilon'$ and $1 - \varepsilon'$. The probability $\varepsilon'$ is defined by

$$\varepsilon' \left( w_L'' + \delta \right) + \left( 1 - \varepsilon' \right) w_L' = w_L' + \delta.$$

Note that since $u_{L,v'} = u_{L,v''}$, the new allocations are incentive compatible and reveal the same amount of information. We have

$$\varepsilon' = \frac{\delta}{w_L'' - w_L' + \delta}, \quad \varepsilon'' = \frac{w_L'' - w_L'}{w_L'' - w_L' + \delta},$$

thus, $\varepsilon' + \varepsilon'' = 1$. Moreover, the mass of agents receiving the different continuation utilities are unchanged. Finally, $\hat{v}'' > \hat{v}'$ implies $w_L'' - w_L' > 0$, therefore, the probabilities are well defined. $\square$

When $K_{t+1}$ is linear, the proof of the proposition follows from these three claims. More specifically, by Claims 1 and 2, we can restrict attention to reporting strategies $\sigma_v \in \{0, 1\}$ with some cut-off $\tilde{v}$ such that $\sigma_v = 1$ for all $v < \tilde{v}$ and $\sigma_v = 0$ for all $v > \tilde{v}$. Since in this case $\max \left\{ \kappa_t \left( v, 1 \right) - \chi_t W_t \left( 1 \right), \kappa_t \left( v, 0 \right) - \chi_t W_t \left( 0 \right) \right\}$ is concave, we can deliver any $w$ by having $\sigma_w = 1$ if $w < \tilde{v}$, and $\sigma_w = 0$ if $w > \tilde{v}$. When $w = \tilde{v}$, continuation utilities are randomized between $\sigma_{\tilde{v}} = 1$ and $\sigma_{\tilde{v}} = 0$. $\blacksquare$