Monetary Policy with

Heterogeneous Agents and Credit Constraints

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Abstract

This paper analyzes the effect of monetary policy in economies with financial market imperfections. This analysis is carried on in a heterogeneous agents framework in which infinitely lived agents face credit constraints and can partially self-insure against income risks by using both financial assets and real balances.

First, we show that credit constraints yield a new theoretical channel through which monetary policy affects the real economy, because of the heterogeneity in money demand. Secondly, we quantify this new channel and we show that inflation has a sizeable positive impact on output and consumption in economies which closely match the wealth distribution of the United States. Thirdly, the average welfare cost of inflation turns out to be similar across complete and incomplete market economies. But wealth-rich and wealth-poor agents are unevenly affected by monetary policy, the latter one benefiting from inflation.

JEL : E2, E5

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1 Introduction

The traditional insight in the textbook monetary literature is that inflation has no real effect in the long run with perfect capital markets, dynastic households and lump-sum taxes, and exogenous labor supply (Lucas, 2000). In contrast to this theoretical well-known result, a large amount of empirical work shows that some low inflation leads to an increase in savings (Loayza,

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et al., 2000), output (Bullard and Keating, 1995) or capital stock (Kahn *et al.* 2001). And it has become common practice for the central banks to target a positive long run inflation rate which ranges between 1 percent and 3 percent (Bernanke and Mishkin, 1997). In order to reconcile this gap between traditional monetary theory and the empirical evidence, recent research has challenged some assumptions under which the monetary superneutrality (or inflation neutrality) result holds. The potential *long-run* real effects of monetary policy has been assessed when inflation induces redistribution of the seigniorage rents across households (Grandmont and Younès, 1973, and Kehoe *et al.*, 1992) or across generations (Weiss 1980 ; Weil, 1991), or if distorting taxes are affected by inflation (Phelps, 1973 and Chari *et al.*, 1996 among others) and eventually if inflation induces some distortions on labor supply .

This paper exhibits a new channel of the non-neutrality of inflation transiting through capital market imperfections. If households can use both fiat money and capital as partial private insurance designs against individual income risks, they can substitute money for financial assets when inflation increases and affects the return on real balances. Yet if asset market imperfections are such that some households are borrowing constrained, then these households can not undertake such a substitution and they will adjust in a different proportion their amount of money compared to unconstrained households. Thus credit constraints induce a heterogeneity in the demand for real balances and capital following a change in the inflation rate. This endogenous heterogeneity in policy rules is at the core of the non-neutrality of money. Since the tightness of credit constraints is a well-established empirical fact (Jappelli 1990, Budria Rodriguez *et al.*, 2002), this channel is likely to have a sizeable quantitative impact on the real economy and on the welfare of households.

To investigate this channel, we model capital market imperfections in a production economy following the approach of Aiyagari (1994). Heterogeneous agents receive idiosyncratic income shocks. They can accumulate interest-bearing financial assets in the form of capital to partially insure against income risks but they face a borrowing constraint. We embed in this framework money in the utility function. Money is praised both for its liquidity service and as a store of value which provides an additional insurance device against labor market risks.

We first provide theoretical evidence that inflation affects aggregate real variables in this incomplete markets framework when borrowing constraints are binding. Inflation gives rise to a heterogeneous substitution effects between interest-bearing assets and real balances across unconstrained households and constrained ones, providing a real channel for monetary policy. The new point is that this real effect is only due to the presence of credit constraints. It shows up even when we shut down the previous traditional real channel of inflation transiting through distorting redistribution of the seigniorage rent, through distorting capital taxes, or through distortion of the labor supply. Moreover, individuals are completely ex-ante identical in our set-up. The heterogeneity in money demand emerges as an endogenous outcome of the presence of borrowing constraints. Thus our non-neutrality result does not hinge on *ad-hoc* assumptions about heterogeneity in preferences.

Second we undertake a quantitative evaluation of the impact of borrowing constraints on the real effect of inflation by calibrating the model on the United States. For that purpose we use a fully-fledged quantitative model with endogenous labor supply and endogenous proportional taxes. Since our channel crucially depends on credit constraints, a key element of our investigation is to study an economy in which the wealth distribution and the fraction of borrowing constrained households closely resembles that of the United States. We then use this set-up to assess the quantitative importance of the role played by credit constraints in the real effect of inflation by running a comparison with corresponding complete market economies.

One of our main result is that credit constraints have a quantitatively sizeable effect. First, we show that borrowing constraints alone can account for a 0.08 percent increase in output following a one point rise in inflation. This result is obtained when we shut down all the traditional channels through which inflation is expected to have a long-run real effect. Namely this framework abstract form potential redistributive effect of the seigniorage rent, the potential distortion on capital tax and the adjustment of labor supply by assuming exogenous hours. Note that in this set-up, inflation would have no long-run real effect on output if the markets were to be complete. Then we show that credit constraints amplifies the real effect of inflation which have been quantified so far through the channels of distorting capital taxes or distorting alteration of labor supply. Regarding distorting taxes on capital first, it has long been recognized that the seigniorage rent could alleviate taxes on capital and induce higher capital accumulation. Yet, this so-called Phelps effect (Phelps, 1973; Chari *et al.* 1996) is quantitatively much more significant in an incomplete market set-up since the presence of borrowing constraint gives rise to precautionary savings motives. A one point rise in the inflation rate triggers an increase in aggregate output by 0.213 percent in the incomplete market set-up against 0.104 in the representative agent economy. Regarding the distorting effect of inflation on labor supply, we show that it matters quantitatively much more in an incomplete market environment. The reason for that is that inflation triggers a much higher level of precautionary saving in presence of borrowing constraints, which leads in turn to a rise in labor productivity and the opportunity to work. By taking into account of the adjustment of labor supply, a one point increase in the inflation rate leads to a 0.44 percent rise in output, which is three times as high as in the corresponding complete markets economy.

Third, we reassess the welfare effect of inflation when credit constraints are taken into account. It first turns out that the average welfare costs of inflation are of the same order under incomplete markets and complete markets. This result holds for similar steady-state comparisons and whatever the assumption about the adjustment of taxes and hours. The key reason for this result is that the sharper increase in output and consumption in the incomplete market framework is offset by an even more pronounced reduction in aggregate real balances compared to the complete market framework. For standard parameterization of the utility function, the positive utility effect of higher consumption is cancelled out by the negative utility effect of lower real balances.

Eventually, we show that inflation has strong welfare redistributive effects which have been ignored so far in the representative agent literature. It turns out that the wealth-poor gain from inflation while the wealth-rich are negatively affected by inflation. This result is mainly due to a general equilibrium price effect. Inflation leads to higher labor productivity and wages at the general equilibrium due to higher steady-state capital. This effect benefits to the wealth-poor whose income is mainly made up of labor income. In contrast the return on financial assets decreases at the general equilibrium, which negatively affects the wealth rich.

Related literature

There are surprisingly few papers analyzing monetary policy with infinitely lived heterogenous agents facing financial credit constraints. To the best of our knowledge, our paper is the first one to provide theoretical and quantitative evidence on the real effect of inflation stemming from credit constraints in a production economy (see also for a theoretical investigation Ragot, 2005).

Some initial papers have studied monetary policy in endowment economy with credit constraints following the seminal articles of Bewley (1980, 1983). But as Bewley's goal was mainly to provide some foundations for the theory of money, this asset was considered as the only store of value. As a consequence, the heterogeneity in money demand and its induced real effect explained above cannot be found in this type of model, since households are not allowed to substitute money for other assets. In the same way, Kehoe, Levine and Woodford (1992) and Imrohoroglu (1992) study the welfare effect of inflation in such endowment economies, but they only measure the redistributive effect of inflation and not its real effect on production.

More recently Erosa and Ventura (2002) analyzed the distributional impact of inflation in an incomplete markets economy but in which credit constraints do not bind in equilibrium. They find that the fraction of wealth held in liquid assets decreases with income and wealth, which is empirically relevant. Yet we prove that this result can be obtained as an endogenous outcome of credit constraints without any specific assumptions about the transaction technology as they did.

Akyol (2004) analyzes the welfare effect of inflation in an incomplete market set-up where credit constraints are binding in equilibrium, but in an endowment economy. Contrary to previous Bewley type models, he takes into account of the possibility to substitute money for other assets. But, this article assumes specific money demand implying that only the high income agents hold money in equilibrium. Furthermore, the analysis is carried on in an endowment economy rather than a production one, excluding any analysis of the long-run real effect of inflation on capital accumulation.

The analysis proceeds as follows. Section 2 first provides a simple model with deterministic individual shock to show analytically the non-neutrality of money transiting only through credit constraints. Section 3 lays out the full model with stochastic individual shocks. Section 4 quantifies the real effect of inflation and its implied welfare gains.

2 A Simple Model

2.1 The model

Although our aim is a quantitative evaluation of the effect of inflation, we first lay out a simplified version of our general model to discuss the main channels through which inflation affects aggregate outcomes. For that purpose, we use a Bewley-style model in which infinitely lived agents face individual income risks and credit constraints. But we make the key assumption that households alternate *deterministically* between the different labor market states. This liquidity constrained model has been used, for instance, by Woodford (1990) to study the effect of public debt and by Kehoe and Levine (2001) to characterize the equilibrium interest rate.

We extend this framework to monetary policy issues by taking into account of the valuation of money in the utility function. We show analytically that the Sidrauski's neutrality result no longer holds when credit constraints are binding in this framework. Inflation affects the long run interest rate, even when the new money is distributed proportionally to money holdings.

Each household can be in two states H or L. In state H (resp. L), households have a high labor endowment e^H (resp. e^L). For the sake of simplicity let us assume that $e^H = 1$ and $e^L = 0$. Households alternate deterministically between state H and L at each period. At initial date, there is a unit mass of two types of households. Type 1 households are in state H at date 1, type 2 households are in state L at date 1. As a consequence, type 1 (resp.. 2) households are in state H (resp. L) every odd period and in state L (resp. H) every even period. All households seek to maximize an infinitely horizon utility function over consumption c^i and real money balances m^i which provide liquidity services. The period utility function u of a type $i = 1, 2$ households is assumed to have the simple form

$$u\left(c_t^i, m_t^i\right) = \phi \ln c_t^i + (1 - \phi) \ln m_t^i$$

where $1 > \phi > 0$ scales the marginal utility of consumption and money. For the sake of simplicity we use a log-linear utility function in this section but the results hold for very general utility functions as shown in Ragot (2005).

At each period $t \geq 1$, type i household can use her revenue for three different purposes. She can first buy an amount c_t^i of final goods. We denote P_t denotes the price of the final good in period t, and Π_{t+1} is the gross inflation rate between period t and period $t+1$, that is $\Pi_{t+1} = P_{t+1}/P_t$. She also saves an amount a_{t+1}^i of financial titles yielding a return of $(1 + r_{t+1})\, a_{t+1}^i$ in period $t+1$, where $1 + r_{t+1}$ is the gross real interest rate between period t and period $t+1$. A borrowing constraint is introduced in its simplest form, and we assume that households can not borrow, $a_t^i \geq 0$. Finally, type i household buys a nominal quantity of money M_t^i, which corresponds to a level of real balances $m_t^i = M_t^i/P_t$. It yields a revenue m_t^i/Π_{t+1} in period $t+1$. In addition to labor income and to the return on her assets, each household receives by helicopter drops a monetary transfer from the State, denoted μ_t^i in nominal terms. Let denote

The problem of the type i household, $i = 1, 2$, is given by

$$\max_{\{c_t^i, m_t^i, a_{t+1}^i\}_{t=1..\infty}} \sum_{t=1}^{\infty} \beta^t u\left(c_t^i, m_t^i\right) \text{ with } 0 < \beta < 1 \tag{1}$$

$$\text{s.t.} \quad c_t^i + m_t^i + a_{t+1}^i = (1 + r_t)\, a_t^i + \frac{m_{t-1}^i}{\Pi_t} + w_t e_t^i + \frac{\mu_t^i}{P_t} \qquad \text{with} \qquad a_t^i, c_t^i, m_t^i \geq 0 \tag{2}$$

where β stands for the discount factor,.a_1^i and $M_0^i = P_0 m_0^i$ are given, and a_t^i and m_t^i.are subject to the standard transversality conditions.

The production function of the representative firm has a simple Cobb-Douglas form $K^\alpha L^{1-\alpha}$ where L stands for total labor supply and K is the amount of total capital which fully depreciates in production. Profit maximization is given by $\max_{K_t, L_t} F(K_t, L_t) - (1 + r_t)\, K_t - w_t L_t$ and yields the standard first order conditions

$$1 + r_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}, \;\; w_t = (1 - \alpha)\, K_t^\alpha L_t^{1-\alpha} \tag{3}$$

In period $t \geq 1$, the financial market equilibrium is given by $K_{t+1} = a_{t+1}^1 + a_{t+1}^2$. The labor market equilibrium reads $L_t = e_t^1 + e_t^2 = 1$. The goods market equilibrium implies $F(K_t, L_t) = K_{t+1} + c_t^1 + c_t^2$.

Finally, let denote $\bar{M}_t$ the nominal quantity of money in circulation and $\Sigma_t = \bar{M}_t/P_t$ the real quantity of money in circulation at the end of period t. The money market equilibrium implies $m_t^1 + m_t^2 = \Sigma_t$ in real terms and $M_t^1 + M_t^2 = \bar{M}_t$ in nominal terms.

Monetary authorities provide a new nominal quantity of money in period t, which is proportional to the nominal quantity of money in circulation at the end of period $t-1$. As a consequence, $\mu_t^1 + \mu_t^2 = \pi \bar{M}_{t-1}$ where the initial nominal quantity of money, $\bar{M}_0 = M_0^1 + M_0^2$ is given. The law of motion of the nominal quantity of money is thus

$$\bar{M}_t = (1 + \pi)\, \bar{M}_{t-1} \tag{4}$$

In order to focus on the specific role of borrowing constraint on the non-neutrality of inflation, it is assumed that monetary authorities follow the "most" neutral rule, which is to distribute by lump sum transfer the exact amount of resources paid by private agents due to the inflation tax. As a consequence, the new money is distributed proportionally to the level of beginning-of-period money balances. In period t, type i agents have a beginning of period quantity of money M_{t-1}^{i}. Hence, we assume that $\mu_t^i = \pi M_{t-1}^{i}$, and the real transfer reads

$$\frac{\mu_t^i}{P_t} = \frac{\pi}{\Pi_t} m_{t-1}^{i} \tag{5}$$

Given initial conditions a_1^1, a_1^2, M_0^1, and M_1^2, and given π, an equilibrium of this economy is a sequence $\{c_t^1, c_t^2, m_t^1, m_t^2, a_{t+1}^1, a_{t+1}^2, P_t, r_t, w_t\}_{t=1\ldots\infty}$ which satisfies the problem of households (1), the first order condition of the problem of the firms (3), and the different market equilibria. More precisely, I focus on symmetric stationary equilibria[1], where all real variables are constant, and where all agents in each state H and L, which will be denoted H and L households, have the same consumption and savings levels. The variables describing agents in state H will be denoted m^H, c^H, a^H, and agents in state L will be described by m^L, c^L, a^L. As a consequence, as the real quantity of money in circulation $\Sigma = \bar{M}_t/P_t$ is constant in a stationary equilibrium, equality (4) implies that the price of the final goods grow at a rate π, and hence $\Pi = 1+\pi$.

2.2 Stationary Equilibrium

With the budget constraint (2), and the amount μ_t^i/P_t given by (5), one finds that the budget constraints of H and L households are given respectively by

$$c^H + m^H + a^H = (1+r)\, a^L + m^L + w \tag{6}$$

$$c^L + m^L + a^L = (1+r)\, a^H + m^H \tag{7}$$

Note that the inflation rate does not show up in these equations since the creation of new money does not introduce any transfer between the two types of households.

Using standard dynamic programming arguments, the problem of the households can be solved easily. This is done in appendix A.

For H households, one finds the following optimal conditions

$$u_c'\left(c^H, m^H\right) = \beta\,(1+r)\, u_c'\left(c^L, m^L\right) \tag{8}$$

$$u_c'\left(c^H, m^H\right) - u_m'\left(c^H, m^H\right) = \frac{\beta}{\Pi} u_c'\left(c^L, m^L\right) \tag{9}$$

[1] In liquidity constraint models, the path of the economy converges toward a steady state, or even begins at a steady state if a period 1 transfer is made to households consistently with steady state values (Kehoe and Levine, 2001)

The first equation is the Euler equation for H households, who can smooth their utility thanks to positive savings. Indeed, H households are the high income households and are never credit constrained. The second equality is the arbitrage equation, which determines the demand for real money balances. H households equalize the marginal cost of holding money in the current period, (i.e. the left hand side of equation 9), to the marginal gain of transferring one unit of money to the following period when they are in state L, (i.e. the right hand side of equation 9). The marginal utility of money shows up here as a decrease in the opportunity cost of holding money. And the gain from money holdings takes into account of the real return $1/\Pi$ of cash .

The solution of the program of L households depends on whether the credit constraints are binding or not. If credit constraints are binding , the solution is $a^L = 0$ and

$$u_c'\left(c^L, m^L\right) > \beta\left(1+r\right)u_c'\left(c^H, m^H\right) \tag{10}$$

$$u_c'\left(c^L, m^L\right) - u_m'\left(c^L, m^L\right) = \frac{\beta}{\Pi}u_c'\left(c^H, m^H\right) \tag{11}$$

The first inequality shows that L households would be better off if they could transfer some income from the next period toward the current one. The second equality involves the same trade-off as the one of H households discussed above. Finally, if credit constraints do not bind for L households, the inequality (10) becomes an equality and $a^L > 0$.

Using expression (8) together with condition (10), one finds that credit constraints are binding if and only if $1+r < 1/\beta$. If credit constraints do not bind, equality (8) and the relationship (10) taken with an equality imply $1+r = 1/\beta$. The following proposition[2] summarizes this standard result.

Proposition 1 *Credit constraints are binding for L households if and only if* $1+r < 1/\beta$*. If credit constraints do not bind then* $1+r = 1/\beta$*.*

When credit constraints are binding, the gross real interest rate $1+r$ is lower than the inverse of the discount factor. As a consequence, there is always capital over-accumulation because of the precautionary motive to save, which is is a standard result in this type of liquidity constrained models (see Woodford 1990 ; Kehoe and Levine, 2001 among others). When credit constraints do not bind, the inflation rate does not affect the long run real interest rate. In this case, inflation only affects monetary variables.

Conditions on the parameters of the model

[2]Note that $1+r$ can not be lower than $1/\Pi$, otherwise the financial market could not clear. As a consequence, an equilibrium with binding credit constraints can exist only if $1/\Pi < 1/\beta$. Moreover, we assume that the surplus left for consumption is positive at the Friedman rule, which implies $\alpha < 1/\Pi$.

2.3 Monetary Policy with Binding credit constraints

In this section we derive the property of the equilibrium when credit constraints are binding in equilibrium for L households, that is when $1+r<\frac{1}{\beta}$. Note that the choice of money balances of H households can be easily written with equations (8) and (9) at the stationary equilibrium, which yields the following equality

$$\frac{m^H}{c^H}=\frac{1-\phi}{\phi}\frac{1}{1-\frac{1}{\Pi}\frac{1}{1+r}} \tag{12}$$

For the H households, the ratio of money over consumption is determined by preference parameters and by the opportunity cost to hold money. To see this, assume that r and π are small, then $1-1/\left(1+\pi\right)\left(1+r\right)\simeq r-(-\pi)$, which is the difference between the real net return on financial titles and the real net return on money or, in other words, which is the nominal interest rate.

By contrast, when credit constraints are binding, that is when $1+r<\frac{1}{\beta}$, one finds, using (8) with (11) :

$$\frac{m^L}{c^L}=\frac{1-\phi}{\phi}\frac{1}{1-\frac{\beta^2}{\Pi}\left(1+r\right)} \tag{13}$$

The equilibrium ratio for L households is not simply determined by the opportunity cost to hold money, but by the difference between consumption in the current period and the return on money holdings two periods ahead. Indeed, the ratio $\beta^2\left(1+r\right)/\Pi$ is the discounted value of one unit of money held in state L, transferred in state H, and then saved on financial market to the next period, where the household is in state L again. When this ratio increases, L households increase the ratio of their money holdings over their consumption. As a consequence, state L households increase the relative demand for money when the real interest increases, contrary to state H households. Indeed, the real interest rate appears as the remuneration of future savings and not as the opportunity cost to hold money. The following proposition summarizes this key property of the model. The proof is left in appendix.

Proposition 2 *If credit constraints are binding in the stationary equilibrium, the real interest rate decreases when inflation increases.*

The non-neutrality of inflation stems from the heterogeneity in money demand. When credit constraints do not bind, all households have the same level of money holdings and thus share the same reaction following a change in inflation. In this case, inflation has no effect on real variables, in lines with the traditional Sidrauski's result. Yet when credit constraints are binding, inflation favors unambiguously capital accumulation and output, which is the traditional Tobin (1965) result. Indeed, the L households decrease their money holdings m^L in a lower proportion than H households do, because money is their only store of value. As a consequence, H households have more resources to save and consume when inflation increases. Indeed, their budget constraints yields $c^H + a^H = w + m^L - m^H$. As an indirect effect, capital accumulation raises w and the incentives to save to smooth consumption.

This simple model has proven that imperfections on financial market give rise to heterogeneity in money demand, which is at the core of the non neutrality of inflation. The next step is to provide a quantitative evaluation of this new channel to assess its magnitude compared to other well known channels through which inflation affects real variables.

3 The General Model

We describe a fully fledged model encompassing more general assumptions on idiosyncratic risks, endogenous labor supply and distorting taxes in order to investigate quantitatively the role of inflation. The economy considered here builds on the traditional heterogeneous agents framework *à la* Aiyagari (1994). But we embed in this framework money in the utility function and monetary policy issues. This section presents the most general model. Different specifications of this model will be used in the simulation exercise to disentangle the various channels through which inflation affects the real economy.

3.1 Agents

3.1.1 Households

The economy consists of a unit mass of *ex ante* identical and infinitely-lived households. They maximize expected discounted utility from consumption c, from leisure and real balances $m = \frac{M}{P}$. Labor endowment per period is normalize to 1, and the working time is simply l. Leisure is thus $1 - l$. For the sake of generality, we follow the literature which introduces directly money m in the utility function of private agents to capture its liquidity services. For the benchmark version of the model, we assume that the utility function has a general CES specification following Chari *et al.* (2000). The utility of agent i is given by:

$$u\left(c_i, m_i, l_i\right) = \frac{1}{1-\sigma}\left[\left(\omega c_i^{\frac{\eta-1}{\eta}} + (1-\omega)\, m_i^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} (1-l_i)^{\psi}\right]^{1-\sigma} \tag{14}$$

where ω is the share parameter, η is the interest elasticity of real balances demand, ψ is the weight of leisure and σ is risk aversion.

Individuals are subject to idiosyncratic shocks on their labor productivity e_t. We assume that e_t follows a three states Markov process over time with $e_t \in E = \left\{e^h, e^m, e^l\right\}$, where e^h stands for high productivity, e^m for medium productivity, e^l for low productivity. The productivity process follows a 3×3 transition matrix[3] Q. The probability distribution across productivity is represented by a vector $n_t = \{n_t^h, n_t^m, n_t^l\}$: $n_t \geq 0$ and $n_t^h + n_t^m + n_t^l = 1$. Under technical conditions, that we assume to be fulfilled, the transition matrix has a unique vector $n^* = \{n^h, n^m, n^l\}$ such that $n^* = n^* Q$. Hence, the n_t converges toward n^* in the long run. n^* is distribution of the population in each state. For instance, n^h is the proportion of the population with a high level of labor productivity. In the general model, there is an endogenous labor supply for each level of productivity.

Markets are incomplete and no borrowing is allowed. In lines with Aiyagari (1994), households can self-insure against employment risks by accumulating a riskless asset a which yields a return r. But they can also accumulate real money assets $m = M/P$, which introduces a new channel compared to the previous heterogeneous agent literature. If the price level of the final good at period t is denoted P_t, the gross inflation rate between period $t-1$ and period t is $\Pi_t = \frac{P_t}{P_{t-1}}$. If an household holds a real amount m_{t-1} of money at the end of period $t-1$, the real value of her money balances at period t is $\frac{m_{t-1}}{\Pi_t}$. As long as $\Pi_t > \frac{1}{1+r_t}$, money is a strictly dominated assets, but which will be demanded for its liquidity services. Households are not allowed to borrow and can not issue some money.

The budget constraint of household i at period t is given by:

$$c_t^i + m_t^i + a_{t+1}^i = (1 + r_t)\, a_t^i + \frac{m_{t-1}^i}{\Pi_t} + w_t e_t^i l_t^i \qquad t = 0, 1, .. \tag{15}$$

with given $(1 + r_0)\, a_0^i$ and m_{-1}^i. The sequence of constraints on the choice variables is

$$a_{t+1}^i \geq 0,\ 1 \geq l_t^i \geq 0, c_t^i \geq 0,\ m_t^i \geq 0 \quad t = 0, 1... \tag{16}$$

The value r_t is the after-tax return on financial assets, e_t^i is the productivity level of the worker at period t, and w_t is the after-tax revenue on labor.

For the sake of realism, we assume that there is a linear tax on private income. The tax rate on capital at period t is denoted χ_t^a and the tax rate on labor is denoted χ_t^w. As a consequence, if $\tilde{r}_t$ and $\tilde{w}_t$ are the revenue of capital and labor paid by the firms, the returns for households satisfy the following relationships

$$\begin{aligned} r_t &= \tilde{r}_t(1 - \chi_t^a) \\ w_t &= \tilde{w}_t(1 - \chi_t^w) \end{aligned}$$

[3]This assumption is based on Domeij and Heatchcote (2003) who found that one needs at least three employment states to match crucial empirical features of the employment process and wealth distribution. See the section devoted to the calibration of the model.

Let denote total wealth in period t by q_t^i. Then

$$q_t^i = (1 + r_t)\, a_t^i + \frac{m_{t-1}^i}{\Pi_t}$$

With this definition, the program of agent i can be written in the following recursive form

$$v\left(q_t^i, e_t^i\right) = \max_{\{c_t^i, m_t^i, l_t, a_{t+1}^i,\}} u\left(c_t^i, m_t^i, l_t^i\right) + \beta E\left[v\left(q_{t+1}^i, e_{t+1}^i\right)\right]$$

$$\begin{aligned} \text{s.t}\;\; c_t^i + m_t^i + a_{t+1}^i &= q_t^i + w_t e_t l_t^i \\ q_{t+1}^i &= (1 + r_{t+1})\, a_{t+1}^i + \frac{m_t^i}{\Pi_{t+1}} \end{aligned}$$

with the sequence of constraints on the choice variables (16) and with the transition probability for labor productivity given by the matrix Q.

Since the effect of inflation on individual behavior heavily depends on whether the credit constraints are binding, we distinguish two cases.

- *Non Binding credit constraints*

In this case, the first order conditions of agent i read as follows

$$u_c'\left(c_t^i, m_t^i, l_t^i\right) = \beta\left(1 + r_{t+1}\right) E\left[v_1'\left(q_{t+1}^i, e_{t+1}^i\right)\right] \tag{17}$$

$$u_c'\left(c_t^i, m_t^i, l_t^i\right) - u_m'\left(c_t^i, m_t^i, l_t^i\right) = \frac{\beta}{\Pi_{t+1}} E\left[v'\left(q_{t+1}^i, e_{t+1}^i\right)\right] \tag{18}$$

$$u_l'\left(c_t^i, m_t^i, l_t^i\right) = -w_t e_t u_c'\left(c_t^i, m_t^i, l_t^i\right) \tag{19}$$

Equality (19) only holds if the solution satisfies $l_t^i \in [0; 1]$. Otherwise, the l_t^i takes on a corner value, and the solution is given by (17) and (18).

Let denote the real cost of money holdings γ_{t+1} by

$$\gamma_{t+1} \equiv 1 - \frac{1}{\Pi_{t+1}} \frac{1}{(1 + r_{t+1})}$$

This indicator measures the opportunity cost of holding money. When the after-tax nominal interest rate r_{t+1}^n, defined by $1 + r_{t+1}^n = \Pi_{t+1}\left(1 + r_{t+1}\right)$ is small enough, then $\gamma_{t+1} \simeq r_{t+1}^n$. With this notation and the expression of the utility function given above, the first order conditions (17) and (18) yield

$$m_t^i = \left(\frac{1 - \omega}{\omega} \frac{1}{\gamma_{t+1}}\right)^{\eta} c_t^i$$

This equality shows that the money demand of unconstrained households is only affected by the substitution effect depending on the opportunity cost of holding money.

- *Binding credit constraints*

When the household problem yields a value for financial savings which is negative, credit constraints are binding, $a_{t+1} = 0$, and the first order condition yields the inequality

$$u_c'\left(c_t^i, m_t^i, l_t^i\right) > \beta\left(1 + r_{t+1}\right) E\left[v_1'\left(q_{t+1}^i, e_{t+1}^i\right)\right]$$

The first order conditions of the constrained problem are given by

$$\begin{aligned} u_c'\left(c_t^i, m_{t+1}^i\right) - u_m'\left(c_t^i, m_{t+1}^i\right) &= \frac{1}{\Pi_{t+1}}\beta E\left[v'\left(\frac{m_t^i}{\Pi_{t+1}}, e_{t+1}^i\right)\right] && (20) \\ u_l'\left(c_t^i, m_t^i, l_t^i\right) &= -w_t e_t u_c'\left(c_t^i, m_t^i, l_t^i\right) && (21) \end{aligned}$$

Money demand has no simple expression in case of binding-constraints. The static trade-off between demand for money and demand for consumption appears at the left hand side of equation (20). If money was not a store of value, this expression would be equal to 0. But, as money allows to transfer revenue to the next period, it creates an additional motive to demand it.

Importantly enough, inflation turns out to have two contrasting effect on the demand for money of borrowing constrained households, what can be seen at the right hand side of equation (20). On the one hand, inflation induces a substitution effect which contributes to decrease the demand for money when inflation increases (represented by the term $\frac{1}{\Pi_{t+1}}$). On the other hand, the inflation rate entering into the value function through a revenue effect, it might induce an increase in demand for money when inflation increases. The core reason for this result is that money is the only store of value for borrowing constrained households. If the function v is very concave, and for realistic values of the parameters, this second effect can dominate, and the demand for money can increase with inflation. We will show in the quantitative analysis that this result holds for the poorest agents. As a consequence, this case proves that the change in money demand yielded by inflation, the so-called *Tobin effect*, can be decomposed into a revenue effect and a substitution effect for the constrained households.

Finally, working hours are determined by the equation (21). If the value of l_t provided by this equality is negative, then $l_t = 0$ and the fist order condition (21) holds with an inequality.

The solution of the program of the households provides a sequence of functions which yield at each date the policy rules for consumption, financial savings, money balances and leisure as a function of the level of labor productivity and wealth:

$$\left.\begin{array}{r} c_t : E \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \\ a_{t+1} : E \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \\ m_t : E \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \\ l_t : E \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \end{array}\right\} t = 0, 1, ...$$

3.1.2 Firms

We assume that all markets are competitive and the only good consumed is produced by a representative firm with an aggregate Cobb-Douglas technology. Let K_t and L_t stand for aggregate capital and aggregate effective labor used in production respectively. It is assumed that capital depreciates at a constant rate δ and is installed one period ahead from production. Since there is no aggregate uncertainty, aggregate employment and, more generally, aggregate variables are constant at the stationary equilibrium

The output is given by

$$Y_t = F\left(K_t, L_t\right) = K_t^{\alpha} L_t^{1-\alpha} \quad 0 < \alpha < 1$$

The effective labor supply is equal to $L_t = L_t^h e^h + L_t^m e^m + L_t^l e^l$, where L_t^h, L_t^m and L_t^l are the aggregate demand for each type of labor. Prices are set competitively:

$$\tilde{w}_t = (1-\alpha)\left(K_t/L_t\right)^{\alpha} \tag{22}$$

$$\tilde{r}_t + \delta = \alpha (K_t/L_t)^{\alpha-1} \tag{23}$$

As high, medium and low productivity workers are perfect substitute with difference productivity, one necessarily gets

$$\tilde{w}_t^h = e^h \tilde{w}_t, \qquad \tilde{w}_t^m = e^m \tilde{w}_t, \qquad \tilde{w}_t^l = e^l \tilde{w}_t \tag{24}$$

The aggregate demand for capital is given by

$$K_t = L_t(\alpha/(\tilde{r}_t + \delta))^{\frac{1}{1-\alpha}}$$

3.1.3 Government

The government levies taxes to finance a public good, which costs G units of final goods at each period. Taxes are proportional to the revenue of capital and labor, with a coefficient χ_t^a and χ_t^w at period t. In addition, the government gets the revenue of the new money created at period t, which is denoted τ_t^{tot} in real term. It is assumed that the government does not issue any debt. The government budget constraint is given by

$$G = \chi_t^a \tilde{r}_t K_t + \chi_t^w \left(L_t^h e_t^h + L_t^l e_t^l + L_t^m e_t^m\right) \tilde{w}_t + \tau_t^{tot} \tag{25}$$

3.1.4 Monetary Policy

The monetary policy is assumed to follow a simple rule. At each period, the monetary authorities create an amount of new money which is proportional with a factor π to the nominal quantity of money in circulation, $P_t\Omega_t = P_{t-1}\Omega_{t-1} + \pi P_{t-1}\Omega_{t-1}$. As is standard in the monetary literature, we assume that the State gets all the revenue from the inflation tax[4], which is a more realistic assumption than the helicopter drops of money. As a result the real quantity of money in circulation at period t is

$$\Omega_t = \frac{\Omega_{t-1}}{\Pi_t} + \pi\frac{\Omega_{t-1}}{\Pi_t} \tag{26}$$

The real value of the inflation tax in period t is

$$\tau_t^{tot} = \pi\frac{\Omega_{t-1}}{\Pi_t} \tag{27}$$

Note that if the real quantity of money in circulation is constant (which is the case in equilibrium), equation (26) implies that $\Pi = 1+\pi$, and hence $\tau = \frac{\pi}{1+\pi}\Omega$, which stands for the standard expression of the inflation tax.

3.2 Equilibrium

Market Equilibria

Let $\lambda_t : E \times \mathbb{R}^+ \longrightarrow [0,1]$ denote the joint distribution of agents over productivity and wealth. Aggregate consumption C_t, aggregate real money holdings M_t, aggregate effective labor L_t^s and aggregate financial savings A_{t+1} are given by

$$\begin{aligned}
C_t &= \int\int c_t\left(e^k, q\right) d\lambda_t\left(e^k, q\right) \\
M_t &= \int\int m_t\left(e^k, q\right) d\lambda_t\left(e^k, q\right) \\
L_t^s &= e^h \int l_t\left(e^h, q\right)\lambda_t\left(e^h, q\right) dq + e^l \int l_t\left(e^l, q\right)\lambda_t\left(e^l, q\right) dq + e^m \int l_t\left(e^m, q\right)\lambda_t\left(e^m, q\right) dq \\
A_{t+1} &= \int\int a_{t+1}\left(e^k, q\right) d\lambda_t\left(e^k, q\right)
\end{aligned}$$

Equilibrium in the final good market implies

$$C_t + K_{t+1} + G_t = Y_t + (1-\delta) K_t \tag{28}$$

Equilibrium in the labor market is

$$L_t = L_t^s$$

Equilibrium in the financial market implies

$$K_{t+1} = A_{t+1} \tag{29}$$

The money market equilibrium is defined by

$$M_t = \Omega_t \tag{30}$$

[4]In practice, the profits of central banks are redistributed to the State and are not used for specific purposes.

where Ω_t is the real quantity of money in circulation at period t.

Competitive equilibrium

A stationary competitive equilibrium for this economy consists of constant decision rules $c(e, q)$, $m(e, q), l(e, q)$ and $a(e, q)$ for consumption, real balances, leisure and financial asset holdings respectively, the steady state joint distribution over wealth and productivity $\lambda(e, q)$, a constant real return on financial asset r, a constant real wage w, the real return on real balances $1/\Pi$, and tax transfers χ^a , χ^w , consistent with the exogenous supply of money π and the government public spending G such that

1. The long run distribution of productivity is given by a constant vector n^*.
2. The functions a, c, m, l solve the problem of households
3. The joint distribution λ over productivity and wealth is time invariant.
4. Factor prices are competitively determined, by equation (22)-(24).
5. Markets clear: equations (28)-(30).
6. The quantity of money in circulation follows the law of motion (26)
7. The tax rates χ^a and χ^w are constant and are defined to balance the budget of the State (25), where the seigniorage rent from the inflation tax τ^{tot} is given by (27).

Note that the equilibrium on the money market and the stationary of the joint distribution imply that the real quantity of money in circulation is constant.

3.3 Calibration

The model period is one year and the model is calibrated on the US economy. Since the primary interest of the paper lies on the interactions between credit constraints, wealth heterogeneity and monetary policy, one key goal is to match the observed distributions of wealth and in particular the share of people borrowing constrained. In what follows we focus on the benchmark incomplete market economy with endogenous prices, proportional taxes, endogenous labor supply for an initial inflation rate of π equals to 2 percent.

Technology and preferences

Table 2 reports the parameters for preferences and technology. The parameters relating to the production technology and the discount factor are standard: capital's share α is set equal to 0.36, the capital depreciation rate is 0.1 and the discount factor is set to 0.96.

We choose parameter values for the utility function (14) as follows. For ω and η we draw on the money demand literature. The interest elasticity η is set to $\eta = 0.5$, which is close to the traditional estimates (e.g. Chari *et al.* 2000, Holman 1998, Hoffman, Rasche and Tieslau 1995), and which can be microfounded in a Baumol-Tobin type model of money demand. The share parameter ω of consumption relative to money is then set to $\omega = 0.98$ in order match the ratio of real balances over GDP. As there is no standard definition of money in this literature (M1 or M2), we use the average value of M1/GDP and M2/GDP which is about 0.30 on the same period. The weight on leisure ψ is set to reproduce a steady state fraction of labor of 33 percent of total time endowment. Eventually we set the risk aversion parameter at a value different to the log utility function but close enough to one in order to match the observed capital-ouput ratio.

Table 1: Benchmark calibration

Parameters	β	α	δ	ω	η	ψ	σ
Values	0.96	0.36	0.1	.98	0.5	2	1.1

Employment Process

Regarding the employment process, the key goal of the calibration is to find a stylized process for wages empirically relevant and which is able to replicate the US wealth distribution - in particular the share of people who are borrowing constrained.

We follow Domeij and Heathcote (2003) who estimated a rather stylized process to match some of these criteria (see Castaneda *et al.* (2003) for an alternative relevant strategy). The authors found that one needs at least three employment states to match two main features of the wealth distribution estimated by Budria Rodriguez *et al.* (2002): the high Gini coefficient of wealth and the fact that the two poorest quintiles of the distribution hold no more than 1.35 percent of the total wealth. Thus the set of employment states is represented by $e = \left\{e^h, e^m, e^l\right\}$ where e^h stands for high productivity, e^m for medium productivity, e^l for low productivity. The ratio between the different productivity levels and the transition probabilities are set in order to match the autocorrelation $\rho_w = 0.9$ and the innovation $\sigma_w = 0.224$ in the individual earnings estimated on the PSID. The implied ratio of productivity values are $e_1/e_2 = 6.06$ and $e_2/e_3 = 5.02$. And the Markov chain consistent with the observed earning process is $p_{1,1} = p_{3,3} = 0.9$ and $p_{2,2} = 0.97$.

$$Q = \begin{bmatrix} p_{1,1} & 1 - p_{1,1} & 0 \\ \frac{1-p_{2,2}}{2} & p_{2,2} & \frac{1+p_{2,2}}{2} \\ 0 & 1 - p_{1,1} & p_{1,1} \end{bmatrix}$$

This specification yields a Gini coefficient of wealth of 0.75, which is quite close to the recent findings of Budria Rodriguez et al. (2002). The Gini coefficient for consumption reaches 0.30 consistently with the findings of Perri and Grueger (2005).

Importantly enough our specification yields an empirically relevant fraction of borrowing constrained households. Some previous estimation concluded that around 20 percent of households were credit constrained (e.g. Japelli, 1990). Recent estimations by Buria *et al.* (2002) on the United States finds that 7.4 percent of people have negative wealth, and are thus potentially concerned by credit constraints. We calibrated the model on this latter smaller value, to avoid any overestimation of the effect at stake. The key point is that this finding is only linked to the introduction of real balances in the traditional Aiyagari model. For instance Domeij and Heathcote (2003) and Heathcote (2005) found that no one is borrowing constrained in their specification. The previous literature generally needs to introduce stochastic discounting factor to match this dimension (Krusell and Smith (1998), Carroll (2000)). By contrast, introducing money in the utility function naturally entails that wealth-poor people need to carry-on real balances next period in order to be able to consume. They would thus set down their level of financial assets equal to zero to be able to keep a positive amount of real balances when they are affected by negative labor productivity shocks.

Table 2 reports the main statistics reproduced by our model under the benchmark specification with endogenous prices, distorting inflation taxes and endogenous labour supply. The benchmark specification matches closely the key observed ratio of capital $K/Y = 3$, of money $(M/P)/Y = 0.35$ and of public spending $G/Y = 0.24$. In the benchmark calibration we assume that the tax rate on capital and labor is the same : $\chi^a = \chi^w \equiv \chi$. The calibration yields an average tax rate on labor and capital $\chi = 0.34$ closed to the observed one (Domeij and Heathcote, 2003). Importantly enough, the benchmark set-up matches the Gini coefficient of wealth and consumption and is able to replicate both the upper tail and the lower tail of the wealth distribution.

Table 2: Benchmark calibration

Values	Data	Benchmark economy
K/Y	3	3
(M/P)/Y	0.30	0.35
G/Y	0.20	0.24
χ	0.33	0.34
Gini Wealth	0.78	0.76
Gini Consumption	0.30	0.30
Wealth 80-100	79.5	80
Fraction with wealth <0	7.4	7.4

4 Results

4.1 Individual policy rules

We start by discussing the impact of inflation on individual policy rules in the benchmark economy with endogenous hours and taxes.

Figure 1 reports the main policy rules in the benchmark economy with an inflation rate at $\pi = 2$ percent. Consumption, real balances and financial assets are an increasing function of labor productivity and current total wealth q made up of financial assets and cash. But due to the presence of borrowing constraints, the value functions and the implied policy rules for consumption and money demand are concave at low value of wealth and productivity. Moreover the policy rule for financial assets held by medium and low productivity workers display kinks at low level of wealth, indicating that these two types of workers are net-dissavers. By contrast high productivity workers are net-savers in order to smooth consumption across less favorable productivity states.

Figure 2 reports the impact of a one percent rate increase in inflation from $\pi = 2\%$ to $\pi = 3\%$ on next period asset holdings and money balances as a function of beginning of period total wealth. The focus is put on the policy rules around the kink where the main non-linearity lies. We focus on the high and the low productivity states, households in the medium state having similar policy rules as the low productivity ones. For high value of productivity, an increase in inflation provides more incentives to save in financial assets at the expense of real money balances whose value has been slashed by inflation. This behavior stands in sharp contrast with that of households in lower productivity states. These households are borrowing constrained on asset holdings at low level of total wealth. In this case they have no other choice than increasing their level of money balances following a rise in inflation in order to sustain their level of consumption. Indeed, money is used as a store of value, and the revenue effect dominates the substitution effect when wealth is low, as explained in the discussion of equality (20). Their level of real money balances decreases only at higher level of total wealth for which credit constraints on financial assets are no longer binding. This contrasted effect suggests that the effect of inflation on the real economy and welfare crucially depends on borrowing constraints.

4.2 Aggregate results

This section quantifies the impact of monetary policy on the real economy and welfare. We look at a policy experiment in which the inflation rate rises by one point from $\pi = 2$ percent to $\pi = 3$ percent. The quantitative theoretical analysis proceeds as follows. We quantify the aggregate impact of inflation depending on the different assumptions on the redistribution of the

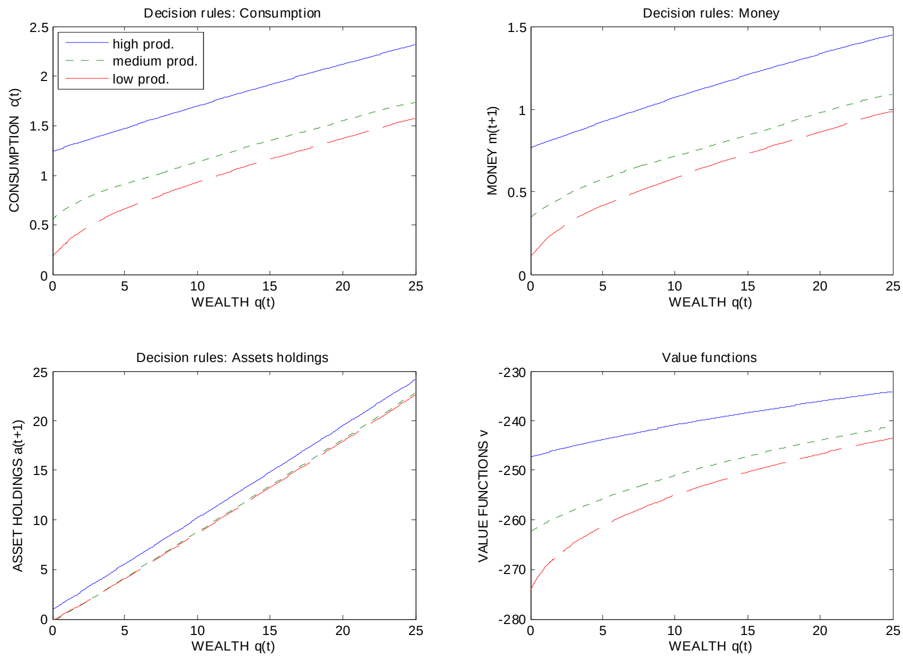

Figure 1: Individual policy rules

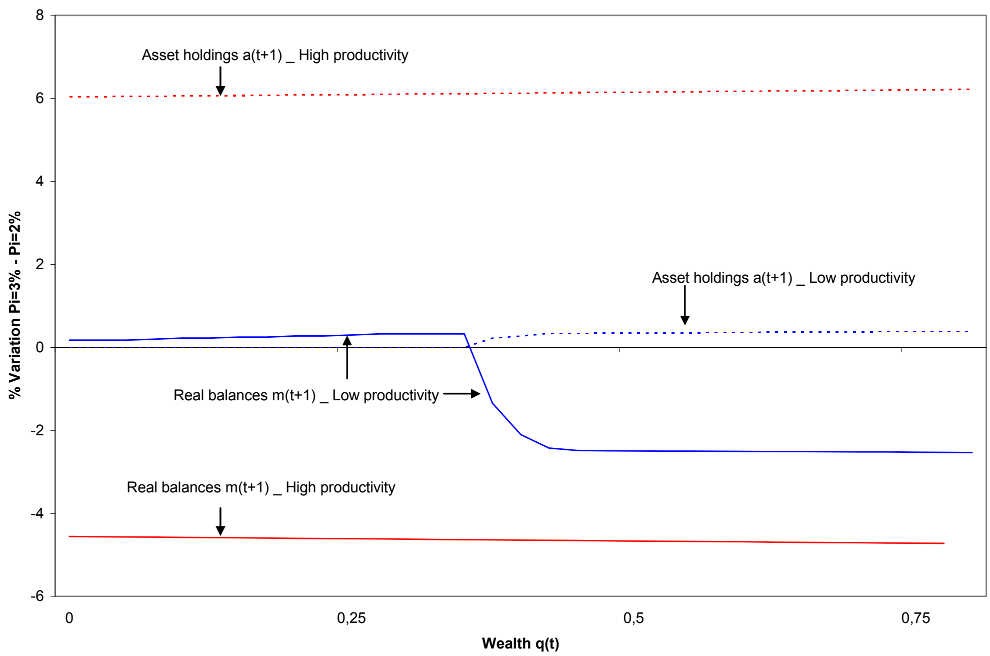

Figure 2: Effect of inflation on individual policy rules

seigniorage rent, the tax structure and the adjustment of labor supply, to be able to disentangle the various effects of inflation in this economy.

First we consider a version of the model in which hours are exogenous, inflation taxes are lump-sum and are redistributed proportionally to the begining-of-period real balances. We abstract thus from any redistribute and distorting issues discussed in the previous literature. Consistently with our theoretical results found in section 2, this set-up allows us to quantify the non-neutrality of monetary policy which *only* transits through credit constraints. This framework is thus mainly illustrative since the neutrality of money would be obtained under these assumptions if markets were to be complete.

Second, we take into account of the traditional redistributive and distorting effects of inflation which interact with credit constraints. Labor supply is still assumed to be exogenous but there are now proportional taxes on labor and capital income. In this case, credit constraints give rise to two new effects of inflation. The redistributive effect is due to the fact that the seigniorage rent is redistributed unevenly across wealth-poor and wealth-rich agents. The distorting tax effect is due to the fact that the seigniorage rent allows a reduction in capital taxes and thus increases the incentives to save. This traditional Phelps effect is amplified by the presence of borrowing constraints through precautionary savings motive. We assess the contribution of credit constraints to these effects by comparing complete markets and incomplete markets with credit constraints. Third, we extend the model by introducing the labor supply decision.

Due to borrowing constraints, inflation gives rise to heterogeneous labor supply depending on the endogenous wealth heterogeneity. We measure this new effect by comparing the incomplete market set-up with the corresponding complete market one with endogenous hours worked and distorting taxation.

For each economy, we change the parameters relating to the household productivity process so that each economy matches the same targeted feature of the US wealth distribution[5]. The calibration given below concerns thus the benchmark model with endogenous labor supply.

4.2.1 Lump-Sum Transfers

Environment

In the first stage of the analysis, we define a special case of our model in which the real effect of monetary policy only transits through credit constraints regardless of other potential

[5] For each environment with exogenous hours, we keep exactly the same parameter values for the incomplete market and the complete market models. Actually, these two set-up provide similar initial steaty-states. By contrast (and as in Heathcote, 2005), we use a smaller labor supply elasticity $\psi = 0.75$ in the representative economy with endogenous hours in order to start from the same steady state values for L, K/Y, M/Y and taxes as the ones which hold in the incomplete market framework.

distortions. For that purpose, we consider the following environment. First, we assume that each household supplies inelastically $l = \bar{l}$ hours of labor. We set $\bar{l} = 0.33$ which corresponds to the steady state value of labor with endogenous labor supply at $\pi = 2$ percent. Second, we assume that there are no taxes on labor and capital, and all (net) transfers are lump sum. Third, the government spending is equal to 0, and the government distributes the new money proportionally to the beginning of period level of real balances held by each household. This environment corresponds to the simple model presented in Section 2 but with a more general labor income process.

The budget constraint (15) of household i rewrites in this case

$$c_t^i + m_t^i + a_{t+1}^i = (1 + \tilde{r}_t)\, a_t^i + \tilde{w}_t e_t^i \bar{l} + \frac{m_{t-1}^i}{\Pi_t} + \tau_t^i$$

where τ_t^i stands for lump-sum transfer of the seigniorage tax, and $\tilde{r}_t$ and $\tilde{w}_t$ are the levels of interest rate and wage paid by the firm (without tax), defined by equations (22) and (23) respectively.

The seigniorage tax is redistributed *ex-post* to each agent as a lump-sum transfer proportional to the beginning of period money holdings

$$\tau_t^i = \frac{\pi}{\Pi_t} m_{t-1}^i$$

As a consequence, the *ex post* individual budget constraint writes

$$c_t^i + a_{t+1}^i + m_t^i = (1 + \tilde{r}_t)\, a_t^i + \tilde{w}_t e_t^i \bar{l} + m_{t-1}^i$$

In this case, inflation no longer shows up in the individual budget constraint. But since the seigniorage tax is redistributed *ex-post*, the inflation rate is still taken into account by households as the anticipated inflation rate affects the arbitrage conditions to hold money.

Aggregate results

Let now consider the aggregate outcomes of a one point rise in the inflation rate from $\pi = 2$ percent to 3 percent in this environment. The aggregate results for the economy with neutral lump-sum taxes and exogenous hours are reported in Table 3 -Line 1. We focus on the main aggregate variables: output Y, capital K, real balance M/P , aggregate consumption C and prices w and r. In this table, we give the percentage change of each variable compared to its value when inflation is set equal to 2%.

With complete markets and non-distorting taxes, inflation has no real effect on stationary real aggregate variables. Each household can adjust in the same proportion her demand for real money and financial asset holdings, leading to a neutral effect of inflation on aggregate consumption, capital and output. The effect of inflation only transits through nominal variables, the aggregate stock of money decreasing by 6.97 percent

By contrast, when markets are incomplete and households face borrowing constraints, those who are credit constrained cannot adjust in the same way their money and capital holdings compared to unconstrained agents as illustrated by figure 2. Constrained agents have no other choice than that of increasing their demand for money to restore their level of real balances and to be able to consume tomorrow. At the other extreme, unconstrained agents increase their level of financial assets whose returns relative to cash is increasing with inflation. As a matter of fact, aggregate capital rises by 0.23 percent, leading to an increase in output and consumption by 0.08 percent and 0.02 percent respectively. Yet since households have more incentive to save in financial assets in the incomplete market set-up, the reduction in real balances is much sharper compared to the complete market set-up.

4.2.2 Redistributive effect of the seigniorage rent

Environment

We now discuss the sensitivity of the role played by borrowing constraints in the non-neutrality of money when we take into account additional distortions in the inflation tax. To that end we introduce proportional taxes in lines with the benchmark incomplete market model described in section 3 and compare the results with a complete market economy. We still shut down the labor supply channel by assuming that the number of hours is fixed at the stationary value $\bar{l} = 0.33$. In this case, the individual budget constraint and the government budget constraint reads respectively as

$$c_t^i + m_t^i + a_{t+1}^i = (1 + \tilde{r}_t(1 - \chi_t^a))a_t^i + (1 - \chi_t^w)\tilde{w}_t e_t^i \bar{l} + \frac{m_{t-1}^i}{\Pi_t}$$

and

$$G = \chi_t^a \tilde{r}_t K_t + \chi_t^w \left(n^h e_t^h + n^m e_t^l + n^m e_t^m \right) \bar{l} \tilde{w}_t + \tau_t^{tot}$$

with $\tau_t^{tot} = \pi \frac{\Omega_t}{\Pi_t}$.

Redistributive distortion of the inflation tax

We first focus on the redistributive effect of the seigniorage rent. In particular we assume that the seigniorage rent is redistributed proportionally to labor income. To isolate this redistributive effect, we need to shut down the Phelps effect transiting through a reduction in capital tax. For that purpose, we assume that the distorting proportional taxes on capital χ^a is not affected by the rise in inflation. This tax is held constant at its stationary value corresponding to an inflation rate of 2 percent. Yet the rise in the seigniorage tax τ_t^{tot} allows a reduction in the proportional tax on labor χ^w. Thus everything works as if the government was engineering a transfers of the seigniorage rent proportionally to labor income. As we assume in this section that labor supply is exogenous, the transfers proportional to labor are not distorting.

Table 3 -Line 2 reports that the tax on labor sharply decreases by 1.04 thanks to higher seigniorage rents. But since the redistribution of the seigniorage rent is proportionally more favorable to high productivity workers, the latter ones have higher incentives to save in order to smooth their consumption. As a matter of fact, the increase in aggregate capital and output - by 0.34 and 0.12 percent respectively - is higher compared to the previous environment with neutral redistribution of the seigniorage rent.

4.2.3 Capital taxation distortion

We now discuss the interplay between credit constraints and distorting taxes on capital. We keep on with the same previous environment with exogenous hours. But we take into account of the adjustment of the tax on capital following a rise in inflation. Due to the seigniorage rent, inflation allows a reduction in the tax rate on capital required to balance the government budget constraint. This effect, traditionally known as the Phelps effect, interacts in our framework with the presence of borrowing constraints which amplifies the incentives to save. We quantify the contribution of credit constraints to this traditional Phelps effect by comparing the incomplete market economy with the complete market set-up.

Table 3-Line 3 first indicates that the tax on capital decreases by 0.94 percent in the incomplete market set-up. This effect provides more incentive to save. Importantly enough, the precautionary saving motive due to the existence of credit constraint amplifies the rise in aggregate capital. Actually the Phelps effect turns out to be twice as high in incomplete market as in complete market set-up, the aggregate capital rising by 0.58 percent in the former case compared to 0.29 percent in the latter one. This allows a proportional increase in output and consumption by 0.15 percent and 0.12 percent in the incomplete market set-up. Conversely, this higher incentive to save with credit constraints leads to a sharper reduction in real money balances in the incomplete market world by 8.78 percent against 6.8 percent in the representative agent economy.

4.2.4 Endogenous labor supply

We end-up the analysis by taking into account of the interplay between borrowing constraints and the labor supply margin. Table 3 - Line 4 compares the benchmark incomplete markets economy described in section 3 with a complete market set-up. Note that taxes on labor and capital income are now both distorting.

The primary channel through which inflation affects labor is by altering the productivity

Table 3: Aggregate impact of inflation: economies with proportional taxes, exogenous hoursy

Economies	Percentage change following a rise in inflation $\pi = 2\% \rightarrow 3\%$								
	Y	K	M/P	C	L	χ^w	χ^a	$\tilde{r}$	$\tilde{w}$
Neutral lump − sum tax									
Exogeneous hours (1)									
Incomplete markets	0.080	0.231	-9.691	0.020	0	0	0	-0.67	0.06
Complete markets	0	0	-6.979	0	0	0	0	0	0
Redistributive distortion									
Exogeneous hours (2)									
Incomplete markets	0.121	0.342	-8.859	0.068	0	-1.04	0	-0.80	0.12
Complete markets	0	0	-6.973	0	0	-0.917	0	0	0
Capital tax distortion									
Exogeneous hours (3)									
Incomplete markets	0.213	0.586	-8.784	0.112	0	-0.940	-0.940	-1.262	0.192
Complete markets	0.104	0.290	-6.886	0.084	0	-0.802	-0.802	-0.465	0.108
Benchmark economy (4)									
Distort.tax − Endo.hours									
Incomplete markets	0.447	0.948	-11.158	0.359	0.166	-1.166	-1.166	-2.946	0.280
Complete markets	0.150	0.323	-6.860	0.106	0.053	-1.365	-1.365	-0.470	0.095

of labor - measured by wages - and thus the marginal rate of substitution between leisure and consumption. As aggregate capital increases, the productivity of labor rises and wages increases by 0.28 percent. This entails a substitution effects in the labor supply which rises by 0.16 percent. Conversely, the rise in labor supply increases capital productivity and provides more incentives to save. This virtuous circle leads to a sizeable increase in aggregate capital and output by 0.94 percent and 0.44 percent respectively. This effect is three times as high as in an complete market economy, allowing much higher steady-state consumption. Yet this sharp substitution between capital and real balances in the incomplete market set-up triggers a reduction in real balances almost twice as high in the environment with borrowing constraints as that in the economy with complete markets.

4.3 Welfare

We conclude this analysis by assessing the welfare costs of inflation in incomplete market set-up compared to the traditional representative agent literature. This analysis is carried on in the benchmark model with endogenous hours and proportional taxes. We use the standard Aiyagari-McGrattan average welfare criterion defined as the expected discounted sum of utilities under the equilibrium stochastic stream of consumption and real balances of infinitely lived agents.

Following Lucas' tradition, we measure the welfare gain of inflation as the percentage of consumption one must give to households living in an environment with low inflation rate to leave them indifferent with living in another economy characterized by higher inflation rate. The monetary policy experiment is the same as the previous one and consists in an increase by one point in the inflation rate from $\pi = 2\%$ to $\pi = 3\%$. Let $c(\epsilon, q)$, $m(\epsilon, q)$ and $l(\epsilon, q)$ be the level of consumption, real balances and leisure of households having a labor productivity ϵ and a level of wealth q. These quantities are defined at the stationary equilibrium under the benchmark level of inflation $\pi = 2\%$ used in the calibration. Let $c^{\Delta\pi}(\epsilon, q)$, $m^{\Delta\pi}(\epsilon, q)$ and $l^{\Delta\pi}(\epsilon, q)$ be the level of these quantities after a change in the inflation rate, and let $\lambda^{\Delta\pi}$ be the new stationary joint distribution after a change in inflation.

The average welfare gain Δ^{av} is thus defined as

$$\int\int u\left((1+\Delta^{av})c(e,q), m(e,q), l(e,q)\right) d\lambda(e,q) = \int\int u\left(c^{\Delta\pi}(e,q), m^{\Delta\pi}(e,q), l^{\Delta\pi}(e,q)\right) d\lambda^{\Delta\pi}(e,q)$$

We also look at the redistributive impact of inflation depending on the level of wealth. More precisely, let denote $\Lambda^{\epsilon}_{X\%}$ the range of financial wealth of the poorest $X\%$ type ϵ workers in the benchmark economy and let define by $\Lambda'^{\epsilon}_{X\%}$ the range of financial wealth of the poorest $X\%$ type ϵ workers in the modified economy. Thus the welfare lost from inflation for the poorest $X\%$ type ϵ workers is given by

$$\int_{\Lambda^{\epsilon}_{X\%}} u\left((1+\Delta_{X,\epsilon})c(\epsilon,q), m(\epsilon,q), l(e,q)\right) dq = \int_{\Lambda'^{\epsilon}_{X\%}} u\left(c^{\Delta\pi}(\epsilon,q), m^{\Delta\pi}(\epsilon,q), l^{\Delta\pi}(e,q)\right) dq$$

where Δ_X is the additional consumption one must give to the the poorest $X\%$ type ϵ workers to make them indifferent between living in the benchmark economy and becoming the poorest $X\%$ type ϵ workers in the modified economy. The welfare cost of inflation for the richest $Y\%$ is defined exactly in the same way.

Table 4 reports the welfare costs generated by a one point increase in the inflation rate from $\pi = 2$ percent to $\pi = 3$ percent. We look both at the average cost and at the cost for the 5 percent poorest and wealthiest households. We also decompose the welfare costs depending on the level of labor productivity by distinguishing the two polar cases of high productivity and low productivity workers.

Table 4 - Col. 1 compares the average welfare cost of inflation under incomplete and complete markets set-up. It turns out that the welfare cost in consumption equivalent of the change in inflation is 0.185% percent in the incomplete market economy. This negative impact is mainly due to the sharp reduction in real balances and the decline in leisure induced by inflation as found in Table 3. This negative effect more than offsets the positive impact of inflation on aggregate capital allowing higher stationary consumption. Yet this welfare cost is slightly smaller than the one which will hold in a complete market economy. In the latter case, inflation triggers a less pronounced positive effect on output and consumption. But the negative utility effect stemming from the reduction of real balances is also lower compared to the incomplete market economy.

Moreover, inflation affects unevenly the welfare of different households, a dimension ignored in the representative agent literature. Table 4 - Col. 2 and 3 show that the wealth-poor gain from inflation while the wealth-rich are hurt by inflation. This effect is mainly due to the price composition effect. As suggested by Table 3, inflation has a significant positive impact on labor productivity and wages by triggering higher capital accumulation. In the benchmark economy, a one point rise in the inflation rate was found to increase wages by 0.28 percent. Conversely the induced higher accumulation of capital entailed a sharp reduction in the interest rate by 2.94 percent. As a matter of fact, inflation increases the welfare of the wealth-poor people whose income is mainly made up of labor income. This effect is all the more important for high productivity workers whose welfare increases by 0.7569 percent against 0.296 percent for the low-skilled worker. By contrast, inflation lowers the welfare of the wealth-rich since their total income is mainly made up of financial assets whose return decreases.

Table 4: Welfare costs of inflation: benchmark economy with proportional taxes and endogeneous hours

Economies	Average (1)	High skill (2)		Low skill (3)	
		Poorest 5%	Richest 5%	Poorest 5%	Richest 5%
Incomplete Markets	-0.185	0.7569	- 0.090	0.296	- 0.189
Complete Markets	-0.192	-0.192	-0.192	-0.192	-0.192

5 Conclusion

This paper has put to the fore a new channel for the non neutrality of money which hinges on credit constraints. Incomplete market and borrowing constraints induce an heterogeneity in households behavior following a change in the inflation rate, because credit constrained households can not substitute away their real balances for financial assets.

First, we have shown that this channel has a quantitative sizeable impact in economies with an empirically relevant wealth distribution. An increase in inflation leads to a substantial rise in long-run output and consumption. Second, the welfare costs of inflation turn out to be on average of the same order in this incomplete market set-up compared to the representative agent framework in a steady-state comparison *à la* Lucas. But inflation has key redistributive effects, some households even gaining from inflation depending on their level of wealth and productivity.

The focus of this paper is on long run steady state inflation. But, a promising route for future research would be to analyze the short run effect of monetary shock in such a model. This framework could provide a new relevant channel in order to account fort the persistence and non neutrality of monetary shocks, as an alternative to traditional sticky prices models. And credit constraints and heterogeneity would offer a suitable framework for studying the short run redistributive effects of monetary policy.

A Solution to the Problem of the Households

Using the Bellman equations, the problem of households can be written in a recursive form. Stationary solutions satisfy, of course, the usual transversality conditions. As a consequence, one can focus on the first order condition of the problem of the households. This one is

$$
\begin{aligned}
V\left(q_t^i, e_t^i\right) &= \max_{\{c_t^i, m_t^i, a_{t+1}^i\}} u\left(c_t^i, m_t^i\right) + \beta V\left(q_{t+1}^i, e_{t+1}^i\right) \\
c_t^i + m_t^i + a_{t+1}^i &= q_t^i + w_t e_t^i + \frac{\mu_t^i}{P_t} \qquad (31) \\
q_{t+1}^i &= R_{t+1} a_{t+1}^i + \frac{m_t^i}{\Pi_{t+1}} \qquad (32) \\
c_t^i, m_t^i, a_{t+1}^i &\geq 0 \qquad (33)
\end{aligned}
$$

with q_1^1, q_1^2 given and with the deterministic change of state $e_{t+1}^i = 0$ if $e_t^i = 1$, and $e_{t+1}^i = 1$ if $e_t^i = 0$. Using (31) and (32) to substitute for c_t^i and q_{t+1}^i, one can maximize only on a_t^i and m_t^i. Using the first order conditions, together with the envelop theorem (which yields in all cases $V'\left(q_t^i, e_{t+1}^i\right) = u_c'\left(c_t^i, m_t^i\right)$), one finds

$$
\begin{aligned}
u_c'\left(c_t^i, m_t^i\right) &= \beta R_{t+1} u_c'\left(c_{t+1}^i, m_{t+1}^i\right) \qquad (34) \\
u_c'\left(c_t^i, m_t^i\right) - u_m'\left(c_t^i, m_t^i\right) &= \frac{\beta}{\Pi_{t+1}} u_c'\left(c_{t+1}^i, m_{t+1}^i\right) \qquad (35)
\end{aligned}
$$

If the previous equations yield a quantity $a_{t+1}^i < 0$, then the borrowing constraint is binding and the solution is given by $a_{t+1}^i = 0$ and $u_c'\left(c_t^i, m_t^i\right) > \beta R_{t+1} u_c'\left(c_{t+1}^i, m_{t+1}^i\right)$ together with (35). In a stationary equilibrium, all H agents become L agents the next period, and the reverse. The H agents are the high revenue agents, and their savings are always higher than the ones of L agents, who have no labor income. As a consequence, credit constraints never bind for H agents. One can rewrite the previous equations using the state of the households instead of their type. With the logarithm utility function, it yields the expressions given in section 2.

B Proof of Proposition 2

In this proof, we assume that credit constraints are binding for L households to derive the equilibrium interest rate. In a second step, Then we check that credit constraints are indeed binding for L agents and not for H agents. The first order conditions of the firm problem yield $1 + r = \alpha K^{\alpha-1}$ and $w = (1-\alpha) K^{\alpha}$.

First, using the first order condition (8), one finds $\frac{c^L}{c^H} = \beta(1+r)$. The equilibrium on the good market yields is $c^H + c^L = K^\alpha - K$. Substituting c^H, w and K by their value one finds

$$
c^L = \beta \frac{1 + r - \alpha}{\beta(1+r) + 1} \left(\frac{\alpha}{1+r}\right)^{\frac{\alpha}{1-\alpha}}
$$

The budget constraint of L agents, given by (7) yields

$$\frac{m^L}{c^L} - \frac{m^H}{c^H}\frac{c^H}{c^L} = \frac{a^H (1+r) - c^L}{c^L}$$

Using the value of the ratio $\frac{c^L}{c^H} = \beta (1+r)$ and the expressions (12) and (13), one finds

$$\frac{\phi}{1-\phi}\left(\alpha \frac{\beta (1+r) + 1}{1+r-\alpha} - \beta\right) = \frac{\beta}{1 - \frac{\beta^2}{\Pi}(1+r)} - \frac{1}{1+r-\frac{1}{\Pi}} \tag{36}$$

The left hand side is decreasing with r. The right hand side is unambiguously increasing in r. One can show that the right hand side is increasing in Π. Indeed, define

$$g(\Pi) = \frac{\beta}{1 - \frac{\beta^2}{\Pi}(1+r)} - \frac{1}{1+r-\frac{1}{\Pi}}$$

and define the function h such that

$$h(y) = \frac{y^3 (1+r)^3}{\left(1 + r - \frac{y^2}{\Pi}(1+r)^2\right)^2} \tag{37}$$

The function h is positive and increasing in its argument. Now, the derivative $g'(\Pi)$ can be written as $g'(\Pi) = \frac{1}{\Pi^2}\left(h\left(\frac{1}{1+r}\right) - h(\beta)\right)$. As credit constraints are binding, $\frac{1}{1+r} > \beta$, and hence one finds that $g'(\Pi) > 0$.

As a consequence, when the equation (36) has a solution r,by the theorem of implicit function it is a decreasing function of Π. A solution r of (36) is an equilibrium interest rate if $1 + r < \frac{1}{\beta}$. Here, we simply assume that the values of the parameters are such that it is the case. This is true for instance for $\beta = 0.96$, $\phi = 0.5, \alpha = 0.33.$ and $\Pi = 1.02$.

References

Aiyagari, S. R., 1994, Uninsured Idiosyncratic Risk and Aggregate Saving, *Quarterly Journal of Economics*, 109(3), 659-684.

Aiyagari, S. R., McGrattan, E.R., 1998, The optimum quantity of debt, *Journal of Monetary Economics*, Elsevier, 42(3), 447-469.

Akyol, A., Optimal monetray policy in an economy with incomplete markets and idiosyncratic risk, *Journal of Monetary Economics*, 51, 1245-1269.

Bernanke, B. S. and Mishkin, F. S., 1997, Inflation targeting: A new framework for monetary policy?, *Journal of Economic Perspectives*, vol. 11, p. 97-116.

Bewley, T., 1980, The optimum quantity of money, In: Kareken, J.H, Wallace, N., (Eds), *Models of Monetary Economies.*

Bewley, T., 1983, A difficulty with the optimum quantity of money, *Econometrica*, 54, 1485-1504.

Bullard James and John Keating, 1995, The long-run relationship between inflation and output in postwar economies, *Journal of Monetary Economics,* vol. 36, 477-496.

Budria Rodriguez, S., Diaz-Gimenez, J., Quadrini, V., and Rios-Rull, J.V, 2002, Updated facts on the U.S distribution of Earnings, Income and Wealth, *Federal Reserve Bank of Minneapolis Quarterly Review*, 26(3), 2-35.

Carroll, C., 2000. Requiem for the Representative Consumer? Aggregate Implications of Microeconomic Consumption Behavior, *American Economic Review*, vol. 90(2), 110-115.

Castaneda, A., Diaz-Gimenez, J., and Rios-Rull, J.V, 2003, Accounting for earnings and wealth inequality, *Journal of Political Economy*, 11(4), 818-857.

Chari, V.V., Kehoe, P.J., and McGrattan, E.R., 2000, Sticky Price Models of the Business Cycle : Can the contract Multiplier Solve the Persistence Problem?, *Econometrica*, 68(5), 1151-1179.

Chari, V.V., Christiano, L.J., and Kehoe, P.J., 1996, Optimality of the Friedman Rule in Economies with Distorting Taxes, *Journal of Monetary Economics*, 37(2), 203-223.

Domeij, D., and Heatchcote, J. 2004, On the distributional effects of reducing capital taxes, *International economic review*, 45(2), 523–554.

Erosa, A. and Ventura, G., 2002, On Inflation as a regressive consumption tax, *Journal of Monetary Economics*, 49, 761-795.

Grandmont, Jean-Michel and Younes Yves (1973), 'On the efficiency of a monetary equilibrium', *Review of Economic Studies,* Vol. 40, pp. 149-165.

Heathcote, J., 2005, Fiscal Policy with Heterogenous Agents and Incomplete Markets, *Review of Economic Studies*, 161-188.

Hoffman, D.L. R.H. HRasche and M.A. Tieslau, 1995, The stability of Long-Run Money Demand in Five Industrial Countries, *Journal of Monetary Economics*, 35(2), 317-339.

Holman, Jill A. 1998, "GMM Estimation of a Money-in-the-Utility-Function Model: The implications of Functional Forms", *Journal of Money Credit and Banking,* Vol. 30, No. 4, 679-698.

Huggett, M, 1997, The One Sector Growth Model with Idiosyncratic Shocks : Steady States and Dynamics, *Journal of Monetary Economics,* 39, 385-403.

Kehoe, T.J. and Levine, D.K., 2001, Liquidity constrained vs. debt constrained markets, *Econometrica*, 69, pp. 575-598.

Kehoe, T.J., Levine, D.K., and Woodford, M., 1992, 'The optimum quantity of money revisited, in *Economic Analysis of Markets and Games*, ed. by Dasgupta, D. Gale, O. Hart, and E. Maskin, Cambridge, MA: MIT press, 501-526.

Krueger, D., and Perri, F., 2005, Does income inequality lead to Consumption inequality? Evidence and Theory?, Forthcoming *Review of Economic Studies.*

Krusell, P. and Smith, A.A. (1998), Income and wealth heterogeneity in the macroeconomy, *Journal of Political Economy*, 106(5), pp. 867-896.

Imrohoroglu, A., 1992, The welfare cost of inflation under imperfect insurance, *Journal of Economic Dynamics and Control*, 16, 79-91.

Japelli, T., 1990, Who is Credit Constrained in the US Economy , *The Quarterly Journal of Economics*, 105(1), 219-234.

Khan Mohshin, Abdelhak Senhadji and Bruce Smith (2001), Inflation and Financial Depth, IMF Working Paper WP/01/04.

Kehoe, T. and Levine, D.K., 2001, Liquidity Constrained vs. Debt Constrained Markets, *Econometrica*, 69: 575-598.

Loayza, Norman Klaus Schmidt-Hebbel and Luis Servén (2000). What Drives Private Saving Across the World, *The Review of Economics and Statistics*, vol. LXXXII, N°2, 165-181.

Lucas, R., 2000, Inflation and Welfare, *Econometrica*, 68(2), 247-274.

Perri, F. and Krueger, 2005, D., Does income inequality lead to consumption inequality? Evidence and Theory, *NYU working paper*

Phelps, E.S., 1973, Inflation in the Theory of Public Finance, *Swedish Journal of Economics*, 75 (1), 67-82.

Rios-Rull, J.V., 1996, Life-cycle economies and aggregate fluctuations, *Review of Economic Studies* 63(3), 465-489.

Ragot, X. 2005, The real effect of inflation with credit constraints, *PSE Working Paper.*

Romer, C.D., and Romer, D., 1998, Monetary Policy and the Well-Being of the Poor, *NBER Working Paper*,n° 6793.

Sargent, T., and Wallace N., 1981, Some Unpleasant Monetarist Arithmetic, *Quarterly review Federal Reserve bank of Mineapolis*, 5(3), 1-17.

Sidrauski, M., 1967, Rational Choice and Patterns of Growth in a Monetary Economy, *American Economic Review,* 57(2), 534-544.

Tobin, J. 1965, Money and Economic Growth, *Econometrica*, 33(4), 671-684.

Walsh, Carl E., 2003 Monetary Theory and Policy, MIT Press, Cambridge, Mass.

Weil, P., 1991, Is Money Net Wealth, *International Economic Review*, 32(1), 37-53.

Weiss, L., 1980, The Effects of Money Supply on Economic Welfare in the Steady State, *Econometrica*, 48(3), 565-576.

Woodford, M., 1990, Public Debt as Private Liquidity, *American Economic Review*, 80(2), 382-88.