Labor Market Flexibility and Growth.

Enisse Kharroubi*

May 19, 2006.

Abstract

This paper studies whether flexibility on the labor market contributes to output growth. First I document two stylized facts concerning the relationship between output growth and labor market institutions. Specifically, labor market flexibility is shown to be on the one hand positively associated with higher total factor input (TFI) growth but on the other hand negatively associated with total factor productivity (TFP) growth. Second in a model where both firms and workers face imperfect capital markets, I show that labor market flexibility relaxes firms borrowing constraints but raises firms incentives to invest in low productivity projects. Moreover labor market flexibility has a positive impact on workers precautionary savings and raises thereby the volume of global savings. As a result, the economy exhibits multiple equilibria: one with inflexible labor contracts low savings and low investment but high productivity and high interest rate and one with flexible labor contracts high savings and high investment but low productivity and low interest rate. The model therefore rationalizes the empirical observations on the relationship between labor market flexibility and the different sources of growth. Finally the model shows that the negative effect on TFP can offset the positive effect on TFI, the high flexibility equilibrium being then dominated in terms of both welfare and growth.

Preliminary Version. Comments Welcome.

* Banque de France. Address: 1, rue de la Vrillière 75049 Paris cedex 01. e-mail: firstname.surname(at)banque-france.fr. I thank Gilbert Cette, Henri Pages and seminar participants at PSE. Usual disclaimers apply.

1. Introduction.

Since its creation, the Euro Zone has been lagging behind the United States in terms of output growth. As illustrated in figure 1, from 1992 to 2005, the US business sector has been steadily growing at a faster pace than its Euro Zone counterpart, year 2001 being the sole significant exception. Moreover while this growth gap seemed to be on a disappearing trend by the end on the 1990's, the first years of the 2000's decade have witnessed a resurgence in this growth gap with a steady expansion from 0.7 in 2002 to about 2.5 percentage points in 2005.

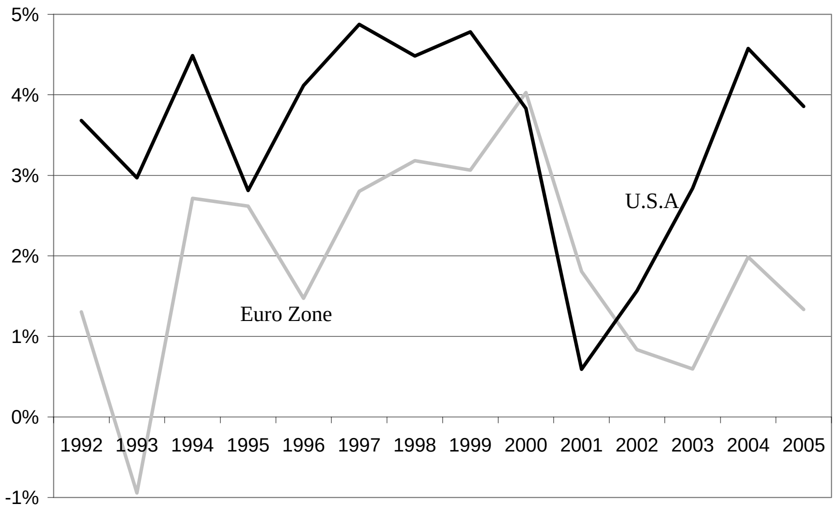

Figure 1: Business Sector GDP Growth. Source: OECD Economic Outlook.

A broader focus on the whole economy (and not only on the business sector) yields a very similar view as to the Euro Zone-USA growth gap: on average, the US grows each year 1.4 percentage point faster than the Euro Zone. Not withstanding this worrying picture for the Euro Zone, a similar pattern emerges from a rapid comparison of the respective productivity growth performances. Comparing the growth rate of output per worker, the growth gap between the US and the Euro Zone is still more than one percentage point in the business sector and 0.8% percentage point for the whole economy.

Why is this so? Why has the U.S. grown, over the last years, significantly faster than the Euro Zone? Where does the growth gap between the Euro Zone and the US come from? Providing an answer to this question is not an easy task: looking at figures for long run fundamental sources of growth, namely investment and employment evolutions, it turns out that employment has been increasing somewhat faster in the US than in Euro Zone (1.4% to 0.8% per year on 1992-2005). Moreover the investment to output ratio has been on average pretty larger in the Euro Zone than in the US (16.1% to 12.9% on 1992-2005). Finally adding the fact that catch-up effects should be larger for the Euro Zone given that its productivity is lower, a rapid calibration of a Solow growth model shows that this figures should predict a productivity growth gap in favor of the Euro Zone not in favor of the US[1]. Still business sector labor productivity growth has been growing on average 1 percentage point faster in the US than in the Euro Zone on 1992-2005[2].

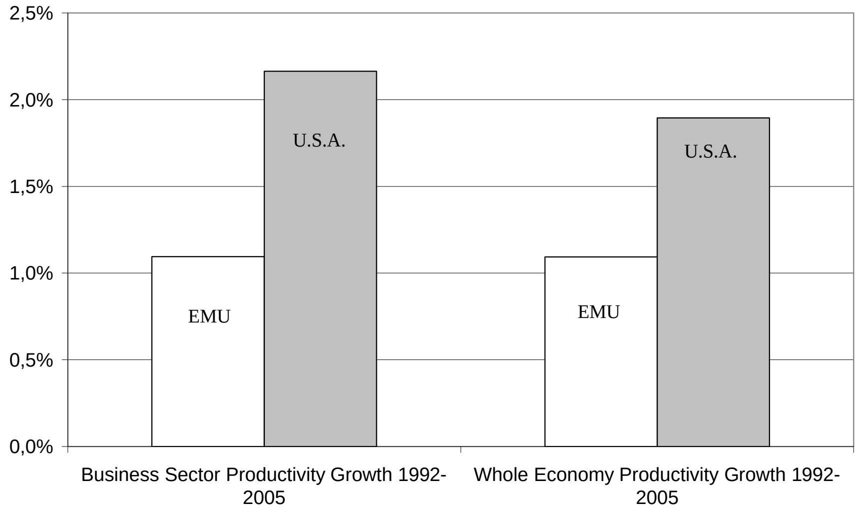

Figure 2: Average Productivity Growth. Source: OECD Economic Outlook.

Therefore if traditional growth determinants cannot account for the output - productivity growth gaps, then this begs the question of where these gaps come from and whether any stark structural difference between these two economies can help understanding this long run growth performance gap? On the list of possible culprits or at least of suspects, the labor market and its regulation have been given very high priority.

[1] Put differently, given the figures for capital accumulation and employment growth in the US and the Euro Zone, the difference in TFP needed to rationalize the difference in growth rates is inconsistent with empirical estimates of TFP growth.

[2] All the figures stated here have been taken out of the OECD economic outlook. The employment variable (ETB) is named "employment of the business sector", the output variable (GDPBV) is named "gross domestic product business sector volume factor cost" and the investment variable (IBV) is named "private non-residential fixed capital formation volume".

Figure 3: Labor Regulation Indexes. Source: Botero et al. [2004].

Indeed, a large number of commentators have pointed in the direction of the labor market, regarded as the driving element of this growth gap. More precisely, the supposed lack of flexibility in Euro Zone labor markets has been set responsible for the poor growth performance relative to the U.S.[3]. However it is important to note that labor market regulation does not have *a priori* any direct impact on growth, at least according to standard growth models because it does not affect directly fundamental sources for capital accumulation such as savings and investment. Nor does the functioning of the labor market affect education or the capacity to carry out research and development activities, which are the primary sources of endogenous long run growth. It therefore remains a question to understand how such a pattern whose influence is mostly indirect (i.e. second order) can have that huge (i.e. first order) impact so as to be a valid explanation for the Euro Zone - US growth gap. In this paper I propose to revisit the interactions between labor market institutions and growth. Specifically, I ask two questions. First can labor market institutions be held responsible for the US-Euro Zone growth gap? In other words, does labor market flexibility enhance growth and how? Second

[3]Recently (march 2006), the president of the ECB, Jean-Claude Trichet, endorsed this view, in an interview declaring "anything that helps raising flexibility is good to fight joblessness in today's world". Both the IMF and the OECD also adhere to a similar belief: "To enjoy strong GDP growth, developed economies need, as a priority, policy frameworks that encourage competitive intensity. This means [...] encouraging labor market flexibility". (Finance & Development, march 2006)."[...] institutional structures and policy settings that favour competition and flexibility in capital and labour markets [...] also make a key difference to growth prospects. In particular, many of our countries need more competitive product markets; labour markets that adjust better and more rapidly to shocks". (The Sources of Economic Growth in OECD Countries [2003]) Last but not least, the Kok report on employment policy (2003) underlines the need for more flexibility in labor markets as a means to enforce the Lisbon agenda designed to make Europe the most competitive economic area in the world.

what are the economic policy recommendations that come out of the set of answers delivered to the first questions?

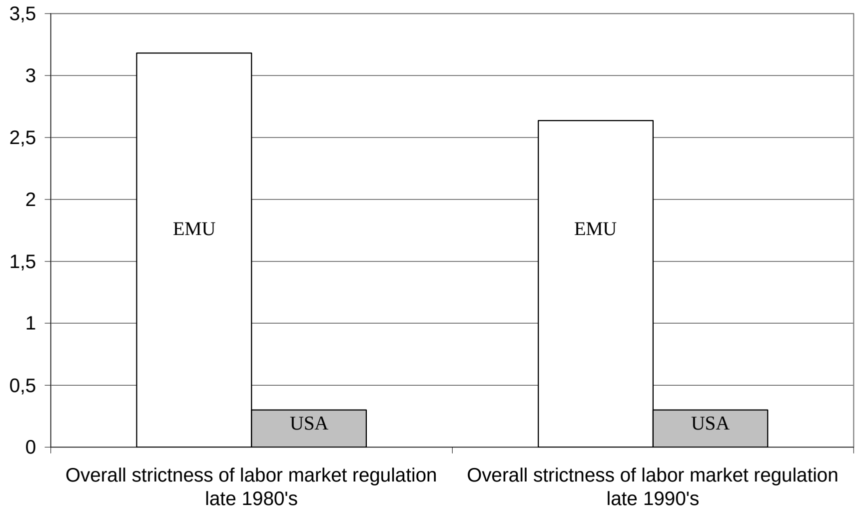

Figure 4: Labor Market Regulation. Source: OECD Employment Outlook.

To do so, I am going to show that labor market institutions namely, labor contracts can affect capital accumulation and growth through the interactions between labor and financial markets at the firm level. Solving the problem of the firm will therefore prove to be helpful in understanding how firms optimally choose their labor contracts, their financial contracts and their technology and whether more flexibility in optimal labor contracts is associated at the macro level with faster capital accumulation, more productive technologies and eventually with higher or lower growth. The basic claim of the model consists in showing that labor market flexibility can help firms reduce borrowing constraints and hence help the economy accumulate production factors at a faster pace. Labor market flexibility is therefore positively associated with growth. However, the model also shows that labor market flexibility gives incentive to firms to invest in less risky but also less productive technologies. Hence labor market flexibility can also be associated with lower growth.

1.1. Decomposing growth and labor market effects.

Output growth or labor productivity growth can be decomposed along the usual growth accounting partition. To grow, an economy can accumulate production factors such as capital and labor. We will refer to this first source of growth as total factor input growth (TFIG). To grow an economy can also improve its technology, i.e. improve the way it combines production factors to produce more output with the same volume of production factors. We will refer to this second source of growth as total factor productivity growth (TFPG). We consider a sample of fifteen European countries and the US on the 1981-2004 period. We get data on TFI and TFP from the GGDC Total Economy Growth Accounting Database[4]. As to labor market flexibility indicators we consider the Botero et al. [2004] database[5]. Then we carry out a simple exercise. We divide our country sample into two categories, countries with low employment protection or employment rigidity and countries with high employment protection or high employment rigidity. In each of these two sub samples, we compute the average TFI growth rate and the average TFP growth rate and compare the figures obtained for each category of countries. It appears that average total factor input growth for the 1981-2004 period has been larger in countries with low employment rigidity (low ER) while total factor productivity growth has been larger in countries with large employment rigidity (large ER).

	Low ER	Large ER
TFIG	1,38%	1,14%
TFPG	0,92%	1,01%

Table 1. Source: Author's calculations[6].

While labor market flexibility is correlated with the different sources of growth according to table 1, there exists a number of other structural conditions which affect knowledge and production factors accumulation. On top of these structural conditions comes the quality of the financial sector or financial development. Conducting a similar exercise to what has been done higher, it appears that financial development measured from the Djankov et al. [2006] database[7] (CP stands for creditor protection) is always associated with larger growth both in terms of total factor productivity and total factor input.

[4]More details about the database, the sample and the variables can be found at the address http://www.ggdc.net/index-dseries.html#top.

[5]More details on these data can be found on the web site http://www.doingbusiness.org/ExploreTopics/HiringFiringWorkers/

[6]A country is defined as belonging to the low (resp. large) ER index category if and only if it ER index is lower (resp. larger) than the median ER index in the sample. Although the reader may legitimately question the statistical signifinance of these differences, the evidence shown simply aims at being suggestive of the labor market regulation differentiated impact.

[7]More details on these data can be found on the web site http://www.doingbusiness.org/ExploreTopics/GettingCredit/

	Low CP	Large CP
TFIG	1,12%	1,40%
TFPG	0,62%	1,28%

Table 2. Source: Author's calculations[8].

Now as these two sets of comparisons are only indicative, we go one step further and carry out regressions on the respective determinants of TFI and TFP growth.

	TFIG	TFPG	TFIG	TFPG
Lagged dep. variable			0,46***	0.31***
Employment Rigidity	-1,07**	0,76*	-0,83*	0,23*
Creditor Protection	-4,97	17,76***	3,33	10,04***
Time effects	yes	yes	yes	yes
N×T	15×25	15×25	15×24	15×24

Table 3. Source: Author's calculations[9].

The results are similar to the first ones: labor market flexibility is associated every thing else equal with higher total factor input growth but lower total factor productivity growth. Therefore it seems that countries with more flexible (less regulated) labor markets tend to accumulate production factors at a higher speed. However it also seems that countries with more flexible (less regulated) labor markets tend to improve more slowly their technology[10].

1.2. Mechanism of the model.

We model labor market flexibility as the possibility for firms to propose wage contracts contingent to the ex post marginal productivity of labor. When the labor market is flexible, then firms do not provide any

[8] A country is defined as belonging to the low (resp. large) CP index category if and only if it CP index is lower (resp. larger) than the median CP index in the sample.

[9] On the first row appear the left hand side variables. On the first column appear the right hand side variable with "lagged dep. variable" representing the lagged dependent variable. All estimations contain time effects and have been carried out under the assumption of heteroscedactic residuals. The time span in 1981-2004. Countries in the sample are : Austria, Belgium, Germany, Denmark, Spain, Finland, France, Great Britain, Greece, Ireland, Italy, Netherlands, Portugal, Sweden, USA.

[10] This conclusion is however subject to a serious data limitation: there is no available data on time variations in labor market regulation: labor market regulation indicators are only available on a cross-country base. It therefore constitutes a caveat that it is still hard to deal with.

insurance to workers against ex post fluctuations in labor productivity: labor compensation is then contingent to ex post effective labor productivity. On the contrary, when the labor market is said to be inflexible, then firms provide insurance to workers against ex post fluctuations in labor productivity and labor compensation is then related to the ex ante average and not the ex post effective productivity of labor. Put differently, labor market flexibility is inversely related to the degree of wage insurance provided by firms to workers[11].

In a simple model with risk neutral firms and risk averse workers, firms should optimally provide fixed wage contracts to workers with full insurance against ex post fluctuations in labor productivity. Now let us make two assumptions. First firms face capital market imperfections in the form of ex post imperfect enforceability. Second, firms can choose the project they invest in among different technologies with more efficient technologies also embedding more volatile shocks. Then the wage contract they agree upon with workers has an influence on their borrowing capacity. Namely if firms provide contingent wage contracts -(part of) labor productivity risk is transferred to workers- then that can raise firms profits before debt repayments in the bad states of the world and thereby raise firms borrowing capacity[12]. As soon as firms marginal productivity of capital is larger than the risk free interest rate, then the policy consisting for firms to provide contingent wage contracts in order to alleviate borrowing constraints can raise expected profits. However the wage contract firms agree upon with workers also has an influence on the technology they invest in. When a firm decides to propose contingent wage contracts, it benefits on the one hand from an increase in its borrowing capacity while on the other hand, it has to pay a premium on its wage bill. Then if the firm were to invest in a highly productive technology, it would loose the gain in terms of increased borrowing capacity since its productivity in the bad state is very low (due to the correlation between average productivity and productivity volatility) while it would still pay a premium on its wage bill. Consequently,

[11] One may argue that although wage flexibility is important, it remains a second order issue relative to employment flexibility as long as workers are more concerned with loosing their jobs than undergoing a wage cut. Although this point is well-taken, it is important to note that under the assumption that a walarasian spot labor market exists at each date, wage risk and employment risk are isomorphic at the aggregate level since at any date, the labor market balances supply from (previously sacked) workers and demand from firms, the wage rate being the equilibrium variable. Therefore under the assumption that a set of complete labor markets exists the dichotomy between wage and employment flexibility is irrelevant and one can focus on wage risk as long as it simplifies the analysis. The model focusing on "wage risk" is in particular more tractable insofar as one can rely on a static ex ante ex post model while the "employment risk" model definitely needs some type of time-to-build technology as to define a non degenerate choice for firms between short and long term labor contracts.

[12] Equivalently, when firms adopt a contingent labor compensation scheme, this reduces the risk premium of its financial liabilities, their volume being given.

the choice to provide workers with contingent contracts would certainly be costly while it would not generate any gain for the firm. Therefore firms optimally invest in less risky and less productive technologies when they provide contingent wage contracts. On the contrary, firms which propose uncontingent wage contracts choose to invest in more volatile and therefore more productive technologies. Put differently, when firms can choose the optimal degree of insurance they provide workers with, then there exists a complementarity between providing more contingent wage contracts and investing in safer, yet less productive technologies. As a result, labor market flexibility is associated with higher growth through more rapid capital accumulation. But it also associated with lower growth through lower average total factor productivity.

Now from a macroeconomic point of view, the capacity of firms to choose their optimal wage contract creates a strategic complementarity. On the one hand, firms borrowing capacity is negatively related to the degree of wage insurance in contracts provided to workers first because transferring risks to workers raises firms profits before debt repayments and second because firms invest in less productive and less risky projects. On the other hand, assuming that workers do not have access to insurance markets, i.e. instruments to hedge labor income fluctuations, and assuming that workers consumption is not perfectly substitutable in time, then due to precautionary motives, workers savings are negatively related to degree of wage insurance in contracts provided by firms. Under the above assumptions, workers prefer to increase their ex ante savings so as to benefit from a larger ex post consumption in case a bad state of nature happens. Therefore both firms' demand for capital and workers' savings are decreasing functions of wage insurance and the economy can end up with two stable equilibria.

In the first equilibrium, firms provide uncontingent wage contracts to workers and invest in relatively productive technologies. As a result, they face tight borrowing constraints. Because firms provide wage insurance to workers and invest in relatively productive technologies, workers savings are low due to the absence of wage income fluctuations and deu to relatively high future labor income. If the capital supply i.e. workers' savings, is sufficiently low, then the equilibrium interest rate is large. With a large equilibrium interest rate firms optimally provide an uncontingent wage contract because they do not need to borrow a large amount of capital due decreasing marginal returns on capital. Consequently, they also invest in relatively risky and productive technologies.

In the second equilibrium, firms provide contingent wage contracts to workers and invest in relatively unproductive technologies. As a result, they face slack borrowing constraints. Because firms provide workers with contingent contracts and invest in relatively unproductive technologies, workers savings are high due to the presence of wage income fluctuations and due to relatively low future labor income. If the capital supply i.e. workers' savings, is sufficiently large, then the equilibrium interest rate is low. With a low equilibrium interest rate firms optimally provide a state contingent wage contract because they need to borrow a large amount of capital due to decreasing marginal returns on capital. Consequently, they also invest in relatively safe and unproductive technologies. However when the economy exhibits multiple equilibria, we show that both the Pareto optimal equilibrium and the high growth equilibrium correspond to the low flexibility-high insurance equilibrium.

1.3. Related literature.

To be written.

1.4. Road map of the paper.

The paper is organized as follows. The following section lays down the model and its main assumptions. Section 3 describes the different strategies agents adopt as regards the labor and the capital market. In section 4, we build the general equilibrium of the economy. The individual and social optimality properties for the different possible equilibria are derived in section 5. The main results of the model as regards growth and labor market flexibility can then be found in section 6. Conclusions are eventually drawn in section 7.

2. The framework.

We consider a single good economy with three types of agents, entrepreneurs, lenders and workers. All agents live for two periods t and $t+1$. There is a continuum of unit mass of each of type of agent.

2.1. Workers.

At time t, workers have a labor endowment equal to one but no capital endowment. Their preference writes as

$$U_w = (c_t)^{\beta} (c_{t+1})^{1-\beta} \tag{2.1}$$

Workers borrow capital to finance their time t consumption c_t. They also provide their labor endowment to firms. At time $t+1$, they use their labor income to finance their time $t+1$ consumption c_{t+1} and pay back the loans contracted at time t. Let us note w_s, a worker's time $t+1$ labor income when state s happens at time $t+1$, then the budget constraints each worker faces write as

$$\begin{aligned} c_t &\leq d_t \\ c_{t+1} &\leq w_s - (1+r)\, d_t \end{aligned} \tag{2.2}$$

where d_t is the amount of debt a worker contracts at time t and r is the interest rate on period t loans due at time $t+1$. Workers' program therefore writes as

$$\begin{aligned} &\max_{c_t; c_{s,t+1}} (c_t)^{\beta} E (c_{s,t+1})^{1-\beta} \\ &\text{s.t. } c_{s,t+1} \leq w_s - (1+r)\, c_t \end{aligned} \tag{2.3}$$

2.2. Entrepreneurs and lenders.

Entrepreneurs and lenders do not have any labor endowment but they have a capital endowment k at time t. Their preference writes as

$$U_e = (b_{t+1})^{\beta} (c_{t+1})^{1-\beta} \tag{2.4}$$

where b_{t+1} represents the time $t+1$ bequest an entrepreneur makes to its off-spring and c_{t+1} represents the time $t+1$ consumption. lenders can lend their capital k on the capital market. Entrepreneurs have access to a set of constant returns to scale technologies. Noting k the capital stock invested (be it entrepreneurs own funds or financial liabilities entrepreneurs have contracted) and l the number of workers hired, entrepreneurs' technologies write as

$$y_s = A_s k^{\alpha} l^{1-\alpha} \tag{2.5}$$

Entrepreneurs' technologies are subject to a macroeconomic shock s, There are two states of nature, a good $s = h$ and a bad one $s = l$ with $A_h > A_l$. Both states of nature are equiprobable. We adopt the following notations: we note A the mean of A_s, $A = \frac{A_h + A_l}{2}$, σ the standard deviation of A_s, $\sigma = \frac{A_h - A_l}{2}$, and δ the ratio between the standard deviation σ and the mean A. Finally we assume that $\frac{\partial A}{\partial \delta} > 0$: some projects are on average more productive than others. However they also embed more volatile shocks.

An entrepreneur is faced with the following budget constraints: At time t, it can invest its own capital and borrow from capital markets to invest in its firm. Similarly, at time t lenders can lend their capital on the loan market. At time $t+1$, entrepreneurs and lenders divide their final income between consumption and bequest to the next generation of entrepreneurs. Noting s the volume of capital invested at time t, and k_i agent i initial capital endowment, an entrepreneur or a lender faces the following budget constraints:

$$\begin{aligned} s_t &\leq k_i \\ c_{t+1} + b_{t+1} &\leq \left(1 + \rho_{i,s}\right) s_t \end{aligned} \tag{2.6}$$

where $\rho_{i,s} = r$ if the agent i is a lender and $\rho_{i,s}$ is the firm's return on asset in state s if agent i is an entrepreneur. Entrepreneurs and lenders program therefore writes as

$$\begin{aligned} &\underset{b_{t+1};c_{t+1}}{max} \left(b_{t+1}\right)^{\beta} \left(c_{t+1}\right)^{1-\beta} \\ &\text{s.t. } c_{t+1} + b_{t+1} \leq \left(1 + \rho_{i,s}\right) k_i \end{aligned} \tag{2.7}$$

2.3. Markets.

At the beginning of each period, there are two different markets which open one after the other. The first market on which transactions take place is the capital market. On this market, entrepreneurs and workers sign one period contracts with lenders. We assume that entrepreneurs face ex post imperfect enforceability. They can default on their claims be they on the labor or on the capital market at some cost. Assuming that these costs are sufficiently large, no entrepreneur will find useful to issue contingent debt contracts because the interest rate charged on such contracts would be too large compared to the gain in terms of increased borrowing capacity. We therefore focus on risk free debt contracts[13]. The risk free interest rate is noted r. The second market on which transactions take place is the labor market. The labor market is competitive. At the end of the period, firms pay wages to workers and financial contracts are paid back.

An entrepreneur profits in state s write as

$$\pi_{1,s} = y_s\left(d, l\right) - w_s l - \left(1 + r\right) d$$

where d is the volume of capital the entrepreneur has borrowed from the capital market and l is the number of workers he has hired. Since transactions are imperfectly enforceable, firms can always retain a fraction τ of their output and abstract from paying the totality of their wage bill and their debts. In this case conditional on state s happening, they earn

$$\pi_{2,s} = \left(1 - \tau\right) y_s\left(d, l\right)$$

with $\tau \leq 1$. To be incentive compatible the face value of the entrepreneur financial liabilities $\left(1 + r\right) d$ and the wage bill $w_s l$ must be such that the cost to pay back one's liabilities is lower than the cost to default. Then a firm liabilities must be such that

$$\left(1 + r\right) d + w_s l \leq \tau y_s\left(d, l\right) \tag{2.8}$$

[13] See appendix 7.3 for a formal examination of the uselessness of contingent contracts.

3. Agents decisions.

3.1. Workers optimal consumption choices.

The problem for workers consists in choosing the optimal consumption path $(c_t; c_{s,t+1})$ given the interest rate on the capital market r, and the wage contract $\{w_s\}_s$ they have agreed on with entrepreneurs. Their program writes as

$$\begin{aligned} &\max_{c_t;c_{s,t+1}} (c_t)^\beta E_s (c_{s,t+1})^{1-\beta} \\ &\text{s.t. } c_{s,t+1} \leq w_s - (1+r)\, c_t \end{aligned} \tag{3.1}$$

At time t, workers have no resource and are compelled to borrow on the capital market to finance their consumption. At time $t+1$, they use their labor income to finance their consumption and pay back their debts. Noting c_t^* the optimal time t consumption, the first order condition of the problem (3.1) then writes as

$$\frac{\beta w_h - (1+r)\, c_t^*}{(1+r)\, c_t^* - \beta w_l} = \left[\frac{w_h - (1+r)\, c_t^*}{w_l - (1+r)\, c_t^*}\right]^\beta \tag{3.2}$$

In the case where $\beta = \frac{1}{2}$, the last condition simplifies as

$$(1+r)\, c_t^* = \frac{w_h}{w_h + w_l} w_l \tag{3.3}$$

This means that consumers optimal first period consumption is such that its second period cost is equal to a given fraction of the lowest second period wage income. Thereby the optimal time $t+1$ consumption $c_{s,t+1}^*$ is always strictly positive:

$$c_{s,t+1}^* = \frac{w_s}{w_h + w_l} w_s \tag{3.4}$$

One can also note that the optimal first period consumption decreases, every thing else equal, with any mean preserving spread in the wage contract $\{w_s\}_s$. This corresponds to a standard precautionary savings motive: when income volatility increases and in the absence of any financial instrument to hedge income fluctuations, workers decide to reduce the amount of capital borrowed from capital market in order not to compromise their future consumption. The expected indirect utility of consumers then writes as

$$V_w = \frac{1}{2}\left(\frac{w_l w_h}{1+r}\right)^{\frac{1}{2}} \tag{3.5}$$

As expected, consumers' expected indirect utility decreases with the interest rate and increases with the income w_s. Moreover consumers are indifferent between two different wage contracts $\{w_{1,s}\}_s$ and $\{w_{2,s}\}_s$ if and only if for they yield the same level of indirect utility, every thing else equal. This then writes as

$$w_{1,l}w_{1,h} = w_{2,l}w_{2,h}$$

Assuming for instance that $\{w_{1,s}\}_s$ is a fixed wage contract, i.e. $w_{1,l} = w_{1,h} = w$ while $\{w_{2,s}\}_s$ is a strictly contingent contract, i.e. $w_{2,s} = \eta_s w$ with $\eta_l \neq \eta_h$, then the last condition simplifies as

$$\eta_h = \eta_l^{-1} \tag{3.6}$$

3.2. Entrepreneurs and lenders optimal consumption choices.

Once production has taken place and liabilities have been paid back, entrepreneurs and lenders problem consists in choosing the volume of goods they want to devote to bequest and consumption given their final income. More precisely their program writes as

$$\begin{aligned} &\underset{b_{t+1};c_{t+1}}{max} \; (b_{t+1})^{\beta} (c_{t+1})^{1-\beta} \\ &\text{s.t. } c_{t+1} + b_{t+1} \leq \left(1+\rho_{i,s}\right) k_i \end{aligned} \tag{3.7}$$

where $\rho_{i,s}$ is the return on entrepreneurs or lenders assets. Given that entrepreneurs and lenders know the return $\rho_{i,s}$ when they choose how much to consume and how much to bequeath, the optimal bequest b^*_{t+1}

and the optimal consumption c^*_{t+1} write as

$$\begin{aligned} b^*_{t+1} &= \beta\left(1+\rho_{i,s}\right)k_i \\ c^*_{t+1} &= (1-\beta)\left(1+\rho_{i,s}\right)k_i \end{aligned} \tag{3.8}$$

Assuming as previously that $\beta = \frac{1}{2}$, entrepreneurs and lenders expected indirect utility then writes as

$$V_e = \frac{1}{2}E\left[\left(1+\rho_{i,s}\right)k_i\right] \tag{3.9}$$

In the case of a lender the optimal decision consists in lending its capital k on the capital market. On the contrary, in the case an entrepreneur, its problem consists in maximizing its expected profit, i.e. $E\left[\left(1+\rho_{i,s}\right)k_i\right]$. To do so, entrepreneurs take two types of decisions: on the one hand they determine the volume of labor l and the amount of capital d they want to invest. On the other hand, they choose the labor contract $\{w_s\}_s$ they offer to workers and the technology $A(\delta)$ they want to invest in.

3.3. Firms optimal behavior.

Given that firm decisions are sequential, the program of a representative firm can be solved with backward induction. First we determine the strategy of the representative firm as regards the volume of labor it hires, then we turn to the capital demand of the representative firm and finally we determine the optimal wage contract and the optimal technology. Let us consider a firm i which has chosen a given compensation scheme $\{w_l, w_h\}$ when other firms choose to propose an equivalent certain wage rate w. Then assuming that the compensation scheme $\{w_l, w_h\}$ verifies workers participation constraint, i.e. $w_l w_h \geq w^2$, where w is the certain equivalent wage rate proposed on the labor market, firm i program first consists in choosing the number of worker l_i such that it solves

$$\max_{l_i} E\Pi(l_i) = A(\delta)(k_i+d_i)^{\alpha} l_i^{1-\alpha} - Ew_s l_i - (1+r)d_i \tag{3.10}$$

The solution to this problem (firm i optimal demand for labor) then writes as

$$(1-\alpha)\,A\left(\delta\right)\left(k_i+d_i\right)^{\alpha} l_i^{-\alpha} = Ew_s \tag{3.11}$$

Now one can solve the problem consisting for firm i in determining its optimal amount of debt finance d_i. This amounts to solve the following problem

$$\begin{aligned} &\max_{d_i} E\Pi\left(d_i\right) = A\left(\delta\right)\left(k_i+d_i\right)^{\alpha} l_i^{1-\alpha} - Ew_s l_i - \left(1+r\right) d_i \\ &\text{s.t.} \begin{cases} \left(1-\alpha\right) A\left(\delta\right)\left(k_i+d_i\right)^{\alpha} l_i^{-\alpha} = Ew_s \\ \forall s,\ \tau A_s \left(k_i+d_i\right)^{\alpha} l_i^{1-\alpha} \geq w_s l_i + \left(1+r\right) d_i \end{cases} \end{aligned} \tag{3.12}$$

Introducing firm i optimal labor demand (3.11) in both the objective function (3.10) and the borrowing constraints $\tau A_s\left(k_i+d_i\right)^{\alpha} l_i^{1-\alpha} \geq w_s l_i + \left(1+r\right) d_i$ we can rewrite (3.12) as the following problem

$$\begin{aligned} &\max_{d_i} E\Pi\left(d_i\right) = \alpha A\left(\delta\right)\left[\tfrac{1-\alpha}{Ew_s} A\left(\delta\right)\right]^{\frac{1-\alpha}{\alpha}} \left(k_i+d_i\right) - \left(1+r\right) d_i \\ &\text{s.t. } \forall s,\ \left[\tfrac{1-\alpha}{Ew_s} A\left(\delta\right)\right]^{\frac{1-\alpha}{\alpha}} \left[\tau A_s - \left(1-\alpha\right)\tfrac{A(\delta)}{Ew_s} w_s\right]\left(k_i+d_i\right) \geq \left(1+r\right) d_i \end{aligned} \tag{3.13}$$

There are then two different cases: if $\alpha A\left(\delta\right)\left[\frac{1-\alpha}{Ew_s}A\left(\delta\right)\right]^{\frac{1-\alpha}{\alpha}} \leq 1+r$, then firms simply lend their capital on financial markets because lending is more profitable than investing in the firm. Firms expected profits write as $E\Pi^* = \left(1+r\right)k$. As is clear the expected profits of firm i do not depend upon nor on the type of the labor contract nor on the technology chosen. On the contrary when $\alpha A\left(\delta\right)\left[\frac{1-\alpha}{Ew_s}A\left(\delta\right)\right]^{\frac{1-\alpha}{\alpha}} > 1+r$ then firm i optimal expected profits write as

$$E\Pi = \frac{\alpha A\left(\delta\right) - \left[\tau A_j - \left(1-\alpha\right)\frac{w_j}{Ew_s}A\left(\delta\right)\right]}{\left(1+r\right)\left[\frac{Ew_s}{(1-\alpha)A(\delta)}\right]^{\frac{1-\alpha}{\alpha}} - \left[\tau A_j - \left(1-\alpha\right)\frac{w_j}{Ew_s}A\left(\delta\right)\right]}\left(1+r\right)k_i$$

where j is the state of nature for which the borrowing constraint is binding:

$$j = \arg\min_p \left[\tau A_p - \left(1-\alpha\right)\frac{w_p}{Ew_s}A\left(\delta\right)\right]$$

Assuming for now on that $\tau = 1$, noting $w_s = \eta_s w$, and simplifying the last expression, we obtain that the state of nature for which the borrowing constraint is binding is the bad state, i.e. $j = l$, if and only if

$$\eta_l^2 > \frac{1-\alpha-\delta}{1-\alpha+\delta}$$

Then firms expected profits write as

$$E\Pi\left(\delta,\eta\right) = \frac{\left[\delta + (1-\alpha)\left[\frac{2\eta^2}{1+\eta^2} - 1\right]\right](1+r)\,k_i}{\left[\delta + (1-\alpha)\left[\frac{2\eta^2}{1+\eta^2} - 1\right]\right] - \left[\alpha - \frac{1+r}{A(\delta)}\left[\frac{w}{(1-\alpha)A(\delta)}\frac{1+\eta^2}{2\eta}\right]^{\frac{1-\alpha}{\alpha}}\right]}$$

where for now on η stands for η_l. As is clear in this last case, the expected profits of firm i do depend upon on the type of the labor contract and the technology chosen. Firms' expected profits can be positively or negatively related to wage variability since on the one hand wage variability raises labor costs and therefore reduces the firm's productivity while on the other hand, wage variability can ease the firm's borrowing constraints by increasing minimum profits before debt repayments. Similarly, choosing a more productive technology raises expected profits on the one hand because total factor productivity is larger. On the other hand however, choosing a more productive technology means every thing else equal a lower borrowing capacity due to more volatile shocks.

The next propositions then derive the main properties of the optimal wage contract and the optimal technology in this last context.

Proposition 1. *As long as firms credit constraint is binding, the optimal wage contract* $\{w_l^*, w_h^*\}$ *is such that* $w_l^* < w < w_h^*$. *Moreover the difference* $w_h^* - w_l^*$ *decreases with the risk free interest rate* r.

Proposition 2. *As long as* $\log\left(A\left(\delta\right)\right)$ *is concave is* δ, *the optimal technology* δ^* *is such that* δ^* *is a decreasing function of the difference* $w_h^* - w_l^*$.

Proof. c.f. appendix 7.2 for a proof of proposition 1. As concerns proposition 2, the first order condition determining the optimal wage contract η^* writes as

$$\frac{1+r}{A(\delta)}\left[\frac{w}{(1-\alpha)A(\delta)}\frac{1+\eta^2}{2\eta}\right]^{\frac{1-\alpha}{\alpha}}\left[1+\frac{1-\eta^4}{4\eta^2}\frac{\delta+(1-\alpha)\left[\frac{2\eta^2}{1+\eta^2}-1\right]}{\alpha}\right]=\alpha$$

while the first order condition determining the optimal technology δ^* writes as

$$\frac{1+r}{A(\delta)}\left[\frac{w}{(1-\alpha)A(\delta)}\frac{1+\eta^2}{2\eta}\right]^{\frac{1-\alpha}{\alpha}}\left[1+\frac{A'(\delta)}{A(\delta)}\frac{\delta+(1-\alpha)\left[\frac{2\eta^2}{1+\eta^2}-1\right]}{\alpha}\right]=\alpha$$

Therefore firms individually choose wage contracts and technologies such that

$$\frac{A'(\delta)}{A(\delta)}=\frac{1-\eta^4}{4\eta^2}$$
$$\frac{1+r}{A(\delta)}\left[\frac{w}{A(\delta)}\right]^{\frac{1-\alpha}{\alpha}}\left[\frac{1+\eta^2}{2(1-\alpha)\eta}\right]^{\frac{1-\alpha}{\alpha}}\left[1+\frac{A'(\delta)}{A(\delta)}\frac{\delta+(1-\alpha)\left[\frac{2\eta^2}{1+\eta^2}-1\right]}{\alpha}\right]=\alpha$$

Then as long as $\frac{A'(\delta)}{A(\delta)}$ is a decreasing function of δ, i.e. as long as $\log(A(\delta))$ is a concave function of δ, the first condition defines a positive relationship between η^* and δ^* while the second condition defines a negative relationship between η^* and δ^*. These two conditions therefore determine a unique couple (δ^*, η^*) which maximizes firms expected profits. ■

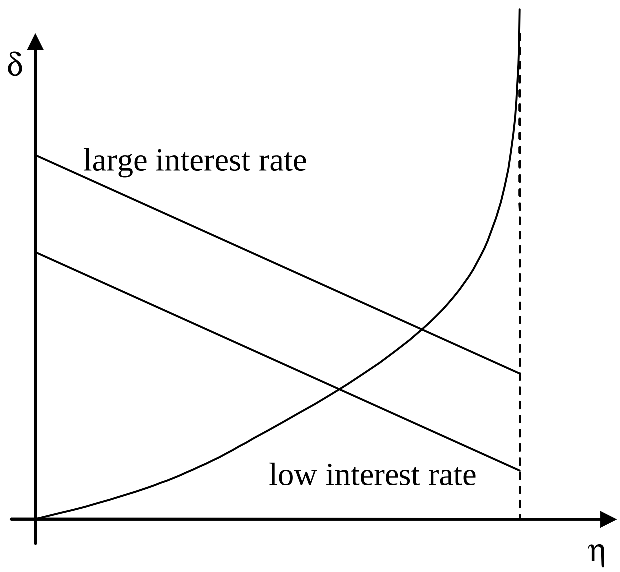

Figure 5: Firm optimal strategies in partial equilibrium.

In this proposition, we have three different properties. The first one states that the situation where firms are not able to borrow capital up to the point where the expected marginal productivity of capital is equal to the risk free interest rate is a necessary and sufficient condition for firms to provide contingent compensation schemes to workers. This property is very natural. Let us consider a firm which provides fixed wage contracts and faces a binding borrowing constraint in the sense that its expected marginal productivity of capital is larger than the interest rate on the capital market when the borrowing constraint is binding. Then on the one hand there is a strictly positive cost to being unable to borrow more capital. On the other hand providing contingent wage contracts could help increase the volume of capital it is possible to borrow while the marginal increase in labor cost is zero with fixed wage contracts since

$$\left.\frac{\partial E w_s}{\partial \eta}\right|_{\eta=1} = \left.\frac{\partial}{\partial \eta}\left(\eta + \frac{1}{\eta}\right)\right|_{\eta=1} = \left.\left(1 - \frac{1}{\eta^2}\right)\right|_{\eta=1} = 0$$

Therefore as soon as firms are credit constrained, they have incentives to provide contingent wage contracts basically because a binding credit constraint is always marginally costly while providing flexible labor contracts is always marginally cost free. Secondly proposition 1 states that a large interest rate reduces firms incentives to provide contingent labor compensation schemes. This is natural since a large interest rate reduces firms demand for capital and therefore reduces the need to provide contingent labor contracts. Finally proposition 2 shows that firms which choose to provide workers with more flexible labor contracts also choose to invest in less productive technologies. Once again this is very natural: if a firm decides to provide flexible labor contracts in order to alleviate its credit constraints, it also undergoes an increase in labor costs due to the risk premium it pays to workers. Now investing in a highly productive technology also means accepting large fluctuations in the firm's productivity. Given that lenders compute the borrowing constraint they impose to firms on the bad state of nature, investing in a highly productive technology and providing flexible labor contracts would mean that the firm would pay for a risk premium on workers wages without benefiting from an increased borrowing capacity since the highly productive technology is very unproductive in the case of a bad shock. Therefore firms prefer to invest in low productivity technologies when they provide flexible labor compensation contracts.

4. The general equilibrium of the economy.

4.1. The equilibrium of the capital market.

Up to now the risk free interest rate has been taken to be exogenous. To determine the equilibrium interest rate that prevails in the economy, one simply needs to equal the supply for capital provided by lenders and the demand for capital expressed by entrepreneurs and consumers. Put differently the equilibrium interest rate is determined through

$$k_l = d + l\,(c_t^*)_1 + (1-l)\,(c_t^*)_2$$

where k_l represents lenders capital supply, d firms aggregate demand for capital, l firms aggregate demand for labor, $(c_t^*)_1$ is the first period consumption of workers hired by entrepreneurs and $(c_t^*)_2$ is the first period consumption of workers who have not been hired by entrepreneurs. Entrepreneur i labor demand l_i and capital demand d_i respectively write as

$$l_i = \left[\frac{(1-\alpha)\,A(\delta)}{Ew_s}\right]^{\frac{1}{\alpha}} (k_i + d_i)$$

$$d_i = \frac{A_l - 2\,(1-\alpha)\,A(\delta)\,\frac{\eta^2}{1+\eta^2}}{(1+r)\left[\frac{Ew_s}{(1-\alpha)A(\delta)}\right]^{\frac{1-\alpha}{\alpha}} - \left[A_l - 2\,(1-\alpha)\,A(\delta)\,\frac{\eta^2}{1+\eta^2}\right]} k_i$$

Given that entrepreneurs are identical, that $w_s = \eta_s w$ with $w_l w_h = w^2$, the equilibrium of the capital market simplifies as

$$k_l - \frac{1}{1+r}\frac{w}{2} = \frac{A_l + 2\,(1-\alpha)\,A(\delta)\,\frac{\eta}{1+\eta^2}\left[\frac{\eta}{1+\eta^2} - \frac{1}{2} - \eta\right]}{(1+r)\left[\frac{Ew_s}{(1-\alpha)A(\delta)}\right]^{\frac{1-\alpha}{\alpha}} - \left[A_l - \lambda\,(1-\alpha)\,A(\delta)\,\frac{\eta^2}{1+\eta^2}\right]} k$$

4.2. The equilibrium of the labor market.

At the equilibrium of the capital market, labor demand balances labor supply. Given that entrepreneurs are all identical, noting k firms aggregate capital stock and d firms aggregate borrowing, the expected wage rate

Ew_s is then equal to the expected marginal productivity of labor

$$Ew_s = (1-\alpha)\, A(\delta)\, (k+d)^{\alpha}$$

and the uncontingent wage rate writes as

$$w = \frac{2\eta}{1+\eta^2} (1-\alpha)\, A(\delta)\, (k+d)^{\alpha} \tag{4.1}$$

4.3. General Equilibrium.

The general equilibrium of the economy corresponds to the situation where all markets, the capital and the labor market, balance supply and demand. To determine the properties of this situations, one simply need to plug the two last expressions in the capital market equilibrium condition, we end up with an equilibrium interest rate r being defined through

$$k_l = \left[A_l - (1-\alpha)\, A(\delta)\, \frac{2\eta^2}{1+\eta^2}\left[1 - \frac{1}{1+\eta^2}\right]\right] \frac{(k+d)^{\alpha}}{1+r} \tag{4.2}$$

where the aggregate volume of capital d firms borrow at the general equilibrium of the economy is such that

$$(1+r)\, d = \left[A_l - \frac{2\eta^2}{1+\eta^2}(1-\alpha)\, A(\delta)\right] (k+d)^{\alpha} \tag{4.3}$$

Finally firm optimal technology is such that

$$\frac{A'(\delta)}{A(\delta)} = \frac{1-\eta^4}{4\eta^2} \tag{4.4}$$

and the optimal labor contract firm propose to workers verifies

$$\frac{1+r}{A(\delta)} \left[\frac{w}{(1-\alpha)\, A(\delta)} \frac{1+\eta^2}{2\eta}\right]^{\frac{1-\alpha}{\alpha}} \left[1 + \frac{A'(\delta)}{A(\delta)} \frac{\delta + (1-\alpha)\left[\frac{2\eta^2}{1+\eta^2} - 1\right]}{\alpha}\right] = \alpha \tag{4.5}$$

Proposition 1. *The general equilibrium of the economy is represented by the vector (η, δ, r, d, w). Firms choose their optimal labor contract η and their optimal technology δ respectively such that (4.5) and (4.4) are verified. The equilibrium interest rate on the capital market r and the volume of capital d firms are able to borrow are respectively such that (4.2) and (4.3) are verified.*

Proof. Evident. ■

Corollary 2. *The equilibrium wage rate on the labor market w is such (4.1) is verified. Using these last expressions and assuming that $A(\delta) = 1 + a^{-1}\delta$ with $a < 1$, the optimal labor contract η verifies*

$$\left[1 + a - \frac{\eta^2}{1+\eta^2}\left(\frac{4}{1-\eta^2} + (1-\alpha)\frac{1+2\eta^2}{1+\eta^2}\right)\right]\left[1 + \frac{1}{\alpha}\left[1 - \left(1-\eta^2\right)\frac{a\left(1+\eta^2\right) + (1-\alpha)\left(1-\eta^2\right)}{4\eta^2}\right]\right] = \frac{\alpha}{2} \tag{4.6}$$

Proof. Introducing the expression for firms optimal technology (4.4) and for the equilibrium wage rate on the labor market (4.1) into the first order condition determining the individual optimal labor contract (4.5), we end up with (4.6). ■

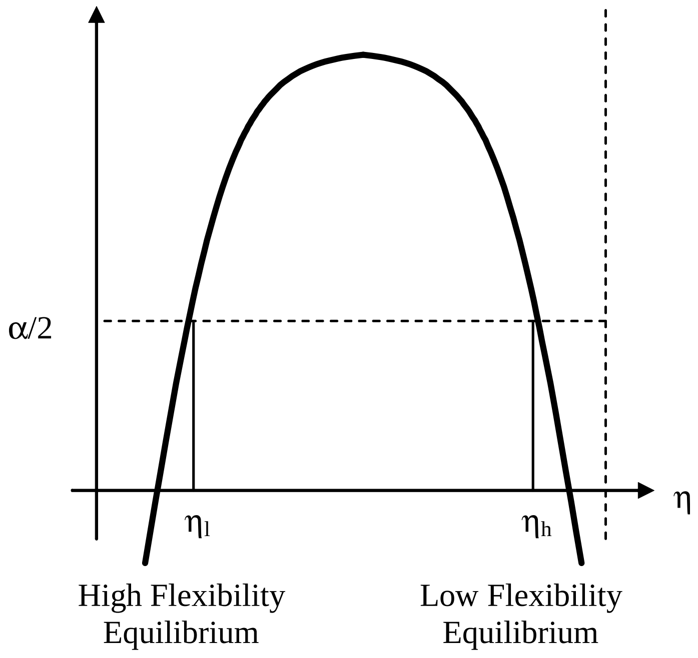

Figure 6: Optimal labor contracts in general equilibrium.

A few remarks here are in order. First firms borrowing capacity decreases with the interest rate. This is due first to larger debt repayments second because firms choose optimally to propose less contingent compensation schemes to workers and third because firms optimally choose to invest in more productive, yet more risky technologies on the other hand. As a result of these three effects, firms demand for capital is walrasian: it decreases with the cost of capital. Second, workers demand for capital decreases with the interest rate as a basic trade-off between contemporary and future consumption. This is due a standard substitution effect. However an increase in the interest rate also has an income effect: it modifies workers future labor income and therefore affects workers contemporary consumption and thereby workers demand for capital. On the one hand firms raise fewer capital when the cost of capital increases. Since capital and labor are complements in firms production function workers demand for capital decreases with the interest rate according to this first effect. On the other hand however, firms invest in more productive technologies when the interest rate increases. This second effect raises every thing else equal, the marginal productivity of labor. Hence workers future labor income is raised and workers increase their contemporary consumption. Therefore the income effect is *a priori* ambigious since labor income can increase or decrease following an change in the cost of capital. However workers contemporary consumption (i.e. demand for capital) is affected in a third manner. When the cost of capital increases, firms provide less contingent labor compensation schemes to workers. Therefore the need for workers to reduce their consumption as a hedging device against labor income fluctuations is reduced. Therefore, workers demand for capital increases as the cost of capital increases according to this last effect. To sum up, workers demand for capital can well be increasing in the cost of capital if the last two effects dominate the first one.

Then if the increase in workers demand for capital compensates for the decrease in firms demand for capital, there can exist a range of interest rates for which the global demand for capital increases with the interest rate due to the fact that the reduction in firms demand for capital is more than offset by the increase in workers demand for capital. As a result, there can be multiple equilibria. In the first one the interest rate is low, firms propose relatively flexible labor contracts to workers and invest in relatively unproductive technologies. Workers have a low contemporary consumption and firms borrow the bulk of available capital on the credit market. In the second equilibrium the interest rate is large, firms propose relatively fixed labor contracts to workers and invest in relatively productive technologies. Workers have a large contemporary consumption and firms borrow only a relatively small share of available capital on the credit market, the bulk of available capital being used to finance workers' consumption.

It is therefore unclear which of the low or the large labor market flexibility equilibrium is the Pareto optimal equilibrium. Nor is it clear if the Pareto optimal equilibrium is the also high growth equilibrium or not. However the model clearly shows that different labor market institutions can emerge and remain existent in a general equilibrium framework as long as some market imperfections are being introduced. The view that the supposed lack of flexibility in Continental European labor markets is an out-of-equilibrium phenomenon, or put differently, is a pure political economy equilibrium is therefore not necessarily completely relevant. Structural cross-country differences in labor market institutions can well be an equilibrium phenomenon entirely based on pure economic mechanisms.

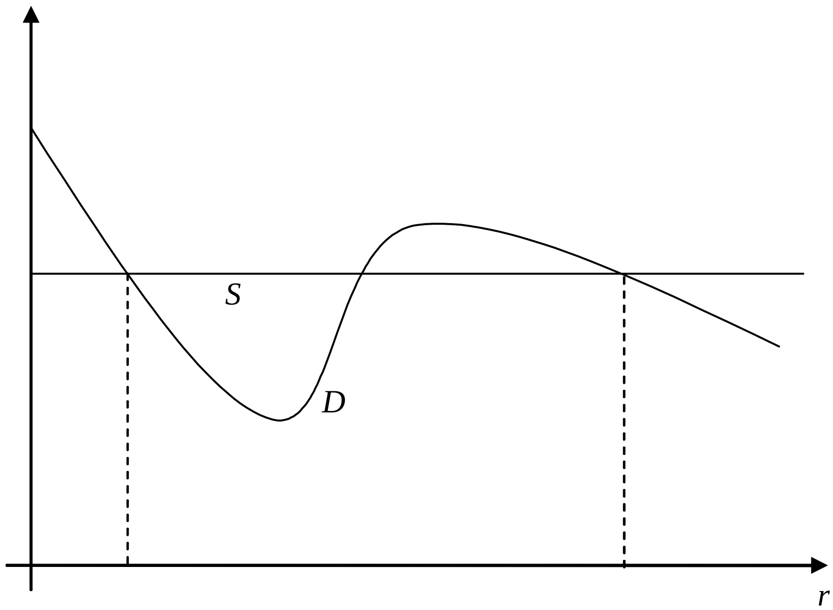

Figure 7: Equilibrium of the capital market.

5. The welfare analysis.

Given that the economy is populated by heterogenous agents, the welfare analysis can be carried out using two different welfare criteria: the utilitarian or the egalitarian social welfare. In the case of the utilitarian welfare criterion, social welfare is simply the sum of individual welfare weighted by each type of agents weight in the total population. Since lenders, workers and entrepreneurs have identical weights in the economy, noting W_{util} the utilitarian welfare criterion, we have

$$W_{util} = V_w + V_f + V_l$$

where V_w represents workers welfare, V_f represents firms welfare and V_l represents lenders welfare. At the general equilibrium of the economy, using in particular expressions (4.1) and (4.2) the different individual welfare functions V_i write as

$$\begin{aligned}
V_w &= \frac{2\eta}{1+\eta^2}\frac{(1-\alpha)}{(1+r)^{\frac{1}{2}}}\frac{A(\delta)(k+d)^{\alpha}}{2} \\
V_f &= \left[\delta + (1-\alpha)\left[\frac{2\eta^2}{1+\eta^2} - 1\right]\right]\frac{A(\delta)(k+d)^{\alpha}}{2} \\
V_l &= \left[1 - \delta - (1-\alpha)\frac{2\eta^2}{1+\eta^2}\left[1 - \frac{1}{1+\eta^2}\right]\right]\frac{A(\delta)(k+d)^{\alpha}}{2}
\end{aligned}$$

Therefore the utilitarian social welfare criterion can write as

$$W_{util} = \left[\frac{\alpha}{1-\alpha} + \frac{2\eta}{1+\eta^2}\left(\frac{1}{\sqrt{1+r}} + \frac{\eta}{1+\eta^2}\right)\right]\frac{(1-\alpha)A(\delta)(k+d)^{\alpha}}{2} \tag{5.1}$$

Proposition 1. *When the economy exhibits multiple equilibria, then the socially optimal equilibrium is the low labor market flexibility equilibrium.*

Proof. At at the general equilibrium of the economy, the interest r, the optimal labor contract η, the volume of debt d firms borrow and firms' optimal technology δ verify the capital market equilibrium condition

(4.2)

$$k=\left[\frac{1-\delta}{1-\alpha}-\frac{2\eta^2}{1+\eta^2}\left[1-\frac{1}{1+\eta^2}\right]\right]\frac{(1-\alpha)\,A\,(\delta)\,(k+d)^{\alpha}}{1+r}$$

Therefore from expression (5.1) the utilitarian social welfare can be simplified as

$$W_{util}\,(\eta,r)=\frac{\left[\frac{\alpha}{1-\alpha}+2\left[\frac{\eta}{1+\eta^2}\right]^2\right]\sqrt{1+r}+2\frac{\eta}{1+\eta^2}}{\frac{1-\delta}{1-\alpha}-2\left[\frac{\eta^2}{1+\eta^2}\right]^2}\frac{k\sqrt{1+r}}{2} \tag{5.2}$$

As is clear this expression is useful since it only depends upon the cost of capital r and the labor contract η which are positively correlated across equilibria: the high labor market flexibility equilibrium is also the low interest rate equilibrium. It helps in particular get rid of the effects that play in opposite direction. Form expression (5.2), it is obvious that a larger interest rate r increases every thing else equal, the utilitarian social welfare criterion. As to the effect of labor market flexibility, the utilitarian social welfare criterion can be written as

$$W_{util}\,(\eta,r)=\frac{N\,(\eta)}{D\,(\eta)}\frac{k\sqrt{1+r}}{2}$$

with

$$\begin{aligned} N\,(\eta) &= \left[\frac{\alpha}{1-\alpha}+2\left[\frac{\eta}{1+\eta^2}\right]^2\right]\sqrt{1+r}+2\frac{\eta}{1+\eta^2} \\ D\,(\eta) &= \frac{1-\delta}{1-\alpha}-2\left[\frac{\eta^2}{1+\eta^2}\right]^2 \end{aligned}$$

As is clear, the numerator $N\,(\eta)$ is strictly increasing in η and r. As to the denominator $D\,(\eta)$, it is a strictly decreasing function of η. Therefore social welfare under the utilitarian criterion is maximized at the low labor market flexibility equilibrium. ■

The intuition for this result is fairly simple: social welfare is maximized at the low flexibility equilibrium because this equilibrium allocates risk to agents which are the least risk averse in the economy and capital to those which have no other means to raise their utility. Put differently from a social point of view, the capital distribution between workers and firms is irrelevant. On the contrary the productivity level of firm investments is relevant and raises social welfare because it raises both firms profits and workers labor income. Since firms make more productive investments when labor market contracts are less flexible, it is natural that the Pareto optimal equilibrium is the low labor market flexibility equilibrium.

Now if we turn to the egalitarian social welfare criterion, it writes as

$$W_{egal} = \max\min\left\{\frac{2\eta}{1+\eta^2}\frac{1}{(1+r)^{\frac{1}{2}}};\frac{\delta}{1-\alpha}+\frac{2\eta^2}{1+\eta^2}-1;\frac{1-\delta}{1-\alpha}-\frac{2\eta^2}{1+\eta^2}\left[1-\frac{1}{1+\eta^2}\right]\right\}\frac{(1-\alpha)\,A(\delta)\,(k+d)^{\alpha}}{2}$$

which according to the capital market equilibrium condition (4.2) writes as

$$W_{egal} = \frac{\max\min\left\{\frac{2\eta}{1+\eta^2}\frac{1}{(1+r)^{\frac{1}{2}}};\frac{\delta}{1-\alpha}+\frac{2\eta^2}{1+\eta^2}-1;\frac{1-\delta}{1-\alpha}-\frac{2\eta^2}{1+\eta^2}\left[1-\frac{1}{1+\eta^2}\right]\right\}}{\frac{1-\delta}{1-\alpha}-\frac{2\eta^2}{1+\eta^2}\left[\lambda-\frac{1}{1+\eta^2}\right]}\frac{1+r}{2}k$$

To be continued.

6. Growth effects of labor market flexibility.

We embed the framework considered in the previous sections into a dynamic model. At each point in time there is a continuum of unit mass of workers, a continuum of mass 2 of agents who can be entrepreneurs or lenders with equal probability[14]. At the beginning of each period, entrepreneurs hire workers and agree on labour contracts with them. They borrow capital from lenders to finance investment and they choose a technology and engage in production. Workers supply labour to entrepreneurs and agree on labour contracts with them. They borrow capital from lenders to finance beginning of period consumption. Lenders lend capital to firms to finance investment. They lend capital also to workers to finance consumption.

At the end of each period, entrepreneurs pay workers according to the labour contracts they agreed upon. They pay back lenders for beginning of period loans and they divide their profits between consumption and bequest. Workers are paid according to the wage contract they agreed upon with entrepreneurs. They pay back lenders for beginning of period loans and consume their labour income net of loan repayments. Lenders are paid back on beginning of period loans extended to workers and entrepreneurs and they divide their final capital income between consumption and bequest.

Noting k_t the capital stock in the economy at the beginning of period t, and k^s_{t+1} the capital stock in the economy at the beginning of period t when state s has happened at time t, the low of motion of the capital stock writes as

$$k^s_{t+1} = \frac{\pi_s\left(k_t\right) + \left(1+r\right)k_t}{2}$$

where $\pi_s\left(k_t\right)$ represents firms profits conditional on state s and $\left(1+r\right)k_t$ lenders profits. If the bad state of nature happens at time t, the capital stock at the beginning of period $t+1$, k^l_{t+1} therefore writes as

$$k^l_{t+1} = \frac{1}{2}\left[A_l\left(\delta\right)\left(\frac{1}{2}k_t + d\right)^{\alpha} - \frac{2\eta^2}{1+\eta^2}\left(1-\alpha\right)A\left(\delta\right)\left(\frac{1}{2}k_t + d\right)^{\alpha}\left[1 - \frac{1}{1+\eta^2}\right]\right] \tag{6.1}$$

The first part of the right hand side $A_l\left(\delta\right)\left(\frac{1}{2}k_t + d\right)^{\alpha}$ represents total output in the economy. The second

[14] This assumption helps simplify the exposition of the model since firms beginning of period aggregate capital stock k_f is always equal to lenders beginning of period aggregate capital stock k_l which is half the economy's beginning of period aggregate capital stock k_t.

part of the right hand side $\frac{2\eta^2}{1+\eta^2}(1-\alpha)A(\delta)\left(\frac{1}{2}k_t+d\right)^{\alpha}$ represents the wage bill distributed to workers. The final part of the right hand side $1-\frac{1}{1+\eta^2}$ represents the share of the wage bill workers dedicate to beginning of period loans repayments. Finally entrepreneurs and lenders bequest a share $\beta=\frac{1}{2}$ of their final wealth and consume a share $1-\beta=\frac{1}{2}$. Similarly if the good state of nature happens at time t, the capital stock at the beginning of period $t+1$, k_{t+1}^h, writes as

$$k_{t+1}^h=\frac{1}{2}\left[A_h(\delta)\left(\frac{1}{2}k_t+d\right)^{\alpha}-\frac{2}{1+\eta^2}(1-\alpha)A(\delta)\left(\frac{1}{2}k_t+d\right)^{\alpha}\left[1-\frac{\eta^2}{1+\eta^2}\right]\right] \tag{6.2}$$

This expression is similar to the above one apart two distinct features. First the technological shock which is good in the latter case and bad in the former case. Second the share of the wage bill workers dedicate to consumption which is large is the latter case and low in the former case. We then establish the following result as regards expected growth.

Proposition 1. *Noting $\theta=\frac{\eta^2}{1+\eta^2}$, the average growth rate of the economy's capital stock writes as*

$$\begin{aligned}E\log\frac{k_{t+1}^s}{k_t} &= \log\frac{(1-\alpha)A(\delta)}{2}+\alpha\log\left(\frac{1+\mu}{2}\right)-(1-\alpha)\log k_t \\ &\quad+\frac{1}{2}\log\left[\frac{1-\delta(\theta)}{1-\alpha}-2\theta^2\right]+\frac{1}{2}\log\left[\frac{1+\delta(\theta)}{1-\alpha}-2[1-\theta]^2\right]\end{aligned} \tag{6.3}$$

where

$$\mu\triangleq 2\frac{d}{k}=\frac{\frac{1-\delta}{1-\alpha}-2\theta}{\frac{1-\delta}{1-\alpha}-2\theta^2} \tag{6.4}$$

and $\delta(\theta)$ verifies

$$\frac{A'(\delta)}{A(\delta)}=\frac{1-2\theta}{1-\theta}\frac{1}{4\theta}$$

Proof. Using the capital market equilibrium condition (4.2) and the equilibrium borrowing constraint for firms (4.3) it is straightforward to obtain the equilibrium firm debt equity ratio (6.4). Then plugging (6.4) into capital accumulation expressions for each state of the world (6.1) and (6.2), the expected capital growth rate expression (6.3) becomes immediate to obtain. ■

The expected growth expression (6.3) can be decomposed along classical growth determinants on the one hand and labor contracts specific effects on the other hand. Among the classical determinants of growth, appears the standard catch-up effect: due to decreasing marginal returns to capital, growth in the capital stock is bound to go to zero as the economy accumulates capital in the absence of any other source of growth.

$$
\begin{aligned}
E\log\frac{k_{s,t+1}}{k_t} &= \underbrace{\log\frac{(1-\alpha)}{2}A(\delta)}_{\text{TFP effect}} + \underbrace{\alpha\left[\log\left(1-\frac{\theta(1-\theta)}{\frac{1-\delta}{1-\alpha}-2\theta^2}\right)\right]}_{\text{TFI effect}} - \underbrace{(1-\alpha)\log k_t}_{\text{Catch-up effect}} \\
&\quad \underbrace{+\frac{1}{2}\log\left[\frac{1-\delta(\theta)}{1-\alpha}-2\theta^2\right]+\frac{1}{2}\log\left[\frac{1+\delta}{1-\alpha}-2\left[1-\theta\right]^2\right]}_{\text{Inter-temporal effect}}
\end{aligned}
$$

Now apart from the standard catch up effect, firms choices as to the optimal wage contract and the optimal technology they adopt generate three different sources of growth. When firms adopt more flexible wage contracts, that helps them increase the volume of capital they can borrow. Hence the volume of firms investment is larger with more flexible labor contracts. The total factor input effect of labor market flexibility is therefore positive and the economy grows faster with more flexible labor contracts. On the contrary when firms adopt more flexible labor contracts, they also optimally choose to invest in less productive technologies. Hence the total factor productivity effect of labor market flexibility is negative and the economy grows slower with more flexible labor contracts. Finally there is a third effect: when firms propose more flexible labor contracts, workers reduce their beginning-of-period consumption and increase their average end-of-period consumption. Therefore when firms propose more flexible labor contracts, the volume of capital workers borrow at the beginning of the period is reduced while the volume of consumption at the end of the period is increased. Now since workers need to pay back loans contracted at the beginning of period, workers beginning of period consumption acts as an investment whose productivity is equal to the interest rate r. On the contrary, workers end-of-period consumption acts to reduce the volume of capital in the economy at the end of the period. Therefore more flexible labor market will tend to reduce capital accumulation because workers are less willing to borrow and more willing to rely on their labor income to finance their consumption. This is the inter-temporal effect.

7. Conclusion.

We have built a model in which the structure of workers compensation and firms productivity are endogenous. This has enabled us to build a theory of growth based on firms choices as to the technological productivity and the structure of workers compensation.

To be completed....

References

[1] Acemoglu, D. and F. Zilibotti (1997), "Was Promotheus unbound by Chance: Risk, Diversification and Growth," Journal of Political Economy, vol. 105, August, pp.709-51.

[2] Aghion, P. and Howitt, P. (1992) "A Model of Growth Through Creative Destruction" *Econometrica,* vol. 60, pp. 323-51.

[3] Alonso-Borrego, C., J. Fernandez-Villaverde. and J.E. Galdón-Sánchez, (2004),"Evaluating Labor Market Reforms: A General Equilibrium Approach", IZA Working Paper 1129.

[4] Azariadis C. (1978). Escalator Clauses and the Allocation of Cyclical Risk, *Journal of Economic Theory*, Vol. 18, No. 1, pp.119-55.

[5] Azariadis, C. and J. Stiglitz (1983) "Implicit Contracts and Fixed-Price Equilibria", *Quarterly Journal of Economics*, Supplément, 1-22.

[6] Baker, G., Gibbs, M. and B. Hölmström. (1994), "The wage policy of a firm," *Quarterly Journal of Economics*, vol. 109, pp. 921-955.

[7] Barro, R. and Sala-i-Martin, X. (1995) "Economic Growth" McGraw Hill. New York.

[8] Bartelsman, E. and Doms, M. (2000) "Understanding Productivity: Lessons from Longitudinal Microdata" *Journal of Economic Literature*, vol. 38, pp. 569 – 594.

[9] Benito A. and I. Hernando (2003), "Labour demand, flexible contracts and financial factors: new evidence from Spain". Banco de Espagna Working Paper 0312.

[10] Bentolila, S. and G. Saint Paul (1992) "The Macroeconomic Impact of Flexible Labor Contracts, with an Application to Spain", *European Economic Review* 36.

[11] Bentolila, S. and G. Bertola (1990), "Firing Costs and Labor Demand: How Bad is Eurosclerosis?" *Review of Economic Studies* 54.

[12] Bernanke, B. and M. Gertler (1989), "Agency costs, net worth and business fluctuations," *American Economic Review*, vol. 79, pp. 14-31.

[13] Bils, M. (1987), "The cyclical behavior of marginal cost and price," *American Economic Review*, vol. 77, pp. 838-55.

[14] Blanchard O. and A. Landier (2002), "The Perverse Effects of Partial Labour Market Reform: Fixed-Term Contracts in France," *The Economic Journal*, 112 (June).

[15] Blanchard O. and J. Wolfers (2000), "The Role Shocks and Institutions in the Rise of European Unemployment: The Aggregate Evidence," *The Economic Journal*, 110 (March).

[16] Botero J.C., S. Djankov, R. La Porta, F. Lopez-de-Silanes and A. Shleifer (2004), "The regulation of labor" *Quarterly Journal of economics*. vol. 119, pp. 1339-82.

[17] Bronars, S. and D. Deere (1991), "The Threat of Unionization, the Use of Debt, and the Preservation of Shareholder Wealth" *Quarterly Journal of Economics*, vol. 106, pp. 231-54.

[18] Burgess S. and D. Mawson, (2003) "Aggregate Growth and the Efficiency of Labour Reallocation" mimeo London School of Economics.

[19] Chevalier, J. and D. Scharfstein (1996): "Capital-market imperfections and counter-cyclical mark-ups: Theory and evidence," American Economic Review, vol.86, pp.703-25 september.

[20] Danthine, J.P. and J. Donaldson (1992), "Risk Sharing and the Business Cycle" *European Economic Review*, vol. 36, pp. 468-75.

[21] Dolado, J., J.C. García-Serrano and J.F. Jimeno (2002), "Drawing Lessons from the Boom of Temporary Jobs in Spain," *The Economic Journal*, 112 (June)

[22] Djankov S., C. McLiesh, and A. Shleifer (2006), "Private Credit in 129 Countries" mimeo Harvard University.

[23] Foster, L., Haltiwanger, J. and Krizan, C. (1998) "Aggregate productivity growth: Lessons from microeconomic evidence" NBER Working Paper 6803.

[24] Gomme, P. and J. Greenwood (1995), "On the Cyclical Allocation of Risk," *Journal of Economic Dynamics and Control*, vol. 19, pp. 91-124.

[25] Gottschalk, P. and R. Moffitt (1994),"The Growth of earnings instability in the U.S. labor market, *Brookings Papers on Economic Activity*, vol.2, pp.217-72.

[26] Greenwald, B., J.E. Stiglitz, and A. Weiss (1984), "Informational imperfections in the capital market and macroeconomic fluctuations," *American Economic Review*, Papers and proceedings, vol.74, pp.194-99.

[27] Guiso, L., L. Pistaferri and F. Schivardi (2001), "Insurance within the firm" CEPR Discussion Paper n°2793.

[28] Gibbons (1998), "Incentives in Organizations", *Journal of Economic Perspectives*, vol.12 pp.115-32.

[29] Ichino A. (1994), "Flexible labor compensation, risk sharing and company leverage," *European economic Review,* vol.38, pp.1411-21.

[30] Hopehayn and Rogerson (1993) "Job Turnover and Policy Evaluation: A General Equilibrium Analysis," *Journal of Political Economy*, 101, 5, 915-938

[31] Hubbard, G.R., 1998, "Capital-Market Imperfections and Investment," *Journal of Economic Literature* 36, 193-225.

[32] Holmström, B. and P. Milgrom (1987), "Aggregation and linearity in the provision of intertemporal incentives," *Econometrica*, vol.55, pp. 303-28.

[33] King, R. and Levine, L. (1993) "Finance, Entrepreneurship and growth" *Journal of Monetary Economics*, vol. 32 pp. 513-42.

[34] Kugler, Adriana and Giovanni Pica, “Effects of Employment Protection and Product Market Regulations on the Italian Labor Market," forthcoming in J. Messina, C. Michelacci, J. Turunen, and G. Zoega, eds., “Labour Market Adjustments in Europe,” Edward Elgar.

[35] Koskela, e. and R. Stenbacka, (2001) ”Equilibrium unemployment with credit and labour market imperfections” CESifo Working Paper 419.

[36] Lamont O. (1996), ”Cash flow and investment evidence from internal capital markets”. NBER Working Paper 5499.

[37] Lucas, R.E. (1977), ”Understanding Business Cycles,” In Brunner, K. and Meltzer, A., eds.

[38] Milgrom P. (1988), ”Employment contracts, influence activities and efficient organization design”, *Journal of Political Economy*, vol. 99, pp. 42-60.

[39] Nelson, R. (1981) ”Research on productivity growth and productivity differences: Deand ends and new departures” Journal of Economic Literature vol. 8 pp. 1137 – 1177

[40] OECD (1999), ”Employment Outlook”, OECD, Paris.

[41] OECD (2003, ”The Sources of Economic Growth in OECD Countries”, OECD, Paris.

[42] Rendon, S. (2005) “Job Creation and Investment in Imperfect Capital and Labor Markets” Mimeo ITAM.

[43] Rodrik, D. (1997), ”Has International Economic Integration Gone Too Far?, Institute for International Economics, Washington, DC.

[44] Rodrik, D. (1998), ”Labor and capital Mobility” Draft paper prepared for the NBER workshop on Trade, Technology, Education, and the U.S. Labor market.

[45] Saint-Paul, G. (1996), “Dual labor markets, a macroeconomic perspective," Cambridge, MIT Press.

[46] Saint-Paul, G. (1999), “Toward a Theory of Labor Market Institutions", UPF Working Paper 433.

[47] Smolny, W. and P. Winker (1999), “Employment Adjustment and Financing Constraints,” Dept. of Economics, University of Mannheim, Discussion Paper, Nr. 573-99, 1999

[48] Stiglitz, J. and A. Weiss (1981), “Credit Rationing in Markets with Imperfect Information,” *American Economic Review*, vol. 71, pp. 393-410.

[49] Stiglitz J. (1994), ”Price rigidities and market Structure” *American Economic Review*, (Papers and Proceedings), vol.74(2), pp.350-55.

[50] Stiglitz J. (1999), ”Toward a general theory of wage and price rigidities and economic fluctuations” *American Economic Review*, (Papers and Proceedings), vol.89(2), pp.75-80.

[51] Tobin, J. (1993), ”Price Flexibility and Output Stability: An Old Keynesian View.” *Journal of Economic Perspectives*, vol.7 (1), pp. 45-65.

[52] Wasmer, E. and P.Weil (2000), ”The Macroeconomics of Labor and Credit market Imperfections”, *American Economic Review*, vol.94, pp. 944-63.

8. Appendix.

8.1. Optimal individual wage contracts. Proof to proposition 1.

If firm i decides to propose a contigent compensation scheme $\{w_l, w_h\}$ such that $w_l = \eta w$, and w is the fixed wage proposed by other firms, then workers participation constraint implies that if $w_h = \eta_h w$ then $\eta_h = \eta^{-1}$. Moreover the bad state of nature determines firm i borrowing constraint if and only if

$$\eta^2 \geq \frac{1-\alpha-\delta}{1-\alpha+\delta}$$

Therefore assume that $\lambda = 1$, expected profits of firm i $E\Pi$ can be written as

$$E\Pi\left(\eta\right) = \max\left[R\left(\eta\right);1\right]\left(1+r\right)k_i$$

where

$$R(\eta)=\frac{\left[\delta+(1-\alpha)\left[\frac{2\eta^2}{1+\eta^2}-1\right]\right]}{\left[\delta+(1-\alpha)\left[\frac{2\eta^2}{1+\eta^2}-1\right]\right]-\left[\alpha-\frac{1+r}{A(\delta)}\left[\frac{w}{(1-\alpha)A(\delta)}\frac{1+\eta^2}{2\eta}\right]^{\frac{1-\alpha}{\alpha}}\right]}$$

and the optimal compensation scheme $\{w_l, w_h\}$ is the solution to the program

$$\begin{gathered}\max_{\eta} R(\eta)\\ \text{s.t. } \eta^2 \geq \tfrac{1-\alpha-\delta}{1-\alpha+\delta}\end{gathered}$$

Deriving the first order condition for the last problem we end up with

$$\frac{1}{4\eta}\frac{\left(1+\eta^2\right)^2}{1-\alpha}\frac{\partial R(\eta)}{\partial\eta}=\frac{1+r}{A(\delta)}\left[\frac{w}{(1-\alpha)A(\delta)}\frac{1+\eta^2}{2\eta}\right]^{\frac{1-\alpha}{\alpha}}\left(1+\frac{1-\eta^4}{4\eta^2}\frac{\delta+(1-\alpha)\left(\frac{2\eta^2}{1+\eta^2}-1\right)}{\alpha}\right)-\alpha$$

Let us then note φ the right hand side variable of the last expression

$$\varphi(\eta)=\frac{1+r}{A(\delta)}\left[\frac{w}{(1-\alpha)A(\delta)}\frac{1+\eta^2}{2\eta}\right]^{\frac{1-\alpha}{\alpha}}\left(1+\frac{1-\eta^4}{4\eta^2}\frac{\delta+(1-\alpha)\left(\frac{2\eta^2}{1+\eta^2}-1\right)}{\alpha}\right)-\alpha$$

Then φ is a strictly decreasing function of η since

$$\frac{\partial\varphi(\eta)}{\partial\eta}\equiv-\frac{1-\eta^4}{4\eta^2}\left[\frac{1}{\alpha}+\frac{1+\eta^4}{\left(1-\eta^2\right)^2}\frac{2}{1-\alpha}\right]\left[\delta+(1-\alpha)\left(\frac{2\eta^2}{1+\eta^2}-1\right)\right]$$

This implies that a necessary and sufficient condition for firms to adopt a contingent compensation schemes writes as $\left.\frac{\partial R(\eta)}{\partial\eta}\right|_{\eta=1}<0$. which simplifies as

$$\frac{1+r}{A(\delta)}\left[\frac{w}{(1-\alpha)A(\delta)}\right]^{\frac{1-\alpha}{\alpha}}-\alpha<0$$

At the equilibrium of the labor market, the wage rate w is such that $w = (1-\alpha) A(\delta)(k+d)^{\alpha}$. The necessary and sufficient condition therefore writes as

$$\alpha A(\delta)(k+d)^{\alpha-1} > 1+r$$

This condition simply states that the expected marginal productivity of capital is larger than the gross interest rate. In other words the amount of debt d firms can borrow is not enough the reach the first best capital stock. This means that the optimal compensation scheme $\{w_l^*, w_h^*\}$ is such that $w_l^* < w < w_h^*$ if and only if firms are credit constraint and cannot issue contingent debt.

Then assuming that $\left.\frac{\partial R(\eta)}{\partial \eta}\right|_{\eta=1} < 0$ (firms are credit constrained), due to the fact that φ is a strictly decreasing function of η and a strictly increasing function of r, the optimal wage $\{w_l^*, w_h^*\}$ is such that $w_h^* - w_l^* = \frac{1-\eta^2}{\eta} w$ is a decreasing function of the interest rate r. In other words a larger interest rate reduces optimal wage procyclicity.

8.2. On the Sub optimality of contingent financial contracts.

In the set up we have adopted, it is straightforward to note that issuing contingent debt is a dominated strategy for firms: A default on financial liabilities implies a default on the wage bill. Therefore any firm which would issue contingent debt would have to pay an infinite premium on its wage bill since workers indifference condition writes as $w_h = w_l^{-1}$ and w_l is zero when a firm issues contingent debt. Now let us assume that firms can pay their wage bill and default on their financial liabilities with a borrowing constraint writing as

$$(1+r) d_i \leq (1-\tau)\left[A_s (k_i + d_i)^{\alpha} l_i^{1-\alpha} - w_s l_i\right]$$

for any state of the world s. Then when firms issue uncontingent debt, then their program is broadly similar to the expressions developed upwards and firms expected profits write as

$$E\Pi = \frac{\alpha - (1-\tau)\left[1-\delta-(1-\alpha)\frac{w_l}{Ew_s}\right]}{\frac{1+r}{A(\delta)}\left[\frac{Ew_s}{(1-\alpha)A(\delta)}\right]^{\frac{1-\alpha}{\alpha}} - (1-\tau)\left[1-\delta-(1-\alpha)\frac{w_l}{Ew_s}\right]}(1+r) k_i$$

when the borrowing constraint is binding in the bad state of nature. On the contrary when the firm wants to issue contingent debt, then its program writes as

$$\max_{l_i} E\Pi(l_i) = \frac{1}{2}\left[A_h (k_i + d_i)^\alpha l_i^{1-\alpha} - w_h l_i - (1+r^*) d_i\right] + \frac{\tau}{2}\left[A_l (k_i + d_i)^\alpha l_i^{1-\alpha} - w_l l_i\right]$$

where r^* is the interest rate on contingent debt. Firms labor demand is then given by

$$\left[\frac{(1-\alpha)(A_h + \tau A_l)}{w_h + \tau w_l}\right]^{\frac{1}{\alpha}} (k_i + d_i) = l_i \tag{8.1}$$

Then since lenders are risk neutral, the risk free interest rate r is such that

$$1 + r = \frac{1}{2}(1 + r^*) + \frac{-(1+r^*)c}{2}$$

where c represents marginal verification costs. Moreover, when firms want to issue contingent debt, there expected profits write as

$$E\Pi(d_i) = \left[(1-\alpha)\frac{A_h + \tau A_l}{w_h + \tau w_l}\right]^{\frac{1-\alpha}{\alpha}} \alpha \frac{A_h + \tau A_l}{2}(k_i + d_i) - \frac{1+r^*}{2} d_i$$

and their demand for capital simplifies as

$$\begin{aligned} &\max_{d_i} E\Pi(d_i) = \alpha\left[(1-\alpha)\tfrac{A_h+\tau A_l}{w_h+\tau w_l}\right]^{\frac{1-\alpha}{\alpha}} \tfrac{A_h+\tau A_l}{2}(k_i+d_i) - (1+r)(1+c)d_i \\ &\text{s.t. } A_l (k_i+d_i)^\alpha l_i^{1-\alpha} - w_l l_i < \tfrac{1+r^*}{1-\tau} d < A_h (k_i+d_i)^\alpha l_i^{1-\alpha} - w_h l_i \end{aligned} \tag{8.2}$$

Introducing the labor demand (8.1) into the constraints of the problem (8.2) we end up with

$$\begin{aligned} &\max_{d_i} E\Pi(d_i) = \alpha\left[(1-\alpha)\tfrac{A_h+\tau A_l}{w_h+\tau w_l}\right]^{\frac{1-\alpha}{\alpha}} \tfrac{A_h+\tau A_l}{2}(k_i+d_i) - (1+r)(1+c)d_i \\ &\text{s.t. } \left[A_l - (1-\alpha) w_l \tfrac{A_h+\tau A_l}{w_h+\tau w_l}\right](k_i+d_i) < 2\left[\tfrac{w_h+\tau w_l}{(1-\alpha)(A_h+\tau A_l)}\right]^{\frac{1-\alpha}{\alpha}} \tfrac{(1+r)(1+c)}{1-\tau} d_i < \left[A_h - (1-\alpha) w_h \tfrac{A_h+\tau A_l}{w_h+\tau w_l}\right](k_i+d_i) \end{aligned} \tag{8.3}$$

Assuming that

$$\alpha\left[(1-\alpha)\frac{A_h+\tau A_l}{w_h+\tau w_l}\right]^{\frac{1-\alpha}{\alpha}}\frac{A_h+\tau A_l}{2}>(1+r)(1+c)$$

the expected profit of the firm write as

$$E\Pi=\frac{\alpha\left(1+\delta+\tau\left(1-\delta\right)\right)-\left(1-\tau\right)\left[1+\delta-\left(1-\alpha\right)w_h\frac{1+\delta+\tau(1-\delta)}{w_h+\tau w_l}\right]}{2\left[\frac{w_h+\tau w_l}{(1-\alpha)(A_h+\tau A_l)}\right]^{\frac{1-\alpha}{\alpha}}\frac{(1+r)(1+c)}{A(\delta)}-\left(1-\tau\right)\left[1+\delta-\left(1-\alpha\right)w_h\frac{1+\delta+\tau(1-\delta)}{w_h+\tau w_l}\right]}\left(1+r\right)\left(1+c\right)k$$

Adopting the follwong notations,

$$b\left(\delta\right)=1+\delta+\tau\left(1-\delta\right)$$

$$B\left(\delta\right)=b\left(\delta\right)A\left(\delta\right)$$

$$c\left(\delta\right)=\frac{1+\delta}{b\left(\delta\right)}=\frac{1+\delta}{1+\delta+\tau\left(1-\delta\right)}$$

we end up with an expression of firms expected profits given by

$$E\Pi=\frac{\alpha-\left(1-\tau\right)\left[c\left(\delta\right)-\left(1-\alpha\right)\frac{w_h}{w_h+\tau w_l}\right]}{2\left[\frac{w_h+\tau w_l}{(1-\alpha)B(\delta)}\right]^{\frac{1-\alpha}{\alpha}}\frac{(1+r)(1+c)}{B(\delta)}-\left(1-\tau\right)\left[c\left(\delta\right)-\left(1-\alpha\right)\frac{w_h}{w_h+\tau w_l}\right]}$$

which is roughly similar to the expected profit expression when firms do not issue contingent debt. (see below)

$$E\Pi=\frac{\alpha-\left(1-\tau\right)\left[1-\delta-\left(1-\alpha\right)\frac{w_l}{Ew_s}\right]}{\frac{1+r}{A(\delta)}\left[\frac{w_h+w_l}{(1-\alpha)(A_h+A_l)}\right]^{\frac{1-\alpha}{\alpha}}-\left(1-\tau\right)\left[1-\delta-2\left(1-\alpha\right)\frac{w_l}{w_l+w_h}\right]}\left(1+r\right)k_i$$

The first order conditions determining the optimal labor contract and the optimal technology then write as

Comparing (8.3) with () it appears that a sufficient condition for the strategy consisting in issuing contingent debt to be Pareto dominated writes as

$$\alpha\left[(1-\alpha)\frac{A_h+\tau A_l}{w_h+\tau w_l}\right]^{\frac{1-\alpha}{\alpha}}\frac{A_h+\tau A_l}{2} \quad < \quad \alpha\left[(1-\alpha)\frac{A_h+A_l}{w_h+w_l}\right]^{\frac{1-\alpha}{\alpha}}\frac{A_h+A_l}{2}$$

$$\frac{1}{2}\frac{1-\tau}{(1+r)(1+c)}\left[\frac{(1-\alpha)(A_h+\tau A_l)}{w_h+\tau w_l}\right]^{\frac{1-\alpha}{\alpha}}\left[A_h-(1-\alpha)w_h\frac{A_h+\tau A_l}{w_h+\tau w_l}\right] \quad < \quad \frac{1-\tau}{1+r}\left[(1-\alpha)\frac{EA}{Ew_s}\right]^{\frac{1-\alpha}{\alpha}}\left[A_l-(1-\alpha)w_l\frac{A}{w}\right.$$

This sufficient condition simplifies as

$$\left[\frac{(1+\delta)+\tau(1-\delta)}{1+\tau\eta^2}\right]^{\frac{1}{\alpha}} < \left[\frac{2}{1+\eta^2}\right]^{\frac{1}{\alpha}}\frac{1+\eta^2}{1+\tau\eta^2}$$

$$\left[\frac{(1+\delta)+\tau(1-\delta)}{1+\tau\eta^2}\right]\left[\frac{1}{2(1+c)}\left[\frac{(1+\delta)}{(1+\delta)+\tau(1-\delta)}\left(1+\tau\eta^2\right)-(1-\alpha)\right]\right]^{\alpha} < \left[\frac{2}{1+\eta^2}\right]\left[(1-\delta)\frac{1+\eta^2}{2}-(1-\alpha)\eta^2\right]$$