Contingent Government Liabilities against Private Expectations in England, 1743-49

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Abstract

England financed its war of the Austrian succession (1743-48) mainly by issuing 3% bonds and 4% callable bonds which were indeed redeemed by Pelham in 1749 through an interest reduction. The implicit policy rules and constraints made the 4% bond a derivative asset of a 3% perpetual. The price data fit with a model of pricing of this derivative. The estimated model shows that the redemption terms were well anticipated, but government bond prices recovered after the war much sooner than expected. Callable bonds enabled the government to exploit that excess of pessimism in reducing its borrowing cost.

Key words: Callable bonds, Derivative Asset Pricing, War Financing, Rational Expectations, Excess-pessimism.

Classification: N23, N43, G14, G18.

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1 Introduction

The financing of wars in 18th century England is one of the most important and glorious events in the history of public finance. It has stimulated a steady flow of studies since then, but most works treat the debt as homogenous or seem to imply, incorrectly, that short-term debt accumulated during wars after which it was refinanced in in long-term debt[1]. This paper analyzes the debt financing of the first of the three wars between 1715 and 1792, the war of the Austrian succession (1743-1748).

In November 1749, Pelham presented his plan to reduce the interest rate on the debt from 4 to 3%. He is given much credit by Dickson (1967) to carry without market disruption a plan that was allegedly in danger. By contrast in France, when the abbé Terray reduced the interest rate on some government liabilities in 1770, the price of the marketed debt collapsed by half. The interest reduction that was a default in France was accepted in England because of the type of debt contract: all long-term debt in England was in callable bonds. One of the main issues addressed here will be how the market in England anticipated the government policy?

The first item in this paper is therefore the description of the market instruments that were issued during the war of the Austrian succession which came at the end of the period in which both the capital market and the credibility of the government were strengthened[2]. The financing of this war set the pattern for the next two, the seven years war (1756-63), and the war of the American independence (1777-83), which deserve a separate study. Using the characteristics of the loan issues, a correct value of the cost of borrowing *ex ante* is computed, taking into account the callable feature of the new loans[3].

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[1]The study by Dickson (1967) on the growth of public deficit financing remains the only detailed description for the period before 1750. (See also Roseveare, 1991).

[2] For the slow decrease of interest rates after 1688, see Stasavage (2003), (2006), and Sussman and Yafeh (2003)). The government borrowed only in time of war and each war was an opportunity to expand the capital market.

[3] The impact of the callable feature on the computation of the cost of borrowing has been emphasized by Harley (1977) and Klovland (1994) for the computation of the long-term rate at the end of the 19th century when the government exercised the call and lowered the interest rate to 2.75% first, and then to 2.5%, the rate in effect today. Harley does not use any model but makes assumptions about the call. Klovland evaluates critically these assumptions and rejects some of them by comparing the *ex post* mean returns of the redeemable and the non redeemable debt for

The dominant financial instruments were long-term bonds with coupons of 3% and 4%. Both were callable bonds: they were legally redeemable at par. In 18th century England, the optimal rule to call the bond immediately when it reaches par (Ingersoll, 1977) could not be applied because of the fixed costs in floating large amount of new debt, and also because of issues of moral hazard and fairness that are discussed in Section 3. The recall of the high coupon debt could not and was not expected to take place at par.

Market participants viewed the 4% debt as a contigent asset that paid a coupon of 4% as long as the price of the 3% was below a specific value which can be taken as the par level. When the 3% reached par and was stable, the 4% debt could be redeemed. In order to save on the transaction cost of refinancing with new 3% bonds purchased by the same individuals holding the 4% debt, the government announced an "interest reduction": the coupon of the 4% was reduced gradually to 3% during a relatively short transition period. That reduction was equivalent to a conversion of the 4% into a 3%, one for one, with an additional payment equal to the present value of the coupons over 3% during the transition (an amount equal to about 4 in 1749). A 4% bond was therefore a derivative financial asset of the 3% bond: all its payments depend solely on the price of the 3%: it would pay 4 pounds per face value of 100 per year until the price of the 3% reached par. Around that date, it was converted into a 3% bond with a additional payment that was not specified at the time of issue but could be anticipated by the market.

A glance at the prices of the 3% and the 4% bonds (traded almost daily) will show that two periods can be identified, before and after April 1748 when peace negotiations started at Aix-la-Chapelle (April 24)[4]. The second period provides a better material for analysis because the problem of expectations by market agents was simpler. Although some military actions (especially at sea) continued until the signing of the treaty in the following October, "the end was in sight". The price of bonds continued to vary randomly but in assessing its future, the trend of interest rates was definitely downwards and the main issue was about the convergence to the peace time level. From the political context, we can assume that the asset prices, subject to shocks, were driven by one random factor, the interest rate that converged randomly to the long-run stable value that enables the government to convert the 4% debt.

some time intervals.

[4]All dates in this paper refer to the Julian calendar, used in England until 1752 and lagging the Gregorian calendar by 11 days.

In Section 4, a one-factor model is fitted to the price observations. It is is based on a discrete time version of the Vasick model (with very short periods of one day), using the tools of modern finance[5]: the interest rate is subject to shocks and has a trend toward the long-term value, around 3%. This model generates a relation between the price of the 4% bond as a function of the price of the 3% bond. In a diagram of points with the two prices as coordinates, the function is represented by a graph that goes through the point of redemption with price 100 for the 3% and 100 plus some premium for the 4%. In addition to the premium, the price function on three other parameters, the rate of convergence to redemption, the variance of the bond price and the risk aversion of agents. These parameters are estimated from the prices of the two bonds between May 1748 and May 1749.

The market predicted accurately the redemption premiumof the interest reduction at least six months before its announcement[6], but it overestimated by a wide margin the length of time for the interest rate to fall back to 3 % and for the redemption of the 4% bonds.

The model generates a distribution of future prices that has to be consistent with the current prices. The expectations of the market were much more pessimistic than the actual turn of events. In October 1748, according to the market prices, the probability that the interest reduction would take place within a year was assessed at less than 1/2 percent and the expectation of the average time to redemption was more than 8 years. Furthermore, policy rules imposed that such a reduction could take place only after the price of the 3% bond had returned to par. But the 3% bond returned to par and the the interest rate was reduced within a year. In this sense, the fall of the price of government debt during the war was “excessive”.

The period 1746-1747 is analyzed in Section 5. For this period, the forecasting problem of the market is more complicated. The issue is not only how fast interest would decrease after the announcement of the peace negotiations in April 1748, but how to predict increases of war expenditures and the amounts of loans in the future. (No significant loan was issued after 1748). On the basis on policy news, the market could anticipate a future rise of war expenditures, borrowing and higher interest rates

[5]Market participants at the time had not taken a course in the modern theory of finance. The evidence shown here is that they did not need to. The modern researcher relies on such a model because he does not have the acumen of money men of 1747.

[6]A simple visual inspection of the price data will show that contrary to the evaluation of Dickson, the plan of Pelham was in no great danger: two months before the official announcement of the interest reduction, it was a foregone conclusion for the market; the price difference between the two assets becomes constant at the value set by the plan.

without any change in the current value of that rate (*i.e.*, a temporarily rising term structure). Such a situation cannot be captured by a one-factor model which imposes a decreasing trend of rates, and the existence of more than one factor is confirmed by the price data. Since the two asset prices are not sufficient for the building of a multi-factor model of asset prices, the information in the prices of the 3% and the 4% bonds will be used in two different ways.

First, for each observation of the pair of prices, one will establish a lower-bound of the expected time to redemption under any expectation about future rates that is consistent with the policy rules about the redemption of the 4% debt. The results will show that the market significantly overestimated the time to redemption throughout the war.

Second, the stochastic model of the previous section will be used to determine for each observation of a pair of prices a value of the short-term interest rate and of the (expected) time to redemption. These two numbers provide an indication about the term structure of future interest rate: a high rate and a short time to redemption imply a steeper curve than a low rate and a long time. The method leads to a sensible interpretation of the evolutions during the war of the short-term interest rate and the expected time until the interest reduction.

The 1749 interest rate reduction applied with uniform terms to both the 14 millions pound of the new debt issued in the war and the 43 millions of the pre-existing debt. Issues related to this old debt are discussed in Section 6. It is shown that, probably because of the failure of an interest reduction in 1737, the market expected that debt to be redeemed later or with a higher premium. This difference in expectations lasted until the very end of 1749.

The analysis of the optimal composition of a new issue is an important problem but it is beyond the scope of this paper. But from the results on expectations and the outcome of events, some assessment of the debt policy of the goverment can be provided, and this assessment is by and large very positive. The callable feature of the debt remarkably enabled the government to turn the excess pessimism of the market ot its advantage. The 4% debt was *de facto* the sum of a perpetual 3% bond and an annuity paying 1 per year until the long-run interest would return to about 3%. As the market believed this date to be distant in the future, more than 10 years, it bought the contingent annuity from the government at a price that grossly exceeded the *ex post* payments by the government. The government won a large bet against the people who paid 12 in 1747 to receive a total of merely 7. If only Terray could have dealt with such contingent annuities.

2 The financial instruments

During the period of peace that followed the wars with Louis XIV, the long-term interest rate decreased regularly from 8% in 1710 to 3% in the mid-thirties. This period of "financial revolution" (Dickson, 1967) has been the focus of a number of works (Roseveare 1991, Stasavage 2003, 2006, Sussman and Yafee 2003). When war resumed in 1743, the long-term interest rate had been around 3% for 10 years and the stage was set for the financing of the next three wars until 1783.

Debt financing in the war of the Austrian succession (1743-1749) is summarized in Figure 1. The price of the 3% annuity (whose features will be discussed below) falls during the war years with record lows around 75, and, a feature which will be central in this paper, recovers rapidly to par at the end of the war. The amounts of the loans (M stands for million pounds), rise gradually. Government borrowing occurs only during the war years and the bulk of borrowing takes place when the long-term interest rate is high.

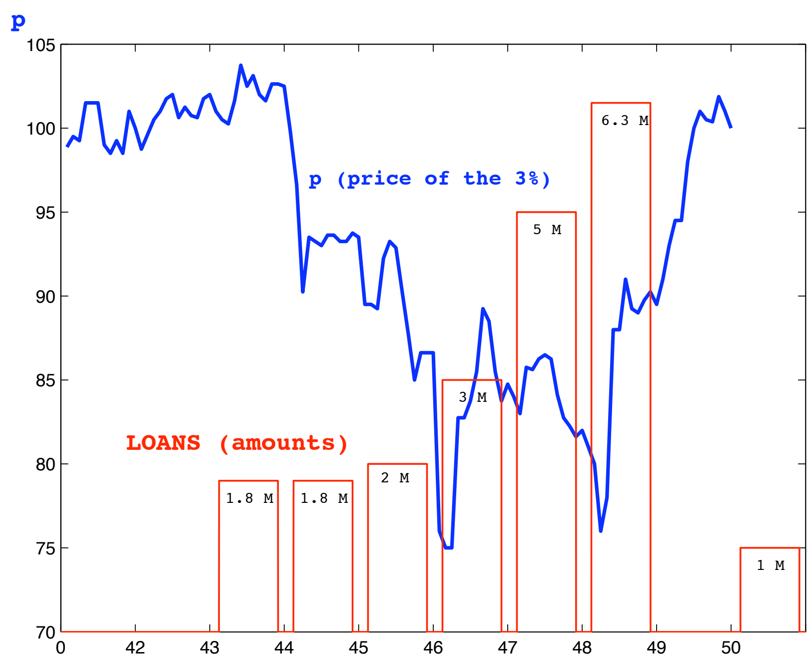

Figure 1: Prices of the 3 % annuity and amounts of loans (1741 - 1749)

Short-term or long-term financing

There is a view that war deficits were financed short-term and refinanced in long-term instruments after the war, when the interest rate was lower. This view is false for the three wars from 1740 to 1783 and is contradicted in this paper. War borrowing relied on a limited set of financial instruments which were long-term. Short-term credit played its standard role over the yearly cycle to match expenditures and ressources, but it did not accumulate to be refinanced at smaller rates after the war. The best known short-term instruments were Navy bills at rates of 5% and higher. Some accumulation of these bills took place, but their refinancing of 3 millions pounds in 1749 was charged on a loan issued under "war conditions" with an interest higher than in the previous year (Grellier, 1812, p. 75). The accounts do not show refinancing of short-term debt after the end of the war, except a small conversion of the Navy victualling bills in 1750 (1 million pounds compared to 20 millions for the war)[7].

An overview of war loans

A war loan issue was an important affair, as it would be today with the privatization of large state companies or IPOs, and it entailed significant fixed costs. Moreover, the large market that we see today in short-term Treasury bills did not exist at the time. Contemporary accounts emphasize that for the government it was essential that each single issue should be "successfull", which meant at the time significantly oversubscribed, as it would be today with investment banks and IPOs.

The process for a new issue began in the late Fall of each war year with the parliamentary session which assessed the amount and the broad terms of the loan. Preparation continued during the winter in discussions with the "moneyed men" and the fine-tuning of the terms of the loans to contemporary market conditions. The subscription was paid in monthly installments of 10 to 20 percent beginning at various times from December to May. Investors seem to have appreciated gambles and a significant fraction of the loan was often raised by lottery tickets with government bonds as prizes. The total value of the prizes was equal to the value of the tickets and could exceed it when special prizes were added in the fine-tuning to secure success of the issue.

The amounts and types of loans[8] are presented in Column 2 and 3 of Table 1. In 1743, a payment of $\mathcal{L}$ 100 would get a 3% annuity with a face value of 100 (Column 3). Likewise in 1744. In 1745, as interest rates were higher (Figure 1), for the same

[7]The accounts do not show the refinancing of a short-term debt after the seven years war. Short-term debt accumulated during the war of the American independence and was refinanced after the peace of Versailles, but it did not represent the major part of debt financing in that war.

[8]The financial instruments of all the loans are easy to trace thanks to a commission of the British Parliament in the late 19th century. Grellier (1812), (see also his 1810 work), provides a number of details on the specifics of the loans, and consolidated data on the British debt in some years. The actual liability of a new loan could be slightly higher than the face value reported in Column 2 and depended on its provisions.

Table 1: New loans 1743-1750

1	2	3	4	5	6	7	8
Date	Amount £ M	Instruments	Rate (%)	Yield (%)	*Ex post* rate	Market prices	Remarks
1743	1.8	100(3%)	3.42	3.42	3.42	3%: 100	1M by subscription, 0.8M by lottery.
1744	1.8	100(3%)	3.33	3.33	3.33	3%: 93	1.2M by subscription, 0.6M by lottery, premium of 3%.
1745	2.0	100(3%), L(1.125)	4.02	4.02	4.02	3%: 89	1.5 M. subscription, 0.5 M lottery, life ann. 4£ 10 for £ 100 in lott.
1746	3.0	100(4%), L(1.5)	5.4	4.81	4.72	3%: 75-83 4%: 91-94	2.5 M subscription, 0.5M lottery, life ann. 9£ for £ 100 in lott.
1747	4.0	110(4%)	4.4	3.73	3.54	3%: 85 4%: 96	plus a 10% premium in bonds (effective rate 4.4 %).
1747	1.0	100(4%)					Lottery
1748	6.3	110(4%)	4.4	3.71	3.51	3%: 80 4%: 90	As the price fell, payment dates were delayed.
1750	1.0	100(3%)				3%: 100	Conversion Navy bills
SUM	20.7						

Source : United Kingom, Parliamentary Papers (1898), Grellier (1810), (1812).

For definitions of the items and descriptions of the computations, see the text.

payment of $\mathcal{L}$ 100, a life-time annuity of $\mathcal{L}$ 1.125 per year (to be written on any person of choice with no distinction for age) was added to the 3% annuity[9].

Life-time annuities represented less than 5 percent of the borrowings during the war: the debt financing of the war of the Austrian succession relied on 3% and 4% annuities. They will be the main focus of this study. In 18th century England, all annuities were perpetuals redeemable at par, unless specified otherwise. This feature is essential in the financial policy of England and contrasts sharply with the loans issued by France at the same time which were mainly in life-time annuities, or with terms of 9 to 32 years (Velde and Weir, 1992).

The 3% annuity

The 3% annuity was the workhorse of debt financing in 18th century England. It was redeemable at par, like any other annuities, but market conditions and policy constraints were such that their probability of redemption was negligible. (Their rate was reduced to 2.5% only at the end of the 19th century after a long period of low rates). The 3% bond had been above par only temporarily. We will see with more details in the next section that a redemption meant in fact a conversion into a debt instrument with a lower coupon when that financial instrument was around par. No debt has ever been issued at less than 3% in 18th century England. In any case, the callable feature of the 3% bonds will be mostly irrelevant for us since we will focus on the conversion from 4% to 3%. In 1751, all the 3% debt was consolidated in *consols*. By an abuse of language, we will use here the term consol for all 3% annuities.

4% annuities

These annuities supported two thirds of the war loans between 1743 and 1748, but before the war, more than 85 percent of the public debt was at 4% (Dickson, 1967). The possibility of redemption was very much taken into account by the market and this feature is central in this paper. A redemption of the debt with a rate higher than 5% had taken place after the wars with Louis XIV, and an attempt had been made in 1737 for the 4% debt. We will examine below the particular features of the redeemability of an 18th century British government bond which are somewhat different from those of a standard callable government bond in the 20th century[10].

[9]As reported Column 8, the loan was partially issued through a lottery: for each say $\mathcal{L}$ 400, the investor would receive a 300 face value of 3% bonds and 10 lottery tickets (at $\mathcal{L}$ 10 each). These tickets would entitle the investor to a life-time annuity of $\mathcal{L}$ 4 and 10 shillings in addition to the prize of the lottery. The prizes were in 3% bonds with fair odds. A similar scheme was used in 1746.

[10] US government callable bonds were available until 1995. Municipal bonds are often callable. Callable corporate bonds are redeemable in the stock of the company.

In the rest of the paper, we will use the notation "bond" for the 4% annuity.

The ex ante cost of loans

The *ex ante* borrowing cost is the interest rate measured by the internal rate of return or yield at the time of issuance of a the loan. The *ex post* cost (the rate the government has ultimately paid) will be considered in a later section. It turned out to be lower because the 4% bonds were recalled earlier than expected, as will be shown later. Two measures are reported in Columns 4 and 5 of Table 1. In Column 4, "Rate" stands for the internal rate of return computed by Grellier (1812) who uses an accounting method. His numbers are useful because he probably had information on additional prizes and life annuities.

The accounting method overstates the *ex ante* borrowing cost because it cannot take into account the redemption features of the assets and it must assume that coupons are paid forever, contrary to maket expectations. For example, in 1747, it generates a rate of 4.4% on an initial investment of 100 which gets a face value of $\mathcal{L}$ 110 of 4% annuities.

In order to compute the long-term internal rate of return of the loan using market prices, we consider the equivalent financing though a 3% annuity. In the 1747 issue, the public should be willing to receive for $\mathcal{L}$ 100 an amount equal to $110 \times 96/85$ of 3% annuities, where 96 and 85 are the prices of the 4% and the 3% annuities (Column 7), which generates a coupon of 3.73. Assuming this coupon to be perpetual, (if not we have an upper-bound), this value is equal to the yield reported in Column 5. Some additional details are presented in the appendix.

The yields in Column 5 illustrate the high cost of life annuities: in 1747 and 1748, when the level of borrowing was at its highest (at 5 and 6 millions), and the long-term rate was as high as at any other time in the war (Table 1 and Figure 1), the yield was significantly lower than in 1745-46 when the amount of the loans was only half. Raising loans through financial instruments actively traded in the market and with a contingent redemption date was much more effective[11] than life annuities *ex ante*. This cost effectiveness will be even stronger *ex post*.

[11] As mentioned above, the french government who was financing the other side of the same wars relied overwhelmingly on life annuities which were expensive. In England also, they were more expensive than marketed bonds. The higher borrowing cost for France has been mechanically assigned to a lower credibility of the goverment but the types of financial instruments and markets may have been more important. An investigation of interactions between financial instruments, markets and institutions in 18th century France remains to be done.

3 Callable bonds as derivatives

In a setting without frictions, the optimal policy for bond calling is to call as soon as its price reaches par[12]. It it is well known however, that both governments and private firms have delayed the redemption of callable bonds. In 18th century England, neither the market nor the government expected that bonds would be redeemed at par. We first describe what was actually done in 1749/1750, and then discuss the constraints on the redemption of callable bonds that the government faced.

3.1 The interest reduction of 1749

In the Fall of 1749, the consol (the 3% annuity) had been around par for a few months. The long-term rate was therefore back to 3%, and was not expected to increase in the near future. The price of the 4% had reached 105. The king in his opening speech to the session of the Parliament, made it official that interest payments should be lowered on the entire 4% debt which greatly exceeded the debt incurred in the last war. The government of Pelham ruled out an immediate reduction of the rate to 3% and fixed the terms at the end of November[13]: the 4% bonds would receive a coupon of 4% for the year of 1750, and then 3.5% for the following 6 years during which they were not redeemable. After 7 years, there would be no distinction between these bonds and the 3% bonds. For a holder of a 4% bond, the interest reduction was equivalent to conversion into a 3% bond with a payment of about $\mathcal{L}$ 4 per bond, paid in installments. That plan was implemented with minor variations for the total debt of 57.7M (millions $\mathcal{L}$) at 4%. By May 1750, only 7M from the 57.7M were not converted. The holders of these bonds were paid off by a new loan.

According to Dickson (1967), the plan was resisted, in particular by the large institutional investors and was apparently in jeopardy during the winter. His account (p. 228-245) cannot be repeated here but its reading may be useful for the discussion of the calling rules and the contradictory evidence by the market which will be examined later.

In 1737, the government had attempted to redeem a fraction of the 4% debt and failed (Dickson, 1967, p. 213), an event which will be discussed below in connection with the market evidence. In 18th century England, the calling of bonds was subject to constraints that were different from those of a modern corporation operating in a large financial market.

[12] Brennan and Schwartz (1977), and Ingersoll (1977) have shown that callable corporate bonds should be called as soon as their value if called is equal to the value if not called.

[13] See the accounts of Grellier (1810), p. 215-221, and Dickson (1967), p. 231-241.

Why not redeem at par?

For U.S. callable Treasury bonds (available up to 1995), the call had to be announced 120 days before. Since the interest rate fluctuates randomly during this period, the optimal price at which the bond should be called is strictly above par, and depends on the volatility of the interest rate)[14]. There was no formal delay for implementation in 18th century England, but a delay and a special compensation at the time of redemption had to take place because the following constraints:

1. The redemption of the 4% debt could be done only by issuing new debt at 3%. However, issuing costs were large. Since the new bond holders would be the same people as for the old bonds, transaction costs would be saved by reducing the interest of the debt but the reduction of the interest rate was not in the debt contract. Hence the government and bond holders were competing to capture the rent of the saved costs. Not surprisingly, the three large companies that represented a dominant fraction of bond holders and had more bargaining power, the South Sea and the East India Company, and the Bank of England, opposed the plan at the beginning. These companies eventually went along but the discussions imposed a delay between announcement and implementation.

2. Bond holders could always reject the conversion at a lower rate and despite the good terms offered by Pelham, some did. Because the government had to provide an interval of time to convert their holding, it had to to provide a premium that would dominate possible price losses of the 3% bond in the short-term. The payment of $\mathcal{L}$ 4 per face value of $\mathcal{L}$ 100 provided some guarantee[15].

3. In 18th century England, the interest rate depended mainly on the government fiscal policy which itself depended on the military policy. Reputation was essential during this period of growth of the public debt. The redemption of callable bonds raised issues of asymmetric information, moral hazard and fairness. An increase of interest rates and a fall of the bond prices soon after the conversion would have raised the suspicion of "inside trading" and a government taking advantage of private information about the future evolution of interest rates. The government needed to give some compensation against a possible capital loss on the newly converted debt,

[14] Bliss and Ronn (1995), (1997), Grau, Forsyth and Vetzal (2003).

[15] The variance of the price changes of the 3% was around 0.2 when the price was around par. One can verify that the variance for a month was around 4 times that value, as expected from uncorrelated changes. A variance of 4 months would be 3.2 which means a standard deviation of 1.8. Twice that deviation means 3.6 which fits with the actual payment of 4 for a probability of a loss smaller than 2%.

at least for the near future after the time of the conversion. Contemporary discussions of the policy emphasize that the 3% annuity had been around par for a few months and were likely to stay at that level in the future. The issue of moral hazard is similar: the government had to avoid any suspicion that it would take advantage of the interest reduction and the lighter burden of the debt to start on new ventures. These arguments strengthen the case for a conversion rate above par as was presented in item 2.

4. The interest reduction raises also an issue of asymmetric information and moral hazard *ex post* with respect to the terms of issuance of the bonds. The 4% bond was called when the government had reached a stable fiscal condition, an event about which there may have been some asymmetric information with bond holders. We will see that the Pelham government had reached this favorable situation much earlier than what the market had anticipated. Did the government issue such bonds with more information than the market? A compensation more generous than the strict terms could alleviate that concern.

5. Finally, there may also have been a perception of fairness. The dividends of peace had come much sooner than expected. Should the goverment capture all these dividends by a strict application of the contract? In the environment of contingent payments without completely specified contracts, the government spent great efforts on good relations with the financial community (the monied men). The government may have wanted to share some of the "peace dividends" with the bond holders (who, it should not be forgotten, had made a fast and large capital gain on their holdings).

Redemption, swap or interest reduction?

Instead of a swap where each 4% bond is converted into a multiple of 3% bond, Pelham implemented a schedule of gradual interest reductions to the level of 3%. Both methods may have the same present value, but two differences should be mentioned.

In the interest reduction, the face value of the loan is not changed, while it increases in the swap. Hence, once the interest reduction has been phased in (seven years here), the service of the debt is smaller after the interest reduction than after the swap. The interest reduction is thus a method to repay some of the debt and saves of the fixed costs of any operation of debt reduction in the future.

Second, agents may have different perceptions in the two methods. The interest reduction may have looked more "natural" than the conversion in multiple of 3% annuities: in November 49, the long-term rate was stable at 3%. It the context of unspecified contingent agreements, perceptions of *ex post* fairness matter greatly and may rest on idiosyncratic and subtle details. Bond holders at 4% seemed to have a special advantage, with some justification, but given the conditions of the time, they "should be satisfied" to enjoy this advantage for a limited time. Under the swap that provides an amount of 3% bonds with a face value higher than 100, the premium may be viewed as an "advantage" gained by holders of 4% redeemables. Furthermore, the rate of the swap may end as a benchmark that could constrain similar redemptions in the future.

3.2 Callable bonds: derivatives on a perpetual

We have seen that the consol (the 3% annuity) can be viewed, in the middle of the 18th century, as a perpetual bond. From the previous discussion of the historical context and events, we can assume that the payoff of bond (the 4% annuity) is contingent on the price of the consol, with a payoff that is defined as follows: let p and q be the prices of the consol and of the bond. The bond pays a coupon of $\mathcal{L}$ 4 per year as long as p is smaller than 100. When p is reaches 100, each bond is converted into one consol with an additional payment of h, (equal to about 4 in 1749)[16]. In summary, *the bond is a derivative asset of the consol*. An central task of the paper is to interpret the observations of the prices p and q using properties of derivative asset pricing.

The prices of the consol and the bond for all years in which they were traded are presented in Figure 2. We can make the following remarks:

(i) For all trading days, the points of coordinates (p, q) are between the two lines $q = p$ and $q = (4/3)p$, and

$$p < q < \frac{4}{3}p.$$

The price of the bond, q, is higher than the consol price p since it pays a higher coupon and is eventually redeemed into a consol with a conversion ratio greater than one. The price q was much lower than $(4/3)p$ which is the price of a non-redeemable bond: expectations about a recall played a major role in the determination of the bond price, at all times during the existence of these bonds.

(ii) The observations after October 2, 1749 are represented with blue stars. There is no apparent discontinuity of the *level* of q in relation to p at the time of the redemption, but there is a discontinuity in the *schedule* between the two prices. The new schedule is a line $q = p + h$, with h equal to about 4: after October 1, two

[16] In the model below, the rule is extended to a probability rule in which the bond is converted into a consol with some probability per period when p is near 100.

months before the official announcement of the interest reduction plan, the market treated that plan as a *fait accompli* for the debt issued during the war: from that date on[17], the premium of the bond over the consol is constant and equal to the value that will hold through 1750. Note also how the relation between q and p seems to "veer" toward the line $q = p + 4$. This relation will be the consequence of rational asset pricing and the model of the next Section will be a tool to show that the terms of the interest reduction of the 4% annuity were accurately anticipated at least since the spring of 1749.

(iii) In any year, the difference $q - p$ is the value of a annuity paying 1 per year until the consol reaches its par value (the time of the interest reduction), with a final payment of 4. The difference evolves randomly over a decreasing trend following the events of the war and the gradual nearing of its end. However, the levels of this difference are high thoughout the war, more than 10 usually: people expected the interest rate to stay above 3% much longer than it actually did.

(iv) On Sunday April 24 1748, the peace conference began at Aix-la-Chapelle. Between the previous Friday and the following Tuesday, in the largest jump of the war (except for the days around Culloden, April 27, 1746), the consol rose from 80 to 85 with further gains immediately after (see the figure). April 1748 marks a sharp difference between two regimes of asset pricing which will be analzyed separately in the next sections.

(v) The beginning of the peace negotiations did not mean the end of uncertainty. The guns of Maurice de Saxe[18] conducting the siege of Maastricht could be heard a dozen miles away[19]. Long lists of ships captured at sea were listed each month in the Gentleman's Magazine until the signing of the peace treaty, October 18, 1748.

[17] The plan was certainly discussed publicly before November 1749. Pelham was against secrecy in the determination of the terms of the interest reduction. We will see in Section 6 that the market treated differently the debt that had been issued before the war.

[18] In a letter to the Comte de Maurepas, Minister of the Marine, the ablest and most successfull general of the time wrote: "In matters political I am only a chatterbox; and if the military situation compels me to discuss them betimes, I don't quote my own opinion as being particularly sound. What I know, and what you ought to know, is that the enemy, however numerous they may be, cannot again penetrate these territories, which I should be very sorry to give up. They are, indeed, a dainty morsel, and when our present woes are forgotten we shall regret having abandoned them. I am ignorant of finance and of our national resources, but I am aware that the rate of interest in England at the close of the last war was only four percent. It has lately reached the unprecedented pitch of twelve percent! As commercial credit is the backbone of England and Holland, I conclude that both are on their last legs. This is not the case with ourselves." (Skrine, 1906, p. 347).

[19] Today, Aachen (Aix-la-Chapelle) in Germany and Maastricht in the Netherlands share the same airport.

Nevertheless, April 1748 had simplified the issues that had to be addressed for the computation of expected future prices: the trend for interest rates was definitely downwards and the main question was how fast they would come down. This new context is reflected in the new relation that appears between the bond and the consol prices in Figure 2. We will see in the next section that this relation can be explained by a model of derivative asset pricing where one factor explains both prices and moves randomly with a trend toward the situation where the consol is back to its par level. Such a model will provide a tool for the measurement of the expectations of contemporaries.

(v) Before 1748, no simple relation appears between the prices p and q in this very complex war[20] between England, France, Holland, Austria, Prussia, Spain and Russia, in which alliances shifted. For example, to a consol price of 85 correspond at least three different levels of the bond price, each with a specific local schedule. The lowest one is observed in the spring of 1748. In the previous year, 1747, another schedule appears with higher bond prices. And in 1746, there is another pattern with even higher bond prices. Before April 1748, the asset prices do not more in a space of dimension one, thus reflecting the uncertainties of the multi-faceted war. The evolution of prices cannot be captured by a one-factor model and this period will be considered in the section after the next one.

4 After April 1748

In 18th century England before the industrial revolution, the credit market was dominated by the government and all shocks to the interest rate were caused by the borrowing policy that was a consequence of wars. There cannot be one simple model that applies to all years of the century. Investors obviously used their knowledge of political and military events to establish their expectations about the future and the prices of the assets, day by day. During the ten years of peace that preceded the war of the Austrian succession, the interest rate was hovering in a narrow band around 3%. Following the onset of the war, the interest rate followed a random path that was subject to intricate fortunes of war, as we have seen. The events of April 1748 altered the context for the formation of expectations and the main issue was the return to peace time conditions similar to those before 1740. This reduction of uncertainty to one dimension can be captured in a one-factor model.

[20] For a descriptive narration, see Browning (1993).

4.1 A derivative asset pricing model of prices and expectations

In the one-factor model, the evolution of prices depends on one factor that evolves randomly over time. One can chose whatever variable as this factor, but since asset prices depends on future interest rates, it is convenient to chose the short-term interest rate.

The short-term rate process

The short-term rate (for one period which is equal to one day) is assumed to follow a process of the Vasicek (1977) type where it varies randomly from period to period with a trend converging to a low value, r^*, which is the peace time interest rate. In order to have a flexible model to be estimated, the process is discretized and the short-term rate is assumed to evolve randomly on a grid of discrete values

$$\mathcal{G} = \{r_1, ..., r_L = r^*, ...r_N\}, \quad \text{with} \quad r_i < r_j \quad \text{for } i < j,$$

and where the grid point r_L is the "long-term stable" value, r^*. The interest rate moves between periods according to a Markov process with some probability to stay constant, to increase, or to decrease by one step on the grid, and a trend towards the long-run value $r_L = r^*$. The probabilities to move from r_i to r_{i+k}, $k \in \{-1, 0, 1\}$ are defined by $m_{i,i+k}$ with

$$\begin{cases} m_{i,i-1} = \alpha + \beta, \quad m_{i,i+1} = \beta, \text{ if } i > L, \\ m_{i,i-1} = \beta, \quad m_{i,i+1} = \alpha + \beta, \text{ if } i < L \\ m_{L,L-1} = \beta, \quad m_{L,L+1} = \beta. \end{cases} \tag{1}$$

The values are adjusted at the boundaries $i = 1$ and $i = N$, and for all i, $m_{i,i} = 1 - m_{i,i-1} - m_{i,i+1}$. Additional details of the model are presented in the appendix.

The consol price

Since the prices of government assets are the discounted values of the coupons and the interest rate follows a Markov process, in each period these prices depend only on the current value of the short-term rate. We have therefore a grid, or vector, of N prices $\{p_i\}_{1\leq i\leq N}$ for the consol (the 3% annuity), and a vector $\{q_i\}_{1\leq i\leq N}$ for the bond (the 4% annuity). In each period where no coupon is paid, assuming risk-neutrality in a first step, the price of the consol satisfies the discounting relation

$$p_i = \frac{1}{1 + r_i}\Big(m_{i,i-1}p_{i-1} + m_{i,i}p_i + m_{i,i+1}p_{i+1}\Big). \tag{2}$$

In the appendix, this equation is adapted to the case when the rate is at a boundary with $i = 1$, $i = N$, and also when a coupon is paid. From this equation, one can compute the price vector $p = \{p_i\}_{1 \le i \le N}$. The grid $\mathcal{G}$ is adjusted such that when the short-term rate is equal to $r_L = r^*$, the value of the consol is at par: $p_L = 100$. Because of uncertainty, r^* is a little smaller than 3%.

Risk-aversion

If agents are risk averse, the previous discounting equation (2) has to be modified. In this model where all the uncertainty arises in one factor, it is assumed that portfolio holders have a marginal utility of wealth that depends only on the state, and that is increases with the current value of the interest rate r_i. (For example, consumption could be smaller at the time of large government borrowings and higher interest rates). This marginal utility λ_i will be assumed to be normalized to 1 for $i \le L$, that is for $r < r^*$, and to be an increasing function of r_i for $i > L$ such that

$$\begin{cases} \lambda_i = 1 & \text{if } i \le L, \\ \lambda_i = 1 + \gamma\Big(\dfrac{i-L}{N-L}\Big)^2 & \text{if } i > L, \end{cases} \tag{3}$$

where the parameter γ will be ajusted to fit the observations.

Equation (2) is replaced now by

$$p_i = \frac{1}{1+r_i}\Big(m_{i,i-1}\frac{\lambda_{i-1}}{\lambda_i}p_{i-1} + m_{i,i+1}\frac{\lambda_{i+1}}{\lambda_i}p_{i+1} + m_{i,i}p_i\Big), \tag{4}$$

which has the same form as the equation (2) provided that M replaced by $\tilde{M}$ with

$$\tilde{m}_{i,i-1} = m_{i,i-1}\frac{\lambda_{i-1}}{\lambda_i}, \qquad \tilde{m}_{i,i+1} = m_{i,i+1}\frac{\lambda_{i+1}}{\lambda_i}, \qquad \tilde{m}_{i,i} = m_{i,i}. \tag{5}$$

The matrix M is the matrix of true probabilities and the matrix $\tilde{M}$ is the matrix of risk-neutral probabilities. When agents are risk-neutral, $\gamma = 0$ and the two matrices are identical.

The bond price

The historical experience in 18th century England is that the redemption of the bond depended on the price of the consol. We can therefore attach to each value of the consol, p_i, a probability of redemption of the bond π_i. The value of this probability will be non-zero only when the consol price is near par or above par, *i.e.*, unless i is near L or above L. Following the events in the Fall of 1749, it will be assumed here that when the redemption is decided and announced, it it implemented at the day of the next coupon payment. That day, the bond is converted to a consol and a lump-sum payment h is made. The lump-sum payment is the present value of the interest that accrued above 3% during the transition period. Following the policy of Pelham and the observations of the prices in 1750, the value of h should be around 4. Here, h will be an important parameter of the model. Its estimation, using prices only until May 1749, will provide a test of the expectations of the market.

The price of the bond satisfies an arbitrage equation that is similar to that of the consol. Consider for example the price $q_i(t)$ when there are t days until the next coupon and the price of the consol is p_i. Assume for simplicity risk-neutrality and let $v_i(t)$ the price of the a bond which is announced to be redeemed in t days while the current consol price is p_i. If in the next period (with $t-1$ days to the coupon), the state is j, the value of the bond is $(1-\pi_j)q_j(t-1)+\pi_j v_j(t-1)$ where π_j is the probability of redemption in state j. A straight arbitrage argument shows in this period with state i, the price of the bond satisfies the equation

$$\begin{aligned} q_i(t) \quad = \quad \frac{1}{1+r_i}\Bigg(\quad & m_{i,i-1}\Big((1-\pi_{i-1})q_{i-1}(t-1)+\pi_{i-1}v_{i-1}(t-1)\Big) \\ & + m_{i,i}\Big((1-\pi_i)q_i(t-1)+\pi_i v_i(t-1)\Big) \\ & + m_{i,i+1}\Big((1-\pi_{i+1})q_{i+1}(t-1)+\pi_{i+1}v_{i+1}(t-1)\Big)\Bigg). \end{aligned} \tag{6}$$

A similar equation is satisfied by $v_i(t)$. Both arbitrage equations are applied by recurrence for half a year which is the time interval between coupon payments. (The equation on the day of the coupon payment is obviously different). The price immediately after a coupon payment is the discounted price after the next coupon payment. A fixed point argument determines that price. The details of the computation are presented in the Appendix.

The relation between the prices of the bond and the consol

In the one-factor model, the prices p and q in any period depend on the state i (each expressed by a vector). Equivalently, one can express the values of q as a function of p. The model thus generates the price of the bond as a derivative of the consol:

$$q_t = \phi(p_t;\zeta), \tag{7}$$

where ζ is a vector of parameters that characterize this function. The graph of such a function, as estimated below, is represented in Figure 3. To summarize, as long as the bond has not been redeemed, a point of coordinates (p,q) moves randomly along the curve generated by equation (7), subject to idiosyncratic variations of the prices.

The parameters of the model

The model depends on vector of 4 parameters $\zeta = (\alpha, h, \beta, \gamma)$ which are ranked by decreasing order of importance. Before the estimation, it is useful to review their impact on the shape of the derivative price schedule $\phi(p; \zeta)$.

1. The rate of convergence of the consol's price to the par is determined by α, perhaps the most important parameter of the model, (equation (1)). A higher α generates a shorter time, on average, to the redemption, thus reducing the value of the contingent annuity, $q - p$, and lowering the schedule of $q = \phi(p; \zeta)$.

2. The premium h paid at the time of the redemption defines the market's expectations about the terms of redemption. A higher premium raises the value of the annuity $q - p$, and has an impact that is similar to a lower rate of convergence. The estimates of α and h will be therefore be positively correlated and this positive correlation is specific to the investors' problem: a higher premium h is roughly equivalent to a delay of the redemption, *i.e.*, a lower value of α.

3. The variance of the consol prices that is generated by the model, over a week and around some arbitrary value $\hat{p}$, is approximated by the equation

$$Var = 7(\alpha - \alpha^2 + 2\beta)\Delta^2 p, \tag{8}$$

   where $\Delta^2 p$ is an average of the terms $(p_{k+1} - p_k)^2$ around $\hat{p}$, and which are generated by the model. We replace $\Delta^2 p$ by its historical average in the interval of prices $(88, 94)$. Given α, equation (8) implies an equivalence between the parameters β and Var. The estimation will be made directly with respect to the paramer Var. Its comparison with the actual variance of the consol prices in the range $(88, 94)$ will provide an independent check of the model.

   An increase of the parameter Var lowers the schedule $q = \phi(p; \zeta)$ because of the concavity of that schedule. We should therefore anticipate a negative correlation between the estimates of Var and the parameter of convergence α.

4. The parameter γ of risk-aversion lowers the value of the short-term interest rate for any given value of the consol: Consider the arbitrage equation (4) between the consol prices in consecutive periods. Risk-aversion lowers the next period value of the consol price if it higher (because of the declining marginal utility of wealth), and increases it if the price falls. In the regime where the bond had not been converted yet, the trend of the asset prices is up. Hence the first effect is stronger than the second and the term in parenthesis in the right-hand-side of (4) is smaller. To keep the equation balance, the interest rate r_i has to decrease.

4.2 Estimation

The sample has $T = 50$ observations of the weekly averages of the consol and the bond prices from the first week of May 1748, immediately after the beginning of peace negotiations in Aix-la-Chapelle, until the last week of May 1749. In July 1749, there is no price quote and after that month the data is probably corrupted by the news about the possibility of an interest reduction (which was announced only in November). The prices are the same as those used in Figure 2, and are adjusted ex-coupon.

A vector of parameters ζ generates a function of the bond price with respect to the consol price, $\phi(p;\zeta)$, that is expressed by the discrete values (p_i, q_i), $i = 1, \ldots, N$ on the grid points in $\mathcal{G}$. These prices are the theoretical asset prices the day after the coupon payment. The maximum likelihood parameter ζ^* minimizes the sum of the squared residuals

$$S(\zeta) = \sum_{t=1}^{T} (q_t - \phi(p_t;\zeta)^2,$$

where the value of $\phi(p_t;\zeta)$ is calculated by linear interpolation with the two grid points closest to p_t. The simultaneous estimation of the four parameters will be remarkable but is subject to strong correlations, as discussed previously, which affect its precision. We have discussed why these correlations must appear in the forecasting problem of contemporary agents since a higher premium h, for example, is a substitute for a delayed interest rate reduction. In order to examine the precision of the model, we also present estimations of reduced sets of parameters were the omitted parameters are fixed at the values obtained in the first estimation.

Estimation of the four parameters

The maximum likelihood estimates of the four parameters, α, h, Var and γ are presented in Table 2. The estimate of h is on target, at 3.85. The estimate of Var is also remarkably close to the actual variance of the observed consol prices in the range $(88 - 94)$ which is equal to 0.41. The model shows some risk-aversion: under the estimated value $\gamma = 0.49$, the marginal utility of consumption when $p = 75$ (its record low), is 10 percent higher than when $p = 100$. The graph of the function $q = \phi(p;\zeta^*)$ with the estimated parameters ζ^* is represented in Figure 3.

The coefficients of correlation of the estimates are above the diagonal in the second part of Table 2. They have the expected signs that were discussed previously. Because of the high correlations, the estimates of the standard deviations of the parameters, on the diagonal, are also high. In order to improve the confidence intervals of the parameters, we now reduce the number of parameters to be estimated.

Estimation with less than four parameters

Since the coefficient of risk-aversion γ is the less critical parameter and its estimated value is so reasonable, we omit it first and set its value at 0.49, the previously estimated value. Subject to this constraint, the new estimates of the remaining parameters are obviously unchanged. The coefficients of correlation are reported in Table 3 and are higher than in Table 2. The standard deviations of the remaining parameters decrease by small amounts.

We next eliminate the parameter Var for two reasons: first the most important parameters for the computation of expectations are the rate of convergence to the redemption α, and the payment at the time of redemption h. Second, we have independent evidence on the variance of the consol prices which matches the estimate of the model. The estimation of the model taking a parameter with fixed values for γ and Var is reported in the second panel of Table 3. The standard errors of α and h are now much smaller, but the correlation between the two parameters is still high at 0.95. We will return to this issue below, after the conversion of the convergence parameter α to the expected time to conversion that is more intuitive.

When the premium h is fixed at its estimated value 3.85, which is very close to its historical value, the last remaining parameter is estimated with high precision, a standard deviation of $\sigma = 0.0004$ for an estimate of 0.0122. Since there is no intuition attached to such a value, the parameter α will be transformed below to the expected time to redemption.

4.3 Inferences from the model

From the estimated model we can measure for different consol prices, the expectations of contemporaries about the terms of the interest reduction, its timing and the implicit short-term interest rate.

The expected terms of the interest reduction

We have seen above that although the plan of interest rate reduction was announced officially at the end of November 1749, the terms of the conversion were integrated in the price of the bond as certain at the beginning of October: from that date on, the difference $q - p$ is constant and equal to about 4, slightly more than the present value of the cumulative coupons in the plan which is 3.7 at the discount rate of 3%. The small difference may be due to non-redemption clause for 7 years. The valuation of the plan should incorporate that clause, hence the correct value for the historical redemption price is about 104.

The value of 104 exceeds the estimated value of 103.85 by a very small amount indeed (Table 3). Note that the estimation uses no observation after May 1749, six months before the announcement of the plan. When we fix the parameters of risk-aversion (which has little impact on the other parameters), the variance parameter Var to its historical value in the price range $88-94$, the standard error for h is equal to 0.24 (Table 3). The 1% confidence interval is therefore between 3.3 and 4.4. This confidence interval can be reduced when we take into account the correlation with the convergence parameter α, as we will see now. To summarize, six months before the announcement of the interest reduction, the market anticipated its term accurately with a precision within one half-yearly coupon of the 4% bond. Symmetrically, Pelham provided what the market expected.

The expected time to redemption and the excess pessimism

Expectations about the redemption can be measured by two indicators, (i) the expected length of time until the redemption of the bond, or (ii) the probability that redemption will take place within some time horizon, say, a year.

(i) In the one-factor model, the expected time until redemption depends only on the current value of the state and can be written as a vector of values $\{\theta_i\}_{1\leq i\leq N}$, where θ_i is the expected time when the price of the consol is p_i. Recall that the redemption policy that was followed in England is represented by a probability π_i of redemption that depends on the current value of the consol π_i. The components θ_i satisfy the equation

$$\theta_i = 1 + m_{i-1,i}(1-\pi_{i-1})\theta_{i-1} + m_{i,i}(1-\pi_i)\theta_i + m_{i,i+1}(1-\pi_{i+1})\theta_{i+1}. \tag{9}$$

This set of equations is the same as in (6) when the rate r_i and the variable v_i are set to 0, and a coupon of 1 per period is introduced in the right-hand side. One solves by the same method as for the bond price q (see the appendix), to find the values $\{\theta_i\}_{1\leq i\leq N}$. Each value θ_i depends only on the state i and can therefore be expressed as a function of the consol price p_i in the same state by a function $\Theta(p)$, with a graph that is presented in Figure 4.

Consider the spring of 1748 when the consol was around 90. People expected that the redemption, with the additional premium of 4, would take place within a time span not shorter on average than 6 years[21]. But the redemption was implemented in only 18 months. Since according to the policy rules, a *necessary* condition for the redemption is the recovery of the consol at par, the market underestimated

[21] If one uses the estimate of α at the upper-bound of its interval of confidence, the expected time is decreased by less than year for p around 90

significantly the speed of recovery of the consol. The low price of the consol p, *and* the high price of the contingent annuity $q - p$, reinforce each other as indicators that the market showed much excess pessimism.

If we take a fixed value of the consol price, we can compute for each value of α an expected time to redemption. The estimation results are therefore easier to interpret if the expected time to redemption (at a given consol price), is substituted for α. Because of the strong correlation between α and h, we consider the joint confidence region for the estimation of the premium and the expected time to redemption (which substitutes for α).

The confidence region at the one percent level[22] of the premium h and the expected time to redemption is represented in Figure 5. The expected time to redemption is computed for a consol price of 89 which was observed in the Fall of 1748, one year before the actual redemption. The range of values for h in the confidence region at $[3.6, 4.2]$ is about the same as the one given by the estimated variance of h. The range of value of the expected time is between 7 and 11 years and if we fix the premium h to its historical value, this range is reduced to about $[8.6, 9.8]$.

(ii) The previous computation of the expected time to redemption generates the mean of the distribution of the times to redemption. The model of derivative asset pricing provides a tool to evaluate the tail of this distribution. Let us ask the following: if the price of the consol is 88.95, as it was in November 1748, ($p_{34} = 88.96$ for the estimated model), what is the probability that the interest reduction will be announced, with the additional payment of 4, within time T?

That probability was computed for a value $\alpha = 0.013$ (an upper-bound at the 1% confidence level[23]), by a large number of simulations of the model. It is a function of the time to redemption, $P(T)$, with a graph presented in Figure 6. For an interval of time of one year, the actual historical value, the probability of redemption is less than 0.2 percent. That probability obviously rises with T. For $T = 2$, the probability is about 6 percent.

[22]The region is computed numerically by computing the Log likelihood function for different points.

[23]This confidence should be computed with fixed $h = 4$. When h is allowed to vary in the estimation, the value of α may be higher (see Figure 5), but a higher h is equivalent to a postponement of the interest reduction. (An increase of h by 1 is equivalent ot a postponement of one year).

The short-term interest rate

The one-factor model generates a relation between the short-term interest rate and the price of the consol which is presented in Figure 4. According to the model, the rate of interest in the Spring of 1748 was between 5 and 6%. This value fits well with other estimates[24]. These rates are somewhat lower than the 12% reported by the maréchal de Saxe (footnote 22), but he may have referred to the earlier winter months and another type of financial instrument.

The impact of a possible default

A positive probability of partial default obviously lowers the price of government liabilities. The possibility of default lowers the price of the consol when it is low, for example at 75, not because people expect a partial default at that price but because they are aware of the random walk of prices and that a lower price today entails a higher probability of reaching an even lower price in the near future with some possible default. A positive probability of default can be introduced in the model for the high values of the interest rate r_i. The impact on the properties of the model is intuitive: it lowers the short-term interest rate for a given level of the consol price, and it increases the variance of the consol (since it increases the price differences between grid points). Numerical experimentations shows that the probability of default on the consol did not add much to the results and this possibility will be ignored here.

A more interesting issue perhaps is a possible default on the redeemable debt. The higher coupon rate may be singled out for a selective reduction of the payments. When a government implements a partial default, the higher interest rate is reduced first, as in 1770 France. The selective default would be a default on the 1 percent premium paid by the bond over the consol. Hence, such a probability would decrease the observed price difference $q - p$. The most robust result of the model is that the market overpriced the bond over the consol in view of the actual redemption date. A probablity of default on the bond would strengthen that result.

[24]Weiller and Mirowski (1990) reports rates around 4% (Figure 1), but they use the India bonds that were very liquid. (See the description of these bonds on their page 5).

5 Before April 1748

Before the congress at Aix-la-Chapelle, uncertainties about future interest rates had a dimension higher than one. For a given consol price, there could be different prices of the bond, depending on the expectations about the time profile of future interest rates. For example, in a war lull with little government borrowing, interest rates could decrease in the short-term but increase in the medium-term. The combination of both effects may have little impact on the consol price but increase the bond price because of a longer expected time to the redemption of the callable bond. In Figure (Figure 2), we can observe multiple bond prices for a given consol price. Because we are restricted to a few prices, we have to proceed by steps. In the first step, we address the issue of the excess expectation of the time to redemption that was one of the striking results of the previous section. This result can be confirmed here with minimal assumptions.

5.1 A lower-bound on the expected time to redemption

Let T be the time of the redemption of the callable bond, as viewed from an arbitrary period taken as 0. It is a random variable that depends on the future events and interest rates (themselves random variables). Let A be the value of the contingent annuity paying $\mathcal{L}$ 1 per year before the 4% bond is redeemed in the random period T, and let δ the price of a zero-coupon bond paying $\mathcal{L}$ 1 in that period T. The value of δ is also the discount factor to the random period T.

The consol is the sum of an amount $a = 3$ of the annuity paying 1 until period T and a (perpetual) consol delivered at T. Likewise, the bond is the sum of an amount $b = 4$ of the same annuity and a consol delivered in period T with an additional lump-sum payment h. The prices p and q of the two assets satisfy therefore the equations

$$p = aA + \delta p^*, \qquad q = bA + \delta(p^* + h), \tag{10}$$

which are solved into

$$A = \frac{qp^* - p(p^* + h)}{bp^* - a(p^* + h)}, \qquad \delta = \frac{bp - aq}{bp^* - a(p^* + h)}. \tag{11}$$

The information conveyed by the prices of the two assets, p and q, is equivalent the values of A δ, which are easier variables to interpret. For the lower bound on the time to redemption, we will use only the information in the annuity valuation, A.

Assume first an arbitrary deterministic path of interest rates and first that agents have a constant marginal utility of consumption. The value of the annuity is equal to

$$A = \frac{1}{1+r_1} + \dots \frac{1}{(1+r_1)\dots(1+r_T)}. \qquad (12)$$

If agents' marginal utility of consumption is increasing with the rate[25], the previous equality is replaced by an inequality, using the same argument as in the previous section. Following the previous discussion of the redemption policy, we can assume that before the redemption, the interest rate is greater than r^*, the rate at which the redemption takes place (near 3%). Therefore, for any interest rate path and value of T, we have the inequality

$$A \leq \frac{1}{1+r^*}\Big(\frac{1-\frac{1}{(1+r^*)^T}}{1-\frac{1}{1+r^*}}\Big) = \frac{1}{r^*}\Big(1-\frac{1}{(1+r^*)^T}\Big),$$

which is equivalent to

$$\frac{1}{(1+r^*)^T} \leq 1 - r^*A.$$

Hence, for any expectations about the future path of interest rates,

$$E\Big[e^{-TLog(1+r^*)}\Big] \leq 1 - r^*A.$$

Using the convexity of the exponential e^{-x} and Jensen's inequality,

$$E[T] \geq \bar{T}(A) = -\frac{Log(1-r^*A)}{Log(1+r^*)}. \qquad (13)$$

For each value set of prices (p, q), which determines A in (11), the previous equation defines a lower-bound on the expected time to redemption. Inversely, a given value of the lower-bound T determines a lower-bound for the contingent annuity

$$A = \frac{1}{r}\Big(1 - \frac{1}{(1+r)^T}\Big),$$

which using (11) is equivalent to

$$q = p\Big(1+\frac{h}{100}\Big) + \frac{1}{r^*}\Big(1-\frac{1}{(1+r^*)^{\bar{T}}}\Big)\Big(1-\frac{3h}{100}\Big). \qquad (14)$$

In Figure 7, loci of constant lower-bounds $\bar{T}$ are represented for the value $h = 4$ that was used in 1749. One can observe the overestimation of the expected time to the redemption of the 4% bonds. In 1747, the lower-bound $\bar{T}$ is 10 years. In 1746, it is 15 years.

[25] One may argue that in this new setting where uncertainty has more than one dimension, agents may use the annuity A as a hedge and apply a discount rate smaller than r^*, for example in a CAPM model with a negative correlation between this annuity and the market. This line will not be pursued here, but one should note that for long intervals of time, the contingent annuity and the 4% bonds were positively correlated. In Figure 2, the consol and the annuity move in the same direction between May and August 1746, and between September 1747 and March 1748.

5.2 Term structures under point expectations

The prices of the consol and the bond provide only partial information on term structure of future interest rates and therefore expectations about future events. We assume first that agents have point expectations about a deterministic path of future interest rates.

As an illustration, compare the prices of the two assets in August 1746 and in January 1749 in Figure 2. At both dates, expectations about the path of future interest rates generated a consol price that was around 89. But these two expectations generated a higher price for the annuity in 1746 (at 13), than in 1749 (at 8). In the second case, the interest rate was higher in the short-run and lower in the long-run: the two effects of the higher rate in the short-run and the lower rate in the long-run neutralized each other for the price of the consol, but for the annuity only the higher rate in the short-run had a impact which was negative. This mechanism is formalized in the next two results[26].

Lemma 1

Under point expectations, a forward-shifting perturbation of the interest rate path that reduces the rate at some date τ and increases it at a date $t > \tau$ (with $t < T$ the redemption date of the callable bond), and keeps the price of the consol unchanged, has a positive effect on the price of the callable bond.

The Lemma can be generalized to any distribution about future interest rates. It provides only a sufficient condition on the term structure for a higher premium $q - p$. No necessary condition can be found without narrowing the types of the possible structures. Assume that the interest rate converges linearly to its stable value r^* and, for simplicity, that time is continuous: the point expectation of the future interest rate r_t is determined by the equation

$$r_t = r(1 - \frac{t}{T}) + r^* \frac{t}{T}, \tag{15}$$

where r_0 is the interest rate in the present (time 0), T is the time of redemption of the

[26] Let p_t and q_t the paths of the consol and the bond prices generated by the interest rate path $\{r_t\}$, and $\pi_t = q_t - p_t$. For any t, $\partial p/\partial r_t = -\beta_{t-1} p_t/(1 + r_t)$, and a similar expression holds for π. Given these expressions, the inequality in the Lemma is equivalent to $q_\tau/p_\tau > q_t/p_t$, or,

$$\frac{\dfrac{b-a}{1+r_{\tau+1}} + \dots \dfrac{b-a}{(1+r_{\tau+1})\dots(1+r_t)} + \dfrac{1}{(1+r_{t+1})\dots(1+r_{\tau+t})}(q_t - p_t)}{\dfrac{a}{1+r_{\tau+1}} + \dots \dfrac{a}{(1+r_{\tau+1})\dots(1+r_t)} + \dfrac{1}{(1+r_{t+1})\dots(1+r_{\tau+t})} p_t} > \frac{q_t - p_t}{p_t},$$

which is equivalent to $(b - a)p_t > a(q_t - p_t)$, or $q_t < (b/a)p_t$, which holds since the right-hand side is equal to the value of the bond when the bond is never redeemed.

bonds and $r^* = 0.03$ is the rate at the time of the redemption. The annuity A and the discount factor δ given in (12) can be expressed in continous time as functions of r and T:

$$A(r,T) = \int_0^T e^{-\int_0^t r_u du} dt, \qquad \delta(r,T) = e^{-\int_0^T r_u du}. \tag{16}$$

Recall that the prices p and q are linear functions of A and δ in (10). From these prices, under the assumption in (15), we can recover the short-term interest rate r and the time to redemption T. The values of r is inversely related to the annuity and the value of T is positively related to the annuity as stated in the next result which is straightforward to prove and which is reciproqual of Lemma 1.

Lemma 2

Under point expectations about future interest rates given in (15), for a given value of p, a higher value of the callable bond q, entails a lower value of the short-term interest rate r and the higher value of the time to redemption T.

The recovering of the short-term rate r and the time to redemption T through the deterministic assumption (15) is now generalized with the stochastic model of the previous section.

5.3 Point estimates of the stochastic model

The computation of the lower-bound of the expected time to redemption in Section 5.1 makes the implicit assumption that all the interest rates above r^* are "end-loaded", just before the time to redemption. (This is a consequence of Lemma 1). Such a term structure is contrary to any actual observation: the expected rate as a function of time may be increasing for a while but it should eventually be decreasing smoothly to the long-term value r^*.

The evolution of the asset prices in the war obviously depended on strong exogenous information that was available to contemporaries, (landing of the Pretender, movements of armies, outcomes of battles, reversal of alliances). In this context, the fitting of a multi-factor model of the term-structure of interest rates with two prices from the 21th century point of view is a futile exercise. We will instead use the stochastic model of the previous section to gain some insight on the evolution of interest rates. Each value of the convergence parameter[27] α generates a schedule $q = \phi(p; \alpha)$, and for each observation (p, q) there is a unique α such that $q = \phi(p; \alpha)$.

[27]The parameters γ and h are set at 0.5 and 4 as in the previous model and the parameter β is adjusted as a function of α to match the variance of the consol prices in the year 1747.

Given this value of α, one can compute for any point (p, q) the short-term interest rate, r, and the expected time to redemption, T (as in Figure 4). The method assumes implicitly expectations with a trend of decreasing future rates. Expectations about a temporarily rising short-rate cannot be identified, but the combination of the information on r and T will be useful, nevertheless. For example, an increase of T with a simultaneous decrease of r is indicative of a higher rate in the near future.

The interpretation of the price observations will be facilitated by the construction of a grid of loci in the space (p, q) with constant rate r and constant expected time T as represented in Figure 8. Comparing the price observations with that grid, some remarks can be made about the evolution of interest rates and expectations during the war.

An account of prices and expectations during the war

Before the war, the consol had been steady around par. The first loan was raised in 1743 with the consol above par. The consol dropped[28] to 90 at the beginning of 1744 when France invaded the Austrian Netherlands[29]. The yield of the new loan was also 10 percent higher than in the previous year (Table 1).

1745 to Winter 1746

In January-March 1745, the consol decreased from 93 to 90 but went back to the level of 93 in May and June. The defeat at Fontenoy (May 11, 1745), a major battle and the first defeat on the continent since the Hundred Years war, did not have an effect on the consol. It was a defeat but not a rout, and it was compensated by good news from America soon after. The high borrowing cost (Table 1) was probably caused more by the type of financing through annuities than by the level of the long-term interest rate, as discussed in Section 2.

The really bad news came during the summer and continued until the winterwhen the Pretender roamed in Scotland while most of the english army was on the continent. The evolution of the price of the consol matches the fortunes and misfortunes of that adventure. The 1746 loan had the highest cost of the war at 4.8%, although the bond prices increased rapidly in May and June (Figure 1). As for the previous

[28]Prices of the 3% annuities from December 1743 to May 1744 were 102.5, 99.25, 96.625, 90.25, 93.5, 93.25.

[29]Some connection between the evolution of bond prices and events is made in this paper but a systematic evaluation would require a separate study. (For examples, see Thompson 1985, Guinnane *et al.* 1994, Ferguson, 2003)

issue, the cost of borrowing may have been increased by the life-annuities[30].

From March 46 on, the new 4% callable was traded and we can use the asset pricing model in Figure 8. The short-term rate r will represent roughly the short-term condition of the credit market while a smaller value of T will represent an improvement in the long-term prospect.

1746

In the Spring of 1746 (April-June), the consol increased while the bond fell slightly: the long rate decreased but the short rate increased, indicating a tightening of the short-term credit market but better prospects for the longer term with a shorter expected time to redemption. Between June to August, the consol increased rapidly from 85 to 90. The mild credit crunch subsided and the short rate moved with the long rate and decreased to around 4%, but the time T increased an indication of higher rates in the future. The situation was the inverse of that in the winter of 1748 that we will see below. From August to November, the long-term outlook continued to deteriorate (with a higher T in Figure 8), but the short-term rate did not increase yet. That rate did increase from 4 to more than 5% between November and February while the prospects about the long-term improved (with the smaller value of T).

1747

For the entire year, the long-term prospect did not change much, but the short-term interest rate increased steadily from 5 to about 6% through the year.

1748

During the winter, the interest rate reached its highest level of the war at more than 7%. (Compare also with an extrapolation of the graph in Figure 8). The increase was very significant with respect to the previous years and the maréchal de Saxe was probably right in his identification of a credit crisis with a high short-term rate and a low consol price. However, investors also knew that the crisis was temporary. In that winter and spring, they expected rates to fall soon in the future and durably: the price difference $q - p$ between the bond and the consol was also at a record low, and from now on it would decrease stochastically until the interest reduction.

[30]The issue of life annuities may also have been a device to broaden the ownership of government debt and the associated political support.

6 Other assets

In 1749, the total debt amounted to $\mathcal{L}$ 70 M with $\mathcal{L}$ 12 M in 3% annuities and the rest in 4% annuities[31]. All these annuities were subject to the same interest reduction, but in that total, about $\mathcal{L}$ 44 M had been inherited from the 20s. The government had attempted an interest reduction in 1737, and failed. Using market prices, we examine market expectations in the 30s and 40s about the redemption of the "old" 4% debt.

Expectations in the late 30' and the failed interest reduction of 1737

The South Sea annuities with an amount of $\mathcal{L}$ 23.6 M, were the main assets in the old debt. They were callable like the bond we have analyzed so far and they were traded. Monthly prices are presented in Figure 9 for the years 1735-1740. We can see in the figure that the 4% debt is well above par and and that the 3% annuity is around par, sometimes much above it. The low level of the interest rate created a situation that was apparently much more favorable for an interest reduction than in November 1749 when the 3% annuity had reached par only in the previous summer. Nevertheless in the Spring of 1737, the attempt at an interest reduction failed. The government seems to have played badly: it attempted the reduction only on some assets in the debt, which stimulates the lobbying by the affected bond holders; no plan for a gradual interest reduction was presented as a compensation for the price well above par. From the prices of the annuities, the contrast with with the Fall of 1749 is striking. A redemption at par would have taken the market by surprise, to say the least, with a price of 107 for the 4% annuity when the 3% was at par.

The years immediately before the war: 1740-1743

From 1740 to 1743, the relation between the prices of the 4% South Sea annuity and the 3% annuity changes remarkably as can be seen in Figure 10: the prices for that period are represented by dots while the 1735-1740 relation is summarized by the regression line of the previous Figure 9. The striking feature is the extraordinary rise of the 4% price with respect to the 3%. The most plausible explanation is that individuals anticipated with some probability, events that would prevent the govenrment to redeem the high interest debt any time soon. Conditions in the credit market were not tight at that time and the interest rate must have decreased in the short-term: despite the uncertainty about the future, the price of the 3% increases moderately over the three years from below to above par.

[31]For a summary of the debt in 1749, see Dickson (1967, p. 232).

The war years: 1743-1749

At the beginning of 1744, with war momentum building up, the expected increase of the interest rate came closer on the horizon. The price of the 3% decreased and stabilized around 90 until August 1745 (Figure 10). Note the high level of the 4% price in relation to the 3%: the expected time of redemption is far off in the future. After the landing of the Pretender in August 1745, the prices of both assets fell significantly. The 3% annuity reached in February 1746 its lowest price in the war (about the same level as in March 1748). It recovered rapidly after Culloden in April 1745.

At the beginning in 1746, the new 4% annuity is introduced. For all years of the war, the South Sea annuity is traded at a price which is above the price of the 1746 annuity by a value which fluctuates between 1 and 3 with an average of about 2. The only exception is in the winter of 1748. The prices of the two annuities between the summer of 1747 and the redemption of 1749 are presented in Figure **??**. After the crisis in the winter of 1748, the South Sea annuity regains rapidly its premium. The market anticipated a pattern somewhat similar to the situation before the war, with a redemption that would come later and/or with compensation larger than for the new debt.

In the Summer of 1749, a few months before the redemption, the market clearly anticipated that the old debt which had resisted the 1737 attempt at redemption, would be treated differently from the new debt that had been incurred during the war (the 4% annuities issued after 1745 that traded at the same price). The prices for the whole year 1749 are presented in Figure 12. From October on, the new 4% is identical to the 3% plus a fixed premium, as we have seen before. The old debt is price above the new by about 2 in October. The reduction of the premium of the old debt is gradual and achieved only at the end of the year. (After January 1, 1750, the old debt is priced like to the new debt).

In view of the market evidence, Pelham did deserve some credit for the interest reduction on the entire 4% debt. Perhaps, his task was made less difficult than in 1737 because the government had able to issue a significant amount of 3% debt during the war whereas in 1737, 3% annuities were a very small part of the public debt. Various lobbying interests may have realized that the coexistence of two large amounts of assets with coupons at 3 and 4% with steady long-term interest rate at 3% was not sustainable. This realization may have been slow in the summer, but by the end of the year, it had been accepted.

7 Conclusion: policy against pessimism

Did the prices of the government bonds drop "too much" during the wars in view of the subsequent evolutions of the short-term rates? The volatility of long-term interest rates in expectations models of the term structure has been analyzed by Shiller (1989). His tests have been applied by Weiller and Mirowski (1990) to long-term bonds in 18th century England, with mixed results[32]. The methodology rests essentially on a comparison between the variance of the short-term rates with that of the long-term rate and the method requires a sufficient number of events. In the 18th century, the sample generates three large fluctuations between 1715 and 1792, one for each war.

This study provides a different type of test about an excess fluctuation. The main issue at the end of war, as attested by the policy of issuing redeemable bonds was the speed of convergence of the bond to its par value. If people expected a slow convergence, then the consol price did not overreact. The information contained in the two prices of the 3% and 4% bonds with the option feature of the latter provides a strong test about two issues.

First, the market was remarkably rational in pricing financial assets consistently, as shown by the good fit of the derivative asset pricing model of the 3% and the 4% annuities. Moreover, the terms of the payoff at redemption were accurately predicted and taken into account long before the redemption.

Second, the market showed an excess pessimism about the recovery of bond prices after the war which is measured in two different ways: the expected time for the bond reaching its par value was strongly overestimated; second the subset of events such that the consol would return to par within a year (which it did) was given a subjective probability of less than 1/4 percent by the market. The price observations provides therefore strong evidence that in the case of the war of the Austrian succession succession, the market overreacted and showed considerable excess pessimism.

In this context, the debt policy during the war of the Austrian succession offers a spectacular illustration of the power of marketed contingent financial instruments. The loans of 1745 and 1746 included non marketed life-time annuities and were more

[32] Note that the fluctuation of the bonds can only be significant downwards because of the callable features of the bonds (which is ignored by Weiller and Mirowski). In the context of the 18th century, the 3% bond could be assumed to be non redeemable, but the previous discussion of the redemption policy shows that if the consol had increased to a stable level above say 105, then the reedemable feature would have depressed it price. The consol still exists today with an interest of 2.5% since 1903, down from 2.75 since 1889, (Miller, 1890, Harley, 1976, Klovland, 1994).

expensive *ex ante* than later loans although the long-term rate was not smaller in 1747-1748 and the amount of the loans more than twice as high.

The contigent feature of the callable bonds enabled the government to take advantage of the excess-pessimism. Simply put, the market paid in April 1746 during the subscription of the 4% bond, a price of $\mathcal{L}$ 12 for the contingent annuity of $\mathcal{L}$ 1 per year until the call of the bond. It was willing to pay that price because it was pessimistic about future interest rate. Its expectation was that the annuity would pay for about 10 years with a final payment of 4. In fact, the annuity lasted only for three years. That price of 12 was much more than the total amount collected on the annuity, ignoring discounting, since the government paid a total of 7 (1 per year from 1746 to 1748, and a total of 4 after). The government was therefore able[33] to bet with great success against the pessimism of the market.

The success of the contingent policy can also be measured by the *ex post* rate of return that the government paid on its loans. This rate (the long-term internal rate of return) is presented in Column 6 of Table 1. Once the government abandoned the inefficient life annuities for the redeemable 4%, it was able to finance the war a long-term rate that even during the worst years, turned out to be only 3.5%.

Finally, the successful betting by the government against pessimistic investors was repeated recently and in a similar context. In the early 1980s, the British government faced adverse expectations of private investors who were pessimistic about the government's conduct and the evolution of interest rates. This time, the ennemy was not France, but inflation. Margaret Thatcher was more confident than the market that she would prevail and her government issued inflation indexed bonds, with coupons linked to the inflation rate. Expecting high coupons for a long time, investors paid high prices to the government, like the buyers of 4% annuities in 1747. Inflation came down much sooner than expected (with some help from Paul Volcker). In the 1980s as in the 1740s, the government won against excessive market pessimism[34].

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[33]The extent of the bet was limited however, perhaps by institutional factor since the government could not issue solely annuities contigent on the consol below par: to each such annuity it had to attach one consol which had indeed a low price because of the pessimism. The government could also have even done better if the loans had been issued with a coupon of 4.5% instead of a 10% premium over par.

[34]If the government had used standard (non inflation linked) bonds, it would have had to pay an average *ex post* real return of 7.7% in the 15 years after 1982 (while the average inflation rate had been 4.3%). On the inflation linked bonds the government paid only 2.8%.

Appendix

1. The borrowing cost

The 1745 loan bundled for each $\mathcal{L}$ 100 a package of one consol at face value $\mathcal{L}$ 100 and a life-time annuity of 1.125. These were not rated by age and we have no information on the age distribution of the subscribers. Grellier reports a rate of 4.02%. There is no need to refine that number which is the true long-run interest cost of the loan but we can extract from that number the implicit maturity of the life-annuity. The maturity that best reproduces Grellier's result is exactly 60 years, which is the number that will kept for the computation of the next yield. Note that this rate of 4.02% is much higher than the market yield of 3% bonds which was around 3.37% (with a price of 89 in Column 6).

The computations of the yields for 1747 and 1748 is explained in the text. In 1746, we have a mix of a callable 4% and a life-annuity. The 4% component is equivalent to a perpetual payment of $3 \times 94/83 = 3.3976 = c$, where 94 and 83 are the prices of the 4% and the 3% annuities in the Spring of 1746, (which are more relevant than the prices at the beginning of the year because the payments for the subscription were made in the spring and the summer). The life-annuity is taken as a payment of 1.5 for 60 years. The yield R is such that $100 = \sum_{1 \leq i \leq 60}(1.5 + c)/(1 + R)^i + (c/R)/(1 + R)^{61}$, which gives a solution of 4.81 %.

2. The one-factor model of the government bond prices: 48-49

The model of interest rate is based on the Vasicek model. It is reasonable to use such a model for regimes of uncertainties around wars[35]. In the class of Vasicek type models, some special cases generate close form solution in continuous time. But these special cases are too constraining for the data. In order to have more freedom for the fitting of the data, we use a discrete numerical model.

As stated in the text, the short-term rate r moves on the grid $\{r_i\}_{1 \leq i \leq N}$ where the function r_i of i is mildly concave, according to a Markov process with the transition probabilities presented in equation (1). Some experimentation showed that the values of α and β could be taken as independent of the state i. The adjustment of the probabilities at the boundaries $i = 1$ and $i = N$ had no impact on the prices. As explained in the text, the value of β will be a function of α such that the variance of

[35]Such a model could not be applied with constant coefficients for large intervals of time that include war and peace. For example, after 1749, the variance of the consol was much smaller than before.

the consol price p that is generated by the model is about the same as the historical value when p is around 91. The transition probabilities in equation (1) define the matrix on transition probabilities $M = [m_{i,j}]$.

The price of the consol

A consol is the sum of an infinite number of zero coupons bonds of different maturities. Each zero coupon bond is priced using risk-neutral probabilities and the price of the bond is the sum of the values of all the zero coupon bonds imbedded in it. At this stage, it is assumed that agents are risk-neutral and that the true probabilities of the random process, in the matrix M are the same as the risk-neutral probabilities. Later, a distinction will be made between these two probabilities.

Coupons were paid twice a year, around the 5th day of January and July. Because the bond price depends on the interest rate which follows a Markov process, the price of the consol in any period depends only on the interest rate in that period and the time to the next coupon payment. Let p be the vector or prices of the consol p_i when the interest is r_i immediately after the payment of the coupon, and $p(t)$ the price t days before the payment of the coupon. Assuming the year to have 366 days, for simplicity, $p(183) = p$. Let u be the column vector of dimension N with all elements equal to 1, and D the diagonal matrix where i-diagonal element is equal to $1/(1+r_i)$.

For any period with t days before payment of a coupon, $1 \leq t \leq 183$,

$$p(t) = DMp(t-1), \quad \text{with} \quad p(0) = 1.5u + p.$$

The matrix $A = DM$ is the matrix of discounted probabilities of the values of an asset in the next period and will play an important role. We also define $J = (DM)^{183}$ the discounted probabilities over the six months time interval between coupon payments.

By iteration of the previous equation, $p(t) = A^t(1.5u + p)$.

Since $p(183) = p$, $p = A^{183}(1.5u + p) = J(1.5u + p)$, and

$$p = 1.5(I - J)^{-1}Ju, \tag{17}$$

where I is the identity matrix of dimension N.

From the value of p we have for any $t \geq 1$,

$$p(t) = 1.5A^t(I - J)^{-1}Ju. \tag{18}$$

On the L grid point, the rate r_L is by definition equal to the long-run value r^* to which the rate converges (if there were no shock). We will always adjust that grid point such that at the grid point L the consol is at par[36]: $p_L = 100$. When the process is deterministic, the value of $r_L = r^*$ is obviously equal to 3%. When the process of the interest rate is random, the long-run value r^* is a little smaller because of the curvature of the consol price: the price is roughly the expectation of the future prices and possible increases of the rate in the future have a stronger (negative) impact on the consol than possible decreases of the rate.

The price of the callable bond

Following the historical policy of Pelham, the redemption of the bond is determined by the following rule: in any period with a consol price p_i, the probability of redemption is equal to π_i: if the redemption takes place, the government is committed to redemption at the date of the next coupon payment; at that date, the bond is transformed into a consol plus an amount h equal to the present value of difference between the coupon payments and 1.5 during the phasing in of the interest reduction. (The value of h is the one that fits best the data after the redemption: $h = 4$).

Let Π be the diagonal matrix with elements π_i. The choice of the π_i will be discussed below. Define the vector

$$w = (1.5 + h)u + p.$$

Using the same method of computation as for the consol price, the price of the bond the day before the coupon payment is

$$q(1) = A\Big((I - \Pi)(2u + q) + \Pi w\Big),$$

where p is the price of the bond immediately after the payment of the coupon.

Let $v(t)$ be the vector-price of the bond with commitment to redeem it by the next coupon date t days ahead. We have

$$v(1) = Aw.$$

The vector-price of the bond with no announcement to redemption yet and with t days to the coupon payment is $q(t)$ which satisfies with $v(t)$ the equations

$$\begin{cases} q(t+1) = A\Big((I - \Pi)q(t) + \Pi v(t)\Big), \\ v(t+1) = Av(t). \end{cases} \tag{19}$$

[36]This modeling choice is validated by the evolution of the consol after the interest reduction of 1749: during the year 1750, the consol always stayed in the interval $[98.5, 101.5]$.

Let $\tilde{A} = A(I - \Pi)$ and $\tilde{J} = \tilde{A}^{183}$. The price of the bond immediately after the payment of the coupon is q and since $q = q(183)$,

$$q = \tilde{J}(2u + q) + \sum_{k=0}^{182} \tilde{A}^{182-k} A\Pi A^k((1.5 + h)u + p).$$

The vector-price of the bond is therefore determined[37] by the equation

$$q = \Big(I - \tilde{J}\Big)^{-1}\Big(2\tilde{J}u + \sum_{k=0}^{182} \tilde{A}^{182-k} A\Pi A^k((1.5 + h)u + p)\Big). \tag{20}$$

The expected time to redemption

Let Θ be the vector $\Theta = \{\theta_i\}_{1 \le i \le N}$. The equation (9) in the text can be written in matricial form

$$\Theta = u + M(1 - \Pi)\Theta,$$

where u is a column-vector of N ones, M is the matrix of elements $m_{i,j}$, I the identity matrix, and Π is the diagonal matrix with diagonal elements π_i. The expected time to redemption is a function of the realization of the interest rate and is expressed by

$$\Theta = (I - M(I - \Pi))^{-1}u. \tag{21}$$

3. Estimation

Let ζ the vector of parameters of the model to be estimated. For a given value of ζ, the vector-price p is determined by (17) and the vector-price q by (20). These vector-price determine points a the function $q = \phi(p; \zeta)$. Given the set actual prices[38] $\{p_t, q_t\}$, $(1 \le t \le n)$, the estimated parameter ζ^* minimizes the sum of the squares

$$\mathcal{S} = \sum_{t=1}^{n} (q_t - \phi(p_t; \zeta))^2.$$

Since the function ϕ is determined by a grid, the value of $\phi(p_t; \zeta)$ is determined by linear interpolation with the two nearest grid points[39].

[37]Since the matrix $I - \tilde{J}$ may be difficult to invert, it is easier to determine the value of q by iteration of the system (19).

[38]Points before May 1747 are omitted. They fit well the curve however. Similarly points after September 1749 are omitted from the sample of estimation since they reflect additional information, as will be discussed later.

[39]Let $p(i) < p_t < p(i+1)$. Then $\phi(p_t) = (q(i)(p_t - p(i)) + q(i+1)(p(i+1) - p_t)/(p(i+1) - p(i))$.

The previous formulae for the prices p and q were established for the the prices the day after the payment of the coupon. For the estimation, all observed prices (from various issues of the Gentleman's Magazine), are adjusted ex-coupon: the pro-rated accumulation of the next coupon, from the date of the last payment, is deducted from the price.

Table 2: Maximum likelihood estimates with 4 parameters

	Estimate	α	h	$Var(p)$	γ
α	.0115	0.0193	-0.7773	-0.8018	0.2215
h	3.85		10.3622	0.9990	-0.7823
$Var(p)$	0.4119			6.3469	-0.7586
γ	0.4834				1.6605

For the meaning of the parameters, see the text. The standard errors are on the diagonal, and the coefficients of correlation are above the diagonal.

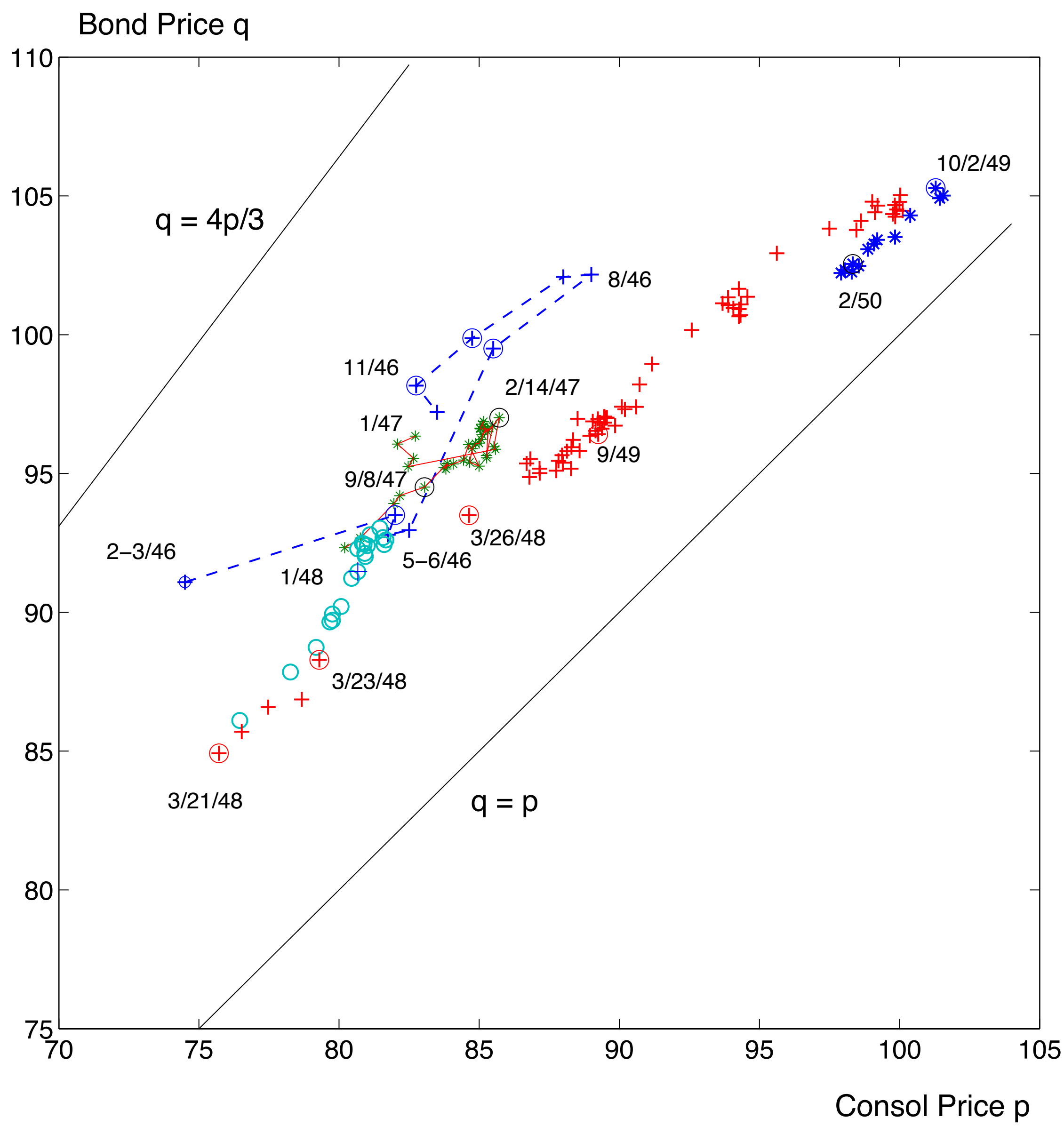
Table 3: Standard errors and correlations with less than 4 parameters

	α	h	$Var(p)$
α	0.0188	-0.9943	-0.9975
h		6.4550	0.9993
$Var(p)$			4.1352

	α	h
α	0.0013	0.9493
h		0.2433

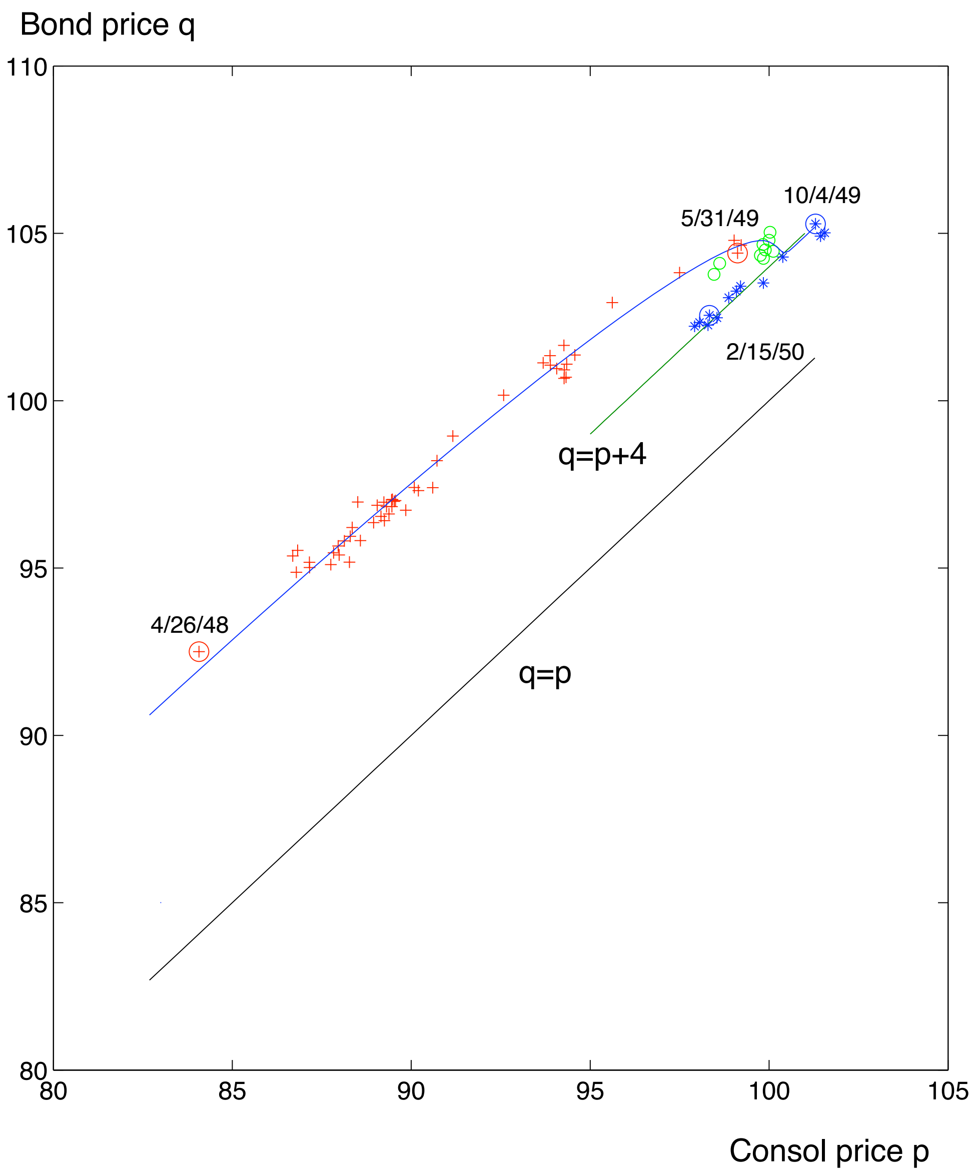
	α
α	0.00042

The notation is the same as in the previous table. Parameters are successively omitted from the estimation and set at the previous estimate. (See the text for comments). The estimates of the remaining parameters are obviously unchanged but the new standard deviations are smaller, and the coefficients of correlation are different.



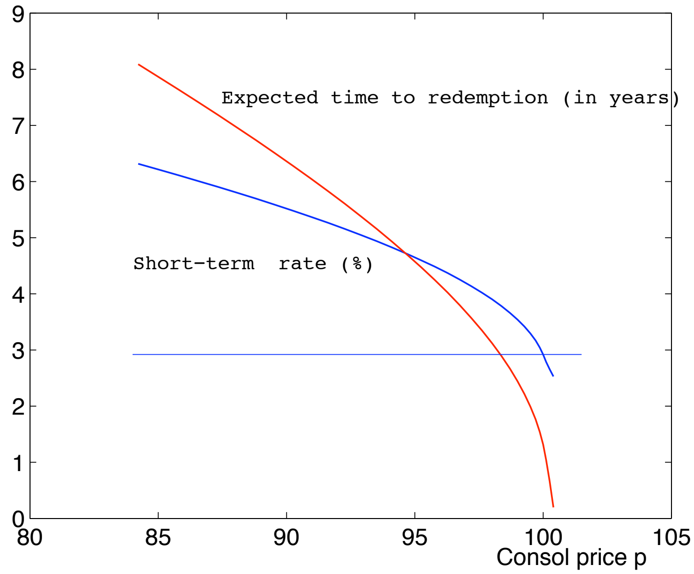
Prices in different time intervals are represented by different symbols (with the indicated dates). The 4% bond is issued in 1746 and the data for 1746 is monthly. All other points are weekly averages of daily prices (when available). Data points in 1746 and in 1747 are joined with a line to highlight the evolution of the point (p, q) over time. Note the price jump at the beginning of the peace negotiations at Aix-la-Chapelle (April 24, 1749). All prices are from the Gentleman's Magazine and are adjusted ex-coupon.

Figure 2: The bond price in relation to the consol price
(February 1746 to February 1750)



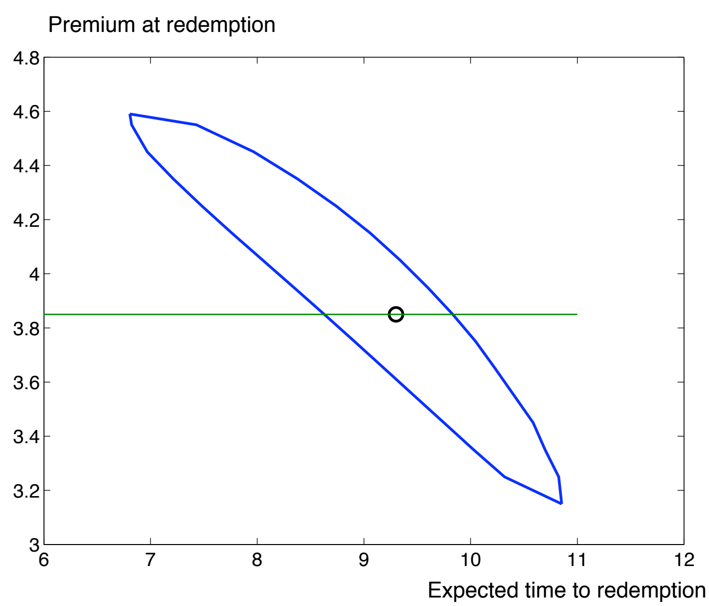
The derivative asset pricing of the bond, in the estimated model, is represented by the plain curve. The points used in the estimation (up to May 1749) are marked by vertical crosses. (The observations after May 1749 are marked by circles and are not part of the estimation sample). The starred points are the observations after October 1, 1749. The short straight line is the consol price plus a premium of 4.

Figure 3: Prices between April 1748 and February 1750



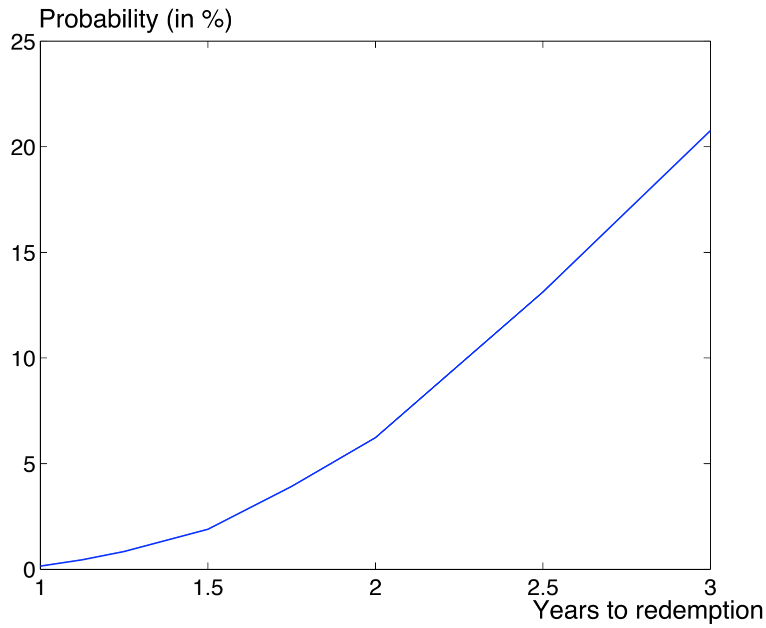
The horizontal line is defined by the long-term rate generated in the model, *i.e.*, the rate when the consol is at par.

Figure 4: Expected time to redemption and short-term rate



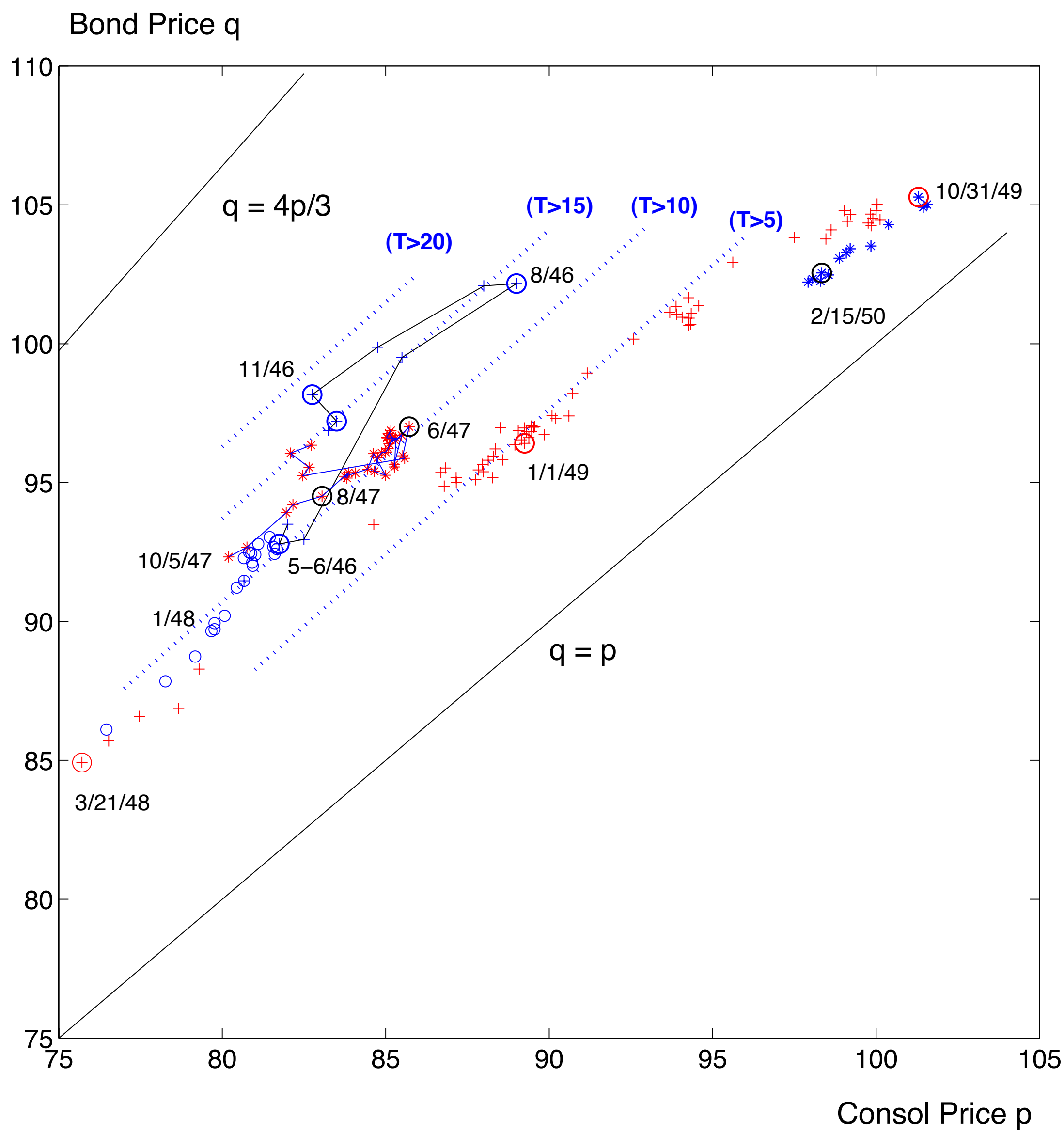
The estimate with four parameters is represented by the circle. The convergence parameter α is converted to the expected time to redemption for a price of the consol equal to 89. The horizontal line corresponds to the estimated premium at redemption.

Figure 5: Confidence region (1%)



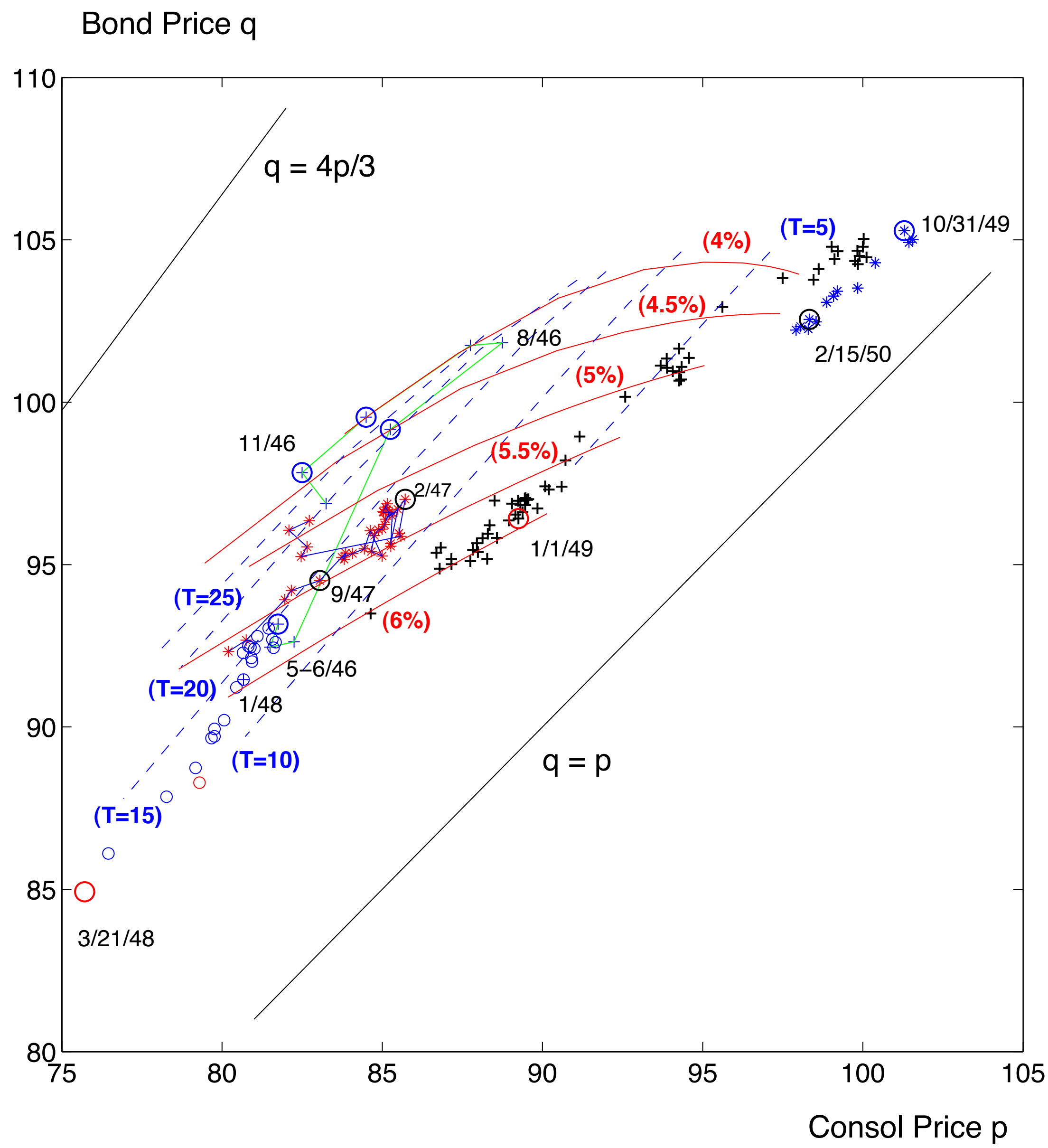
The probability is that the redemption occurs in a time smaller than the value of the horizontal axis when the price of the consol is 89. The actual historical value is 1.

Figure 6: Probability of redemption within a time T



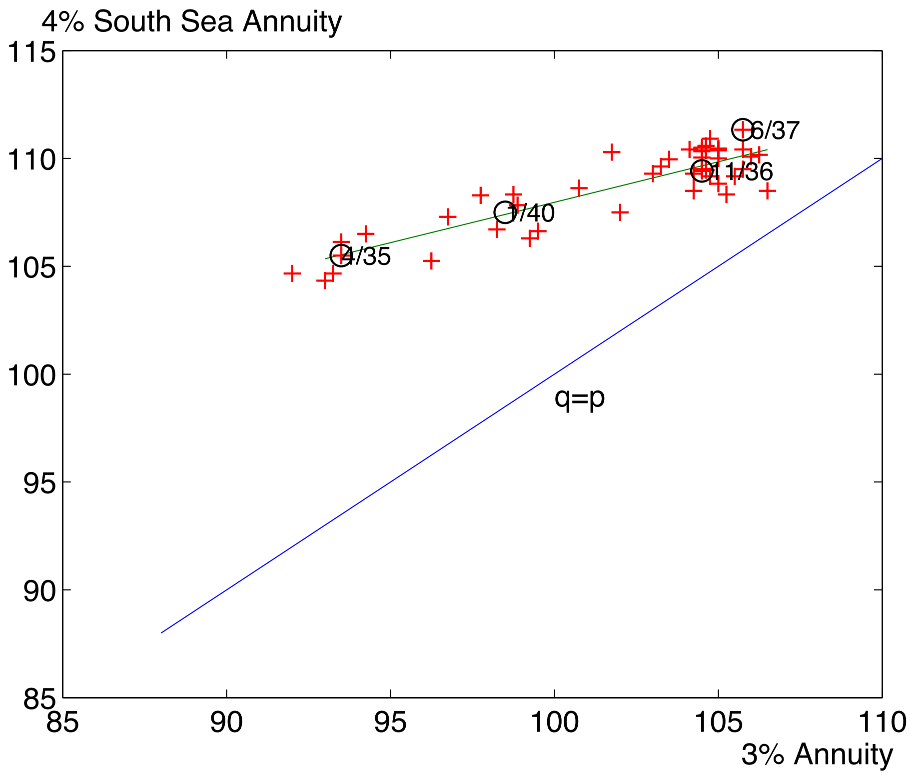
The dotted lines represent points with equal lower-bound expected time to redemption under the rules used in 1749 (in years), with an interest rate not smaller than 3% before the redemption.

Figure 7: Minimum expected time to redemption



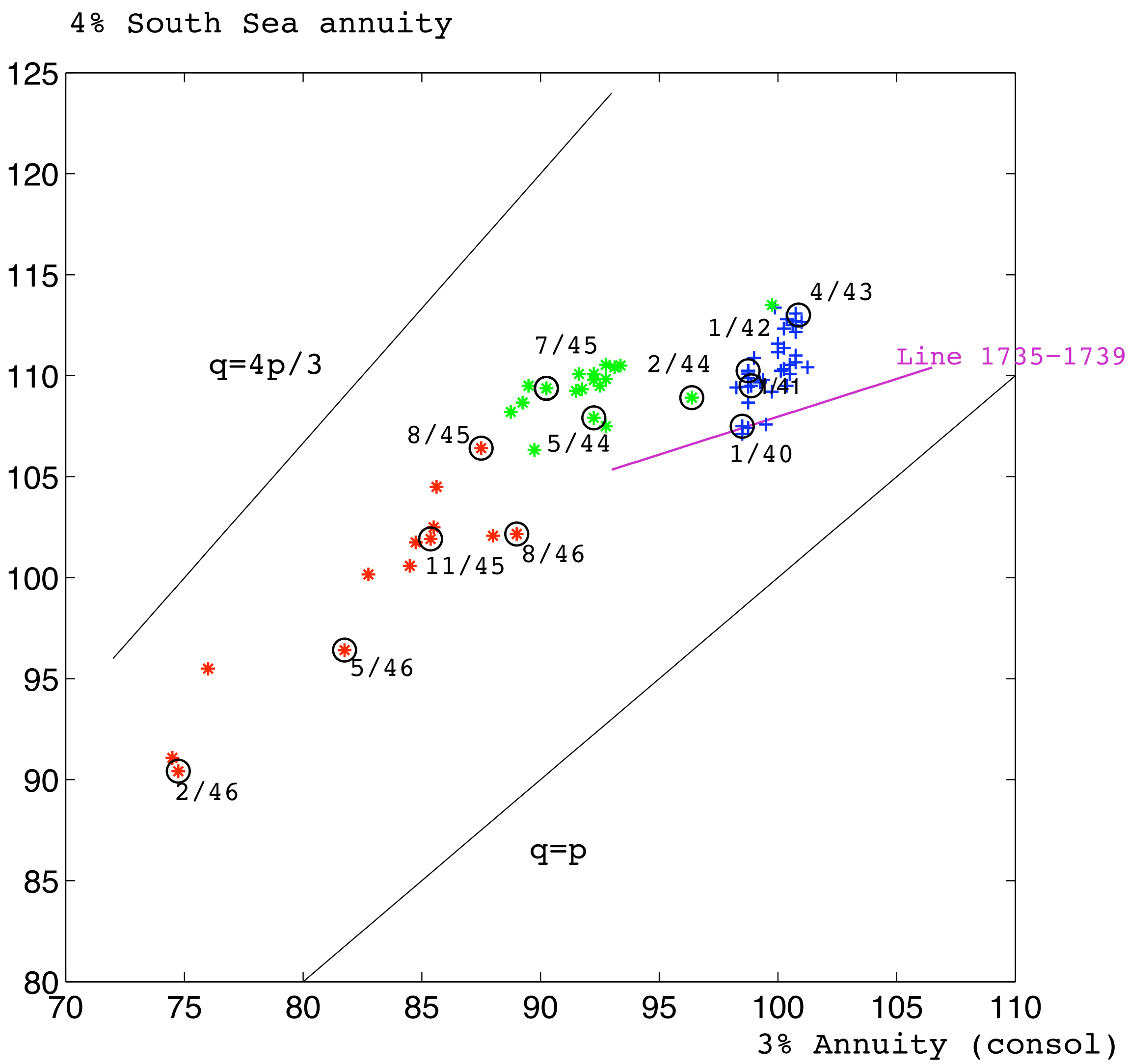
For each point in the diagram, the stochastic model, with a suitable rate of convergence, generates a current value of the short-term rate and the expected time to the call of the bond (maturity). The loci with constant rates and constant maturities are drawn for some values and marked accordingly. The observations are the same as in Figure 2.

Figure 8: Interest rates and expected time to redemption



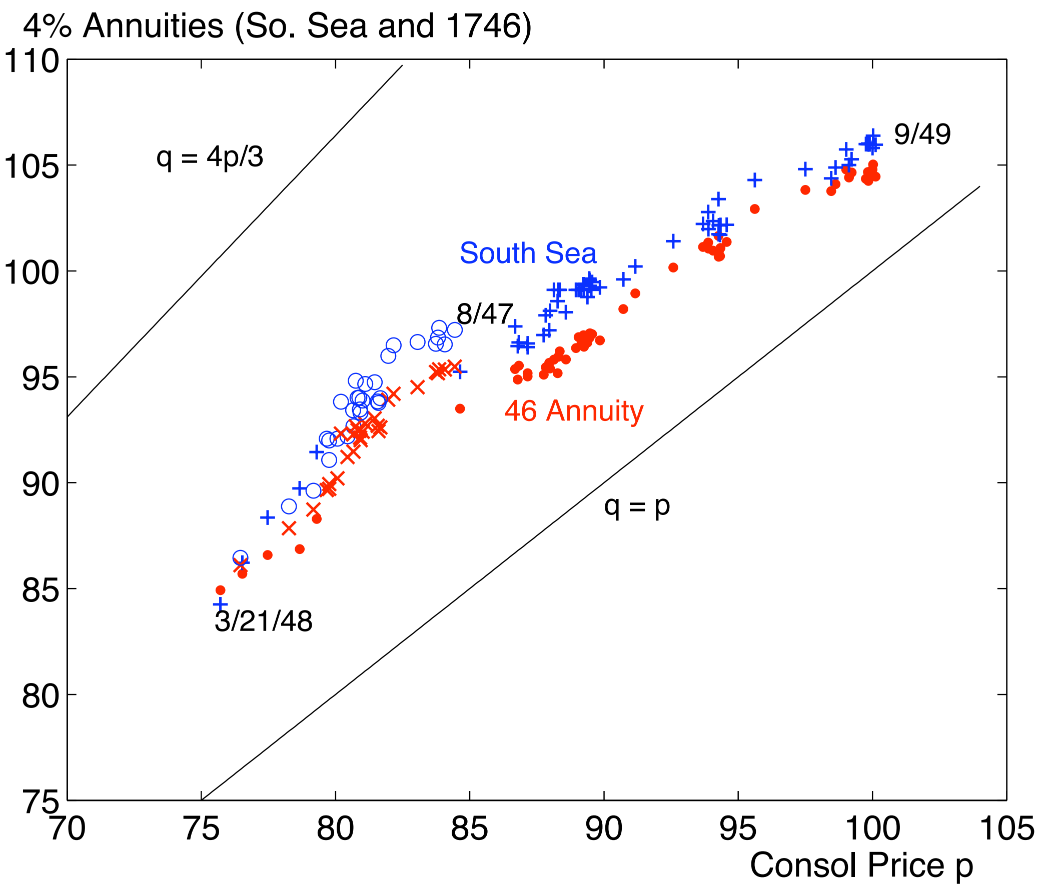
The South Sea annuity is the "New Annuity". The line is obtained by linear regression.

Figure 9: South Sea 4% Annuities 1735-40



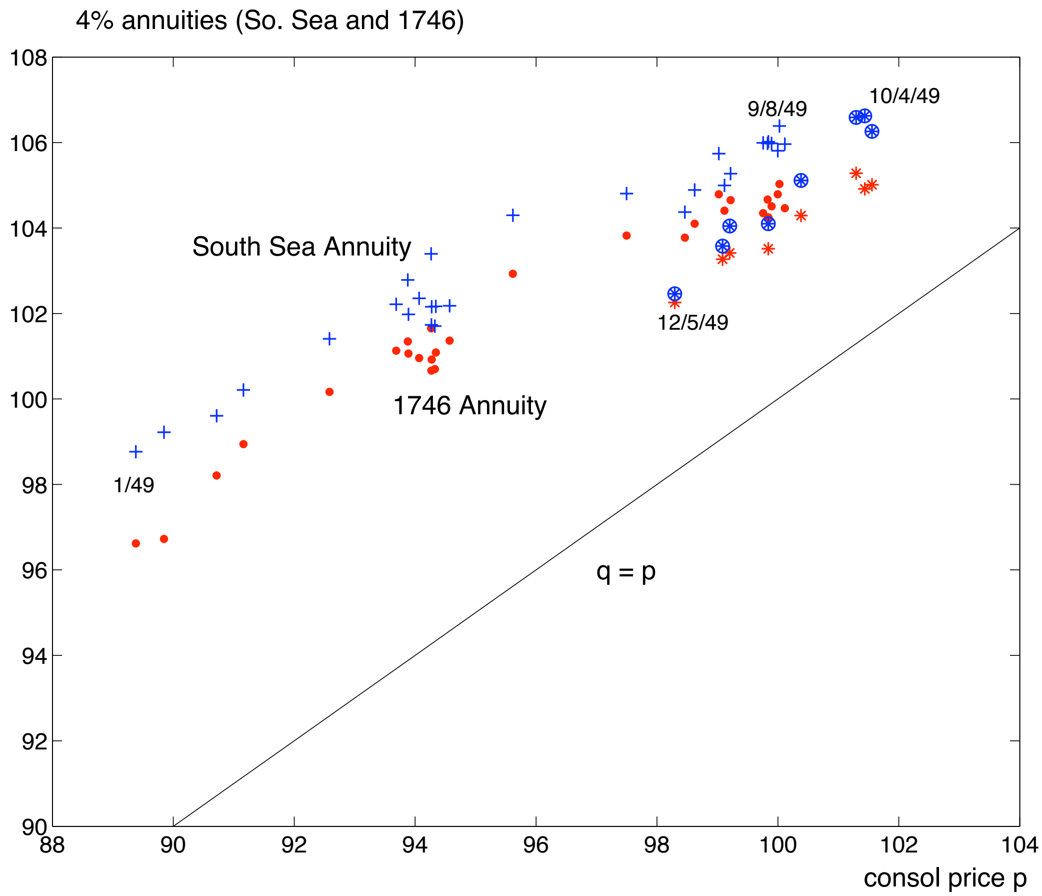
The line segment is the same as in Figure 9 and summarizes the prices in that period. Prices from 1740 to 1743 are represented by dots, from 44 to July 45 by crosses, and from August 45 to the end of 46 by stars.

Figure 10: South Sea 4% Annuities 1740-45



The 1746 annuity is represented by diagonal crosses before March 1748 and by dots afterwards. The South Sea annuity is presented by stars before March 1748 and by vertical crosses afterwards. .

Figure 11: South Sea and 1746 Annuities: 1747-1749



The South Sea annuity is represented by crosses until the beginning of September. After the beginning of October, the 1746 annuity becomes perfect substitute to the consol plus a premium (stars), while the evolution of South Sea annuity is gradual and completed only in December (circled stars).

Figure 12: The redemption of the South Sea Annuities

REFERENCES

Barro, Robert (1987). "Government Spending, Interest Rates, Prices, and Budget Deficits in the United Kingdom, 1701-1918," *Journal of Monetary Economics*, **20**, 221-247.

Bliss, Robert R., and Ehud I. Ronn (1998). "Callable U.S. Treasury Bonds: Optimal Calls, Anomalies, and Implied Volatilities," *Journal of Business*, **71**, 211-52.

Brennan, M. J., and E. S. Schwartz (1977). "Convertible Bonds: Valuation and Optimal Strategies for Call and Conversion," *Journal of Finance*, **32**, 1699-1715.

Buchinsky, Moshe and Ben Polak, (1993). "The Emergence of a National Capital Market in England, 1710-1880," *Journal of Economic History*, **53**, 1-24.

Bühler, Wolfgang, and Michael Schultze (1993). Analysis of the Call Policy of Bund, Bahn, and Post in the German Bond Market. Lehrstuhl für Finanzierung, Universität Mannheim, Working Paper 93-1, in W. Bühler, H. Hax, R. Schmidt, ed. (1999), *Empirical Research on the German Capital Market Edition*.

Bühler, Wolfgang, and Christian Koziol (2004). "Calling convertible Bonds too Late Can be Rational," EFA 2004 Maastricht Meetings Paper No. 4355.

Browning, Reed S. (1993). *The War of the Austrian Succession*, New York : St. Martins Press.

Carlos, A., L. Neal, K. Wandschneider (2007). "The Origins of National Debt: The financing and Re-financing of the War of the Spanish Succession," mimeo.

Chamley, Christophe (1985). "Efficient Taxation in a Stylized Model of Intertemporal General Equilibrium," *International Economic Review*.

Clark, Gregory (2001). "Debt, Deficits, and Crowding-Out: England, 1727-1840," *European Review of Economic History* **5**, 403-4356.

Dickson, P. G. M. (1967). *The Financial Revolution in England*, London.

Ferguson, Niall (2003). "Political Risk and the International Bond Market Between the 1848 Revolution and the Outbreak of the First World War," mimeo, New York University.

The Gentleman's Magazine, Various issues since 1737.

Grau, A. P. A. Forsyth, and K. R. Vetzal (2003). "Convertible Bonds with Call Notice Periods," mimeo.

Grellier, J. J. (1812). *The Terms of All the Loans Which Have Been Raised for the Public Service*, 3rd ed. London,

Grellier, J. J. (1810). *A History of the National Debt*, London.

Guinnane, Timothy W., Harvery S. Rosen and Kristen L. Willard (1994). "Turning Points in the Civil War: Views from the Greenback market," *American Economic Review*.

Harley, C. K. (1976). "Goschen's Conversion of the National Debt and the Yield on Consols," *Economic History Review*, **29**, 101-106.

Heim, C. and Mirowski, P. (1991). "Crowding Out: A Response to Black and Gilmore," *Journal of Economic History*, **51**, 701-6.

Homer, Sidney, and Richard Sylla (1991). *A History of Interest rates*, Rutgers University Press: New Brunswick and London.

Hoppit, J. (1986). "Financial Crises in 18th Century England," *Economic History Review*, **39**, 39-58.

Ingersoll, Jonathan E., Jr. (1977). "An Examination of Corporate Call Policies on Convertible Securities, *Journal of Finance*, **32**, 463-78.

Joslin, D. M. (1960). London Bankers in Wartime, 1739-84, in L. S. Pressnell (ed.), *Studies in the Industrial Revolution*, London: University of London, Athlone Press.

Klovland, Jan Tore (1994). "Pitfalls in the Estimation of the Yield on British Consols, 1850-1914," *Journal of Economic History*, **54**, 164-187.

Miller, A. C. (1890). "The Conversion of the English Debt," *Quarterly Journal of Economics*, **4**, 437-448.

Mirowski, Philip, (1987). "What Do Markets Do" Efficiency Tests of the 18th Century London Stock Market," *Explorations in Economic History*, **24**, 107-129.

Neal, L. (1990). *The Rise of Financial Capital Markets in the Age of Reason*, Cambridge: Cambridge University Press.

Neal, Larry. (2000). "How It All Began: The Monetary and Financial Architecture of Europe During the First Global Capital Markets, 1648-1815," *Financial History Review*, **7**, 117-140.

O'Brien, Patrick (1967). "Government Revenue, 1793-1815: a Study in Fiscal and Financial Policy in the Wars against France," Ph.D. thesis, Oxford university.

———— (1994). "The State and the Economy, 1688-1815," in R. Floud and D. McCloskey (eds) *The Economic History of Britain Since 1700*, vol. 1, Cambridge University Press.

Roseveare, H. (1991). *The Financial Revolution, 1660-1760*, London: Longmans.

Shiller, Robert (1989). *Excess Volatility*, MIT Press.

Skrine, F.H. (1906). *Fontenoy and Great Britains' Share in the Austrian Succession*, Edinburgh : W. Blackwood and sons.

Stasavage, David (2003). *Public Debt and the Birth of the Democratic State*, Cambridge University Press.

Stasavage, David (2006). "Partisan Politics and Public debt: The Importance of the Whig Supremacy for Britain's Financial Revolution," mimeo, NYU.

Sussman, N. and Y. Yafeh (2003). "Constitutions and Commitment: Evidence on the Relation Between Institutions and the Cost of Capital," Hebrew university of Jerusalem.

Thompson, Richard G. (1985). *Expectations and the Greenback Rate, 1862-1878*, New York: Garland.

United Kingdom, Parliamentary Papers (1868-69). *Accounts of Public Income and Expenditure of Great Britain 1688-1868*, Vol. XXXV.

United Kingdom, Parliamentary Papers (1898). *History of the Funded Debt from 1694 to 1786*, Vol. LII.

Vasicek, O. (1977). "An Equilibrium Characterization of the Term Structure," *Jour-*

nal of Financial Economics, **5**, 177-178.

Velde, François R., and David R. Weir (1992). "The Financial Market and Government Debt Policy in France, 1746-1793," *The Journal of Economic History*, **52**, 1-39.

Waldenström, Daniel and Bruno S. Frey. (2006). "Using Markets to Measure Pre-War Threat Assessments: The Nordic Countries Facing World War II," mimeo.

Weiller, Kenneth J. and Philip Mirowski (1990) "Rates of Interest in 18th Century England," *Explorations in Economic History*, **27**, 1-20.