The Price Theory of Two-Sided Markets [*]

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Abstract

I establish a number of baseline positive and normative results in the price theory of two-sided markets building on the work of Rochet and Tirole (2003). On the positive side, I introduce the notion of vulnerability of demand to separate previously confounded effects. I find that competition, price controls and subsidies always reduce the *price level*, defined as the sum of prices on the two sides of the market. However, price controls and competition that are "unbalanced" may raise prices on one side of the market. The normative analysis emphasizes the importance of externalities across the two sides of the market and their impact on socially optimal pricing. The socially optimal price level, which takes an intuitive Ramsey-pricing form, is always below cost. Subsidies may be desirable even if the profits of the firm are disregarded. In determining optimal price balance, seemingly similar welfare criteria generally conflict. Consumers on one side of the market may want to make transfers to the other side in order to thicken their pool of partners. Unbalanced competition that undermines such transfers may harm all parties. A number of implications for policy are discussed.

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1 Introduction

Interest in the economics of two-sided markets has increased dramatically over the last half decade. A series of high stakes, closely fought antitrust cases[1] and regulatory debates[2] have focused policy interest on differences between these industries and standard markets. Antitrust and regulatory activity has been particularly concentrated in the payment (credit and debit) cards, internet service provision, television, newspapers and software industries. Partially in response, a number of theoretical advances have helped focus this attention. Rochet and Tirole (2003 and 2006) demonstrated that "two-sidedness", in a sense they defined, is a feature of many industries beyond those that the notion was initially developed to treat. They laid out a general modeling framework for understanding two-sided markets. Theory and practice have played complementary roles in fostering a recognition that two-sided markets constitute a substantial portion of aggregate economic activity. Furthermore, their prevalence in new and growing economic sectors makes understanding these industries all the more important.

Unfortunately despite this recent attention, we know very little about how to analyze or make policy in two-sided markets. A literature has recently developed illustrating that many intuitions from standard, one-sided markets break down in two-sided markets. This literature, excepting a recent attempt at unification by Rochet and Tirole (2006), has been divided into two separate strands. The first emphasizes the role of fixed membership costs and network membership externalities. The other literature focuses on externalities from the usage of the service and assumes linear (per-transaction) costs and pricing. Building on the model supplied by Rochet and Tirole (2003), this paper adopts the second approach. However, I believe, and hope to show in a revision of this paper, that at least the positive results apply more broadly.

Following the definition supplied by Rochet and Tirole (2006), a two-sided market is one where:

1. There are two distinct groups of consumers (say men and women in the case of a dating website) served by the market.

2. Some part of the value of the service (all in the case of a dating website) to the consumers comes the capacity of the service to connect the two sides of the market.

3. The individual prices charged to each side of the market, and not just the sum of those prices, matter in determining the usage of the service and consumer welfare. In order for the individual prices to matter, it must be the case that Coase's theorem fails in the relationship between the two sides of the market and therefore externalities exist between the two groups of consumers. Each group would like to provide side payments to the other group for joining the market, but is prevented from doing so by social, legal, informational or contractual barriers (in the case of the dating website, compensatory payments for dating are clearly taboo).

Pricing in two-sided markets differs fundamentally from pricing in standard markets. In setting her price, a toy manufacturer considers the structure of her costs, the demand for her product and the competition she faces. In setting the price for a video game console, a manufacturer certainly considers all

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[1]Evans (2003) provies a comprehensive review of antitrust litigation in two-sided markets, including payment cards and software to 2002. Many of the cases he discusses, prominently *USA vs. Visa International, Mastercard International* and the European Union's complaint 29.373 against Visa International remain opened; others have opened since. Two-sidedness also played an important role in the massive United States vs. Microsoft Corporation case.

[2]Prominent among these have been the debate over "net neutrality", discussed more below, and over the regulation of interchange fees on payment cards.

of these factors. However, she must additionally take into account the effect that the her price and the resultant number of console owners has on her capacity to earn profits from a second group of consumers: the video game makers she licences to make games for the system. If the (per-game) profit she earns from game makers increases, she will have an incentive to reduce the price of the console so as to encourage the profitable purchase of games. This "topsy-turvy" effect first identified by Rochet and Tirole (2003) complicates the price effects of competition. One can think of prices in two-sided markets as being roughly like a see-saw suspended by its axle from a rubber-band. Competition applies pressure to the sides of the see-saw, stretching the rubber band and reducing prices; but it may also shift the balance of the see-saw, so that the direction of price effects is unclear.

Similarly, welfare considerations in two-sided markets are complicated by externalities across the two sides. Men may be willing to pay a higher price for using a dating website if this reduces the price for women using the site thereby improving the men's mate selection. Positive externalities across the two sides of the market mean that prices on each side affect the welfare of the other side.

This paper makes three basic arguments: one positive and two normative. On the positive side, I introduce the notion of "vulnerability of demand" to help separate the tendency of competition and price controls to put downward pressure on prices from Rochet and Tirole's topsy-turvy effect. I show that competition, price regulation and subsidies always drive down the *price level*: the sum of prices on the two sides of the market. I use the same set of tools to show that "unbalanced" competition or price controls, which put much greater downward pressure on the prices of one side of the market than the other, tend to increase prices on the unpressured side. I also demonstrate that "balanced" competition and price regulation, as well as any form of subsidy, drive down prices on both sides of the market.

On the normative side I consider the effects of externalities on the socially optimal *level*, as well as *balance*, of prices. Strictly positive externalities on both sides of the market mean that the socially optimal price level is below cost. By analogy to the familiar optimal taxation formulas, the optimal price level is below cost by an amount corresponding to the marginal positive externality. When a monopolist governs the balance of prices, the formula must be adjusted slightly. Subsidies directed at reducing the effective prices faced by consumers can, unlike in standard markets, improve social welfare even if we imagine all of the firm's profits being burned. This effect arises because externalities across the two sides of the market can be substantial for a broad range of demand functions.

In terms of price balance, I take as a starting point the observation, by Rochet and Tirole (2003), that the social and consumer surplus-maximizing price balance involves charging a higher price to the higher average surplus group than a monopolist would choose, as this group gains more external benefits from the addition of partners on the other side of the market. My results detail the effects of this fact in significant additional detail and often conflict with the spirit of their findings. In particular, I show that the seemingly similar welfare criteria of social surplus, consumer surplus and volume maximization agree on price balance only with demand functions in a set of measure 0. Ascertaining the direction of their disagreement only requires knowledge of the relationship between average surplus on the two sides of the market. Under any form of private (monopoly or duopoly) governance, price balance may be so out-of-whack that one side of the market would like to make transfer payments to the other side. While balanced competition and price regulation are always beneficial as they reduce both prices, unbalanced competition can harm *both* sides of the market if it effects a transfer from a low average surplus group to a high average surplus group. Price

discrimination may have important benefits in two-sided markets, as it may facilitate transfer payments from high average surplus groups to low average surplus groups. In fact, price discrimination on one side of the market may benefit the discriminated *against* group.

In order to make the analysis more concrete, I also introduce a tractable and intuitive *linear vulnerability class* of demand functions that provide illustrative examples of the results. I go on to address the policy implications of the analysis. In antitrust, traditional doctrine that focuses on one price at a time is problematic in two-sided markets, given the possibility that competition can raise prices on one side of the market. Regulatory policy is also complicated by Rochet and Tirole's topsy-turvy principle. Additional informational and strategic problems may emerge in the regulation of price balance. Subsidies have a number of substantial benefits in two-sided markets beyond those they offer in standard markets. I conclude by discussing limitations and extensions of the paper. Some of these I plan to address in a future draft; others are left to future research.

The remainder of this paper is divided into seven sections. Section 2 discusses the relationship of my work to the existing literature on two-sided markets. Section 3 introduces and motivates the notion of vulnerability of demand that serves as my primary tool of positive analysis. Section 4 analyzes the positive price effects of competition, price regulation and subsidies. Section 5 addresses a variety of normative issues. Section 6 provides a brief summary of results. Section 7 discusses some of the implications of the analysis for policy. Section 8 concludes.

2   Relationship to the literature

The analysis in this paper builds directly on the canonical model of Rochet and Tirole (2003) [RT2003]. I believe, and hope to show in a future draft of this paper, that my results are substantially more general than this model. However, in its current form this paper is best seen as a framework for analyzing this canonical model. In fact, in the positive analysis that follows I take the first-order conditions that RT2003 shows characterize monopoly optimization and duopoly equilibrium in the model as the starting point for my analysis. In the normative analysis, I use welfare criteria that are either directly taken from RT2003 (in the case of consumer surplus) or are simple extensions of these (in the case of tax-augmented consumer surplus and social surplus). Furthermore the "topsy-turvy" effect that plays a crucial role in my analysis originates with RT2003.

However, it is worth noting that the spirit of my results are somewhat different than those of RT2003. In particular, they argue that "(P)rivate business models do not exhibit any obvious price structure bias (indeed, in the special case of linear demands, all private price structures are Ramsey optimal price structures)." By contrast, my results show that all private price levels are above the social optimum and that clear distinctions between socially and privately optimal price balance can be clearly identified except in extremely special (measure 0) cases. Rochet and Tirole were not wrong, since by "price structure" they refer to price balance given a particular price level and by "obvious price structure bias" they simply mean that (roughly by symmetry) one cannot tell *a priori* which way it is socially optimal to shift prices. Nonetheless the spirit of the policy implications of my work is somewhat different from theirs.

Another paper related to the results presented here is Chakravorti and Roson (2006) which tries to pin down the effects of competition on individual prices on the two sides of the market. By contrast to our results, Chakravorti and Roson (2006) conclude that competition always drives down prices on both sides

of the market. However, the Chakravorti and Roson (2006) argument is flawed[3]. While it is true that competition always reduces the sum of prices on the two sides of the market in their model (to which the arguments here apply), competition may raise prices on one side of the market.

The other paper most closely related to ours is the unifying survey by Rochet and Tirole (2006). They make several contributions that inform my analysis here. First, they provide a general definition of two-sided markets, emphasizing, as I do here, the joint importance of the failure of neutrality in pricing and externalities across the two sides of the market. Second, they develop a canonical model of two-sided markets which incorporates most previous work as a special case. The second point will be extremely important in a future draft of this paper, as I believe my results apply (under some reasonable assumptions) to this broad class of models.

Our work here also relates to the survey by Evans (2003), who cites RT2003 as showing that private price structures are roughly socially optimal. However, he also argues, along the lines of my results, that price level rather than balance is the most policy relevant variable. Evans claims, but does not cite results showing as I do, that competition always reduces price level, but may raise prices on one side of the market. In another finding related to ours, Kind and Nilssen (2003) find that sufficiently unbalanced competition may reduce welfare in a model of advertising. Laffont et al. (2003) also emphasize, along the lines of my welfare results, the potential desire of consumers to make transfer payments. Kaiser and Wright (2005) provide empirical support for the topsy-turvy effect.

3   The "vulnerability" of demand

A primary purpose of this paper is to ask to what extent and in what ways our intuitions about industrial structure, welfare and regulation from standard, one-sided markets carry over into two-sided markets. Therefore it is useful to return briefly to the familiar monopoly pricing problem in standard markets to develop intuition for analyzing two-sided markets and to motivate the primary analytical tool used below understand price dynamics in two-sided markets.

Consider the problem of a monopolist in a standard, one-sided market facing consumer demand $D(\cdot)$ and per-unit cost of production c. The familiar first-order condition is given by:

$$\frac{p}{\eta(p)} = \gamma(p) \equiv -\frac{D(p)}{D'(p)} = p - c \equiv m \tag{1}$$

where $\eta(p)$ represents the elasticity of demand. Thus γ is the ratio of price to elasticity of demand, a sort of price- or value-weighted inverse elasticity. In order to ensure satisfaction of second order conditions, I assume that γ is downward sloping which is equivalent[4] to demand being log-concave. When I consider two-sided markets I assume demand is log-concave on both sides.

Notice that whenever γ is greater than the monopolist's mark-up m, the monopolist has a strict incentive to raise (lower) prices; thus γ seems to capture how "exploitable" or "vulnerable" consumers are at a given price. If demand is more vulnerable than the current mark-up the monopolist is charing, it is in her interest to raise prices; if demand is less vulnerable than the current mark-up, the monopolist is charging, she has

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[3]A simple counterexample is available upon request.

[4]To see this equivalence, note that γ is just the negative inverse of the derivative of the logarithm of D. Therefore its being decreasing is equivalent to demand being log-concave.

overreached and should lower her prices. She maximizes when her mark-up or "exploitation" is exactly equal to the vulnerability of demand. I therefore refer to γ as the consumers' vulnerability of demand or *vulnerability* for short.

In the next section, I will use the vulnerability to analyze the effects of competition on prices in two-sided markets. In order to understand the connection between these results in two-sided markets and our intuition in standard markets, it is useful to ask what vulnerability can tell us about the effects of competition on prices in one-sided markets.

Consider two firms selling differentiated but competing substitutes in a one-sided market. We assume that the firms are symmetrically differentiated: if the demand for firm 1's product when firm 1 charges price p and firm 2 charges price p' is $D(p, p')$ then the demand for firm 2's product when firm 1 charges p' and firm 2 charges p is $D(p, p')$. Furthermore, each firm faces linear cost c of production. For expositional clarity, I look for a symmetric Bertrand equilibrium (as I do in my analysis of two-sided markets); that is a Bertrand equilibrium where both firms charge the same price $\bar{p}$. By analogy to the problem of the monopolist analyzed above, if $D(p, \bar{p})$ is log-concave in its first argument, then it will be optimal for firm 2 to charge $\bar{p}$ if and only if:

$$\gamma_o(\bar{p}) \equiv \frac{p}{\eta_o(\bar{p})} = \bar{p} - c \equiv \overline{m}$$

where own-price elasticity of demand $\eta_o(p) \equiv -\frac{pD_1(\bar{p},\bar{p}))}{D(p)}$ and $D_1(\cdot, \cdot)$ denotes the derivative of demand with respect to its first argument. Thus symmetric Bertrand equilibrium occurs at the price that equates margin to *own-price vulnerability of demand*, the analog of vulnerability of demand in a duopoly setting.

Alternatively, we could consider a monopolist[5] owning both firms. Assuming, again for expositional simplicity, that the monopolist (optimally) sets symmetric prices for the two symmetric services, she faces precisely the same problem as discussed above, except that demand is now $2D(p) \equiv 2D(p, p)$. So again if

$$\gamma(p) = -\frac{D(p)}{D'(p)} = -\frac{2D(p)}{2D'(p)} = -\frac{2D(p)}{2\Big(D_1(p) + D_2(p)\Big)}$$

the monopolist maximizes (assuming that $D(p)$ is log-concave) where:

$$\gamma(p) = m$$

To avoid confusion with own-price vulnerability, let me refer to γ as *total vulnerability of demand*. Now note that by the definition of the two firms' products being substitutes, we have that $D_2(\cdot, \cdot) > 0$, so for all p we have:

$$\gamma_o(p) = -\frac{D(p)}{D_1(p)} < -\frac{D(p)}{D_1(p) + D_2(p)} = \gamma(p)$$

Because $D_1(p) < 0$ is the dominant term and the addition of D_2 reduces its magnitude. Thus own-price vulnerability of demand always lies below total vulnerability of demand. Because $p - c$ is obviously increasing in p, this means that $m = \gamma(p)$ always occurs at a higher level[6] of margin (price) than $m = \gamma_o(p)$.

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[5]Or an efficient, price-setting cartel.

[6]Formally, if $p^\star$ solves $p^\star - c = \gamma_o(p^\star)$ then clearly $\gamma(p^\star) > p^\star - c$. Because $\gamma(p)$ is declining and $p - c$ is increasing in p,

Thus, competition reduces prices relative to those charged by a monopolist. The intuition is very familiar: when a firm faces competition, raising prices is more dangerous (demand is less vulnerable) because the competitor, who does not simultaneously raise her prices, will steal some customers. Therefore a competitor will have a greater incentive to hold down prices than a monopolist; when demand is less vulnerable due to competition, it cannot be exploited to the same extent as under monopoly.

This analysis immediately raises the question of whether the same reasoning applies in a two-sided market. The following section demonstrates that in fact it does when one considers the price level, rather than the individual prices, and constructs a vulnerability of demand aggregated across the two sides of the market.

4 Positive Analysis

In this section I consider the price effects of competition in a two-sided market. Rather than deduce the first order conditions governing monopoly optimization and Bertrand equilibrium in a two-sided market, I take these as given and use them as starting points of analysis. This approach is useful because I believe that the arguments used to analyze these first order conditions are more general than the RT2003 model from which they originate. I hope to prove in a future draft that the arguments used to analyze the first-order conditions apply significantly more generally. Furthermore, analyzing the first-order conditions directly allows me to abstract away from a number of complex details of the structure of demand and competitive interactions that obscure the main arguments.

4.1 Competition and the Price Level

Recall that in the model I consider, a monopolist owns two platforms and operates them for their joint profits. The two platforms are symmetrically differentiated and I consider only symmetric pricing strategies. I denote by p_M^B and p_M^S the prices charged by a monopolist for the use of each of the two platforms to the buyer's and seller's side of the market. If $D^B(p_M^B)$ is total demand on the buyers' side of the market and $D^S(p_M^S)$ is total demand on the sellers' side of the market then the monopolist's profits are:

$$(p_M^S + p_M^B - c)D^B(p_M^B)D^S(p_M^S)$$

Rochet and Tirole (2003) show that the first order conditions for the maximization of the monopolist's profits are given by:

$$m_M \equiv p_M^S + p_M^B - c = \gamma^B(p_M^B) = \gamma^S(p_M^S) \tag{2}$$

Where

$$\gamma^i(p_M^i) \equiv -\frac{D^i(p_M^i)}{D^{i\prime}(p_M^i)}$$

is declining in its argument by my assumption of log-concave demand. Thus the monopolist's optimization is analogous to the case of a standard market. The monopolist sets her mark-up equal to the

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it must be that the $p^{\star\star}$ that solves $\gamma(p^{\star\star}) = p^{\star\star} - c$ has $p^{\star\star} > p^{\star}$. For a clearer and more detailed argument, see the proof of the two-sided case in the following section.

vulnerability of demand on each side of the market. Thus she, just as in a standard market, equates vulnerability to mark-up; however she also ensures that vulnerability of demand is the same on both sides of the market. Intuitively, if vulnerability of demand is higher on one side of the market than the other, then by raising prices to the more vulnerable side and lowering them to the less vulnerable side the monopolist can raise her volume while leaving her mark-up unchanged by balancing the market. Equating vulnerability on the two sides of the market thus corresponds to the monopolist trying to optimally "get both sides on board".

The simultaneity of the monopolist's price level and balance decisions makes understanding the monopoly pricing problem more complex than in standard markets. It is therefore useful to construct an analog to the single vulnerability in standard markets by composing together the vulnerability of demand on the two sides of the market into a *vulnerability level of demand* that is a function of the *price level*, the sum of the prices on the two sides of the market: $q_M \equiv p_M^B + p_M^S$. Using this notation, we can rewrite equation 2:

$$m_M \equiv q_M - c \;=\; \gamma^B(p_M^B) = \gamma^S(q_M - p_M^B) \tag{3}$$

I define the vulnerability level in an intuitive manner: for a given price level q_M the vulnerability level is the vulnerability at which the monopolist achieves her optimal balance between buyers' and sellers' prices, given that price level. Formally let $\overline{\gamma}(q_M) \equiv \gamma^B\left(\overline{p}_M^B(q_M)\right)$ where $\overline{p}_M^B(q_M)$ solves $\gamma^B(p_M^B) = \gamma^S(q_M - p_M^B)$ given q_M. Thus if I can show certain properties of $\overline{\gamma}$, the monopolist's optimal choice of price level is given by $q_M - c = \overline{\gamma}(q_M)$ just as in a standard market.

First, however, I must show that if demand on both sides of the market is concave, then the vulnerability level behaves roughly like a standard vulnerability in a one-sided market with log-concave demand. Namely, I want that $\overline{\gamma}$ be a well-defined function, that it be always positive and, crucially, that it be downward sloping (a sort of demand level log-concavity). So long as any monopoly optimum exists for any price level[7], all of these properties are obvious.

Because γ^B, γ^S are both downward sloping (as demand on both sides of the market is log-concave), $\gamma^S(q_M - p_M^B)$ is upward sloping in p_M^B given q_M and $\gamma^B(p_M^B)$ is downward sloping in p_M^B. Thus given any q_M, $\gamma^S(q_M - p_M^B)$ and $\gamma^B(p_M^B)$ have a unique crossing (as a strictly decrease function and a strictly increasing function cannot intersect more than once) and therefore (so long as an optimum exists for any price level) have exactly one point of intersection[8] for any price level q_M. Furthermore, clearly $\overline{\gamma} > 0$ as $\gamma^B, \gamma^S > 0$.

Finally, it is easy to see that $\overline{\gamma}$ is downward sloping. Note that $\overline{p}^B(q)$ is defined implicitly by solving $\gamma^B(p^B) = \gamma^S(q - p^B)$. Thus by the implicit function theorem and the fact that $\gamma^{i'} < 0$ for both i, we have that:

$$\overline{p}^{B'}(q) = \frac{\gamma^{S'}\left(q - \overline{p}^B(q)\right)}{\gamma^{B'}\left(\overline{p}^B(q)\right) + \gamma^{S'}\left(q - \overline{p}^B(q)\right)} > 0$$

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[7]Formally, $\exists s, t : \gamma^B(t) < \gamma^S(s)$ and $\exists s', t' : \gamma^S(s') < \gamma^B(t')$.

[8]Under the hypotheses stated in footnote 5, let $P_M^{B'}(q_M) \equiv max\{t, q_M - s\}$ and let $p_M^{B''}(q_M) \equiv min\{t', q_M - s'\}$. Then clearly if we let $h(q_M, p_M^B) \equiv \gamma^B(p_M^B) - \gamma^S(q_M - p_M^B)$ the equation has a solution where $h(p_M^B) = 0$. Furthermore clearly $h(p_M^{B'}(q_M)) < 0$ and $h(p_M^{B''}(q_M)) > 0$. But h is continuous as it is the difference of two continuous functions (as I assumed demand is continuously differentiable). Thus by the intermediate value theorem, there exists a value $p^B(q_M)$ solving the equation $\gamma^B(p_M^B) = \gamma^S(q_M - p_M^B)$ for any price level q_M.

Thus $\bar{p}^B$ is increasing in the price level, and because γ^B is declining, $\overline{\gamma}(q)$ is decreasing. Thus, under some weak technical conditions, the properties of the vulnerability on each side of the market transfer over to the vulnerability level, enabling me to apply the same techniques I used in standard markets to analyze the price level in two-sided markets. Our strategy is thus to separate the monopolist's pricing decision into two tractable components:

- A price level decision, entirely analogous to that in a standard market.

- A price balance decision determined by equating vulnerabilities on the two sides of the market.

I harness this division to first isolate and analyze the effects of competition on the price level. Then I reintroduce the effects of competition on price balance in order to get a sense of the potential consequences of competition for the individual prices on the two-sides of the market. It is worth noting that my approach here contrasts with that of RT2003, which does not consider the effect of competition or policy on the price level or individual prices, instead focusing on how price balance, given a particular price level, differs across different industrial organizations.

In order to address competition, however, one first needs to ask what conditions characterize Bertrand equilibrium. We therefore consider the two platforms, owned above by a single monopolist, being run separately in competition. I consider Bertrand equilibria where each platform charges the same price usage price p_C^B to the buyers and the same price p_C^S to the sellers. Because of the complex dynamics of competition in two-sided markets, I avoid here the derivation of the following conditions characterizing a symmetric equilibrium and instead refer to RT2003 who show that for $\left(p_C^B, p_C^S\right)$ to form a Bertrand equilibrium it is necessary that:

$$m_C \equiv p_C^S + p_C^B - c = \gamma_o^B(p_C^B, p_C^S) = \gamma_o^S(p_C^S, p_C^B) \tag{4}$$

where γ_o^i is the own-price vulnerability of demand on side i; again I do not formally define this object here as there are several possible definitions that yield the requirement that Bertrand equilibrium satisfy equation 4 above. For maximal generality[9], I allow γ_o^i to depend on prices on each both of the sides of the market. The single, intuitive hypothesis I require[10] is that the two products be substitutes in the sense that $\gamma_o^i(p^i, p^j) < \gamma^i(p^i)$. That is, demand on each side of the market is less vulnerable (more elastic) under competition than under monopoly for any combination of prices on the two sides of the market. Given how little I focus on the derivation of the conditions characterizing competition and monopoly, it is useful to think of these conditions as the hypotheses of my theorems (the starting point of my analysis) rather than as results.

Proposition 1. *Maintain the above assumptions and suppose that the monopolist's (unique) optimal price level $q_M^\star = p_M^{B\star} + p_M^{S\star}$ and any (of potentially many) Bertrand equilibrium price levels $q_C^\star = p_C^{B\star} + p_C^{S\star}$ satisfy the first order conditions above.*

Then any price level consistent with Bertrand equilibrium $q_C^\star$ is strictly less than $q_M^\star$, the unique optimal monopoly price level.

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[9]In RT2003 only seller's side own-price vulnerability depends on both prices, and buyer's side own-price vulnerability is downward sloping. Because I do not need either of these properties, I do not invoke them.

[10]This easily holds in the case of RT2003 as $\gamma_o^S(p^S, p^B) = \sigma(p^B)\gamma^S(p^S)$ where $\sigma \in (0, 1)$ and $\gamma_o^B(p^B) < \gamma^B(p^B)$ by the substitutability of the two platforms.
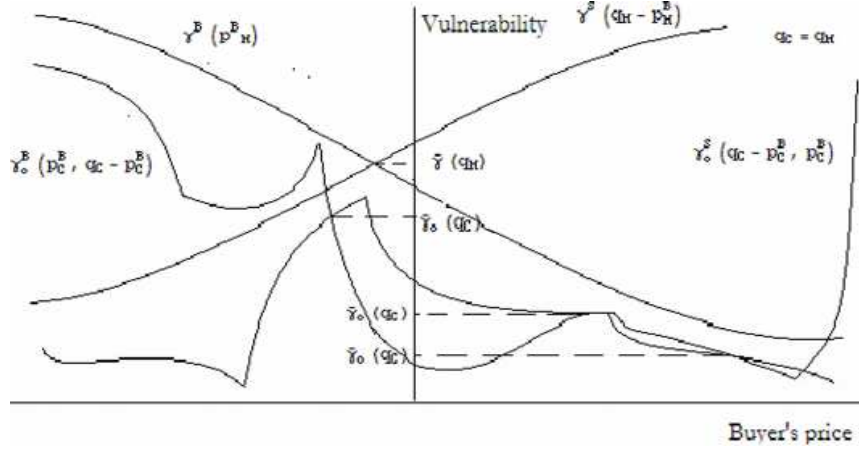
Figure 1: All values of own-price vulnerability level lie below total vulnerability level.

Proof. The proof proceeds in a manner entirely analogous to my argument in the preceding section about the effects of competition on prices in a standard, one-sided market. There I argued that because vulnerability is lower under competition, competitors will have an incentive to charge lower margins than monopolists. In order to make the same argument here, I must establish the sense in which vulnerability is lower under competition than under monopoly. In particular I use the notion of vulnerability level discussed and derived in the case of monopoly above. For clarity, I now refer to this as total vulnerability level and derive its Bertrand analog, own-price vulnerability level, which is relevant in the case of competition.

Let the *own-price vulnerability level of demand* $\overline{\gamma}_o(q) \equiv \gamma_o^B(\overline{p}^B(q))$ where $\overline{p}^B(q)$ solves $\gamma_o^B(p^B, q - p^B) = \gamma_o^S(q - p^B, p^B)$ given q. Note that I assume nothing whatsoever about the shape of γ_o^B and γ_o^S so $\overline{\gamma}_o$ may well be undefined[11] or multi-valued. Our claim is simply that *if* there is a Bertrand equilibrium *every one* must have a lower price level than optimal monopoly pricing.

Now I show that the own-price vulnerability level is always below the total vulnerability level; that is all values of $\overline{\gamma}_o(q) < \overline{\gamma}(q)$ the unique total vulnerability level. Figure 1 demonstrates the basic reasoning. Formally, if both γ_o^i lie below both γ^i, it is clear that two own-price vulnerabilities are each "trapped beneath" their respective total vulnerabilities and therefore that the intersections of the two own-price vulnerabilities must always lie below the intersection of the two total vulnerabilities. Formally, suppose that, in contradiction of the claim, there is a value of $\gamma_o(q) \geq \gamma(q)$. Then $\exists \overline{p}^B, \overline{p}^{B'} : \gamma_o^B(\overline{p}^B, q - \overline{p}^B) = \gamma_o^S(q - \overline{p}^B, \overline{p}^B) \geq \gamma^B(\overline{p}^{B'}) = \gamma^S(q - \overline{p}^{B'})$. Now note that $\gamma^B(\overline{p}^B) > \gamma_o^B(\overline{p}^B, q - \overline{p}^B) \geq \gamma^B(\overline{p}^{B'})$ by hypothesis. By log-concavity γ^B is decreasing, so it must be that $\overline{p}^{B'} > \overline{p}^B$. But note too that by the same argument, $\gamma^S(q - \overline{p}^B) > \gamma_o^S(q - \overline{p}^B, \overline{p}^B) \geq \gamma^S(q - \overline{p}^{B'})$ so, as γ^S is also decreasing by log-concavity, $\overline{p}^B > \overline{p}^{B'}$. But clearly these contradict one another. Thus any value of $\overline{\gamma}_o(q) < \overline{\gamma}(q)$.

Now I show, along the lines of reasoning in standard markets, that $\overline{\gamma}_o$ lying below $\overline{\gamma}$ implies that any intersection between $\overline{\gamma}_o(q)$ and $q - c$ must occur at a lower price level than the unique intersection of $\overline{\gamma}(q)$

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[11]Weak conditions on γ_o^i, such as continuity and positivity, ensure that, given the existence of a monopoly optimum, a Bertrand equilibrium also exists.

and $q - c$. Note that this second intersection is, in fact, unique and exists by precisely the same arguments as in standard markets. One can see that any intersection of $\overline{\gamma}_o(q)$ with $q - c$ must occur at a lower price level than this unique intersection by exactly the same reasoning in standard markets. Suppose that, to the contrary of this claim, there is a value $\exists q > q'$ such that q equates some value of $\overline{\gamma}_o(q)$ with $q - c$ and q' equates $\overline{\gamma}(q')$ with $q' - c$. Clearly for this value of $\overline{\gamma}_o(q), \overline{\gamma}_o(q) = q - c > q' - c = \overline{\gamma}(q')$. But I showed $\overline{\gamma}$ is decreasing so $\overline{\gamma}(q') > \overline{\gamma}(q)$. But by hypothesis $\overline{\gamma}(q) > \overline{\gamma}_o(q)$ so $\overline{\gamma}(q) > \overline{(q')}$. But this is clearly a contradiction, establishing that any price level equating $q - c$ with $\overline{\gamma}_o(q)$ is less than the level equating $q - c$ with $\overline{\gamma}(q)$.

But clearly any price level satisfying the conditions characterizing Bertrand equilibrium must equate some value of $\overline{\gamma}_o(q)$ with $q - c$ and the unique monopoly optimum equates $\overline{\gamma}(q)$ with $q - c$. Thus $q_M^\star > q_C^\star$ for any Bertrand equilibrium price level $q_C^\star$.

$\square$

The intuition behind the proof is precisely the same as in a standard market. A competitor, unlike a monopolist, has to worry about her opponent stealing her customers when she unilaterally raises prices; therefore, the demand she faces is less vulnerable and she sets lower prices. With respect to the price level (with price balance being chosen optimally) the same reasoning applies to the two-sided market competitor's incentives relative to those of a two-sided market monopolist. If own-price vulnerability is lower on both sides of the market than total vulnerability, it is intuitive that the overall own-price vulnerability level should be lower than the total vulnerability level, for any given price level.

4.2 The Balance of Competition

We may be interested in more than merely the price level; the individual prices on the two sides of the market may have important normative and policy implications. Therefore one needs to consider the effects of competition on price balance, as well as on price level. While we know competition always reduces the price level, it may be that competition is so much more intense for one side of the market than for the other that the resultant adverse shift in price balance more than offsets, for one side of the market, the effect of competition on the price level. Competition tends to reduce price on both sides of the market; but this reduction in prices has a secondary effect. When prices fall on one side of the market, this reduces the incentive the firms have to attract consumers on the opposite side of the market; therefore, it tends to incentivize the firms to raise prices on the other side of the market. It may be that, for one group of consumers, this topsy turvy effect outweighs the first effect of competition to reduce prices on each side.

4.2.1 Completely unbalanced competition raises prices for the less competed-for side of the market

We know that the direct effect of competition is captured by its tendency to reduce the vulnerability of demand. Thus we should expect that one side of the market may face an increase in prices if its own-price vulnerability is very close to its total vulnerability and if own-price vulnerability is significantly lower than total vulnerability on the other side of the market. A limiting case of this logic is what one might call completely unbalanced competition. Formally, I call competition *completely unbalanced* if $\gamma_o^i(p^i, p^j) = \gamma^i(p^i), \forall p^i, p^j$ but $\gamma_o^j(p^i, p^j) < \gamma^j(p^j), \forall p^i, p^j$. Interestingly, my analysis below will indicate that

in addition to being a natural limiting case of competition[12], completely unbalanced competition ends up encompassing an important form of price regulation, price regulation on one side of the market, leaving the other side unregulated. The following proposition establishes the price effects of such competition.

Proposition 2. *Suppose that competition is completely unbalanced. Then for any Bertrand equilibrium:*

$$p_C^{j^\star} < p_M^{j^\star}$$

but

$$p_C^{i^\star} > p_M^{i^\star}$$

That is, competition reduces prices on the competed for side of the market, but raises it on the other side of the market.

Proof. Note that the logic of Proposition 1 shows that any Bertrand equilibrium price level $q_C^\star$ lies strictly below[13] $q_M^\star$. At any Bertrand equilibrium and at the monopoly optimum, (own-price and total, respectively) vulnerability on side i of the market is equated to margin. Thus:

$$\gamma^i\left(p_C^{i^\star}\right) = \gamma_o^i\left(p_C^{i^\star}, p_M^{i^\star}\right) = q_C^\star - c < q_M^\star - c = \gamma^i\left(p_M^{i^\star}\right)$$

So $\gamma^i\left(p_M^{i^\star}\right) > \gamma^i\left(p_C^{i^\star}\right)$. But recall that γ^i is decreasing so it must be that $p_C^{i^\star} > p_M^{i^\star}$. In order for this to be the case and to still have $q_M^\star > q_C^\star$ it clearly must be the case that $p_C^{j^\star} < p_M^{j^\star}$ completing the proof. $\square$

The intuition behind the proof is simple. Completely unbalanced competition only puts pressure on prices on one side of the market. Without offsetting pressure on the other side of the market, the topsy-turvy effect implies that prices must rise on the other side of the market, even as the price level falls. In the physical analogy of the see-saw, completely unbalanced competition is like adding a weight to only one side of the see saw, which will tend to stretch the rubber band (lowering the level of prices); but it will also tend to raise the other side of the see-saw (prices on the other side of the market).

While the proof only considers this extreme case of perfectly unbalanced competition, it is clear by smoothness that there will be less extreme cases in which competition is simply very unbalanced and that these may also lead one side of the market to face higher prices under competition than under monopoly. Characterizing the exact set of conditions under which competition leads to an increase in price on one side of the market remains a difficult question[14]. Nonetheless, there is a strong intuition that competition

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[12]Due to space constraints, I do not include here an example of completely unbalanced competition arising from the primitive demand model of RT2003. However, examples are available. Intuitively, completely unbalanced competition favoring the sellers occurs when the products are perfectly differentiated for the buyers on the margin at the equilibrium price, but most buyers would prefer to use at least one platform rather than none. This occurs easily when services on one platform are worth some constant amount more to all buyers on their preferred platform. Completely unbalanced competition favoring the buyers is more difficult to construct, and requires taking a limit where marginal substitutability of the products gets very large at the equilibrium prices, but the number of consumers willing to switch cards if forced to is small. Then by taking the limit as the number of these "switchers" gets small, one obtains, in the limit, completely unbalanced competition favoring the buyers. Of course, these are both "limiting cases", but because the inequalities in the proposition are strict, the proposition continues to hold away from the limit.

[13]This is not precisely true. Proposition 1 assumed that both own-price vulnerabilities lied strictly below total vulnerability; here both own-price vulnerabilities lie only weakly below total vulnerability and one lies strictly below. It can easily be seen that the same logic carries through with only one strict inequality.

[14]Some heuristic investigation seems to yield a few hints towards a more general characterization. First, the relationship

is likely to lead to unbalanced price reductions, and even increases in prices to one side of the market, when the platform services are much more substitutable for one group of consumers than for the other. Therefore the effects of competition are likely to be unbalanced when the incentives created by competition are unbalanced.

4.2.2 Perfectly balanced competition reduces prices on both sides of the market

However, it is certainly not the case that competition always leads to such seemingly perverse outcomes. Competition may, and likely often does, lead to a reduction in prices on both sides of the market. So long as it is not the case that competition is much more intense for consumers on one side of the market than for those on the other, the topsy-turvy effect is not very large and is easily dominated by the general tendency of competition to reduce vulnerability and therefore prices. This is particularly clear when competition is *perfectly balanced*; that is, competition leads to no shifts in the price balancing incentives of the firms. Again, interestingly, in addition to providing intuition about the effects of competition, the perfectly balanced case turns out to encompass another interesting for of price regulation, namely a control on the price level leaving price balance unregulated. Formally, I call competition *perfectly balanced* if $\gamma_o^i(p^i, p^j) = \alpha \gamma^i(p^i), \forall p^i, p^j$ for both i and for a common $\alpha \in (0, 1)$. The following proposition shows the effect of this other extreme[15] form of competition.

Proposition 3. *Suppose that competition is perfectly balanced. Then we have $p_C^{i^\star} < p_M^{i^\star}$ for both i. That is, competition reduces prices on both sides of the market.*

Proof. First note that if $\overline{p}^B(q)$ solves $\gamma^B(p^B) = \gamma^S(q - p^B)$ for a given q then clearly it also solves $\gamma_o^B(p^B, q - p^B) = \gamma_o^S(q - p^B, p^B)$ for each q as:

$$\gamma_o^B\left(\overline{p}^B(q), q - \overline{p}^B(q)\right) = \gamma^B(\overline{p}^B(q)) = \gamma^S(q - \overline{p}^B(q)) = \gamma_o^S\left(q - \overline{p}^B(q), \overline{p}^B(q)\right)$$

Thus, given the price level, the price charged on each of the two sides of the market is the same under perfectly balanced competition as under monopoly; that is, perfectly balanced competition leaves the dynamics of price balance unchanged. Now from Proposition 1 we know that perfectly balanced competition reduces the price level. Note that $\frac{\partial\left(q - \overline{p}^B(q)\right)}{\partial q} = 1 - \overline{p}^{B'}(q)$. Therefore in order to show that competition reduces both prices one need simply show that $\overline{p}^{B'}(q) \in (0, 1)$ for all values of q. Recall that, from above:

$$\overline{p}^{B'} = \frac{\gamma^{S'}}{\gamma^{B'} + \gamma^{S'}}$$

But $\gamma^{S'}, \gamma^{B'} < 0$ by log-concavity. Thus $\overline{p}^{B'} \in (0, 1)$ completing the proof.

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between how far vulnerability on each side of the market falls in response to an increase in competition seems to be important. That is suppose that in a simple case $\gamma_o^i(p^i, p^j) = \alpha^i \gamma^i(p^i), \forall p^i, p^j$ where $\alpha^i \in (0, 1)$. Then competition seems to be imbalanced in favor of side i against side j sufficiently to cause it to raise j's price when $\log(\alpha^i) \ll \log(\alpha^j)$. Second, the slope of the vulnerability functions appear to matter. The side of the market with a less steep vulnerability function seems to be disfavored in terms of the effects of competition on price balance. That is, suppose that we are again in the situation where $\gamma_o^i(p^i, p^j) = \alpha^i \gamma^i(p^i), \forall p^i, p^j$. Suppose too that the slope of γ^i is β^i. Then competition will tend to be unbalanced in favor of side i of the market if $\beta^i > \beta^j$. These intuitions can likely be formalized, but it seems unlikely that a fully general characterization will be possible, especially when γ_o^i is allowed to depend on the prices on the other side of the market.

[15]Despite this being an extreme, simplifying example, RT2003 show that a Hötelling model of product differentiation gives rise to perfectly balanced competition.

□

Intuitively when competition is perfectly balanced, the dynamics of setting (privately) optimal price balance are not changed by the introduction of competition; that is, there is no topsy-turvy effect. Because the balance dynamics have not changed, it cannot be optimal for the competitors to set a higher price to one side of the market than was charged under monopoly, given that they are forced by competition to set a lower price level. In my physical analogy, perfectly balanced competition is like applying weights to both sides of the see-saw: the rubber band stretches, reducing both prices, but the see-saw's balance does not shift. This intuition, that a change in policy or industrial organization that reduces the price level while leaving the dynamics of price balance fixed will lower prices to both sides of the market, will be important to our analysis of price regulation, subsidies and taxes.

The results above are a mixed bag for our intuitions from standard markets. While, as one would expect, competition does drive down prices in the sense of reducing the overall price level, one should not expect that the individual prices on the two sides of the market will both be reduced by competitive pressures. In fact one price may rise as a consequence of competition. The analysis also provides some intuition as to when competition may have such unbalanced effects; namely when the competing platforms offer services that are significantly more substitutable for one group of consumers than the other. However, it is unlikely that one will generally be able to predict with much accuracy the effect of competition on individual prices. This has important implications for the welfare effects of competition and for the design of antitrust policy. The first of these is discussed in the following section. Policy considerations are deferred to Section 7.

4.3 An Application to Price Regulation

Before I move on to considering the welfare effects of competition, it is useful to consider another classic policy solution to the problem of market power: price regulation. For various reasons (usually the primary being economies of scale) economists have sometimes advocated the regulation of prices charged by monopolies as preferable to antitrust policy directed to introducing increased competition. Incentive[16] and political economy problems often present practical challenges to the implementation of price regulation. Nonetheless, it is useful to understand the abstract price theoretic effects of such policies to form clearer analytical intuitions about the similarities and differences between policy in one- and two-sided markets. Furthermore, as discussed in section 7, there are a number of recent policy debates centered on the price regulation of two-sided markets.

Considering a two-sided market with a monopolist providing services (on one or two platforms), there are a few types of price regulation that seem potentially interesting:

1. *Unilateral Price Controls (UPC)*: one option a regulator of a two-sided market monopolist has is to impose a price control on one side of the market and to leave the other side unregulated. That is, the regulator might require that the monopolist charge[17] a price no higher than $\overline{p}^i$ to side i of the market, but put no regulation on the prices charged to side $j \neq i$. I will assume that such regulation

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[16]Laffont and Tirole (1993) provide an excellent summary of theoretical research on incentive problems in regulation.

[17]In the case when the monopolist operates two platforms for their joint profit, this regulation would apply to both platforms.

is binding (otherwise it is uninteresting); that is, I assume that if the monopoly's optimal price is $p^{i^\star}$ then $\overline{p}^i < p^{i^\star}$.

2. *Price Level Controls (PLC)*: another option a regulator might exercise is to leave the individual prices on the two sides of the market unregulated, but place a cap on the price level the monopolist may charge. That is the regulator would require that $p^B + p^S \leq \overline{q}$, but permit the monopolist to charge any $p = (p^B, p^S)$ satisfying this requirement. Such a policy will be binding if the monopolist's optimal price level $q^\star < \overline{q}$.

3. *Price Balance Controls (PBC)*: some regulators might be more interested in the balance of prices[18] between the two sides of the market than in the level of prices. Therefore, the regulator might require that, holding the current price level $q^\star$ constant, the monopolist should raise the price to side i of the market from $p^{i^\star}$ to $\underline{p}^i$ and lower the price on side j of the market from $p^{j^\star}$ to $\overline{p}^j$. Such a policy is a PBC if $\overline{p}^j + \underline{p}^i = q^\star = p^{i^\star} + p^{j^\star}$. The policy is binding so long as $p^\star = (p^{i^\star}, p^{j^\star}) \neq (\underline{p}^i, \overline{p}^j) = \overline{p}$. A natural price control that falls in this domain would be to restrict the monopolist from price discriminating between the two sides of the market.

4. *Full Price Controls (FPC)*: the most robust approach a regulator might take is to place binding price controls on both of the individual prices. That is the regulator might choose prices $\overline{p}^B < p^{B^\star}$ and $\overline{p}^S < p^{S^\star}$ and require that the monopolist charge prices to the buyers and sellers, respectively, no higher than these.

The welfare implications of each of these policies is crucial and is considered below. However, also interesting are the *positive* effects of such policies on prices. For PBCs and FPCs these are immediately clear, the same is not the case for UPCs and PLCs. Namely, one might wonder what effect UPCs have on prices on the other side of the market and what effect PLCs have on price balance and whether PLCs may raise prices for one side of the market, as competition can. To maintain the focus of this section on the positive aspects of price theory in two-sided markets, I defer the discussion of PBCs and FPCs entirely to the following section and here focus only on UPCs and PLCs. This division works well for another interesting reason. Despite the rather stylized nature of the examples of competition provided in the preceding subsection, these extreme cases end up having direct relevance to the understanding of UPCs and PLCs. Surprisingly, PLCs can be seen as a special case of perfectly balanced competition and UPC's can be seen as a special case of completely unbalanced competition. Thus these seemingly unusual competitive circumstances are not only useful for demonstrating the possibility of certain competitive outcomes, but also for understanding the consequences of regulatory policies.

4.3.1 Unilateral price controls raise prices on the other side of the market

First consider the case of unilateral price controls (UPCs). Such price controls restrict the monopolist's ability to price on one side of the market, but not on the other side. This bears a striking similarity to the notion of completely unbalanced competition, in which competition only hinders the firms' pricing power on one side of the market. This connection can also be seen formally. Suppose that the (total)

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[18]While such an exclusive focus on balance seems quite odd, it has become a preoccupation of payment card industry regulators and antitrust authorities.
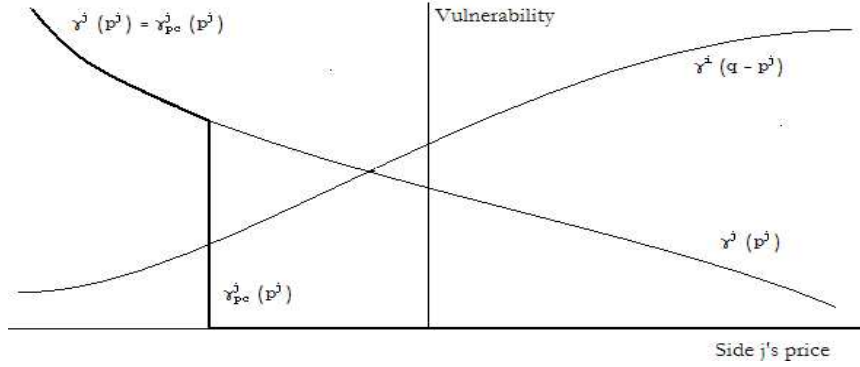
Figure 2: Monopolist's balancing decision unconstrained and under a unilateral price control.

vulnerability of side j of the market is given by $\gamma^j(p^j)$ and that a regulator imposes a UPC at prices $\bar{p}^j$ on prices for side j of the market. It is intuitive (and established formally in the appendix) that the monopolist's optimization subject to this price control is the same as her optimization without the control, except that $\gamma^j(p^j)$ is replaced by the correspondence:

$$
\gamma_{pc}^j(p^j) = \begin{cases} \gamma^j(p^j) & p^j < \bar{p}^j \\ [0, \gamma^j(p^j)] & p^j = \bar{p}^j \\ 0 & p^j > \bar{p}^j \end{cases}
$$

This function is pictured above in figure 2. With some slightly technical alterations covered in the appendix, one can see that this fits into the framework of Proposition 2: the price control, like unbalanced competition, reduces the vulnerability of demand on one side of the market, leaving the other side unaltered. Therefore, we know immediately that a (binding) unilateral price control reduces prices on the regulated side of the market and the overall price level, but increases the price on the unregulated side. Just as in the case of unbalanced competition, the intuition is the topsy-turvy effect, that lowering prices on one side of the market reduces the monopolist's incentive to restrain prices on the other side of the market. This result is stated formally in the following proposition.

Proposition 4. *Suppose that a regulator imposes a (binding) unilateral price control on side j of the market at price $\bar{p}^j$. Let $p^\star = (p^{i^\star}, p^{j^\star})$ with associated price level $q^\star$ be the monopolist's unconstrained optimal prices and let $p_{pc}^\star = (p_{pc}^{i^\star}, p_{pc}^{j^\star})$ with associated price level $q_{pc}^\star$ be the monopolist's optimal prices subject to the price controls. Then $p_{pc}^{j^\star} = \bar{p}^j$, $p_{pc}^{i^\star} > p^{i^\star}$ and $q_{pc}^\star < q^\star$.*

Proof. See appendix. □

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4.3.2 Price level controls reduce prices on both sides of the market

Now consider the case of a price level control (PLC). Just as in the case of perfectly balanced competition, a PLC does nothing to shift the dynamics of price balance between the two sides of the market. Rather, it simply forces the price level to fall. Therefore, again modulo some technical considerations addressed in the appendix, (binding) PLCs reduce prices on both sides of the market. This result is stated formally in the following proposition.

Proposition 5. *Maintain the notation of Proposition 4, but now suppose the regulator imposes a (binding) price level control at price level $\bar{q}$. Then $p_{pc}^{i\star} < p^{i\star}$ for both i and $q_{pc}^{\star} = \bar{q}$.*

Proof. By definition $\bar{q} < q^{\star}$, so by the sufficiency of the first order conditions, it must be the case that the monopolist chooses to charge $\bar{q}$. Now given that she chooses price level $\bar{q}$, my earlier discussion indicates that she charges individual prices $\bar{p}^{B}(\bar{q})$ and $\bar{q} - \bar{p}^{B}(\bar{q})$ to the two sides of the market. But from the proof of Proposition 3 both of these are decreasing in q and thus because $\bar{q} < q^{\star}$ the result follows immediately. $\square$

Thus the framework developed in the preceding two sections to address competition can easily be extended to understanding the price effects of these two types of price control. Furthermore, this extension provides a better intuition as to the meaning of "balanced" and "unbalanced" competition. Sometimes competition will tend (like a price level control) to put pressure mostly on the price level and have little effect on price balance; I call such competition balanced and it tends to reduce both prices. Other times competition will tend to put much more pressure on prices on one side of the market than those on the other side (like a unilateral price control); I call such competition unbalanced and it tends to raise prices on the unpressured side of the market, while lowering prices on the pressured side of the market and the overall price level. More general competition can be viewed as similar to some combination of a PLC and a UPC on one side of the market; the particular mix will depend on how substitutable the services of the two platforms are for one another to each side of the market.

4.4 An Application to Taxes and Subsidies

Another simple application of the framework developed here is to understanding the positive effects of various subsidies the government might give in a two-sided market. A basic feature of two-sided markets is the non-neutrality in allocation of prices between the two sides of the market. Therefore it is useful to consider whether this failure of price neutrality carries over to the analysis of taxation and subsidies in two-sided markets. Does it matter whether subsidies in a (monopolistic) two-sided market are given to buyers, sellers or the monopolist who serves the market? Second, given the potentially perverse effect of some price controls and forms of competition, it is worth asking what effects subsidies (or taxes) can have on the (effective) prices charged by a monopolist in a two-sided market. The following proposition provides a simple answer to these questions.

Proposition 6. *Suppose that a policy maker considers the possibility of providing a subsidy (charging a tax) of σ (-σ) to the buyers, sellers or monopolistic firm in a two-sided market.*

1. *Assuming the size σ of the subsidy is the same regardless of where it is given, all of these policies are payoff- and effective (cum-subsidy) price-equivalent to all agents at the optimal monopoly response prices.*

2. *Furthermore, if $\sigma > 0$ then the effective (cum-subsidy) price $p^{i^\star}_{sub}$ faced by either side i at the monopoly optimum with the subsidies is always strictly less than the monopoly optimal price $p^{i^\star}$ faced by that side of market without subsidies. When a tax is imposed ($\sigma < 0$) the opposite result obtains.*

Proof. First I want to show that the monopolist's problem is independent of where in the market the subsidy is given. To do this, I show that a subsidy to either side of the market is equivalent to a subsidy to the firm. If the firm is given a per-unit subsidy of σ, this is of course equivalent to reducing its cost by σ. Thus, given the subsidy, the monopolist solves:

$$p^i + p^j - (c - \sigma) = \gamma^i(p^i) = \gamma^j(p^j) \tag{5}$$

If side i of the market receives a subsidy σ this is equivalent to to reducing the effective price this side faces. Thus with a subsidy of σ to side i of the market the monopolist solves:

$$p^i + p^j - c = \gamma^i(p^i - \sigma) = \gamma^j(p^j) \tag{6}$$

Now if denoting by $\bar{p}^i \equiv p^i - \sigma$ the effective price paid by side i of the market. Then rewriting equation 6 as:

$$\bar{p}^i + p^j - (c - \sigma) = \gamma^i(\bar{p}^i) = \gamma^j(p^j)$$

Thus the monopolist's problem is precisely the same as under a direct subsidy to the firm, except that now price is called the effective price. Nevertheless consumers face the same effective prices, the monopoly receives the same effective prices, the monopoly earns the same profits and the welfare of all groups is identical.

Now I want to show that any of these forms of subsidies leads to lower prices on both sides of the market. To do this, I define $q^\star(\sigma)$ as the optimal monopolist price level for a given subsidy σ. Then $q^\star(\sigma)$ is defined implicitly by:

$$q^\star(\sigma) - (c - \sigma) = \gamma^i\left(\bar{p}^i\big(q(\sigma)\big)\right) = \gamma^j\left(q(\sigma) - \bar{p}^i\big(q(\sigma)\big)\right)$$

where $\bar{p}^i(q)$ is defined as earlier. Thus by the implicit function theorem (dropping arguments):

$$q^{\star\prime} + 1 = \gamma^{i\prime}\bar{p}^{i\prime}q^{\star\prime}$$

Recall from above $\bar{p}^{i\prime} = \frac{\gamma^{j\prime}}{\gamma^{i\prime}+\gamma^{j\prime}}$ so:

$$1 = q^{\star\prime}\left(\frac{\gamma^{i\prime}\gamma^{j\prime}}{\gamma^{i\prime}+\gamma^{j\prime}} - 1\right)$$

$$q^{\star\prime} = \frac{\gamma^{i\prime}+\gamma^{j\prime}}{\gamma^{i\prime}\gamma^{j\prime} - \gamma^{i\prime} - \gamma^{j\prime}} < 0$$

since $\gamma^{i\prime}, \gamma^{j\prime} < 0$. Thus $q^{\star\prime} < 0$. Recall from Propositions 3 and 5 that prices on both sides of the market are decreasing in the price level (if price balance is, as here, held constant). The result is that an increase in the subsidies decreases the effective prices faced by both sides of the market. Precisely the

reverse reasoning shows the result on taxes.

$\square$

Despite the lack of initial price neutrality in the monopolist's decision making in a two-sided market, the optimization of the monopolist internalizes any subsidies anywhere in the market equally. That is, the lack of incidence neutrality in a two-sided market is eliminated by the monopolist's privately optimal governance. If subsidies to the buyers lead to an effective buyers' price below the monopolist's optimal choice, she can always raise prices to the buyers and lower then to the sellers. This also hints at why subsidies always reduce prices on both sides of the market: because subsidies to the buyers raises the price received by the monopolist (for a given effective price she charges to the buyers) the topsy-turvy effect leads the monopolist to reduce prices to the sellers as well. Because subsidies do not shift the dynamics of price balance, their tendency to reduce the (effective) price level in fact brings down both of the individual prices on the two sides of the market, just as price level controls and perfectly balanced competition do. While the analogy is not quite as direct as in the case of price level controls, subsidies can also be viewed as related to perfectly balanced competition.

Thus the positive analysis of subsidies (or taxes) in two-sided markets accords well with our standard intuitions: physical allocation is irrelevant to economic allocation and subsidies reduce effective (cum-subsidy) prices. Both of these facts will be crucial to my normative analysis of subsidies, presented below.

5 Normative Analysis

This section is devoted to understanding the effects of various policies and industrial structures on welfare in two-sided markets. The crucial unifying theme that runs through all of the analysis is that of the externalities inherent to two-sided markets. Men who use dating websites benefit from more women using the website in ways that those women are not fully compensated for. Therefore policies that reduce prices paid by these women not only benefit the women, but also the men who now have more potential partners. This section is devoted to understanding the effects of such externalities on welfare and policy analysis in two-sided markets.

5.1 A Framework for Welfare Analysis

I adopt a simple and intuitive framework for welfare analysis under monopoly. I also consider the effects of competition on welfare. However, I abstract away from some important issues in competition, particularly the welfare effects associated with the choice of platform. I exploit a clever simplifying assumptions of RT2003 to focus on the welfare effects of prices and policies on *participation* in the market, leaving the effects of prices and policies on the *selection* of platform for future research. While this limits the robustness of my welfare results, there are informal reasons[19] to suspect they are even stronger when these assumptions are relaxed and thus I believe it provides a useful first pass at understanding the effects of competition policy in two-sided markets. In the case of monopoly, to which much of the analysis below is devoted, these assumptions provide more a more accurate guide to understanding welfare in two-sided markets.

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[19]That is, if there are coordination and platform selection problems, then competition may lead to a wider range of welfare effects than under the RT2003 model.

The crucial assumption, common to most of the literature on two-sided markets, is that of *multiplicative demand*. Formally, I assume that the total demand for services in the market (under monopoly) is given by $Q(p^B, p^S) = D^B(p^B)D^S(p^S)$ where p^B is the price charged to the buyers and p^S is the price charged to the sellers. This assumption is substantive in a few important ways:

1. It embodies the basic two-sided market externality (that one side of the market benefits from having more partners on the other side of the market) in the simplest possible way. Namely, it assumes that the benefit of the service to one side of the market is proportional to the number of participants on the other side of the market. For the welfare analysis that follows this is the crucial assumption and, I believe, a relatively innocuous one, primarily intended to make welfare calculation tractable.

2. It assumes relative independence of decision making on the two sides of the market. That is, it assumes that the decision of buyers to use the service is relatively independent of the decision of sellers to use the service. It therefore abstracts away from coordination problems between the two sides of the market, instead focusing on participation externalities.

Under the assumption of multiplicative demand, welfare criteria are easy to derive. The development here is take from Rochet and Tirole (2003). Transactions in two-sided market improve the welfare of consumers on each side of the market and of the shareholders of the firm serving that market. The second of these is easiest to treat. Given a set of prices $p = (p^B, p^S)$ and an associated price level $q \equiv p^B + p^S$ the profit earned by the firm serving the market is clearly $\pi^{firm} = (q - c)D^B(p^B)D^S(p^S)$. I will consider also a situation where the monopolistic firm is operated by a benevolent social planner, in which case profit (or loss) can be interpreted as the revenue raised from (or the cost of subsidies to) the market. As Rochet and Tirole calculate, the expressions for consumer surplus on each side of the market are similarly intuitive. The *conditional consumer surplus*, the value to consumers conditional on using the service, on side i of the market is given by $V^i(p^i) \equiv \int_{p^i}^{\infty} D^i(p)dp$. I assume that this surplus, on each side of the market, is log-concave. This assumption plays precisely the same role that log-concavity of demand does in the monopolist's optimization: namely it guarentees the sufficiency of first-order conditions and the existence of optima. *Unconditional consumer surplus*, the expected surplus earned, on side i of the market is given by:

$$D^j(p^j)V^i(p^i) \tag{7}$$

This is precisely the same as the expression for consumer surplus in a standard market, except that it is multiplied by the demand for the service on the other side of the market, incorporating the fundamental two-sided externality. For brevity, I will refer to conditional consumer surplus on side i of the market as "surplus on side i" and unconditional consumer surplus by its full name. In the analysis that follows, I will consider three welfare criteria in addition to the unconditional surplus of the two separate sides of the market. The first, and perhaps most compelling, is total (unconditional) social surplus:

$$\pi^{soc} = D^B(p^B)V^S(p^S) + D^S(p^S)V^B(p^B) + (p^B + p^S - c)D^B(p^B)D^S(p^S) \tag{8}$$

Second and also quite familiar is total (unconditional) consumer surplus, which I will refer to as consumer surplus:

$$\pi^{con} = D^B(p^B)V^S(p^S) + D^S(p^S)V^B(p^B) \tag{9}$$

Finally I will consider what I refer to as *tax-augmented consumer surplus* which takes into account the costs of subsidies to consumers. Suppose that the government offers a per-transaction subsidy of σ^S to the sellers and σ^B to the buyers. In this case the tax-augmented consumer surplus is given by:

$$\pi^{tax} = D^B(p^B - \sigma^B)V^S(p^S - \sigma^S) + D^S(p^S - \sigma^S)V^B(p^B - \sigma^B) - (\sigma^S + \sigma^B)D^B(p^B - \sigma^B)D^S(p^S - \sigma^S) \quad (10)$$

Of course, under such a subsidy scheme, consumer surplus on side i of the market also changes to $D^j(p^j - \sigma^j)V^i(p^i - \sigma^i)$.

5.1.1 Linear Vulnerability Class of Demand Functions

Throughout the normative analysis that follows it is often useful to temporarily restrict attention to a tractable, intuitive class of demand functions in order to provide examples of and intuitions for the propositions. Therefore, I briefly discuss here the *linear vulnerability class* of demand functions which are particularly tractable in analyzing two-sided markets.

Suppose that demand is of the form:

$$D(p) = \begin{cases} \frac{(a-p)^\alpha}{b} & p \le a \\ 0 & p > a \end{cases}$$

Then restricting attention to $p \le a$:

$$D'(p) = -\frac{\alpha(a-p)^{\alpha-1}}{b}$$

and:

$$\gamma(p) = -\frac{D(p)}{D'(p)} = \frac{a-p}{\alpha}$$

Note that if $\gamma' < 0$ then $\alpha > 0$ so log-concavity requires that $\alpha > 0$. Furthermore, note that demand is only positive (and decreasing in price) if $b > 0$. Also, note that vulnerability is linear in price; thus, I call this class of demand functions linear vulnerability[20]. The parameters a and b of this demand class have clear interpretations: a is the maximum price any consumer is willing to pay for the good and b is the inverse of the size of demand. However, α is a bit trickier. One useful way to conceive of α is by considering the relative curvature of demand:

$$\frac{D''(p)}{D'(p)} = \frac{\alpha(\alpha-1)(a-p)^{\alpha-2}}{\alpha(a-p)^{\alpha-1}} = \frac{\alpha-1}{a-p}$$

Thus normalizing price by how far it falls below the "intercept" a where demand is equal to 0, then $\alpha - 1$ measures the relative curvature of demand. If $\alpha > 1$ demand is convex and the higher that α gets, the more convex demand is. If $\alpha < 1$ demand is concave and the closer to 0 that α is the more concave is demand. Clearly if $\alpha = 1$ demand is linear.

It is also useful to calculate the consumer surplus of this demand class:

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[20]Note that if vulnerability is linear and demand is log-concave and decreasing, it is necessary, as well as sufficient, that demand be of this form if it has linear vulnerability.

$$V(p) = \int_p^a \frac{(a-p)^\alpha}{b} dt = \frac{(a-p)^{1+\alpha}}{b(1+\alpha)}$$

Thus average surplus (as discussed below) is:

$$\overline{V}(p) = \frac{V(p)}{D(p)} = \frac{a-p}{1+\alpha}$$

Note that because $V'(p) = -D(p)$, this shows that surplus is automatically log-concave in this class, given the restrictions we have placed on the parameters. Finally, I will later be interested in the relationship between vulnerability and average surplus. Note that:

$$\frac{\overline{V}(p)}{\gamma(p)} = \frac{\alpha}{1+\alpha}$$

Thus average surplus, relative to vulnerability, is bounded between 0 and 1. For some examples later, I will need that average surplus is greater than vulnerability. In order to obtain this, in the appendix I present an extension of the linear vulnerability demand class. The intuitive idea behind this extension is to imagine that in addition to the linear vulnerability consumers there is a small number of consumers that take a very large, fixed benefit from using the service. I take a limit as their benefit gets large and the number of the consumers gets small, while maintaining that the monopolist prefers to serve the linear vulnerability consumers, not just this small mass of high benefit consumers. This allows us to raise average surplus, relative to vulnerability[21]. In particular, a member of this extended class can be constructed at monopoly optimal prices $p_M^\star = \left(p_M^{i\star}, p_M^{j\star}\right)$ such that at prices[22] $p = \left(p^i, p^j\right)$ the vulnerability of demand is:

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[21]Note that this extended demand class is not smooth (and does not satisfy log-concavity in spirit even if it were smooth). However, it is easy enough to work with that it is worth checking the second-order conditions by hand, as discussed briefly in the next footnote and extensively in the appendix. I believe, but have not been able to show, that there exist log-concave demands with average surplus greater than vulnerability for at least some prices; but there do not exist demands with log-concave surpluses satisfying this property. To see that this is the case, note that the derivative of the logarithm of V is:

$$-\frac{D}{V}$$

And the derivative of this is:

$$-\frac{D'V + D^2}{V^2} = -\frac{D'}{V} - \frac{D^2}{V^2} = \frac{1}{\gamma \overline{V}} - \frac{1}{\overline{V}^2}$$

The sign of this expression is determined by the negative of the sign of (as $\overline{V} > 0$):

$$1 - \frac{\overline{V}}{\gamma}$$

Thus log-concavity of V requires that $\overline{V} > \gamma$ and thus it is necessary to step at least *somewhat* outside the class of demand functions I focused on for technical reasons below in order to generate some of the effects I want.

[22]If the market is being served by a monopolist, one has to be careful to ensure that at the new prices the monopolist still finds it optimal to serve the linear vulnerability consumers (and therefore to treat a demand function from this extended linear vulnerability class like one from the linear vulnerability class). In the applications of this class below, one always has to check that this is the case. Details on the derivations of these constraints are provided in the appendix. For now, I just note what they are. If the market is served by a monopolist, she will only choose to serve the linear vulnerability customers (as opposed to simply extracting all the surplus of the small mass of fixed-value consumers) if her optimal prices given the serves the linear vulnerability customers $p_M' = \left(p_M^{i'}, p_M^{j'}\right)$ have $p_M^{i'} < p_M^{i\star}$. In many of the cases I consider below, the monopolist will not have full control over her prices and so some of theses checks may be unnecessary. Furthermore, this analysis arises from the fact that in the appendix I construct the extended class to have the maximum possible average surplus, subject to the constraint that the monopolist chooses to serve the linear vulnerability consumers. This yields a particularly intuitive formula for average surplus (relative to vulnerability), but could be substantially relaxed and still yield $\overline{V}^i > \gamma^i$ so long as α_i is high

$$\gamma^i(p^i) = \frac{a_i - p^i}{\alpha_i}$$

just as in the standard, linear vulnerability case, but:

$$\frac{\overline{V}^i(p^i)}{\gamma^i(p^i)} = \frac{\gamma^i\left(p_M^{i\star}\right)D^i\left(p_M^{i\star}\right)}{\gamma^i(p^i)D^i(p^i)} + \frac{\alpha_i}{1+\alpha_i}$$

As a result, at the monopoly optimal prices:

$$\frac{\overline{V}^i\left(p_M^{i\star}\right)}{\gamma^i\left(p_M^{i\star}\right)} = 1 + \frac{\alpha_i}{1+\alpha_i} > 0$$

Note that γ^i and D^i are both declining in price, so for any price higher than $p_M^{i\star}$ and for some range of prices (potentially quite large if α_i is near 1) below $p_M^{i\star}$ this demand function has $\overline{V}^i > \gamma^i$.

5.2   Socially Optimal Price Balance

In order to accurately understand the welfare effects of various policies in two-sided markets, it is important to determine a natural benchmark against which various price outcomes will be judged. The most natural standard, of course, is that dictated by one of the common welfare criteria (social, consumer or tax-augmented consumer) discussed above. I now set out to determine what pricing scheme is optimal under each of these welfare standards. For maximum clarity, I separate, as in the positive analysis, optimal pricing into two components: an optimal price balance, holding the price level constant, and an optimal price level, holding the determinants of price balance constant. This subsection considers the first question; the following subsection addresses the second and applies the theory of optimal price level to the problem of subsidies in two-sided markets.

5.2.1   Socially Optimal Price Balance and Consumer Welfare Optimal Price Balance

The first analysis of socially optimal price balance in two-sided market is due to RT2003. They demonstrate that, supposing the price level is fixed at $\overline{q}$, the balance of prices that maximizes *consumer surplus* is given by:

$$\overline{V}^B(p^B)\gamma^B(p^B) = \overline{V}^S(p^S)\gamma^S(p^S) \tag{11}$$

Where $\overline{V}^i(p^i) \equiv \frac{V^i(p^i)}{D^i(p^i)}$ is the *average surplus* on side i of the market. By my assumption of log-concavity of surplus on each side of the market, average surplus is declining in price. Consumer surplus maximizing prices are related to the monopoly optimal prices which solve $\gamma^B(p^B) = \gamma^S(p^S)$, but also take into account the average surplus on the two sides of the market. In particular, as will be discussed further below, consumer welfare maximizing prices tend to be higher for the side of the market with greater average surplus than do monopoly optimal prices. The intuition for this comes from the existence of externalities in two-sided markets. Men on a dating website would like to, if they could (socially or legally), make side payments to women to attract more to the website; so too would women like to make payments to men.

enough, thereby easing the problem of the monopolist's choice to serve or not serve the linear vulnerability consumers. All of this is discussed extensively in Appendix F.

The structure of prices can implicitly allow such explicitly forbidden payments by charging more to the side of the market that would like to make greater payments (the side with higher average surplus) and using this increased price to cross-subsidize the other side of the market. Now of course both sides of the market would like to make payments so that this shift in price balance only represents the *net* payment consumers would like to make. However, constrained to a particular price level, it is (consumer welfare) optimal to adjust prices to reflect these optimal net side payments between the two sides of the market.

Now I consider the problem of socially optimal price balance, using total social surplus, rather than just consumer surplus, as our welfare criterion. Substituting $p^S = q - p^B$ into the expression from 8, total social surplus is:

$$\pi^{soc} = D^B(p^B)V^S(q - p^B) + D^S(q - p^B)V^B(p^B) + (q - c)D^B(p^B)D^S(q - p^B) =$$

$$\pi^{con} + (q - c)D^B(p^B)D^S(q - p^B) \propto \lambda\pi^{con} \pm (1 - \lambda)D^B(p^B)D^S(q - p^B)$$

$\pm$ is $+$ and $\lambda \equiv \frac{1}{1+q-c}$ when $q \geq c$ and $\pm$ is $-$ and $\lambda \equiv \frac{1}{1+c-q}$ when $c > q$. Thus when the price level is weakly above cost, maximizing social surplus is equivalent to maximizing a convex combination of consumer surplus and volume (the goal of the firm), where the weight on volume is proportional to $q - c$. When the price level is strictly below cost, on the other hand, maximizing social surplus is equivalent to a convex combination of maximizing consumer surplus and minimizing volume. Thus if price level is above cost, the social surplus optimum will lie somewhere between the consumer surplus optimum and the volume maximizing prices that would be chosen by the monopolist. This follows from the well-known fact that the maximizer of a convex combination of concave functions is a convex combination of their individual maximizers. On the other hand, if the price level is below cost[23], then the social optimum will be further away from volume maximization than the consumer welfare optimum is. To see this more clearly, I now derive the expression for the social surplus maximizing prices. To do this differentiate π^{soc} with respect to p^B:

$$D^{B'}(p^B)V^S(q - p^B) - D^{S'}(q - p^B)V^B(p^B) + (q - c)\Big[D^{B'}(p^B)D^S(q - p^B) - D^{S'}(q - p^B)D^B(p^B)\Big]$$

Some simple algebra allows the sign of the above expression to be determined by the sign of:

$$\lambda\Big[\gamma^B(p^B)\overline{V}^B(p^B) - \gamma^S(q - p^B)\overline{V}^S(q - p^B)\Big] \pm (1 - \lambda)\Big[\gamma^B(p^B) - \gamma^S(q - p^B)\Big]$$

where the $\pm$ has the same interpretation as above. Thus, just as discussed above, the incentives of the social surplus maximizing planner are a convex combination of those of the consumer surplus maximizing planner and those of the monopolist. Furthermore, equating this derivative to 0 is sufficient[24] to maximize

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[23]While this case may seem odd, I show below that it is in fact socially optimal.

[24]This is only obvious when $q \geq c$. On the other hand, when $q < c$ then it will only hold when $\lambda\overline{V}^i \geq 1 - \lambda$. This condition is the same as $V^i(p^i) \geq c - q$, which holds at optimal prices for any price level above the optimal price level, as is shown below in the section on optimal price level. However, we might worry about what happens away from the optimal prices. Note that given a price level, increasing one price implies decreasing the other. Thus if $\lambda\overline{V}^i \geq 1 - \lambda$ for both i and j given q, then suppose that one raises p^i from $p^{i^\star}$ to $p^{i'} > p^{i^\star}$ and $\lambda\overline{V}^i(p^{i'}) < 1 - \lambda$. Then $\big[\lambda\overline{V}^i(p^{i'}) - (1 - \lambda)\big] < 0$ and $\big[\lambda\overline{V}^j(q - p^{i'}) - (1 - \lambda)\big] > 0$ as $\overline{V}^j(q - p^{i'})$ is declining. Thus the first order condition given below still suffices as the marginal incentive for the social

social surplus (given a price level), since γ^i and $\overline{V}^i$ are both strictly decreasing and everywhere positive. Setting this derivative to 0 and rewriting it in a form parallel to the conditions prescribing consumer welfare-optimal price and volume maximizing prices:

$$\left[1 - \lambda + \lambda \overline{V}^B(p^B)\right]\gamma^B(p^B) = \gamma^S(p^S)\left[1 - \lambda + \lambda \overline{V}^S(p^S)\right] \tag{12}$$

when $q > c$. In this case, social welfare maximization is a combination of volume/profit maximization and consumer surplus maximization, where the weight on the former is proportional to $q - c$. When $q = c$ the firm earns no profits or losses and thus the social surplus maximizing prices are identical (given the price level) to the consumer surplus maximizing pries. When $q < c$ the the social surplus maximizing prices (given the price level) are further away from volume maximization than consumer surplus maximization:

$$\left[\lambda \overline{V}^B(p^B) - (1 - \lambda)\right]\gamma^B(p^B) = \gamma^S(p^S)\left[\lambda \overline{V}^S(p^S) - (1 - \lambda)\right] \tag{13}$$

5.2.2   Necessary conditions for agreement among welfare criteria

From the above it is clear that, in general, there will be conflict between the objectives of maximizing social surplus, consumer surplus and volume, objectives which tend to coincide in standard markets, including standard multi-product pricing problems[25]. Thus it is important to distinguish between objectives when devising policy. Surprisingly, though, RT2003 show that in the case when demand on both sides of the market is linear, consumer surplus maximizing prices and volume maximizing price balance[26] (given a particular price level) are the same[27]. One might wonder how general this result is. Should we expect that, constrained to a price level, a monopolist's goals roughly coincide with social goals[28]? That is, we clearly know that monopolists charge a price level higher than is optimal, but do they systematically tend to distort prices? How general is Rochet and Tirole's double-linear demand example? The following proposition demonstrates that this intuition is quite special indeed and that there is a conflict among these criteria in general.

Proposition 7. *Suppose that $p^{i\star}(q)$ solves volume (monopolist's profit) maximization[29] $\gamma^i(p^i) = \gamma^j(q - p^i)$ given q.*

Then $p^{i\star}(q)$ also solves for an open interval $N_{\overline{q}}$ of value of q about $\overline{q}$:

1. *Consumer welfare maximization: $\overline{V}^i(p^i)\gamma^i(p^i) = \overline{V}^j(q - p^i)\gamma^j(q - p^i)$*

2. *Social welfare maximization: $\left[\lambda \overline{V}^i(p^i) \pm (1 - \lambda)\right]\gamma^B(p^i) = \gamma^j(q - p^i)\left[\lambda \overline{V}^j(q - p^i) \pm (1 - \lambda)\right]$*

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planner, given q is still to lower p^i and raise p^j if p^i is above its optimal level given q. Now this only holds for price levels at or above the optimal price level. But this optimal price level is below cost, so it seems reasonable to restrict my attention to considering price levels at or above the optimal price level.

[25]See, for example, Laffont and Tirole (1993), section 3.1.

[26]Rochet and Tirole solve, equivalently if $q > c$, for monopoly optimal prices. However, here I want to consider cases when the price level may be at or below cost.

[27]Given that social surplus optimal prices lie "between" these when $q > c$, it is trivial to see that in this case, such price balance is also social surplus optimal.

[28]Rochet and Tirole (2006) imply that this is the case, claiming that "price balance is less likely to be distorted by market power than price level."

[29]The same reasoning shows that social surplus maximization and consumer surplus maximization agree only under the same conditions.

If and only if:

1. *Average surplus is identical on the two sides of the market:* $\overline{V}^i\left(p^{i\star}(\overline{q})\right) = \overline{V}^j\left(q - p^{i\star}(\overline{q})\right)$

2. *The slope of vulnerability is identical on the two sides of the market* $\forall q \in N_{\overline{q}}$: $\gamma^{i\prime}\left(p^{i\star}(q)\right) = \gamma^{j\prime}\left(q - p^{i\star}(q)\right)$

Proof. First note that if $p^{i\star}(q)$ solves $\gamma^i(p^i) = \gamma^j(q - p^i)$ given q then its solving

$$\overline{V}^i(p^i)\gamma^i(p^i) = \overline{V}^j(q - p^i)\gamma^j(q - p^i)$$

is clearly equivalent to solving:

$$\left[\lambda\overline{V}^i(p^i) \pm (1 - \lambda)\right]\gamma^B(p^i) = \gamma^j(q - p^i)\left[\lambda\overline{V}^j(q - p^i) \pm (1 - \lambda)\right]$$

By simple subtraction and division so long as $\lambda \neq 0$ (which is the case by the definition of λ). Thus it suffices to consider when $p^{i\star}(q)$ solves the first of these equations. $p^{i\star}(q)$ solving this first equation is equivalent to it solving:

$$\overline{V}^i(p^i) = \overline{V}^j(q - p^i)$$

since by assumption $\gamma^i\left(p^{i\star}(q)\right) = \gamma^j\left(q - p^{i\star}(q)\right)$. This yields the first condition in the proposition. For the second, note that for $\overline{V}^i\left(p^{i\star}(q)\right) = \overline{V}^j\left(q - p^{i\star}(q)\right)$ to continue to hold in some neighborhood around an $\overline{q}$ where it holds, it must be that:

$$\frac{d\left[\overline{V}^i\left(p^{i\star}(q)\right) = \overline{V}^j\left(q - p^{i\star}(q)\right)\right]}{dq} = 0$$

in this neighborhood. Calculating this derivative (dropping arguments):

$$\overline{V}^{i\prime} - \overline{V}^{j\prime} = \frac{D^{j2} + D^{j\prime}V^j}{D^{j2}}p^{j\star\prime} - \frac{D^{i2} + D^{i\prime}V^i}{D^{i2}}p^{i\star\prime} = \frac{\overline{V}^i}{\gamma^i}p^{i\star\prime} - \frac{\overline{V}^j}{\gamma^j}p^{j\star\prime}$$

Now recall from earlier that:

$$p^{i\star\prime} = \frac{\gamma^{j\prime}}{\gamma^{i\prime} + \gamma^{j\prime}}$$

for both i. If the hypotheses hold then clearly $\frac{\overline{V}^i}{\gamma^i} = \frac{\overline{V}^j}{\gamma^j} > 0$. So the sign of the expression is determined by:

$$\gamma^{j\prime} - \gamma^{i\prime}$$

Which is equated to 0, completing the "only if" direction of the proof. The reverse direction obtains from checking that the argument flows uninterrupted in the opposite direction.

<div align="right">□</div>

Thus only when average surpluses happen to coincide when vulnerabilities do *and* any reduction in the price level is evenly split between the two sides of the market (vulnerability have the same slope on both sides of the market) will the welfare criteria agree at more than a point. To see how special the resultant conditions are, the linear vulnerability case proves illustrative.

Corollary 1. *If both demands are of the linear vulnerability form $D^i(p^i) = \frac{(a_i - p^i)\alpha_i}{b_i}$ for both i then the hypotheses of Proposition 7 are satisfied for some $q < a_i + a_j$ only if*

$$\alpha_i = \alpha_j$$

That is, on a set of measure 0 under the Lebesgue measure over $(\alpha_i, \alpha_j) \geq 0$.

Conversely, suppose that $\alpha_i = \alpha_j$. Then for every $c < q < a_i + a_j$ the hypotheses of Proposition 7 hold.

Proof. Recall from above that in the linear vulnerability demand class $\overline{V}^i(p^i) = \frac{a_i - p_i}{1 + \alpha_i} = \frac{\alpha_i}{1 + \alpha_i} \gamma_i(p^i)$. Thus the first condition (in the second set of conditions) in Proposition 7 is satisfied at the monopoly optimum (where $\gamma^i = \gamma^j$) if and only if $\alpha_i = \alpha_j$. Furthermore, note that the slope of γ^i is everywhere $-\frac{1}{\alpha_i}$ so in this case the second condition is automatically satisfied for all q such that $p^{i\star}(q) \leq a_i$ and $q - p^{i\star}(q) \leq a_j$. But of course so long as $a_i + a_j > q > c$ the monopolist will always do better if demand is strictly positive than if it is 0. So $p^{i\star}(q) \leq a_i$ and $q - p^{i\star}(q) \leq a_j$ are equivalent to $c < q < a_i + a_j$, which completes the proof. $\qquad\square$

RT2003's bilaterally linear demand example clearly falls within this class with $\alpha_i = \alpha_j = 1$. Thus their linear result should be seen as extremely special (occurring on a set of Lebesgue measure 0 in the linear vulnerability class), though slightly more general than bilateral linearity of demand. The corollary furthermore gives a sense of when welfare criteria tend to conflict. If the convexity (concavity) of demand differs on the two sides of the market, then so will average surpluses relative to vulnerability. Thus[30], only in the very special case when the "shape" of demand coincides on the two sides of the market will volume (constrained monopolist profit maximization), consumer surplus and social surplus maximization coincide given a particular price level. Given this general conflict, the following analysis, with a few illustrative exceptions, focuses on the maximization of social surplus, as this seems the most strongly justified of the criteria. Hopefully, though, the preceding discussion gives some sense of how results might differ if consumer surplus had been my focus.

5.3   Subsidies and the Socially Optimal Price Level

One important element that distinguishes my normative analysis here from that of RT2003 is that they only consider "price structure", that is price balance constrained to a particular price level, rather than the socially optimal setting of price level. Some questions of specific interest to me are:

- Is the socially optimal price level in two-sided markets "at-(marginal)-cost" just as in standard one-sided markets?

- Can we say that preventing the elevation of the price level above cost associated with market power is systematically more (or less) important in two-sided markets than in standard markets?

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[30]This only holds for the linear vulnerability case, but I believe the intuition is general.

- Do subsidies have additional benefits in two-sided markets with (or even without) market power over their benefits in standard markets?

- Can subsidies improve tax-augmented consumer welfare in two-sided markets, as we know they never can in standard markets?

This subsection is addressed to answering these questions.

5.3.1 Socially optimal price level

First I consider the socially optimal price level, the price level maximizes that social welfare π^{soc}. Of course the social welfare-maximizing price level will depend on how, given a price level, prices are charged to the two sides of the market. In fact, if a change in the price level influences the balance of prices, we know, from my analysis of competition, that prices may rise on one side of the market even as the price level falls. Here, however, I want to abstract away from issues of price balance. Therefore I suppose that prices on each side of the market are increasing functions $p^i(q)$ of the price level. This covers several interesting cases:

- A benevolent social planner[31], operating a firm in a two-sided market, and seeking to maximize social welfare subject to the constraint that the price level is q will always optimally charge $p^i(q)$ that is increasing in q.

- Recall from my analysis of price regulation, subsidies and balanced competition that a monopolist maximizing profit subject to a constraint on the price level will charge prices to each side of the market that are decreasing in the required price level.

- This likely applies to duopoly and other environments where firms compete, but the demonstration of this is beyond the scope of this paper.

- Generally so long as a mechanism for decreasing the price level does not alter the dynamics of price balance, it will tend to reduce prices on both sides of the market.

So long as prices are an increasing function of the price level, the following proposition establishes a baseline result about the socially optimal price level.

Proposition 8. *Suppose a benevolent social planner seeks to maximize social welfare π^{soc} subject to the constraint that the individual prices on the two sides of the market are given by $p^B(q)$ and $p^S(q) = q - p^B(q)$, strictly increasing functions over which the social planner has no control. Then all of the following statements hold:*

1. *The price level $q^\star$ chosen by the social planner is strictly below cost c.*

2. *The marginal value to the social planner at any price level $q \geq c$ (and some $< c$) of decreasing the price level is strictly positive.*

3. *A marginal increase in the price level over cost causes a first-order harm to social welfare.*

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[31]This follows from precisely the same argument as with monopoly governance of price balance, as the functions on both sides of the equation in the social planner's balance problem are decreasing in their arguments by our assumptions.

Proof. Recall that total social surplus is given by:

$$D^B(p^B)V^S(p^S) + D^S(p^S)V^B(p^B) + (p^B + p^S - c)D^B(p^B)D^S(p^S)$$

or in this context:

$$D^B\left(p^B(q)\right)V^S\left(p^S(q)\right) + D^S\left(p^S(q)\right)V^B\left(p^B(q)\right) + (q - c)D^B\left(p^B(q)\right)D^S\left(p^S(q)\right)$$

Differentiating this with respect to q yields (dropping the arguments of the functions for spatial economy):

$$D^{B'}p^{B'}V^S - D^S p^{S'} D^B + D^{S'}p^{S'}V^B - D^B p^{B'} D^S + D^S D^B + (q - c)\left[D^{B'}p^{B'}D^S + D^{S'}p^{S'}D^B\right]$$

By assumption $p^{i'} > 0$ and $D^{i'} < 0, D^i > 0$ so the last term of the above expression is weakly negative for $q \geq c$. Furthermore clearly $p^{S'} + p^{B'} = 1$ by construction. Therefore the whole expression is at most:

$$D^{B'}p^{B'}V^S + D^{S'}p^{S'}V^B - D^B D^S + D^S D^B = D^{B'}p^{B'}V^S + D^{S'}p^{S'}V^B$$

But clearly $D^{i'} < 0$, $p^{i'} > 0$ and $V^i > 0$ so the above expression is strictly negative. This expression is exactly the magnitude of negative externalities (across the two sides of the market) that is associated with an increase in the price level. This establishes that the socially optimal price level cannot be weakly above cost and the other parts of the proposition. $\qquad\square$

In a two-sided market there are strictly positive externalities associated with the entrance of consumers on one side of the market. Whenever the price level is not below cost, the cost of serving an additional transaction is weakly negative. On the other hand, the marginal improvement in consumer surplus is not just, as in standard markets, given by the resources transferred to consumers by the price fall. Rather, the reduction in prices also has a direct positive externality on the other side of the market equal to $-D^{i'}(p^i)V^j(p^j)$; this implies that all three statements hold. Thus even when pricing is at cost (as one might expect under some form of perfect competition), policy, such as subsidies, that might reduce the price level further are socially desirable. Furthermore, just as we would be more concerned with market power in a sector of the economy that produced significant positive externalities than in one in which consumption is entirely private, we should likely pay special attention to policies designed to temper market power in two-sided markets. Market power, in addition to its standard, second-order distortive effect also causes a direct, first-order harm to welfare because even when the price level is at cost it is already away from the optimum.

Furthermore, it is useful to more explicitly characterize the socially optimal price level. In order to do this, one must make some assumption about how price balance is determined. A particularly natural and interesting case is when price balance is also chosen to be a social optimum. That is, in the problem of setting prices to maximize social surplus, the price level is characterized by the following theorem:

Proposition 9. *Let* $p^{\star\star}(q) = \left(p^{i^{\star\star}}(q), p^{j^{\star\star}}(q)\right)$ *be the socially optimal prices for a given price level q. Then the socially optimal price level is given by:*

$$q^{\star\star} = c - \overline{V}^i\left(p^{i\star\star}(q)\right) \tag{14}$$

For both sides i of the market.

Proof. See appendix. □

Intuitively, the optimal price level is below cost exactly by the amount of the positive externality that one side of the market confers, on the margin, to consumers on the other side. That is, the optimal subsidy (when the firm is run by optimal social price balancing planner) is equal to the marginal positive externality of the subsidized action at the optimum. This condition essentially just applies the theory of optimal taxation[32] and subsidies with externalities to the context of two-sided markets. To get a better sense of the comparative statics of the optimal price level with respect to the shape of the demand function, the following corollary specializes to the linear vulnerability of demand case.

Corollary 2. *Suppose that demand on both sides of the market is of the linear vulnerability form. Then:*

$$q^{\star\star} = a_i + a_j - \frac{a_i + a_j - c}{1 - \nu}$$

where

$$\nu \equiv \frac{1}{2 + \alpha_i + \alpha_j}$$

Proof. See appendix. □

Note that $q^{\star\star}$ is clearly decreasing in ν so long as $a_i + a_j > c$, which I assume, as otherwise the problem is uninteresting. Thus holding constant the "intercept" of demand, the optimal amount by which the price level should fall below cost is negatively related to $\alpha_i + \alpha_j$; that is, the more concave demand is, the more subsidies it is optimal for a social planner administering a two-sided firm to provide. This makes sense as the more concave demand is the higher[33] is average surplus at given prices.

5.3.2 Socially optimal subsidies

We know from Proposition 7 that subsidies to any group in a two sided market obey the hypotheses of Proposition 8. Therefore, as long as the effective price level is (weakly) above cost, an increase in subsidies is always beneficial (on the margin) to social surplus. However, it is not clear what happens when the effective price level falls below cost. That is, what is the criterion for determining the global socially optimal subsidy? The formula given above for the optimal price level depended on the social planner controlling price balance, as well as price level. Therefore it is not directly useful here. Because, from Proposition 7, subsidies can have no direct effect on a monopolist's choice of price balance, I must separately consider the optimal subsidy, taking into account how the monopolist chooses price balance given a particular price level. Optimal subsidies are characterized by the following proposition.

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[32]In fact, an interesting direction for future research would be the direct application of the theory of Ramsey pricing policy in two-sided markets. Even though my results here bear a striking resemblance to standard Ramsey pricing results, my approach to their derivation is less systematic.

[33]Average surplus is $\frac{a-p}{1+\alpha}$.

Proposition 10. *Let $p^{i^\star}(q)$ denote the monopolist's optimal choice of side i's price in the price balance problem when her incentive is to optimally charge price level q, given the subsidy she receives. Assume the following technical condition*[34] *sufficient for the existence of the optimal subsidy:*

$$\left(\overline{V}^i - \overline{V}^j\right)\left(\gamma^{i''}\gamma^{j'} < \gamma^{j''}\gamma^{i'}\right) - \left(\gamma^{i'} + \gamma^{j'}\right)\left(\frac{\overline{V}^i\gamma^{i'}}{\gamma^i} + \frac{\overline{V}^j\gamma^{j'}}{\gamma^j}\right)$$

where each function is evaluated at $q - p^{i^\star}$ for all functions on side j of the market and $p^{i^\star}$ for all functions on side i of the market. Then the social surplus optimal subsidy to a monopolist is the one that induces the monopolist to charge an effective price level $q^\star$ defined implicitly by:

$$q = c - \frac{\overline{V}^j\left(q - p^{i^\star}(q)\right)\gamma^{j'}\left(q - p^{i^\star}(q)\right) + \overline{V}^i\left(p^{i^\star}(q)\right)\gamma^{i'}\left(p^{i^\star}(q)\right)}{\gamma^{j'}\left(q - p^{i^\star}(q)\right) + \gamma^{i'}\left(p^{i^\star}(q)\right)} = c - \overline{V}^j\left(q - p^{i^\star}(q)\right)p^{i^{\star'}}(q) - \overline{V}^i\left(p^{i^\star}\right)\left(1 - p^{i^{\star'}}(q)\right)$$

Proof. See appendix. □

On its face, this result is a bit more cumbersome than the expression for the socially optimal price level when price balance is determined optimally. This makes sense because the social planner's problem is more difficult when she needs to consider how her subsidies are divided between participants on the two sides of the market. However, the optimum has a relatively clean form. Enough subsidies should be given so that the price level falls below cost by (roughly) an amount equal to the marginal externalities that would accrue if the price level were further reduced by a small amount. That is, the condition is essentially just an extension of the logic of Proposition 9, adding the fact that prices are not balanced socially optimally, so the optimal price level must take into account a weighted average of the average surpluses on the two sides of the market to capture the external effect of a reduction in the price level. Note that it is not clear whether the optimal price level in a market where balance is governed by a monopolist is above or below that in a market where balance is chosen socially optimally. On the one hand, when a monopolist governs the market, there may be more bang-for-the-buck in reducing the price level, since there are unrealized external benefits that the monopolist has not balanced. As a result one might think the optimal price level under monopoly is lower than under socially optimal price balance. This effect is reflected in the fact that, because the monopolist does not balance average surpluses, $\overline{V}^i + \overline{V}^j$ is higher under monopoly price balance than under optimal price balance. On the other hand, the monopolist does not allocate the reduction in the price level optimally, so subsidies may be "wasted" on her. Hence the optimal price level with a monopolist governing balance may be higher than under socially optimal price balance. This effect is reflected in the fact that $p^{i^{\star'}}(q)$ is not chosen so as to maximize the right hand side of the expression in Proposition 10, as it would be under socially optimal governance. In order to get more intuition for this unwieldy expression, I now specialize to the linear vulnerability case in the following corollary.

Corollary 3. *Suppose that demand on both sides of the market is of the linear vulnerability form. Then the technical condition in Proposition 10 is always satisfied and:*

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[34]This condition is not necessary for this existence, but is a standard sufficient condition. It is satisfied quite often (including always in the linear vulnerability and always when demand on the two sides of the market are sufficiently similar). One way to interpret it is that the relative curvature of vulnerability be not too much greater on the higher average surplus than the higher average surplus side of the market.

$$q^\star = a_i + a_j - \frac{a_i + a_j - c}{1 - \eta}$$

where

$$\eta \equiv \frac{\alpha_i \alpha_j}{(\alpha_i + \alpha_j)^2} \left[\frac{1}{1 + \alpha_i} + \frac{1}{1 + \alpha_j} \right]$$

Proof. See appendix. □

This condition bears a striking resemblance to the solution for the socially optimal price level under socially optimal price balance, except that ν is replaced by η. This shows how the two more general conditions are closely related to one another. The interpretation of the comparative statics of η are similar as well. Suppose that one constrains $\alpha_j = \theta \alpha_i$ for some $\theta > 0$ and allow α_j to vary; then α_i parameterizes the convexity of demand. Calculations in the appendix show that η is decreasing in α_i so, again as with ν, the socially optimal drop in the price level below cost is decreasing in α_i so that the more concave is demand the more that the price level should optimally fall below cost. The following corollary is useful for comparing the socially optimal price level given socially optimal price balance with the socially optimal price level under monopolistic price balance.

Corollary 4. *Suppose that demand on both sides of the market is of the linear vulnerability form. Then*

$$q^\star \geq q^{\star\star}$$

where the expression holds with equality if $\alpha_i = \alpha_j$ and with strict inequality otherwise.

Proof. See appendix. □

Thus, at least within the linear vulnerability class, the socially optimal price level under monopoly governance of the price balance is always above that under socially optimal governance of the price balance. In this case the intuition about the monopolist not optimally allocating reductions in the price level dominates; only when $\alpha_i = \alpha_j$ and the monopolist automatically sets the socially optimal price balance is the optimal price level the same under the two forms of governance. It is not clear how general this intuition is and this remains an interesting topic for future research.

5.3.3 Subsidies can improve pure tax-augmented consumer welfare

One common criticism of subsidies as a solution to monopoly distortions is that they are sops to the owners of the firm. More precisely, it is easy to show[35] that if the social welfare function puts no weight on the firm's profits, then subsidies are never desirable in the absence of externalities. The firm absorbs more than all of the efficiency gains from subsidies; subsidies act as a more-than-efficient transfer of wealth from the government to the firm. Economists will disagree about whether this argument is reasonable

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[35]This is obvious. Note that by the envelope theorem, a small subsidy only benefits consumers as a transfer to them from the state, and therefore is welfare neutral under tax-augmented consumer surplus assuming no reaction by the firm. However because the subsidy moves the effective price faced by consumers below the firm's optimal choice, the firm will raise prices harming tax-augmented consumer welfare. For larger subsidies, the argument becomes even stronger, as the payment of subsidies increases demand, costing the government more in subsidy payments. For formal analysis, see any industrial organization text, for example Tirole (1988), section 1.1.

and there are certainly other reasons to be wary of subsidies. Nonetheless it is worth considering whether the same argument might apply to two-sided markets. Disregarding the profits earned by the firm, is it ever socially useful to institute subsidies? The following Proposition establishes that in two-sided markets, unlike standard markets, subsidies may in fact improve pure tax-augmented consumer surplus.

Proposition 11. *Let the monopoly optimal prices be* $p^\star = \left(p^{B\star}, p^{S\star} \right)$. *Then there is a subsidy that improves tax-augmented consumer surplus (social surplus with no weight on the monopolist's profits) if[36]:*

$$\gamma^{S'}\left(p^{S\star}\right)\gamma^{B'}\left(p^{B\star}\right)\gamma^{i}\left(p^{S\star}\right) < \overline{V}^{B}\left|\gamma^{B'}\left(p^{B\star}\right)\right| + \overline{V}^{S}\left|\gamma^{S'}\left(p^{S\star}\right)\right|$$

For either i.

Proof. See appendix. □

Again, in order to make this a bit more concrete and connect it more directly to the shape of demand I specialize to the linear vulnerability case.

Corollary 5. *Suppose that demand on both sides of the market is of the linear vulnerability form. Then the condition of Proposition 11 is equivalent to:*

$$1 < \alpha_i \alpha_j \left[\frac{1}{1+\alpha_i} + \frac{1}{1+\alpha_j} \right]$$

Which in turn is equivalent to demand being convex if $\alpha_i = \alpha_j$ *and occurs when* α_i *is sufficiently convex with* $\alpha_j = \theta \alpha_i$ *for some* $\theta \in (0,1)$.

Proof. See appendix. □

There are a few things worth taking away:

1. There are additional benefits from subsidies in two-sided markets, above those in standard markets. It is possible for subsidies to be socially desirable even under the extreme assumption of entirely neglecting the profits of the monopolist. Thus subsidies may be significantly more appealing in two-sided markets than in standard markets. The reason is simple. Increased demand on one side of the market yields externalities to the other side of the market which a monopolist does not absorb.

2. The argument of the first point does not only happen in "weird" or specially constructed cases, but is true of a reasonable range of demand functions. I only consider a specific, easy to work with case, since broader analysis is even messier, but the intuition from this cases is that subsidies are particularly valuable to tax-augmented consumer welfare when demand is sufficiently convex. It seems unlikely that this simple convexity condition is sufficient (or necessary) for subsidies to be tax-augmented consumer surplus improving for a broader range of demand functions, or for asymmetric demands. However, the basic intuition that convexity of demand yields large average surplus relative to vulnerability and its derivative and therefore makes subsidies more attractive for improving tax-augmented consumer surplus seems to have broader applicability.

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[36]I believe, but have not been able to show, that this condition is necessary as well as sufficient for the existence of a tax-augmented consumer surplus improving subsidy.

3. It is interesting here that the intuition is very different than in the determination of socially optimal price level; in that case, the more *concave* was demand the more subsidies should be given. Here the more *convex* is demand the more desirable subsidies are if one does not value the welfare of the firm's shareholders. Evidently the more concave is demand the more that subsidies act as transfers to the firm. Thus which shape of demand is more encouraging of subsidies depends crucially on how one feels about the tendency of subsidies to act as transfers to the firm's shareholders.

4. There are two main intuitions in the proof of the theorem that are briefly worth considering. First, and this applies equally to standard markets[37], the amount of a subsidy that is passed on to consumers is inversely proportional to the rate at which the vulnerability (level) of demand is falling in price (which, in the linear vulnerability case is proportional to the concavity of demand). Intuitively, if vulnerability is steep, a monopolist will face a much higher vulnerability after subsidies are given to the two sides of the market and will react by undoing most of the the resultant decrease in effective prices. Second, the higher average surplus is relative to vulnerability, the more externalities the government gets for its buck. The less concave demand is the higher average surplus is relative to vulnerability.

Thus, both because they are social surplus enhancing even when the price level is at cost and because they can improve even a narrow definition of social welfare that excludes the profits of the firm, subsidies may have a more important role to place in policy in two-sided markets than in standard markets. A more detailed analysis of the policy implications of these results appears in section 7.

5.4 Competition and Price Regulation

Having discussed some more foundational questions about socially optimal pricing and to subsidies, I now return to three applications: antitrust, price regulation and price discrimination. First I consider the welfare effects of competition and price regulation, two of the most important policy levers economists consider to alleviate the harms caused by market power. From my positive analysis, the effects of competition and price regulation are intimately related to one another in two-sided markets. However, in two-sided markets, welfare analysis of competition is substantially more complex than the analysis of welfare under monopoly. The crucial problem is that competition may affect the selection of platforms on which the participants interact and not merely the fact of a transaction occurring.

To abstract away from these platform selection problems to focus on the welfare implications of platform participation, I make the rather extreme assumption of RT2003 in analyzing welfare. Namely, I assume that the platforms are only differentiated for one side of the market and that, at the symmetric price equilibria I consider, this side of the market chooses which platform is transacted on. This implies that welfare is only determined by participation and that the surplus on the side of the market for which the platforms are differentiated is just given by the surplus consumers on that side derives from the better of the two platforms for them. Thus, as RT2003 show, one can use precisely the same welfare analysis as above to treat the comparison between monopoly ownership of two platforms and duopoly ownership as I did to treat the effects of various policies on welfare under monopoly with a single platform. Furthermore,

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[37]As discussed in a footnote in the appendix which proves this proposition, vulnerability therefore offers a substantial simplification of standard formulae for the incidence of taxes and subsidies.

there is a clean, logical separation of the positive dynamics of competition, which relate to the degree of substitutability of the two platforms for the two sides of the market and are discussed above, and the analysis of welfare, which depends only on the resultant prices and is analyzed here. Thus, it suffices here to consider the possible effects on welfare of the different possible price consequences of competition and regulation.

5.4.1   A fall in both prices is welfare enhancing

The simpler of the cases I considered above was that of perfectly balanced competition or price level controls. I proved that these always reduce prices on both sides of the market. Therefore, by Proposition 8, so long as a price level control is not set below cost, both perfectly balanced competition and price level controls improve social surplus, consumer surplus and (it can easily be seen in the proof) welfare on both sides of the market. This simple, clean result is a significant contrast to the case when competition or price controls raise prices on one side of the market.

5.4.2   If one price rises, anything is possible

When competition is unbalanced, or when a unilateral price control is imposed, the price may rise on one side of the market while the price on the other side (and in fact the price level) fall; I call this situation an *unbalanced reduction in the price level*. In this situation, essentially anything goes. There the possible welfare effects of such an unbalanced reduction in the price level may have are various. It may be significantly more welfare-enhancing for both sides of the market than a balanced reduction in the price level or it may actually be welfare reducing! The critical point is that unbalanced reductions in the price level have two effects:

1. First, they reduce the price level which, by Proposition 8, is always welfare enhancing.

2. Second, they shift price balance. There is no *a priori* guarantee that such a shift will be either welfare-enhancing or welfare-reducing. Nor is there a guarantee as to whether this effect will be larger or smaller than the price level effect.

Whenever the fall in the price level is relatively small and the imbalance in the price level reduction is large, the second effect dominates. The welfare consequences of such change can differ broadly. The crucial determinant is what the welfare effects of transfer payments from one side of the market to the other are. If one side of the market would like (on average) to make transfers to the other side to encourage them to use the service more and the other side of the market would like (on average) to accept these payments, then an unbalanced price reduction that raises prices to the high average surplus group and reduces them for the low average surplus group is particularly attractive and the reverse is particularly unattractive. The following lemma provides a classification of the welfare effects of pure side payments which will be useful in understanding the welfare effects of unbalanced reductions in the price level and therefore of unbalanced price regulation (unilateral price controls) and competition. Before stating the lemma, it is worth clearly defining what I mean by a "transfer payment". I will say that side i of the market makes an ϵ (pure) transfer payment to side j of the market if side i's price p^i rises by ϵ and side j's price p^j falls by ϵ.

Lemma 1. *Let $\overline{V}^i$ and γ^i be the average surplus and vulnerability respectively on side i of the market at the monopoly optimal prices $\left(\gamma^i(p^i) = \gamma^j(p^j)\right)$. Then, for a small enough (case-dependent, not uniform) ϵ transfer payments starting at the monopoly optimal prices, the following classification of the welfare effects holds:*

1. *When $\overline{V}^i > \overline{V}^j$ total consumer (and social) surplus is improved by a transfer to side j from side i and decreased by the opposite transfer. If $\overline{V}^i = \overline{V}^j$ total consumer and social surplus are decreased by any transfer.*

2. *When $\overline{V}^i > \gamma^i = \gamma^j$, surplus on side i of the market is increased by a transfer from side i to side j; that is, side i of the market benefits from a transfer from side i to side j and is harmed by the reverse transfer.*

3. *When $\overline{V}^i < \gamma^i = \gamma^j$, side i benefits from a transfer from side j to side i and is harmed by the reverse transfer.*

4. *When $\overline{V}^i = \gamma^i = \gamma^j$, side i is harmed by any transfer.*

Proof. See appendix. □

The lemma provides a local, but full, characterization of transfers between the two sides of the market. The intuition behind it is simple. The condition on consumer and social surplus maximization demanding transfers from the high average surplus side of the market to the low average surplus side of the market is just an application of the theory of socially optimal price balance discussed above. It states that to move in the direction of average surplus weighted vulnerability equation from simple vulnerability equation one must make transfers from the higher average surplus group to the lower average surplus group. The intuition behind the average surplus versus vulnerability condition is also simple. At monopoly optimal prices, vulnerability on the two sides of the market is equated. When vulnerability is low, a transfer to the other side of the market is valuable as it raises demand on the other side of the market by a relatively large amount. When average surplus is high, transfers from side i to the j side of the market are valuable to the average member of side i. Thus when average surplus on side i is high relative to vulnerability, side i will want to make transfers to side j. The conditions with equalities follow from the second order conditions discussed earlier.

Thus, a certain transfer may improve or reduce consumer and social welfare, depending on whether it is from or to the higher average surplus group. Transfers may also be beneficial (surplus improving) or harmful (surplus reducing) to either side of the market, depending on the relationship between vulnerability and average surplus at the monopoly optimal prices. Almost nothing is ruled out *a priori*[38], except obvious

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[38]However, it is worth wondering whether all the situations described Lemma 1 are possible. That is, it may be that parts of Lemma 1 are irrelevant as no demand function and cost will lead them to occur. Therefore, it is worth showing that for certain parameters of (an extension of) the now familiar linear vulnerability form each of the possible situations occurs. This issue turns out to be a bit technically tricky and requires relaxing the assumption of log-concavity of surplus. This discussion is therefore deferred to the appendix. It is worth noting though that demand functions (for which monopolist optimization is well-defined) spanning the full range of hypothesis of Lemma 1 exist. However, it is not clear whether smooth log-concave demand functions with $\overline{V} > \gamma$ exist; they do not exist in the linear vulnerability case and it is shown in the appendix that no demand with log-concave surplus can have this property. Therefore, the appendix steps outside of this class of demand functions.

contradictions, such as both sides of the market benefiting from a transfer and yet consumer surplus declining.

Now I use Lemma 1 to analyze the effects of unbalanced reductions in the price level. Globally analyzing the effects of an unbalanced reduction in the price level on welfare is tricky and not terribly instructive: the combination of the price level effect and the slippery price balance effect makes the detailed characterization and prediction of the effects of such a reduction unreliable. Instead to ease analytical clarity, I simply consider small, sufficiently unbalanced reductions in the price level in order to make the basic point that an unbalanced reduction in the price level may have a wide variety of effects. That is, I focus here on a case where the price balance effect dominates the price level effect and both effects are local (the first-order approximation of the price balance effects from Lemma 1 is roughly accurate). To do this, we define the notion of a *sufficiently small and unbalanced reduction in the price level (SSURPL)*. Suppose that monopoly optimal prices are $p^\star = \left(p^{i\star}, p^{j\star}\right)$ and the prices that prevail after a fall in the price level are $p' = \left(p^{i'}, p^{j'}\right)$. Then the price level reduction is a SSURPL *favoring side i of the market* if $p^{j'} > p^{j\star}$ and $p^{i'} + p^{j'} < p^{i\star} + p^{j\star}$ and the following three numbers are sufficiently[39] small:

1. $p^{i\star} - p^{i'}$

2. $p^{j'} - p^{j\star}$

3. $\frac{p^{i\star} + p^{j\star} - p^{i'} - p^{j'}}{p^{j'} - p^{j\star}}$

I also say that two price level reductions $p' = \left(p^{i'}, p^{j'}\right)$ and $p'' = \left(p^{i''}, p^{j''}\right)$ are *equivalent* if $p^{i'} + p^{j'} = p^{i''} + p^{j''}$; that is they have the same associated price level. Finally, a price level reduction $p' = \left(p^{i'}, p^{j'}\right)$ is *balanced* if $p' \leq p^\star$. With these definitions, I can now provide a (partial) classification of the welfare effects of SSURPL's parallel to that of Lemma 1.

Proposition 12. *Maintain the definitions from Lemma 1. Then:*

1. *When $\overline{V}^i > \overline{V}^j$ total consumer (and social) surplus is improved more by a SSURPL favoring side j than by any equivalent balanced reduction in the price level. A SSURPL favoring side i reduces consumer and social surplus.*

2. *When $\overline{V}^i > \gamma^i = \gamma^j$, surplus on side i of the market is increased by a SSURPL favoring side j of the market more than it is by any equivalent balanced price level reduction. That is, side i of the market "benefits" a SSURPL disfavoring it. A SSURPL favoring side i harms side i.*

3. *When $\overline{V}^i < \gamma^i = \gamma^j$, side i benefits from a SSURPL favoring side i more than from any equivalent balanced price level reduction and is harmed by a SSURPL favoring side j.*

Proof. See Appendix.

□

There are two extreme cases[40] worth considering that illustrate the range of possible effects:

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[39]That is to say that Proposition 13 will hold if all of these numbers are less than some $\epsilon > 0$.

[40]Note that, as discussed above, these extreme cases require the relaxation of log-concavity of surplus. Therefore they may not seem extremely realistic. However, it is certainly possible to have the somewhat less extreme case of a SSURPL reducing social welfare or increasing it more than any balanced price level reduction within the log-concave demand and surplus setting.

1. In the good scenario, the unbalanced reduction in the price level shifts price balance so as to increase prices to the side of the market with very high average surplus (greater than vulnerability) and reduce prices to the side of the market with very low average surplus (less than vulnerability). This will occur in the case of unbalanced competition if the platforms are much more substitutable for the lower average surplus group than for the higher average surplus group. In the case of price regulation, it will result from a unilateral price control imposed on the lower average surplus side of the market. In this case, such an unbalanced reduction in the price level improves welfare much more than a balanced reduction would. That is, it improves the welfare of consumers on *both sides* of the market more than an equivalent balanced reduction would. The crucial reason is that it allows, through prices, consumers on one side of the market to make net side payments they would like to consumers on the other side of the market, as well as reducing the price level. Suppose men on a dating website gain more on average from the site than women using it do. Then unbalanced competition in the women's favor or a unilateral price control on women benefits men even more than balanced competition or price level controls would, as it allows them to make socially (and perhaps legally) prohibited payments to the women to enhance their pool of possible mates.

2. In the bad scenario, the reverse occurs. Unbalanced competition "favors", or a unilateral price control is imposed on, the high average surplus group. In this case, *both sides* of the market may be worse off than they were in the absence of any intervention, despite the resultant reduction in the price level. If, in my recurrent dating website example, a unilateral price control is imposed on the men's side of the market, the price to women rises and the men, despite paying a lower price lose a large part of the value of the service to them. The few remaining women, on the other hand, pay higher prices and earn less surplus, despite the increased number of men chasing them. In this case, unbalanced competition or unilateral price controls can have a perverse effect.

Between these two extreme cases lie a variety of intermediate scenarios in which, for example, the price balance effect is negative, but not bad enough to offset the welfare gains from a reduction in the price level; or the price balance effect is positive, but not so positive that both sides of the market are better off than they would have been under a balanced price level reduction.

When competition or price regulation[41] is unbalanced it may be exceptionally beneficial or positively harmful, even in a very strong sense. This depends crucially on whether the intervention raises prices to the side of the market with higher average surplus, in which case it is extremely beneficial, or to the side of the market with lower average surplus, in which case it may actually be harmful. For strategic and informational reasons discussed briefly in Section 7, it may be very difficult for a regulator to know which is the case.

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[41]It is worth noting that I do not here actually construct examples, based on primitive competition or vulnerability functions that show that various price controls or forms of competition can lead to particular unbalanced reductions of the price level. It should be clear, though, that the effect of competition on prices can span the full range of unbalanced price reductions I discuss, as it is the substitutability of the platforms for different groups of consumers that determines the effects of competition, rather than the total vulnerability of demand. Similarly, unilateral price controls may have essentially any effect on prices as the derivative of vulnerability can be separated from its level and associated average surplus. While I do not endeavor to make these arguments formal here, for brevity's sake, arbitrary examples can be constructed, even within the extended linear vulnerability class of demand functions.

5.4.3 Identifying welfare-improving price balance controls

In addition to providing a foundation for my analysis of the welfare effects of unbalanced (unilateral) price controls and competition, Lemma 1 also augments our understanding of another sort of natural price control, pure price balance controls. Suppose that a social planner considers instituting a policy that shifts the balance of price but leaves the price level (roughly) unchanged. Lemma 1 then provides a characterization of the welfare effects of such a policy, as such policies are exactly government mandated transfer payments from one side of the market to the other. Such price balance controls generally[42] will be able to improve consumer and social surplus, at least on the margin starting at monopoly optimal prices. In some cases, such price balance controls can even improve welfare on both sides of the market simultaneously!

However, from a policy perspective what is most important is whether such beneficial price balance controls can be identified. It may be that there exists some price balance control that is socially beneficial, but determining which way a social planner should move prices requires knowing the intricate third and fourth derivatives of demand and therefore it is in practice infeasible to identify such policies. However, Lemma 1 states that, if our goal is to improve consumer or social surplus (at the margin), the informational demands of designing such a policy are actually quite minimal *so long as the social planner starts at the monopoly optimal prices, given a particular price level.* Given this assumption, the social planner can simply adjust price balance by reducing prices for the lower average surplus side of the market and raising it for the higher average surplus side. Thus all the social planner need have is a reasonable sense of which side of the market earns higher average surplus (benefits more on average from the service) at current prices, so long as those current prices are monopoly prices. While this seems to recommend strongly in favor of price balance controls, subsubsection 7.3.2 discusses informally the strategic and information problems raised by the necessity that current prices be monopoly optimal prices.

5.5 Price Discrimination

As is clear above, welfare considerations in two-sided markets are significantly complicated by the presence of externalities across the two sides of the market. These externalities can lead to surprising and counter-intuitive conclusions. One area in which this is particularly striking is price discrimination. The following discussion highlights some of the potential implications of two-sided externalities for price discrimination. Here I focus on third-degree (market segmenting) price discrimination, as I consider it the most practically relevant and theoretically interesting[43] case.

5.5.1 Additional benefits of price discrimination in two-sided markets

Recall that in a standard, one-sided markets (third-degree) price discrimination has one basic social welfare benefit and one primary social welfare cost. On the upside, price discrimination allows low-value consumers to participate in a market they would otherwise have been priced out of. On the downside it inefficiently raises prices to the high value group of consumers, driving some that gain a quite high benefit from

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[42]When it is not the case that average surpluses on the two sides of the market are equal at the monopoly optimal prices. See Proposition 7.

[43]For reasons that will become apparent below.

participation out of the market. Additionally, price discrimination acts as a transfer from consumers to the monopolist.

While all of these considerations are still relevant in two-sided markets, my preceding analysis suggests some important differences. Before discussing these, it is worth briefly clarifying what is meant by third-degree price discrimination in a two- sided market. This concept of course does not refer to the ability of the monopolist to charge different prices to the two sides of the market; this flexibility is a basic feature of all of my preceding analysis. Rather, by third-degree price discrimination I mean the capacity of the monopolist to separate out two groups of consumers on one side of the market and charge differential prices to each group. The effects on welfare of price discrimination in two-sided markets may be significantly more beneficial than in standard markets:

1. In a standard market, price discrimination always harms the market segment that pays a higher price under discrimination than in its absence. In a two-sided market that need not be the case. Recall from Lemma 1 that if consumers have higher average surplus than vulnerability at the monopoly optimal prices, then this set of consumers would like to make transfers to the other side of the market. Price discrimination may facilitate such transfers. If there are two groups of consumers on one side of the market, one with high average surplus one with low average surplus, it may be optimal for the monopolist to charge them a relatively low price if only uniform pricing is available. However, under price discrimination the monopolist may be able to extract more of the high average-surplus group's surplus. If this occurs, then the topsy-turvy effect implies that the monopolist will tend to lower prices to the consumers on the other side of the market. This, in turn, may make the discriminated-against higher average surplus group better off than they were in the absence of discrimination, because of the external benefits that thereby accrue to them.

2. In a standard market, price discrimination (like subsidies) acts as a large transfer from consumers to the shareholders of the firm[44]. In a two-sided market the picture is more complicated, as it was in my analysis of the effect of subsidies on tax-augmented consumer welfare above. In particular, the response of a two-sided market firm to the increased profits it earns from price discrimination will be to reduce prices on the opposite side of the market. It thus passes on some of the additional profits it makes to consumers on the other side of the market and, perhaps more importantly provides more partners for the consumers on the discriminated side of the market to transact with.

3. The effect of price discrimination on volume is ambiguous in standard market, and so it is on the discriminated side of the market in two-sided markets. However, usually price discrimination tends to increase volume [45]. When this is the case, price discrimination has an additional benefit in two-sided markets over those in standard markets: it provides more partners for consumers on the non-discriminated side of the market to connect with.

4. Finally, it seems intuitively plausible that if price discrimination is made possible on both sides of the market, the monopolist will tend to move towards charging the two sides of the market an average price closer to their average surplus than before the price discrimination was possible. In this case,

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[44]For an extensive discussion of classic results on price discrimination see Tirole (1988), chapter 3. Section 3.2 is particularly germane here.

[45]See Tirole (1988), 137-40.

the possibility of price discrimination tends to move prices in the two-sided market closer to the socially optimal price balance.

Of course, this discussion is extremely hand-waving and informal. It seems that each of these effects might potentially move in the opposite direction. It could be, for example, that price discrimination reduces volume, harming consumers on the opposite side of the market or actually shifts price balance away from equalizing average surplus on the two sides of the market. Thus a more detailed, formal analysis of the welfare implications of price discrimination seems like an important topic for future research. Nonetheless it seems intuitive to think that price discrimination will generally lead a monopolist to extract an amount closer to the average surplus from a group of consumer than uniform pricing does. To the extent that price discrimination has this effect, it will tend to align the incentives of the monopolist more closely with social incentives

As a first stab at a more formal analysis of the welfare effects of price discrimination, in the next subsection I use the (extended) linear vulnerability class of demand functions to provide an example of a market where price discrimination increases the surplus of all groups of consumers, as well as (of course) the firm.

5.5.2 An example of benefits to discriminated-against group

Due to time constraints, the construction of this example is left to a future draft of this paper. However, from some preliminary sketches I strongly believe such a case exists.

6 Summary of Results

It is worth briefly summarizing the results above. On the positive side I found, using the tool of the vulnerability of demand to separate the "competitive pressure" effect from the topsy-turvy effect, that:

1. All forms of competition, price regulation and subsidies reduce the price level.

2. Unbalanced competition and unilateral price regulation raise prices on the "unpressured" side of the market.

3. Balanced competition, price level controls and subsidies reduce prices on both sides of the market.

On the normative side I found, by considering the effect of "two-sided" externalities and introducing the linear vulnerability demand class, that:

1. The welfare criteria of volume, consumer surplus and social surplus generally conflict in the choice of optimal price balance.

2. The socially optimal price level (under virtually any governance of price balance) is below cost.

3. The socially optimal price level under socially optimal price balance is below cost by exactly the amount of the marginal externality across the two-sides of the market.

4. The socially optimal price level under monopoly governance of the price balance is similar to that under socially optimal governance, with an adjustment for the monopolist's failure to optimize balance.

5. Subsidies can improve social welfare under monopoly ownership even ignoring profits of the firm.

6. It may be that consumers on one side of a two-sided market would like to make side payments to consumers on the other side of the market. Even when this is not the case, such transfers can improve consumer welfare. Price balance controls that allow these transfers can be identified from the average surplus on the two sides of the market and can improve consumer welfare (perhaps even on both sides of the market).

7. Unbalanced competition and unilateral price controls may be positively harmful (relative to no intervention) or may be even more beneficial than balanced competition or price level controls depending on the balance of average surplus between the two sides of the market.

8. Price discrimination may have surprising additional benefits in two-sided markets.

7   Implications for Policy

Much of the preceding analysis has important implications for policy, particularly in the areas of antitrust, subsidies and regulation. In this section I briefly highlight some of these. Detailed analysis of the design of policy in two-sided markets, taking into account the informational limitations on policy makers, is left to future research.

7.1   Why Does Policy in Two-Sided Markets Matter?

Past work on two-sided markets has suggested that policy in two-sided markets may not be as effective or important[46] as in standard markets because:

1. There may be less difference[47] between monopolistic and socially optimal pricing in two-sided markets than in standard markets.

2. The set of possible consequences[48] of policy in two-sided markets is much larger than in standard markets, so intervention is more dangerous in two-sided markets than in standard markets.

   Thus, previous studies have argued it may be best to take adopt a stance of benign neglect towards two-sided markets on the theory that policy makers should first do no harm and also on the theory that there may not be very much good that can be done.

   My analysis here suggests very different conclusions. Far from being unimportant or ineffective, the results above suggest two reasons why activist policy is if anything more attractive in two-sided markets:

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[46]Evans (2003) argues that "There is no reason to believe that anticompetitive problems are more or less prevalent in two-sided markets than in other ones." This is a convex combination of the Rochet-Tirole-Wright perspective I discuss below and my view.

[47]RT2003.

[48]Wright (2004)

1. First, because the socially optimal price level is always below cost in two-sided markets, market power, in addition to its classic distortive effects, also causes first order harm. Therefore, we should expect[49], if anything, that market power is more harmful in two-sided markets than in standard markets.

2. Second, because there are two dimensions (level and balance) along which prices may be distorted by the difference between private and social interests, there is a greater scope for socially beneficial intervention in two-sided markets than in standard markets.

Thus it would seem that policy, and particularly optimal policy, should be a topic of significant interest in two-sided markets. The following subsections discuss briefly how the analysis above might help guide this policy process.

7.2   Antitrust

Antitrust policy seeks to deter anticompetitive practices by firms and "combination(s)... in restraint of trade"[50]. The economic substance of this goal is a topic that has been extensively debated since the passage of the Sherman Act. However, two related, but distinct, interpretations of antitrust laws are most common today:

1. Under one view[51], perhaps most dominant, antitrust law empowers competition authorities to prosecute any act by firms that alters the organization of an industry so as to substantially harm social welfare through the increase of market power while simultaneously profiting the firm or firms.

2. Under a second, narrower but intimately related view[52], antitrust law empowers competition authorities to prosecute any act by firms that alters the organization of an industry so as to substantially reduce competition (increase market power) through some "anti-competitive" means.

These two closely related views differ only when competition fails to align with social welfare, as I have shown may be the case in a two-sided market. Regardless of which view one takes, the preceding analysis provides guidance on how antitrust policy should be implemented. Under the first view, it helps address the normative question: "(When) does a reduction in competition or the use of anti-competitive practices harm social (or consumer) welfare?" Under the second view, it helps address the more positive question: "From what price signals can one infer a reduction in competition or anti-competitive behavior?" I first consider this second, narrower question, then the broader normative one and finally turn to the ways in which the view presented here conflicts with current antitrust doctrine and case law.

7.2.1   Price level must be used to identify anti-competitive behavior

Because of the close association between prices and market power; and because of the role of prices in mediating the harms caused by market power, violations of antitrust law (collusion and anticompetitive

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[49]While this intuition seems clear, a formal statement of what exactly this means (what does one hold constant in the comparison?) is left to future research.

[50]For example, this quote is drawn from the Sherman Antitrust act in the United States, the text of which is reprinted in Hovenkamp (1994).

[51]One influential exponent of this view is Posner (2001)

[52]Hovenkamp (1994) implicitly adopts and defends this second approach.

practices) are often inferred from changes in price. Mergers are often approved or barred on the basis of whether the merged firm would have an incentive to raise prices, and exclusionary practices are often identified with pricing below cost. The analysis above indicates that in order for such standards to serve as a reasonable proxy for the competitiveness of a two-sided market, they must focus on the overall price level across the two-sides of the market, rather than on individual prices. Competition may actually raise prices for one side of the market; it may also lead to "below cost" (even negative) pricing to one side of the market. Therefore collusion and anticompetitive practices cannot, even in theory, be identified by an increase in prices on one side of the market and exclusionary conduct cannot be inferred from pricing below cost on one side of the market without taking into account the pricing decision of the firm on the other side of the market.

7.2.2   Competition is correlated, but imperfectly, with welfare

The usefulness of the price level in identifying practices and industrial organizations that are "pro-competitive" does *not* necessarily indicate that any reduction in price level (or rise in competition) is necessarily socially beneficial. In fact, our normative analysis indicates that competition may actually *harm* all parties in a two-sided market.

However, the permissive attitude towards market power this seems to suggest is too simplistic. My analysis showed that when competition is sufficiently balanced, it always increases social welfare and this improvement is important for the reasons specified above. When competition is sufficiently unbalanced, the perverse welfare-reducing effects of competition may result; but this is only one possibility. Unbalanced competition may, in fact, be more beneficial than balanced competition. Therefore, it seems that if an antitrust authority is attempting to maximize social welfare in ignorance of demand on the two sides of the market, she would likely conjecture that in expected terms unbalanced competition is approximately as beneficial as balanced competition. My analysis does not indicate that antitrust policy ignorant of demands should steer clear of two-sided markets. Further, more detailed analysis of policy would be required to establish this formally, the intuition should be that antitrust enforcement is not only on average beneficial in two-sided markets, but particularly important in such markets.

On the other hand, there may be some situations where a competitive authority has strong reasons to believe that a merger to monopoly[53] will end a very unbalanced competition that was holding down the price of the group in the market with higher average surplus. In this case, one should expect the authority to be more likely to permit a merger to monopoly than in a standard market. However, it seems that the informational demands to offer support for such forbearance are quite high. Thus in the absence of information, an antitrust authority focused on expected social welfare (and not on obeying a Hippocratic oath) will tend to vigorously enforce antitrust policy, guided by the correct inferential tools described above, unless she has a detailed knowledge of the demand structure in a particular market.

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[53]In principle, one should expect these results to generalize to mergers from N to $N - k$ firms. Showing that this holds is a goal for a future version of this paper.

7.2.3 Flaws in current antitrust doctrine

Current antitrust case law and doctrine at many competition authorities[54] holds for the most part that if anticompetitive harms are alleged to one group of consumers (in terms of raised prices) they cannot be offset by related actions of the firm in reducing prices to another set of consumers. Violating the law in one area, so the reasoning goes, is not compensated by a firm finding it optimal to concurrently reduce prices elsewhere. While such reasoning is sound when the two sets of consumers are in different markets, it is flawed in a two-sided market, where the prices on the two sides of the market are inextricably and negatively linked by the topsy-turvy effect. Thus to be internally coherent, antitrust doctrine must be updated to deal with two-sided markets. Otherwise antitrust law will generate many false negatives and positives: false negatives because harms from increased prices will only be perceived one one side of the market, even when both groups of consumers may be harmed due to external effects; false positives because competition that drives up prices for one group of consumers while lowering them for the other may be seen as "anticompetitive" or collusive.

7.3 Regulation

An alternative policy tool analyzed above, somewhat less popular in recent years, is regulation. In two-sided markets, regulation of both price level and price balance have the potential to improve welfare. However, as always, informational and strategic problems may complicate implementation. A detailed analysis of these problems is beyond the scope of this paper, but in what follows I address three issues:

1. What does my analysis tell us about the likely price consequences of certain existing or considered policies?

2. Are there strategic and informational concerns in regulatory implementation of two-sided market regulation that are new and qualitatively different than those in standard markets?

3. Without a detailed analysis, what should heuristically guide the decisions of a regulator of two-sided markets?

7.3.1 The consequences of unilateral price controls

Existing price regulation of two-sided markets is almost exclusively unilateral. The reason behind these policies is an analogy to standard markets: classical economics tells us that welfare can often be improved by placing a price control on a monopolist. Without a knowledge of two-sided markets, the consumers on one side of the market who feel abused by a monopolist's market power may find it natural to lobby regulators to place controls on the prices they face, with no references to prices faced by consumers on the other side of the market. The most prominent example of such regulation are the policy of "net neutrality"[55]

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[54]Evans (2003) extensively discusses current antitrust case law and doctrine related to two-sided markets.

[55]Wu (2004) provides an informative, legally-oriented survey of the net neutrality issues. A more opinionated, but also more economics-oriented, overview is provided by Yoo (2005).Technically net neutrality prohibits internet service providers from discriminating among content providers in terms of the access they afford them to consumers. Of course, a firm which cannot exclude non-payers will never be able to charge for the use of the service it provides. Another complication arises from the fact that content providers do pay for access to the net; but those who they pay are not necessarily the same internet service providers that provide access to consumers on the other side of the market. The net neutrality debate is essentially over whether this second group should be allowed to charge content providers for access to the consumers they serve, in addition to other internet service providers charging the content providers for the posting of their content.

maintained by the Federal Communications Commission in the US that restricts the ability of internet service providers (ISPs) to charge content providers (like Google and Yahoo!) for transmitting data to consumers. Instead, the ISPs are only allowed to charge consumers for access to the internet. Another well-known example is the regulation of "interchange fees" charged by payment cards[56] to merchants when cards are used at their stores.

My analysis above has one important implication for such unilateral price controls: they will tend to raise prices (or reduce services) on the other side of the market. In the case of net neutrality this is particularly vivid: the only way for ISPs to be incentivized to lay cable into the houses of poor, rural families may be for them to have the capacity to extract profits from content providers. On other hand, unilateral price controls do bring down the price level. Considering the "mean" of all of the possible welfare cases, these policies likely improve social welfare, if distributional considerations are ignored, but also involve a large transfer from one side of the market to the other side. Often (as in credit cards or ISPs) one side of the market is merchants or businesses and it is likely that if unilateral price controls are enacted they will tend to favor this side of the market, given the collective action problems faced by disparate consumers[57]. Of course, this neither condemns nor supports unilateral price controls, per se, but it does suggest that other, potentially more "balanced", "pro-consumer" price regulation (which has the additional benefit of being guaranteed to be welfare improving) is available: namely, price level controls. The precise way such controls might be implemented is better left to future analysis.

7.3.2 Strategic and informational problems with balance and full regulation

Strategic and informational problems in price regulation are familiar to economists[58]. In two-sided markets, achieving optimal price level runs into precisely the same difficulties as regulating prices in a standard market might: a regulator may have trouble identifying the firm's cost, making the regulatory process a strategic one. Because these concerns are familiar, there is no need to discuss them here. However, some additional problems distinct to two-sided markets do arise in the implementation of controls on price balance (or in full price controls).

Identifying socially optimal price balance, either to design price balance controls or as part of a regime of full price controls, requires the social planner to know the vulnerability of demand and average surplus, as a function of price, on both sides of the market. This is almost certainly more than any social planner could realistically hope to know accurately. However, the preceding analysis offered a potential out: moving prices locally in favor of the side of the market with lower average surplus improves welfare so long as one starts at the monopoly optimal prices.

There is a basic problem, though, in actually implementing such a policy. A monopolist aware of the fact that a policy maker is likely to shift prices against the side of the market with higher average surplus is likely to set prices even lower to this group than is optimal for himself in anticipation of this policy. Thus

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[56]In fact, the interchange fess act as transfer payments to banks which issue credit cards. If one believes, as most of the literature has maintained, that banks exercise some market power over the consumers they issue cards to, this is essentially equivalent to a unilateral price control, where the bank, rather than the credit card administration company which is often not-for-profit, is viewed as the monopolist. Evans and Schmalensee (2005) provide an overview of the regulation of interchange fees and economic research on this aspect of the classic two-sided market: payment cards. More on the "vertical relationship" will be available soon in a paper I am writing.

[57]Cite some political economy.

[58]Laffont and Tirole (1993)

a naive policy maker following such a prescription is likely to do more harm then good, potentially even moving prices further away from socially optimal balance. In fact, a commitment problem might arise in which a social planner is better off committing not to adjust price balance even though at any point in time it is better (myopically) for her to follow the average surplus-based rule. In anticipation of this, the monopolist optimally offsets prices further against the socially optimal balance, allowing the social planner to move price balance back towards her optimal choice of prices.

This strategic informational problem invites a few possible responses:

1. Abandon altogether the notion of price balance controls or full price controls. Given that identifying which side of the market to impose a unilateral price control on suffers from the same problem, these may also fall under the same criticism. Given that, modulo the familiar strategic problem, at least in terms of direction, price level controls always improve welfare on both sides of the market, one might decide to focus on controlling the price level. Given the potential commitment problem involved, it would be important that this policy be credible.

2. Try to design more sophisticated policies designed to take account of these strategic concerns. For example, if a social planner is really at monopoly optimal prices before instituting a price balance control, she should observe a reduction in the volume of transactions in the move to the new price balance. If she does not she can infer she is being "tricked" by the monopolist and should move price balance further until she observes a reduction in volume as a consequence of her actions. Such a strategy surely has problems associated with it, but it might be possible to design a strategy for regulating price balance that is game theoretically sound. The analysis of such sophisticated policies is far beyond the scope of this paper and is a potentially interesting topic for future research.

3. Finally, one might hope that given the lack of focus to this point on the standards proposed here for determining socially beneficial price balance regulations that firms have not yet factored into their prices their potential strategic relationship with regulators. Therefore, it might be possible to "surprise" firms and use data collected before the circulation of this paper and related research to gain some sense of socially beneficial price balance controls in particular markets. However, this strategy at best can work over a short time horizon and only in relatively stable, well-established markets.

Which strategy seems most reasonable is beyond the scope of this paper, but is an important policy problem and an interesting question for future research.

7.3.3 Towards a theory of optimal price regulation in two-sided markets

Given the lack of formal analysis of the policy problem here, we cannot hope to offer an implementable, general theory of optimal price regulation in two-sided markets. However, a few points are worth keeping in mind:

1. Price level controls[59] have the attractive feature that, so long as they do not cause the firm(s) to discontinue service, they improve welfare on both sides of the market.

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[59]Given their novelty, the design and implementation of price level controls for particular industries is presents an interesting problem for policy makers. For example, in the payment cards industry it might require that banks pass on a certain fraction of any interchange fees they receive to card-holders.

2. If it is difficult to identify the socially optimal price balance (or at least which direction it is optimal to move prices in), price balance regulations have the unattractive feature that they will always reduce the monopolist's volume and therefore profits. While this certainly is acceptable if an improvement in social welfare results, given the difficulties discussed above this may provide a (weak) reason to focus on regulation of the price level.

3. Unless they are significantly cheaper or easier to implement, or the policy maker has a strong belief that one side of the market has lower average surplus than the other, it seems that there is not an intellectual coherent reason why a policy maker would want to implement unilateral price controls. Thus my analysis here suggests that previous price regulation of two-sided markets was flawed due to confusion about the nature of two-sided markets.

4. The attractiveness of price balance controls (or full price controls) will depend on the knowledge the policy maker has of demand, rather than of cost.

7.4   Subsidies

The positive, normative and policy analysis above suggests that subsidies may be quite an attractive policy option in two-sided markets. Subsidies have a number of additional benefits in two-sided markets over their merits in standard markets:

1. Subsidies may (for a relative wide range of demand functions) improve even tax-augmented consumer surplus.

2. Subsidies are social optimal even when pricing is at cost, so there is little danger of a social planner "overshooting" by much.

3. Subsidies are guaranteed to improve social welfare and do not have the price level regulation problem of risking a shutdown in or capital flight from the industry.

Thus my analysis indicates that subsidies may be an interesting, under-appreciated policy tool in two-sided markets.

7.5   Price Discrimination

Because the section on price discrimination is not complete, it is probably best not to say too much here. It is worth noting, however, that price discrimination may be a way to come closer to socially optimal price balance, if it allows the monopolist to extract most surplus on both sides of the market. Perfect price discrimination, of course, is not very attractive as it involves a large transfer to the share holders of the firm. However, licensure auctions for a monopoly in a two-sided market coupled with government-aided price discrimination may have greater attraction in a two-sided market than in a standard market, given the additional complexities of pricing in two-sided markets which other forms of policy may be insufficient to optimize.

8   Conclusion

This paper establishes a number of baseline intuitions to guide the theoretical understanding and policy analysis of two-sided markets. Just like the corresponding baseline price theory of standard markets, however, it only provides a first swipe at industrial organization questions in two-sided markets. Many topics remain for future research. I plan to address a few in a future revision of this paper. Others are best left to future research.

8.1   Extensions for future work

In a future revision of this paper I hope to extend the results obtained in two ways. First, I plan to address the question of price discrimination in a more precise manner. Second, and more importantly, I hope to show that the results here apply more generally than the RT2003 framework. I particular I would like to show that:

1. The positive results on competition apply to all existing models of duopoly competition that consider symmetric equilibria in two-sided markets and to characterize some more primitive conditions on the structure of demand and market interactions under which they hold.

2. There are conditions under which any model falling within the broad class of canonical models considered by Rochet and Tirole (2006) obey the positive results on subsidies and price regulation. I believe, from some preliminary work, that these results are extremely general.

3. The welfare results are a bit more special, but I would like to incorporate into the paper a better sense of how sensitive they are to the RT2003 assumptions.

4. The matching and platform selection assumptions of RT2003 can be generalized while still obtaining the same positive results.

5. My results on competition can be extended to the case of mergers from N to $N - k$ firms (generic mergers).

6. My results hold in N-sided market, so long as the framework is extended to the N sides appropriately.

From some preliminary work I believe all of these extensions are possible, but I have not worked them out fully, and so I am not certain of their validity.

8.2   Limitations and broader extensions

The analysis above has a few substantial limitations:

1. It abstracts away from game theoretic coordination and bootstrapping issues across the two sides of the market in order to focus on price theory.

2. It abstracts away from platform selection and coordination failure issues in its welfare analysis of competition.

3. It only considers symmetric competitive equilibria.

4. It relies heavily on the multiplicative form of externalities in two-sided markets.

5. The policy analysis does not consider informational and strategic constraints in policy carefully.

Addressing any of these deficits is a project for future research as they are too broad to consider here. Other interesting topics for future research related to the analysis here include:

1. Applications of the notion of vulnerability of demand to other problems in industrial organization.

2. Empirical work testing positive predictions and measuring normatively/policy relevant variables like average surplus.

3. Theoretical investigation of collusion and anticompetitive practices in two-sided markets.

4. More careful analysis of the effect of the micro-structure of the relationship between the two sides of the market on welfare analysis.

Overall, two-sided markets continue to present an interesting combination of difficult theoretical challenges and immediate policy relevance.

Appendix A: Proof of Propositions 4

Proof of Proposition 4. Consider the monopolist's price balance decision, given a price level. Note that if $\gamma^j(p^j) > \gamma^i(q - p^j)$ then the monopolist has a strict incentive to increase prices. Thus suppose that $\gamma^j(\overline{p}^j) > \gamma^i(q - \overline{p}^j)$; then the monopolist, given a particular price level, always maximizes by setting $p^j = \overline{p}^j$. Furthermore, to maximize profits the monopolist also must set $q - c = \gamma^i(q - p^j)$, for precisely the same reasons as the original derivation of the first-order conditions characterizing the monopoly optimum (in either the one- or two-sided case). Thus I can proceed just as in my analysis of unbalanced competition. First I construct the "price controlled vulnerability level" and show that it always lies (sometimes weakly) below $\overline{\gamma}(q)$ and therefore the monopoly optimal price level is lower under the price control than under no regulation. This in turn implies that $\gamma^i_{pc}(p^{i\star}_{pc}) < \gamma^i(p^{i\star})$ and therefore that $p^{i\star}_{pc} > p^{i\star}$ as γ^i is decreasing and $q^\star - c = \gamma^i(p^{i\star}), q^\star_{pc} - c = \gamma^i(p^{i\star}_{pc})$ by the monopolist's optimization conditions.

Let me consider $\overline{\gamma}_{pc}(q)$. Let $p^j(q)$ solve $\gamma^j(p^j) = \gamma^i(q - p^j)$ given q. Then I can define $\overline{\gamma}_{pc}(q) \equiv \gamma^i(q - \min\{\overline{p}^j, p^j(q)\})$. Note that clearly $q - \min\{\overline{p}^j, p^j(q)\} \geq q - p^j(q)$ so that $\gamma^i(q - \min\{\overline{p}^j, p^j(q)\}) \leq \gamma^i(q - p^j(q))$ and thus $\overline{\gamma}_{pc}(q) \leq \gamma(q)$ by the definition of $\overline{\gamma}(q)$ from the earlier proofs. Thus $q^\star_{pc} \leq q^\star$, again by the reasoning of the earlier proof. Finally, I need to show that $q^\star_{pc} \neq q^\star$.

To see this, note that I assumed that the price control was binding, that is $\overline{p}^j < p^{j\star}$. But, by definition, $p^{j\star}$ solves $\gamma^i(q^\star - p^j) = \gamma^j(p^j)$ given $q^\star$ and this $p^j(q^\star) = p^{j\star}$. Thus:

$$\overline{\gamma}_{pc}(q^\star) = \gamma^i\left(q^\star - min\{\overline{p}^j, p^j(q^\star)\}\right) = \gamma^i\left(q^\star - min\{\overline{p}^j, p^{j\star}\}\right) = \gamma^i(q^\star - \overline{p}^j) < \gamma^i(q^\star - p^{j\star}) = \overline{\gamma}(q^\star)$$

Furthermore, $q^\star - c = \overline{\gamma}(q^\star)$ so $q^\star - c > \overline{\gamma}_{pc}(q^\star)$ and thus $q^\star$ does not solve $q^\star - c = \overline{\gamma}_{pc}(q^\star)$. Thus $q^\star_{pc} \neq q^\star$. So it must be the case that $q^\star_{pc} < q^\star$ which is what remained to be shown.

$\square$

Appendix B: Proof of Proposition 9 and Corollary 2

Proof of Proposition 9. I maximize social surplus, taking i's price as constant (optimally given by my price balance calculation) to derive the first-order condition for maximization of social surplus with respect to price level. Again social surplus is given by:

$$D^i(p^i)V^j(q - p^i) + D^j(q - p^i)V^i(p^i) + (q - c)D^i(p^i)D^j(q - p^i)$$

Taking the derivative with respect to the price level q and dropping arguments for spatial economy yield:

$$-D^i D^j + D^{j\prime} V^i + D^i D^j + (q - c)D^{j\prime} D^i = D^{j\prime} V^i - (c - q)D^{j\prime} D^i = \frac{D^{j\prime}(q - p^i)}{D^i(p^i)}\left[\overline{V}^i(p^i) + q - c\right]$$

which is clearly negative if

$$q < c - \overline{V}^i(p^i)$$

and positive if:

$$q > c - \overline{V}^i(p^i)$$

as $\overline{V}^i(p^i)$ is independent of q, $D^{j\prime} < 0$ and $D^i > 0$. Thus the satisfaction of the first-order condition is sufficient for maximization. This first order condition is:

$$\overline{V}^i\left(p^i\right) = c - q$$

which is what I sought to show. Clearly this argument holds for either i, which completes the proof. □

Proof of Corollary 2. Recall that in the linear vulnerability class:

$$\overline{V}^i(p^i) = \frac{a_i - p^i}{1 + \alpha_i}$$

At the social optimum, my proof above of Proposition 9 shows that average surpluses are equated on the two sides of the market[60]:

$$\frac{a_i - p^i}{1 + \alpha_i} = \frac{a_j + p^i - q}{1 + \alpha_j}$$

$$p^{i\star\star}(q) = (1 + \alpha_i)\frac{a_i - a_j + q}{2 + \alpha_i + \alpha_j}$$

$$\overline{V}^i\left(p^{i\star\star}(q)\right) = \frac{a_i - (1 + \alpha_i)\frac{a_i - a_j + q}{2 + \alpha_i + \alpha_j}}{1 + \alpha_i} = \frac{a_i + a_j - q}{2 + \alpha_i + \alpha_j}$$

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[60]Note that this only holds at the socially optimal price level, but this is exactly how use it here.

Using my definition of ν, the socially optimal price level solves:

$$q = c - \nu(a_i + a_j - q)$$

$$q^{\star\star} = a_i + a_j - \frac{a_i + a_j - q}{1 - \nu}$$

Note that $\nu > 0$ as $\alpha_i, \alpha_j > 0$ so the comparative statics results quoted in the next paragraph hold as well.

$\square$

Appendix C: Proof of Proposition 10 and Corollaries 3 and 4

Proof of Proposition 10. Recall from the proof of Proposition 8 that the derivative of social surplus with respect to the price level is given by:

$$D^{i'}p^{i'}V^j + D^{j'}p^{j'}V^i + (q - c)\left[D^{i'}p^{i'}D^j + D^{j'}p^{j'}D^i\right]$$

Where functions on side i are evaluated at $p^i(q)$, those on side j are evaluated at $p^j(q)$ and the prices p^i and p^j are functions of q. The sign of this expression is the same as that of:

$$-\overline{V}^j\gamma^j p^{i'} - \overline{V}^i\gamma^i p^{j'} + (c - q)\left[\gamma^j p^{i'} + \gamma^i p^{j'}\right]$$

Now $\gamma^i, \gamma^j > 0$ and at the monopoly's optimal price balance they are equated. So the expression for the sign of the above expression can again be simplified to:

$$-\overline{V}^j p^{i'} - \overline{V}^i p^{j'} + (c - q)\left[p^{i'} + p^{j'}\right] = -\overline{V}^j p^{i'} - \overline{V}^i p^{j'} + c - q$$

Recall that

$$p^{i'}(q) = \frac{dp^i}{dq} = \frac{\gamma^{j'}}{\gamma^{j'} + \gamma^{i'}}$$

So now I can rewrite my expression for the sign of the derivative of social welfare with respect to the price level as:

$$-\frac{\overline{V}^j\gamma^{j'} + \overline{V}^i\gamma^{i'}}{\gamma^{i'} + \gamma^{j'}} + c - q$$

The standard sufficient condition to ensure that the first order condition will be sufficient for maximization is that this expression above be decreasing. Taking its derivative with respect to q yields shows that the sign of the derivative is determined by:

$$\left(\gamma^{j''} + \gamma^{i''}\right)\left(\overline{V}^j\gamma^{j'} + \overline{V}^i\gamma^{i'}\right) - \left(\gamma^{j'} + \gamma^{i'}\right)\left(\left[\overline{V}^{j'} + 1\right]\gamma^{j'} + \gamma^{j''}\overline{V}^j + \overline{V}^i\gamma^{i''} + \left[\overline{V}^{i'} + 1\right]\gamma^{i'}\right)$$

or equivalently:

$$\left(\overline{V}^i - \overline{V}^j\right)\left(\gamma^{i''}\gamma^{j'} - \gamma^{j''}\gamma^{i'}\right) - \left(\gamma^{i'} + \gamma^{j'}\right)\left(\frac{\overline{V}^i\gamma^{i'}}{\gamma^i} + \frac{\overline{V}^j\gamma^{j'}}{\gamma^j}\right)$$

which is precisely what I hypothesized was negative. Thus under my hypotheses, the first order condition suffices and the socially optimal price level (the socially optimal subsidy achieves this price level) is given by:

$$q = c - \frac{\overline{V}^j\left(q - p^{i\star}(q)\right)\gamma^{j'}\left(q - p^{i\star}(q)\right) + \overline{V}^i\left(p^{i\star}(q)\right)\gamma^{i'}\left(p^{i\star}(q)\right)}{\gamma^{j'}\left(q - p^{i\star}(q)\right) + \gamma^{i'}\left(p^{i\star}(q)\right)} = c - \overline{V}^j\left(q - p^{i\star}(q)\right)p^{i\star'}(q) - \overline{V}^i\left(p^{i\star}\right)\left(1 - p^{i\star'}(q)\right)$$

Which is what I wanted to show.

$\square$

Proof of Corollary 3. Note that the technical condition is automatically satisfied if vulnerability is linear as the LHS is 0 and the RHS is strictly positive as $\gamma^{i'}, \gamma^{j'} < 0$ and $\overline{V}^i, \overline{V}^j, \gamma^i, \gamma^j > 0$.

To show that the formula provided is correct, note that $p^{i\star}(q)$ solves:

$$\frac{a_i - p^i}{\alpha_i} = \frac{a_j + p^i - q}{\alpha_j}$$

$$p^{i\star}(q) = \frac{\alpha_j a_i - \alpha_i a_j + \alpha_i q}{\alpha_i + \alpha_j}$$

Using the expression for $\overline{V}^i(\cdot)$ and recalling that $\gamma^{i'} = -\frac{1}{\alpha_i}$:

$$\frac{\overline{V}^j\left(q - p^{i\star}(q)\right)\gamma^{j'}\left(q - p^{i\star}(q)\right) + \overline{V}^i\left(p^{i\star}(q)\right)\gamma^{i'}\left(p^{i\star}(q)\right)}{\gamma^{j'}\left(q - p^{i\star}(q)\right) + \gamma^{i'}\left(p^{i\star}(q)\right)} = \frac{\frac{a_j - \left(q - p^{i\star}(q)\right)}{\alpha_j(1+\alpha_j)} + \frac{a_i - p^{i\star}(q)}{\alpha_i(1+\alpha_i)}}{\frac{1}{\alpha_i} + \frac{1}{\alpha_j}} =$$

$$\frac{\frac{1}{\alpha_i+\alpha_j}\left[\frac{1}{1+\alpha_i} + \frac{1}{1+\alpha_j}\right]}{\frac{\alpha_i+\alpha_j}{\alpha_i\alpha_j}}(a_i + a_j - q) = \eta(a_i + a_j - q)$$

So $q^\star$ solves:

$$q = c - \eta(a_i + a_j - q)$$

$$q^\star = a_i + a_j - \frac{a_i + a_j - c}{1 - \eta}$$

Which establishes the proposition. In order for the comparative statics results to hold I need that $\eta \in (0, 1) \forall \alpha_i, \alpha_j > 0$ and that constraining $\alpha_j = \theta\alpha_i$ that η be decreasing in $\alpha_j \forall \theta > 0$.

Let me consider these in turn. First note that $\eta > 0$ obviously. To show $\eta < 1$:

$$\eta = \frac{\alpha_i\alpha_j}{(\alpha_i + \alpha_j)^2}\left[\frac{1}{1 + \alpha_i} + \frac{1}{1 + \alpha_j}\right] < \frac{2\alpha_i\alpha_j}{\alpha_i^2 + 2\alpha_i\alpha_j + \alpha_j^2} < 1$$

where the two inequalities follow from $\alpha_i, \alpha_j > 0$.

Now consider the second. Constraining $\alpha_j = \theta\alpha_i$ then:

$$\eta = \frac{\theta\alpha_i^2}{\alpha_i^2(1+\theta)^2}\left[\frac{1}{1+\alpha_i}+\frac{1}{1+\theta\alpha_i}\right] = \frac{\theta}{(1+\theta)^2}\left[\frac{1}{1+\alpha_i}+\frac{1}{1+\theta\alpha_i}\right]$$

which is clearly decreasing in α_i because $\theta > 0$.

$\square$

Proof of Corollary 4. The expression obviously holds with equality if and only if $\nu = \eta$ and hold with strict inequality if and only if $\eta > \nu$. After some algebraic simplification:

$$\frac{\eta}{\nu} = \frac{\alpha_i\alpha_j(2+\alpha_i+\alpha_j)^2}{(1+\alpha_i)(1+\alpha_j)(\alpha_i+\alpha_j)^2}$$

How this compares to 1 is exactly how the top expression compares to the bottom expression. That is, the fraction will be less than 1 exactly when:

$$\alpha_i\alpha_j(2+\alpha_i+\alpha_j)^2 < (1+\alpha_i)(1+\alpha_j)(\alpha_i+\alpha_j)^2$$

A whole lot of tedious algebra yields that this occurs exactly when:

$$(\alpha_i-\alpha_j)^2(1+\alpha_i+\alpha_j) > 0$$

which is true so long as $\alpha_i \neq \alpha_j$. Clearly when $\alpha_i = \alpha_j$ it is the case that $\nu = \eta$.

$\square$

Appendix D: Proof of Proposition 11 and Corollary 5

Proof of Proposition 11. First recall that I showed above that:

$$p^{i^{\star\prime}} = \frac{\gamma^{j\prime}}{\gamma^{i\prime}+\gamma^{j\prime}}$$

where, as throughout the rest of this proof when arguments are dropped, the i functions are evaluated at $p^{i^\star}(q)$ and the j functions are evaluated at $p^{j^\star}(q) = q - p^{i^\star}(q)$. Now I am interested in the effect of subsidies on the prices on the two sides of the market, which I can determine by considering the effects of subsidies on the price level and then using the above decomposition to determine the effect on the individual prices. Let $q^\star(\sigma)$ be the optimal price level given a particular subsidy level.

Then recall from the proof of Proposition 6 that $p^{i^{\star\prime}}$ is given by[61]:

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[61]In the case of a standard, one-sided market with vulnerability γ the same argument yields $p^{\star\prime} = -\frac{1}{1+|\gamma\prime|}$ and thus the ratio of the part of the subsidy absorbed by the monopolist to that kept by the consumers is $|\gamma\prime|$; that is, consumers keep more of the subsidy when $|\gamma\prime|$ is low. This offers, in my opinion, a substantial simplification over the traditional elasticity-based incidence formula:

$$\frac{\eta^2(p)}{\eta(p)-\eta^2(p)-\eta\prime(p)p}$$

and is easier to interpret. Note that, as proven above $\overline{\gamma}\prime = \frac{\gamma^{B\prime}\gamma^{S\prime}}{|\gamma^{B\prime}|+|\gamma^{S\prime}|}$ so that the formula above for derivative of the price level with respect to the subsidy in the two-sided case is just the direct analog of the standard case, where vulnerability level substitutes for vulnerability as in my analysis of competition.

$$q^{\star\prime} = \frac{\gamma^{i\prime} + \gamma^{j\prime}}{\gamma^{i\prime}\gamma^{j\prime} - \gamma^{i\prime} - \gamma^{j\prime}} = -\frac{|\gamma^{i\prime}| + |\gamma^{j\prime}|}{\gamma^{i\prime}\gamma^{j\prime} + |\gamma^{i\prime}| + |\gamma^{j\prime}|}$$

as $\gamma^{i\prime}, \gamma^{j\prime} < 0$ by log-concavity. Now I can proceed to analyze the effect of subsidies on tax-augmented consumer surplus. The expression for tax-augmented consumer surplus given in equation 10 is:

$$D^i V^j + D^j V^i - \sigma D^i D^j$$

Differentiating with respect to the amount of the subsidy:

$$D^i p^{i\star\prime} q^{\star\prime} V^j + D^j p^{j\star\prime} q^{\star\prime} V^i - D^i D^j q^{\star\prime}\left(p^{i\star\prime} + p^{j\star\prime}\right) - D^i D^j - \sigma\left(D^i p^{i\star\prime} q^{\star\prime} D^j + D^j p^{j\star\prime} q^{\star\prime} D^i\right)$$

Dividing by $D^{i\prime} D^{j\prime} > 0$, using the definition of γ^i, γ^j and the fact that $p^{i\star\prime} + p^{j\star\prime} = 1$, the sign of the expression is determined by:

$$-\overline{V}^j \gamma^j p^{i\star\prime} q^{\star\prime} - \overline{V}^i \gamma^i p^{j\star\prime} q^{\star\prime} - \gamma^i \gamma^j\left(1 + q^{\star\prime}\right) + \sigma q^{\star\prime}\left(\gamma^j p^{i\star\prime} + \gamma^i p^{j\star\prime}\right)$$

Thus there exists a socially beneficial subsidy if:

$$-\overline{V}^j \gamma^j p^{i\star\prime} q^{\star\prime} - \overline{V}^i \gamma^i p^{j\star\prime} q^{\star\prime} - \gamma^i \gamma^j\left(1 + q^{\star\prime}\right) > 0$$

At the monopoly optimal prices $p^\star = \left(p^{i\star}, p^{j\star}\right)$. Plugging in the expression for $p^{i\star\prime}, p^{j\star\prime}$ and $q^{\star\prime}$ we have this as equivalent to:

$$\frac{\gamma^{j\prime}\gamma^{i\prime}\gamma^j\gamma^i}{\gamma^{i\prime}\gamma^{j\prime} - \gamma^{i\prime} - \gamma^{j\prime}} < \frac{\overline{V}^i|\gamma^{i\prime}|\gamma^i + \overline{V}^j|\gamma^{j\prime}|\gamma^j}{\gamma^{i\prime}\gamma^{j\prime} - \gamma^{i\prime} - \gamma^{j\prime}}$$

The denominator of both sides is clearly positive as $\gamma^{i\prime}, \gamma^{j\prime} < 0$. Thus multiplying by the denominator and plugging in the points of evaluation yields that the inequality is the same as

$$\gamma^{j\prime}\gamma^{i\prime}\gamma^j\gamma^i < \overline{V}^i|\gamma^{i\prime}|\gamma^i + \overline{V}^j|\gamma^{j\prime}|\gamma^j$$

But of course at monopoly optimal prices $\gamma^i = \gamma^j > 0$ so we have this inequality being equivalent to:

$$\gamma^{j\prime}\gamma^{i\prime}\gamma^i < \overline{V}^i|\gamma^{i\prime}| + \overline{V}^j|\gamma^{j\prime}|$$

Which proves the proposition.

$\square$

Proof of Corollary 5. Plugging in the expressions for the linear vulnerability case yields:

$$\frac{1}{\alpha_i \alpha_j}\frac{a_i - p^i}{\alpha_i} < \frac{1}{\alpha_i}\frac{a_i - p^i}{1 + \alpha_i} + \frac{1}{\alpha_j}\frac{a_j - p^j}{1 + \alpha_j}$$

But at the monopolist's optimal prices, $\frac{a_i - p^i}{\alpha_i} = \frac{a_j - p^j}{\alpha_j}$ so:

$$\frac{1}{\alpha_i \alpha_j} \frac{a_i - p^i}{\alpha_i} < \frac{1}{\alpha_i} \frac{\alpha_i}{1 + \alpha_i} \frac{a_i - p^i}{\alpha_i} + \frac{1}{\alpha_j} \frac{\alpha_j}{1 + \alpha_j} \frac{a_i - p^i}{\alpha_i}$$

$$1 < \alpha_i \alpha_j \left[\frac{1}{1 + \alpha_i} + \frac{1}{1 + \alpha_j} \right]$$

Which is the condition in the corollary. When $\alpha_i = \alpha_j = \alpha$ this simplifies to:

$$1 < \frac{2\alpha^2}{1 + \alpha}$$

$$0 < 2\alpha^2 - \alpha - 1$$

$$0 < (2\alpha + 1)(\alpha - 1)$$

Which is satisfied (for $\alpha > 0$) if and only if $\alpha > 1$, which is the same as demand being convex (on both sides of the market). If instead one constrains $\alpha_j = \theta \alpha_i$ for some $\theta \in (0, 1)$ then:

$$1 < \theta \alpha_i^2 \left[\frac{1}{1 + \alpha_i} + \frac{1}{1 + \theta \alpha_i} \right]$$

Note that when $\alpha_i = 1$ the right hand side expression is:

$$\frac{\theta}{2} + \frac{\theta}{1 + \theta} < \frac{1}{2} + \frac{1}{2} = 1$$

as $\theta < 1$ so that when demand on side i is linear, the inequality is not satisfied. On the other hand in the limit as $\alpha_i \to \infty$ clearly the quadratic term dominates and the right hand side goes to ∞. Finally, note that the expression is strictly increasing in α_i by taking its derivative with respect to α_i:

$$\frac{2 + \alpha_i}{(1 + \alpha_i)^2} + \frac{2 + \theta \alpha_i}{(1 + \theta \alpha_i)^2} > 0$$

Thus the inequality is satisfied for sufficiently large $\alpha_i > 0$; that is, sufficiently convex i side demand, completing the proof of the corollary.

$\square$

Appendix E: Proof of Lemma 1 and Proposition 12

Proof of Lemma 1. Let me consider each part of the lemma in turn. First consider the effect of a transfer on consumer (and social) surplus. Given price level $q^\star$ the consumer surplus is given by:

$$V^i(p^i)D^j(q - p^i) + V^j(q - p^i)D^i(p^i)$$

Taking the derivative with respect to p^i yields the effect on consumer surplus of a small transfer from side i to side j (dropping arguments):

$$-D^i D^j - D^{j'} V^i + D^i D^j + D^{i'} V^j$$

On the other hand, social surplus is given by:

$$V^i(p^i)D^j(q-p^i) + V^j(q-p^i)D^i(p^i) + (q-c)D^i(p^i)D^j(q-p^i)$$

The effect of a small transfer from i to j on social surplus is:

$$-D^i D^j - D^{j'} V^i + D^i D^j + D^{i'} V^j + (q-c)\left[D^{i'} D^j - D^i D^{j'}\right]$$

The signs of these expressions respectively are given by (after dividing through by $D^{i'} D^{j'} > 0$):

$$\gamma^i \overline{V}^i - \gamma^j \overline{V}^j$$

$$\gamma^i \overline{V}^i - \gamma^j \overline{V}^j + (q-c)\left[\gamma^i - \gamma^j\right]$$

But at monopoly optimal prices, $\gamma^i = \gamma^j$ so these both have sign determined by:

$$\overline{V}^i - \overline{V}^j$$

Thus consumer and social welfare increase from an transfer from side i to side j that is sufficiently small if $\overline{V}^i > \overline{V}^j$ and decrease from such a transfer (increase from the opposite transfer) if $\overline{V}^j < \overline{V}^i$. Furthermore both of these problems are concave by my assumption of log-concavity of demand and surplus, so when $\overline{V}^j = \overline{V}^i$ and there is no first order effect, the second order effect dominates and must be negative, the proof of the first part of the Lemma.

The last parts of the lemma are even simpler. Consumer surplus on side i of the market is:

$$V^i(p^i)D^j(q-p^i)$$

Taking the derivative with respect to p^i and dropping arguments yields:

$$-D^i D^j - D^{j'} V^i$$

Dividing again by $D^{i'} D^{j'}$ shows that the sign is determined by:

$$-\gamma^i \gamma^j + \gamma^i \overline{V}^i$$

Or, recalling that at monopoly pries $\gamma^i = \gamma^j$:

$$\overline{V}^i - \gamma^j = \overline{V}^i - \gamma^i$$

Thus welfare on side i of the market is increased (decreased) by a small transfer to (from) side j of the market if $\overline{V}^i > \gamma^i$ and the reverse is true of the reverse transfer. Furthermore the problem again concave at the optimum by log-concavity of the two functions, so again the second order effect is negative, establishing the rest of the lemma.

$\square$

Proof of Proposition 12. I only prove the second part of the proposition; the rest follows easily using the same strategy. Namely I showed that a SSURPL favoring side j of the market may harm consumers on

side j of the market. Let Δq represent the reduction in the price level associated with this SSURPL and let Δb be the increase in i's price resulting from this SSURPL. Then letting $p^\star = \left(p^{i^\star}, p^{j^\star}\right)$ represent the monopolist's optimal prices before the SSURPL and let $p' = \left(p^{i'}, p^{j'}\right)$ represent the prices after the SSURPL $p^{i'} = p^{i^\star} + \Delta b$ and $p^{j'} = p^{j^\star} - \Delta b - \Delta q$. Our proof strategy is to consider the SSURPL in two stages. First, prices on side j of the market fall by Δq. Then there is a pure transfer from side i to side j of the market. I show that if $\overline{V}^j\left(p^{j^\star}\right) > \gamma^j\left(p^{j^\star}\right)$ then the welfare gain to side i during the first stage is more than offset by the welfare loss during the second stage if Δq and Δb are chosen sufficiently small.

Consumer surplus on side j of the market is:

$$D^i(p^i)V^j(p^j)$$

The derivative of this expression with respect to p^j is:

$$-D^i(p^i)D^j(p^j) < 0$$

Thus the welfare gain from a reduction of Δq in p^j is:

$$\Delta\pi_{PL}(\Delta q) = -\int_{p^{j^\star}}^{p^{j^\star}-\Delta q} D^i(p^i)D^j(p^j)dp^j = \int_{p^{j^\star}-\Delta q}^{p^{j^\star}} D^i(p^i)D^j(p^j)dp^j > 0$$

Note that by finiteness of demand, $\Delta\pi_{PL}(0) = 0$; also $\frac{\partial\Delta\pi_{PL}}{\partial\Delta q} = D^i(p^i)D^j\left(p^{j^\star} - \Delta q\right) \in \mathbb{R}_+$ so $\Delta\pi_{PL}$ is smoothly increasing in Δq.

Now suppose I consider the welfare effects of a pure transfer from side i to side j of the market starting at prices $p = (p^i, p^j)$. If the transfer is of size Δb then the resultant surplus on side j of the market is:

$$\pi(\Delta b, p^i, p^j) = D^i(p^i + \Delta b)V^j(p^j - \Delta b)$$

Taking the derivative with respect to Δb and dropping arguments yields:

$$\frac{\partial\pi}{\partial\Delta b} D^{i'}V^j + D^iD^j$$

Some simple division shows the sign of this expression is determined by:

$$\gamma^i - \overline{V}^j$$

Now note that at $p^\star$ by definition:

$$\gamma^i = \gamma^j < \overline{V}^j$$

By assumption. Thus, by differentiability of $\overline{V}^j$ and γ^i, for sufficiently small Δq:

$$\gamma^i\left(p^{i^\star}\right) - \overline{V}^j\left(p^{j^\star} - \Delta q\right) < 0$$

Thus for sufficiently small $\Delta q, \Delta b$:

$$\frac{\partial\pi}{\partial\Delta b}\left(\Delta b, p^{i^\star}, p^{j^\star} - \Delta q\right) < 0$$

Let:

$$\Delta\pi_{PB}(\Delta q) \equiv \min_{\Delta b}\left[\pi\left(\Delta b, p^{i^\star}, p^{j^\star} - \Delta q\right) - \pi\left(\Delta b, p^{i^\star}, p^{j^\star}\right)\right] < 0$$

Clearly $\Delta\pi_{PB}(0) < 0$ and thus by continuity of $\Delta\pi_{PB}$, $\{\Delta\pi_{PB}(\Delta q) : \Delta q \in [0,m)\}$ is bounded away from 0 for some $m > 0$. Note that the total welfare effect on side j of the market of the SSURPL is:

$$\Delta\pi_{PL}(\Delta q) + \Delta\pi_{PB}(\Delta q)$$

If Δb is chosen appropriately. Now because the second term is bounded below and away from 0 and the first term is smoothly decreasing to 0 as $\Delta q \to 0$ it is clear that for small enough $\Delta q > 0$ this expression is negative. Thus the SSURPL defined by Δq, $\Delta b \equiv argmin_{\Delta b}\left[\pi\left(\Delta b, p^{i^\star}, p^{j^\star} - \Delta q\right) - \pi\left(\Delta b, p^{i^\star}, p^{j^\star}\right)\right]$ for some $\Delta q > 0$ reduces welfare on side j of the market, even though it reduces the price level and enacts a transfer to side j of the market. Precisely the same logic can be used to establish all of the other parts of the proposition.

$\square$

Appendix F: An Extension of the Linear Vulnerability Demand Form

In this section I construct an extension of the linear vulnerability demand form in order to allow for average surpluses in excess of vulnerability. In doing so, my goal is to maintain the tractability of linear vulnerability in calculating optimal prices, while increase the range of possible average surpluses. To do this, I consider adding a small number of consumers λ_n to the market that have a high valuation for the service β_n. Formally:

$$D_n^i(p^i) = \begin{cases} 0 & p^i > \beta_n \\ \lambda_n & \beta_n \leq p^i > a_i \\ \lambda_n + \frac{(a_i - p^i)^{\alpha_i}}{b_i} & p^i \geq a_i \end{cases}$$

Where $\beta_n > a_i$ and $\lambda_n > 0$ for all n and $\alpha_i, b_i > 0$.

It is clear that this piecewise demand function is not log-concave...in fact, it is not even smooth! Therefore, I need to be careful to construct the demand so that the monopolist will find it optimal to equate vulnerability on side i of the market to that on side j, rather than simply extracting all the surplus of the small mass of high-value consumers on side i. To do this, let γ_n^i be the vulnerability associated with the above demand and let γ^j be the vulnerability on side j of the market. Furthermore, let $p_n^\star(q) = \left(p_n^{i^\star}(q), p^{j^\star}(q)\right)$ be the prices equating vulnerability (optimizing for the monopolist[62]) on the two sides of the market given price level q. Then monopolist will choose to serve the linear vulnerability consumers on side i of the market if and only if:

$$\lambda_n\left(\beta_n + p^{j^\star}(q) - c\right) \leq (q - c)\left[D_n^i\left(p^{i^\star}(q)\right)\right] = (q - c)\left[\lambda_n + \frac{(a_i - p^{i^\star}(q))^{\alpha_i}}{b_i}\right]$$

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[62]That is, assuming $q > c$ and that q is not so large that no allocation in prices across the two sides of the market generates positive demand.

$$\lambda_n \left(\beta_n - p^{i^\star}(q) \right) \leq \frac{\left(a_i - p^{i^\star}(q) \right)^{\alpha_i}}{b_i} (q - c)$$

$$\lambda_n \leq \frac{\left(a_i - p^{i^\star}(q) \right)^{\alpha_i}}{\left(\beta_n - p^{i^\star}(q) \right) b_i} (q - c)$$

Thus if I choose the mass to maximize the average surplus subject to the constraint that it is optimal for the monopolist to still serve the linear vulnerability consumers when the consumers have value β_n then:

$$\lambda_n \equiv \frac{\left(a_i - p^{i^\star}(q) \right)^{\alpha_i}}{\left(\beta_n - p^{i^\star}(q) \right) b_i} (q - c) = \frac{\left(a_i - p^{i^\star}(q) \right)^{\alpha_i}}{\left(\beta_n - p^{i^\star}(q) \right) b_i} \gamma^i \left(p^{i^\star}(q) \right)$$

as the monopolist's optimization equates vulnerability to margin. If one chooses some sequence of $\beta_n \to \infty$ then clearly $\lambda_n \to 0$ and $D_n^i(p^i) \to D^i(p^i) = \frac{(a_i - p^i)^{\alpha_i}}{b_i}$ which is just the linear vulnerability form. However, average surplus (at this monopoly optimal price level) is given by:

$$\overline{V}_n^i \left(p^{i^\star}(q) \right) = \frac{\lambda_n \left(\beta_n - p^{i^\star}(q) \right) + \frac{\left(a_i - p^{i^\star}(q) \right)^{\alpha_i+1}}{b_i(1+\alpha_i)}}{\lambda_n + \frac{\left(a_i - p^{i^\star}(q) \right)^{\alpha_i}}{b_i}} = \frac{\left(a_i - p^{i^\star}(q) \right)^{\alpha_i} \gamma^i \left(p^{i^\star}(q) \right) + \frac{\left(a_i - p^{i^\star}(q) \right)^{\alpha_i+1}}{b_i(1+\alpha_i)}}{\lambda_n + \frac{\left(a_i - p^{i^\star}(q) \right)^{\alpha_i}}{b_i}} \to$$

$$\left(a_i - p^{i^\star}(q) \right) \left[\frac{1}{\alpha_i} + \frac{1}{1 + \alpha_i} \right] \equiv \overline{V}^i \left(p^{i^\star}(q) \right)$$

as $D_n^i(p^i) \to D^i(p^i) = \frac{(a_i - p^i)^{\alpha_i}}{b_i} \implies \gamma_n^i(p^i) \to \gamma^i(p^i) = \frac{(a_i - p^i)}{\alpha_i}$. Thus we have:

$$\frac{\overline{V}^i \left(p^{i^\star}(q) \right)}{\gamma^i \left(p^{i^\star}(q) \right)} = 1 + \frac{\alpha_i}{1 + \alpha_i}$$

Thus average surplus is higher and in particular it may be greater than vulnerability (it always is at the optimal prices for which it is constructed). The final thing that remains to be considered is over what range of prices this demand function is valid (i.e. continues to have the sufficiency of the first-order conditions for monopolist maximization). This is not relevant for all of our applications, in some of which the monopolist no longer has full control over prices on side i of the market, but it does arise in some applications. This is a some what complicated issue, so an extended discussion is omitted here[63], but there is a simple way to see that this is not a significant problem.

Defining each λ_n not to satisfy the inequality tightly, but rather to be half the level satisfying the inequality with equality, the monopolist strictly prefers to serve the linear vulnerability consumers and

$$\frac{\overline{V}^i \left(p^{i^\star}(q) \right)}{\gamma^i \left(p^{i^\star}(q) \right)} = \frac{1}{2} + \frac{\alpha_i}{1 + \alpha_i}$$

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[63]But is available upon request.

Thus in this case it will be true that for some open set of prices around these monopoly optimal prices the conditions required for the monopolist to continue to serve the linear vulnerability customers will continue to be satisfied and if $\alpha_i > 0$ (demand is convex) still $\overline{V}^i > \gamma^i$. The whole point of this exercise was to show that this outcome was feasible while maintaining monopolist optimization; thus I have shown this can occur for some dense open set about the monopoly optimal prices used for the construction of the demand form. Given that in all of the applications I consider "sufficiently small" changes in price, this suffices to show the possibility of the cases I am interested in. Looking for a nicer, smooth and tractable class of demand functions with $\overline{V} > \gamma$ remains an interesting question for future research.

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