A Malthusian Model with Seasonality*

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Abstract

This study shows that *seasonality*, i.e. making a distinction between cropping and non-cropping agricultural activities, is vital for understanding not only the existence of labour surplus in a Malthusian environment but also conflicting effects of technological progress on land and labour productivity.

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"The discussion of low or zero marginal productivity in agriculture suffers from a neglect of the seasonal differences in employment and wages. Many off-season operations are in fact required in order to obtain higher crop frequency through labor intensive methods alone, and so may well appear to be of very low productivity if viewed in isolation from their real function."

Ester Boserup in 'Agricultural Growth and Population Change,' *The New Palgrave*, 1987

1 Introduction

Throughout the pre-industrial era, higher land productivity has paved the way for higher population density. But what happened to the productivity of labour in pre-industrial agriculture when land productivity went up? From a theoretical viewpoint, two theses compete: a 'decline thesis', stating that labour productivity was negatively correlated with land productivity, and a 'rise thesis,' stating the opposite (Hunt, 2000). The 'decline thesis' is mainly associated with work by Ester Boserup, who makes a strong case that agricultural intensification raised labour costs per unit of food produced (Boserup, 1965). The 'rise thesis,' by contrast, can be deduced by looking at estimates of England's population distribution (Table 1): Correcting for food imports and the inclusion of virgin land, a nearly fourfold increase of England's population level between 1500 and 1800 gives proof that land productivity went up over the period. At the same time, the halving of the share of England's population employed in agriculture over the same period suggests that agrarian labour productivity went up as well.[1]

< Table 1 about here >

The purpose of the current paper is to show that the two theses can be understood within the context of a single theory. More generally, the paper offers a theoretical frame-

[1] Food imports, which by the beginning of the nineteenth century represented about one fifth of all English food consumption (Crafts, 1985), clearly is not enough to accommodate the growth of England's population over the period; nor is it enough to explain the drastic change in England's occupational structure.

work for analysing the effects of technological progress on land and labour productivity in a Malthusian environment. Combining elements from Boserup's hypothesis about agricultural development with building blocks from the Malthusian model, the present work demonstrates that *seasonality*, i.e. making a distinction between cropping and non-cropping agricultural activities, is vital for understanding not only the existence of labour surplus in pre-industrial agriculture (as is well-documented by Karakacili, 2004) but also conflicting effects of technological progress on land and labour productivity.

The theory presented below encompasses the conventional Malthusian feature, consistent with findings by Clark (2007) and others, that pre-industrial technological progress appears, over the long run, to result in population increase rather than increase in per capita income. In addition, however, the theory also comprises the conventional Boserupian feature that agricultural intensification raises labour costs per unit of food produced. According to the current study, therefore, technological progress not only makes living-conditions more crowded without improving standards of living (a Malthusian feature), it also force people to work harder (a Boserupian feature)—conclusions that seem consistent with stylised historical facts proposed by Voth (2003).

Yet, not all types of technological progress will entail this outcome. In line with the Malthusian model, technological progress, when it occurs in relation to cropping (high-season) activities, will accommodate a larger population but, over the long run, will not affect income per capita. Unpredicted by the Malthusian model, however, technological progress, when it occurs in relation to non-cropping agricultural (low- or off-season) activities (i.e. seeding, weeding, harrowing, hoeing etc), will not affect the size or growth rate of population, or the per capita income.

These ambiguous effects of technological progress reach on also to labour surplus and labour productivity. If technological progress occurs in relation to cropping activities, then it diminishes labour productivity and thus comes at a cost to labour surplus. On the other hand, if technological progress occurs in relation to non-cropping agricultural activities, then it increases labour productivity and thus expands labour surplus. Ultimately, therefore, what happens to agrarian labour productivity under agricultural intensification (higher land productivity) depends on the relative growth of technology in cropping and non-cropping activities, respectively.

The current work relates to three strands of literature. Firstly, it links to a bulk of theoretical papers that explore the process of industrialisation and the transition from stagnation to growth. These includes Galor and Weil (2000), Goodfriend and McDermott (1995), Hansen and Prescott (2002), Jones (2001), Kögel and Prskawetz (2001), Lucas (2002), Tamura (2002) and Weisdorf (2007). The current paper contributes to this field of study from two perspectives: in part by introducing seasonality aspects to the production of agrarian output, in part by demonstrating, by contrast to the existing studies, that agrarian labour productivity may indeed decline along with the process of industrialisation.

Secondly, the current work connects to a line of studies that combine Malthusian and Boserupian features. Such a synthesis has been addressed, among others, by Kremer (1993), Kuznets (1973), Lee (1986,1988), Simon (1977) and Strulik (1997). The existing literature builds primarily on Boserup's assertion that population growth drives technological progress (rather than the reverse), an approach not adopt in the current study. Instead, emphasis here is put on another leading notion advanced by Boserup, namely that agricultural intensification raises labour costs per unit of food produced.

Finally, the present work relates to a field of study within development economics on labour surplus. In an attempt to explain how industrial capital accumulation takes place without affecting labour wages, the so-called *surplus labour theory* (first proposed by Lewis (1954) and further developed by Fei and Ranis (1964) and other) rests on an assumption of "unlimited" labour supplies in agriculture. The current work contributes to this particular field of study by demonstrating how the introduction of seasonality aspects helps motivating the existence of surplus labour in agriculture.

The paper continues as follows. Section 2 constructs a simple theoretical framework—a Malthusian model with seasonality—suitable for analysing the effects of technological progress on living standards, population size/density and agrarian land and labour productivity. Section 3 provides a comparative static analysis demonstrating how rates of labour productivity may go up or down with land productivity, depending on the relative rate of growth of technology in cropping and non-cropping agricultural activities, respectively. Finally, Section 5 concludes.

2 The model

Consider a one-sector economy that produces a single type of good, namely food. Food production is a two-step procedure: first, food is cultivated (an intermediate goods activity), then cropped (a final goods activity). Economic activities extent over infinite (discrete) time. Unless explicitly stated, all variables are considered in period t.

There are two types of seasons: high (or peak) seasons and low (or off) seasons. The fraction $\gamma \in (0,1)$ measures the total length of the high seasons to the entire year. The size of γ may be affected by geography, climate, crop types etc, but throughout the analysis is taken to be exogenous (or already optimally chosen).

Intermediate goods production (i.e. non-cropping agricultural activities such as seeding, weeding, harrowing, hoeing etc) takes place exclusively during the low seasons. Final goods production (i.e. cropping activities) takes place only during the high seasons.

2.1 Demography

In period t, the economy's adult population (synonymous to its labour force) consists of N_t identical individuals. At the beginning of each period, a new generation replaces the old one. Evolution in the size of the population from one period to the next is thus given by

$$N_{t+1} = n_t N_t,$$

where n_t is the number of surviving offspring of any given adult individual in generation t. Since all individuals are identical, n_t is the gross rate of growth of population.

Suppose that surviving offspring are 'checked' (in a Malthusian sense) by the resources of their parents. More specifically, the number of surviving offspring is an increasing function of an adult's income, measured by w, i.e.

$$n_t = n\left(w_t\right), \tag{1}$$

where $n(\cdot)$ is assumed to be continues and monotonic, with $n(0) = 0$ and $n(\infty) > 1$. It follows that there exists a unique level of income per capita at which the population level is constant (i.e. where $n = 1$). Throughout, this income level will be referred to as *the level of subsistence.*

2.2 Output

As explained above, the production of output is a two-step procedure, involving first the production of intermediate goods, then the production of final goods.

Intermediate goods production technology Intermediate goods production is subject to constant returns to land and labour. As is common in the related literature (e.g. Galor and Weil, 2000), the use of capital in production is suppressed throughout the analysis. The intermediate goods production function (superscript L for *low-season* agricultural activities) is

$$Y^L = A^L \left((1-\gamma) e^L N\right)^{\alpha} X^{1-\alpha}, \quad \alpha \in (0,1) \tag{2}$$

where $1-\gamma$ is the total length of the low seasons to the entire year; $e^L \in [0,1]$ is the share of total low-season labour resources employed during the low season; X are units of land put under cultivation (land is in fixed supply, hence no subscript t); and A^L measures total factor productivity in intermediate goods production. The net rate of growth of low-season total factor productivity between any two periods is given by $g_{A^L} \geqq 0$.

Final goods production technology Final goods production is also subject to constant returns to land and labour. The final goods production function (superscript H for *high-season* agricultural activities) is

$$Y^H = A^H \left(\gamma e^H N\right)^{\beta} X^{1-\beta}, \quad \beta \in (0,1), \tag{3}$$

where $e^H \in [0,1]$ is the share of total high-season labour resources employed, and A^H is total factor productivity in final goods production, which has a net growth rate between any two periods of $g_{A^H} \geqq 0$.

High-season effort and wages Following Boserup (1975), who holds that high-season labour supply is normally a 'bottleneck' in development, the high-season labour supply will set the upper limit to agricultural output (hence the term *peak season*). As will become apparent below, this implies that final output is limited, not by how much food can be grown, but by how much food can be cropped.

If high-season labour is a bottleneck, then the total high-season labour resources are employed in final goods production, i.e. $e^H = 1$. Rewriting (3), the economy's total output is thus

$$Y^H = A^H (\gamma N)^\beta X^{1-\beta} \tag{4}$$

As is normally assumed in the related literature (e.g. Galor and Weil, 2000), there are no property rights over land, so land rent is set to zero. Hence, individuals receive the average product, and income per capita, measure by w, thus equals output per capita, so that

$$w \equiv \frac{Y^H}{N} = A^H \gamma^\beta \left(\frac{X}{N}\right)^{1-\beta}. \tag{5}$$

Low-season effort and labour surplus By definition, unharvested goods decay by the end of the high season. This means that no more low-season labour resources are employed during the low season than it takes to grow the total amount of final goods that can be cropped. Equating (2) and (3), therefore, it follows that the share of low-season labour resources employed during the low season is

$$e^L = \left(\frac{A^H}{A^L}\frac{\gamma^\beta}{(1-\gamma)^\alpha}\left(\frac{N}{X}\right)^{\beta-\alpha}\right)^{1/\alpha} < 1. \tag{6}$$

Consistent with Boserup's position that high-season (rather than low-season) labour supply is a bottleneck, it is assumed throughout the analysis that the inequality in (6) is not violated. Empirically, this assumption also finds support in studies done, for example, by Allen (1988), Campbell and Overton (1991), Clark (1991), Karakacili (2004) and Voth (2000). Karakacili (2004), for instance, makes a strong case that agrarian labour surplus existed in the pre-industrial era, even at a time thought to have had the lowest agrarian labour productivity rates, namely shortly prior to the Black Death. Similarly, Voth's (2000) finding—that English farmers, even over a period where the share of labour employed in agriculture was dropping, were capable of increasing their annual working hours by 45 percent between 1760 and 1800—provide evidence of the existence of pre-industrial agrarian labour surplus.

Land and labour productivity Following Hunt (2000), land productivity, denoted D, is calculated as output per unit of land, i.e.

$$D = \frac{wN}{X} \tag{7}$$

Labour productivity is calculated as output per unit of land divided by labour input per unit of land. Using (4)–(6), labour productivity, denoted E, can thus be written as

$$E = \frac{D}{\left(\gamma + (1-\gamma)\, e^L\right) N/X} = \frac{w}{\gamma + (1-\gamma)\, e^L}. \tag{8}$$

3 Analysis

In the following, we want to analyse the effects of technological progress on a variety of variables in steady state. In steady state, all dynamic variables grow at a constant rate (possibly zero). First, we identify the wage rate, the population density, and the land and labour productivity in steady state. Then, we explore the effects on these variables of technological progress.

To simplify matters, and without loss of generality, we can use the parametrisation $n(w) = w$ to analyse steady state and stability conditions. Using (5), the gross rate of growth in income per capita from one period to the next is

$$\frac{w_{t+1}}{w_t} = \frac{A^H \gamma^\beta \left(\frac{X}{N_{t+1}}\right)^{1-\beta}}{A^H \gamma^\beta \left(\frac{X}{N_t}\right)^{1-\beta}} = \left(\frac{1}{w_t}\right)^{1-\beta}.$$

In steady state, the rate of growth of population, and, therefore, that of income per capita, is constant. Setting $w_{t+1} = w_t$, we obtain a unique, non-trivial steady state level of income per capita at

$$w^* = 1. \tag{9}$$

Since in steady state (marked with asterisks) the population level is constant ($n = w = 1$ so $N_{t+1} = N_t$), income per capital in steady state is at *the level of subsistence.*

Notice that stability of the steady state requires that $\partial w_{t+1}/\partial w_t \left|_{w_t=w^*}\right. < |1|$. This condition is fulfilled when $\beta < 1$, i.e. with diminishing returns to labour in the production of output, which we have assumed above.

Using (5), it follows that the steady state population level (or density, when setting $X \equiv 1$) is

$$N^* = \left(\gamma^\beta A^H\right)^{1/(1-\beta)} \equiv N^*(\underset{+}{\gamma}, \underset{+}{A^H}), \quad X \equiv 1. \tag{10}$$

Notice that high-season technological progress makes living-conditions more crowded, i.e. increases population density. Low-season technological progress, on the other hand, has not effect on the density of population.

Furthermore, inserting (10) into (6), the share of labour resources employed during the low season in steady state is

$$e^{L*} = \left(\frac{\left(\gamma^\beta A^H\right)^{\frac{1-\alpha}{1-\beta}}}{(1-\gamma)^\alpha A^L}\right)^{1/\alpha} \equiv e^{L*}(\underset{+}{\gamma}, \underset{+}{A^H}, \underset{-}{A^L}) \tag{11}$$

It follows that high-season technological progress is labour-demanding, i.e. requires a more extensive use of labour resources. Low-season technological progress, however, has the opposite effects.

Finally, inserting (9) and (10) into (7), land productivity in steady state can be written as

$$D^* = \left(\gamma^\beta A^H\right)^{1/(1-\beta)} = D^*(\underset{+}{\gamma}, \underset{+}{A^H})$$

while, combining (8) and (11), it follows that labour productivity in steady state is

$$E^* = \frac{1}{\gamma + \left(\frac{\left(\gamma^\beta A^H\right)^{\frac{1-\alpha}{1-\beta}}}{A^L}\right)^{1/\alpha}} \equiv E^*(\underset{-}{\gamma}, \underset{-}{A^H}, \underset{+}{A^L}). \tag{12}$$

Notice first that only technological progress in cropping activities gives rise to higher land productivity. Notice also the ambiguous effects of technological progress on the labour productivity: technological progress in non-cropping activities increases labour productivity; by contrast, however, technological progress in cropping activities, somewhat surprisingly, reduces labour productivity.

The latter result is due to diminishing returns to labour, which implies that population growth ultimately brings down wages to the level of subsistence. Because high-season technological progress demands more work-effort during the low season, people end up working harder without getting higher wages. As a result, rates of labour productivity decline with technological progress in cropping activities.

And now for the question that we set out to answer in the introduction: Did labour productivity go up or down with higher land productivity? To answer the question, consider equation (12). Higher land productivity implies that there is technological progress in cropping activities (i.e. that A^H goes up). Whether labour productivity goes up or down with higher land productivity thus depends on whether

$$g_{A^L} \gtrless \frac{1-\alpha}{1-\beta} g_{A^H}.$$

That is, if technological progress in cropping activities appears at a faster rate than the progress in non-cropping, or, more specifically, if $g_{A^H} > \frac{1-\beta}{1-\alpha} g_{A^L} > 0$, then labour productivity goes down with higher land productivity, and vice versa.

4 Conclusion

This study demonstrates that seasonality is vital for understanding conflicting effects of technological progress on land and labour productivity in a Malthusian environment. The analysis shows that labour productivity in agriculture goes up or down with higher land productivity, depending on the relative rate of growth of technological progress in cropping and non-cropping activities, respectively.

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TABLE 1

Estimated English Population Distribution

Year	Total (mio.)	Agri. (mio.)	Agri. (pct.)
1500	2.5	1.85	74.0
1600	4.4	3.03	68.8
1700	5.2	2.86	55.0
1750	6.0	2.70	45.0
1800	9.1	3.23	35.5

Source: Allen (2000)