The Great Escape?

A Quantitative Evaluation of the Fed's Liquidity Facilities*

Marco Del Negro, Gauti Eggertsson, Andrea Ferrero, Nobuhiro Kiyotaki

Federal Reserve Bank of New York and Princeton University

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Abstract

We introduce liquidity frictions into an otherwise standard DSGE model with nominal and real rigidities, explicitly incorporating the zero bound on the short-term nominal interest rate. Within this framework we ask: Can a shock to the liquidity of private paper lead to a collapse in short-term nominal interest rates and a recession like the one associated with the 2008 U.S. financial crisis? Once the nominal interest rate reaches the zero bound, what are the effects of interventions in which the government exchanges liquid government assets for illiquid private paper? We find that the effects of the liquidity shock can be large, and show some numerical examples in which the liquidity facilities prevented a repeat of the Great Depression in 2008-2009.

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Key Words: Financial crisis, liquidity shocks, financing constraints, liquidity facilities, zero lower bound

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1 Introduction

In December 2008, the federal funds rate collapsed to zero. Standard monetary policy through interest rate cuts had reached its limit. Around the same time, the Federal Reserve started to expand its balance sheet. By January 2009, the overall size of the Fed's balance sheet exceeded \$2 trillion, an increase of more than \$1 trillion compared to a few months earlier (Figure 1). This expansion mostly involved the Federal Reserve exchanging government liquidity (money or government debt) for private financial assets through direct purchases or collateralized short-term loans. These direct interventions in private credit markets were implemented via various facilities, such as the Term Auction Facility, the Primary Dealer Credit Facility, and the Term Securities Lending Facility.[1] In broad terms, these facilities can be thought of as non-standard open market operations, whereby the government exchanges highly liquid government paper for less liquid private paper. Alternatively, one can think of them, broadly speaking, as non-standard discount window lending, which provides government liquidity using private assets as collateral. This paper studies the quantitative effects of these liquidity policies on macroeconomic and financial variables.

Ever since the famous irrelevance result of Wallace (1981), the benchmark for many macroeconomists is that non-standard open market operations in private assets are irrelevant. Eggertsson and Woodford (2003) show that this result extends to standard open market operations in models with nominal frictions and money in the utility function, provided that the nominal interest rate is zero. Once the nominal interest rate reaches its lower bound, "liquidity" has no further role in this class of models, or in most other standard models with various types of frictions, such as Rotemberg and Woodford (1997) or Christiano et al. (2005).

In this paper, we depart from Wallace's irrelevance result by incorporating a particular form of credit frictions proposed by Kiyotaki and Moore (2008) (henceforth, KM). The KM credit frictions are of two distinct forms. First, a firm that faces an investment

[1] See Armantier et al. (2008), Adrian et al. (2009), Fleming et al. (2009), and Adrian et al. (2011) for details about the various facilities.

opportunity can borrow only up to a fraction of the value of its current investment. This friction is a relatively standard financing constraint.[2] Second, a firm that faces an investment opportunity can sell only up to a certain fraction of the "illiquid" assets on its balance sheet in each period. In the model, these illiquid assets correspond to equity holdings of other firms. More generally, we interpret these illiquid assets as privately issued paper such as commercial paper, bank loans, mortgages, and so on. This friction is a less standard "resaleability" constraint.

In contrast to private assets, we follow KM and assume that government paper, i.e., money and bonds, is not subject to the resaleability constraint. This assumption gives government paper a primary role as "liquidity". In this world, Wallace's irrelevance result no longer applies because the composition of liquid and illiquid assets in the hands of the private sector affects the equilibrium, and the government can change this composition. The assumption of limited resaleability of private paper and the role of government paper as liquidity gives a natural story for the crisis of 2008 and the ensuing Fed's response. In our study, the source of the crisis of 2008 is a shock to the resaleability of private paper. Suddenly, the secondary markets for this type of assets froze. We think of this shock as capturing a central aspect of the crisis.

We embed the KM credit frictions in a relatively standard dynamic stochastic general equilibrium (DSGE) model along the lines of Christiano et al. (2005) and Smets and Wouters (2007). The model features nominal and real frictions, such as price and wage rigidities and aggregate capital adjustment costs. Conventional monetary policy is implemented via variations in the nominal interest rate according to a standard interest rate policy rule that is constrained by the zero bound. Non-conventional policy consists of open market operations in private assets that increase the overall level of liquidity in

[2]This constraint is similar to the collateral requirement in Kiyotaki and Moore (1997). Earlier contributions (Kocherlakota (2000), and Cordoba and Ripoll (2004)) argued that collateral constraints have a limited quantitative role in amplifying macroeconomic fluctuations. This result is, however, conditional on the fundamental shocks that drive the business cycle. Financial constraints do amplify shocks that shift the demand of collateral (Liu et al. (2010)). Nezafat and Slavik (2010) find that such shocks are capable of generating asset price volatility in the model comparable to that of the aggregate stock market in the data.

the economy. We use the more than \$1 trillion intervention by the Fed to calibrate the non-standard policy reaction function of the government.

Our main result is that both the financial shock and the liquidity policy can have a quantitatively large effect. A calibrated shock to the resaleability constraint makes output and inflation drop by about the same magnitude as in the data. The impact of the policy intervention is substantial. In our baseline scenario, absent non-standard open market operations, output and inflation would have dropped by an additional 50%. Our quantitative results depend crucially on the expected duration of the crisis. Had private agents expected a more persistent freeze in the private paper market, the economy may have suffered a second Great Depression in the absence of interventions. With intervention, the economy "escapes" from a repeat of the Great Depression in some of our numerical examples (hence, the title of the paper). The reason is that liquidity policies can have especially large effects at zero interest rates, a result reminiscent of the case of the multiplier of government spending in Eggertsson (2009) and Christiano et al. (2009a).

Nominal rigidities and the zero bound play a crucial role in our analysis. Under flexible prices, the KM financial frictions can only account for a drop in investment. In this case, aggregate output is almost unchanged because consumption makes up for the fall in investment. The consumption boom requires the real interest rate to be negative in order to induce people to spend more. But the real interest rate can hardly fall if the nominal interest rate cannot turn negative and prices are sluggish. As a consequence, with nominal rigidities and a plausible specification of monetary policy that incorporates the zero lower bound, the freeze in the private paper market triggers a drop not only in investment, but also in consumption and aggregate output.

Unconventional policy can alleviate the crisis by targeting directly the source of the problem, which is the loss of liquidity of the private paper. By swapping partially illiquid private paper for government liquidity, thus making the aggregate portfolio holdings of the private sector more liquid, the intervention lubricates financial markets, reducing the fall in investment and consumption. Importantly, we are not assuming that the policy intervention violates any of the private sector resaleability constraints. Instead, the intervention only increases the overall supply of government paper by purchasing private paper in the open market while respecting all individual resaleability constraints.

This paper belongs to the strand of literature introducing financial frictions in monetary DSGE models, such as Bernanke et al. (1999), Christiano et al. (2003), Christiano et al. (2009b), Goodfriend and McCallum (2007), Brunnermeier and Sannikov (2009), and Curdia and Woodford (2009a). Recently, Gertler and Karadi (2011), Gertler and Kiyotaki (2010), and Curdia and Woodford (2009b) have also analyzed the role of nonconventional central bank policies during the Great Recession. The key difference in our paper is that we study the type of frictions suggested in KM, which allows us to characterize the crisis as a drying-up of liquidity in the secondary markets. In our view, a reduction in the resaleability of private papers represents a relatively natural story for the shock that triggers the downturn and the under-utilization of the factors of production.[3] Additionally, this approach provides an immediate rationale for thinking about the liquidity interventions.

Our main focus is on the Great Recession, which according to the National Bureau of Economic Research dates began in December 2007 and ended in June 2009, with the focal point being the default of Lehman Brothers in September 2008. Although the market for mortgage-backed securities stopped working well in August 2007, our paper concentrates on the events that followed the default of Lehman. The Fed facilities that we evaluate in this paper were started in December 2007 and were escalated with the collapse of Lehman in the fall of 2008, when the fed funds rate ultimately reached zero.[4]

Before going further, we should emphasize a few important limitations of our analysis. While we believe our approach is a natural first cut, we should stress that the

[3]Ajello (2011) estimates that a financial intermediation shock in a model with KM-type financial frictions accounts for 40% of output and 55% of investment volatility.

[4]Our analysis does not extend to the large-scale asset purchase program ("Quantitative Easing II") implemented during the fall of 2010 in response to the further weakening of economic activity. The differences in liquidity between private and public assets do not apply to this type of intervention. The "preferred habitat" theory can provide a rationale for this type of asset purchase program based on the notion of limits to arbitrage (Vayanos and Vila (2009)). Chen et al. (2011) embed a preferred habitat framework in an otherwise standard DSGE model to estimate the effects of asset purchase programs on macroeconomic variables.

liquidity constraints proposed by KM are "reduced form." Consequently, our model is silent on whether the Fed's interventions can affect the incentive structure of the private sector. This aspect is certainly important, as the private sector response may lead to an endogenous change in the liquidity constraints that we currently take as given.[5] More generally, we abstract from the costs of intervening, which can take many forms. Therefore, our paper has only positive, not normative, content: We show that liquidity interventions can be quantitatively important for macroeconomic stability in the short-run. Our findings suggest that understanding the consequences of these policies for the incentive of the private sector should be a high priority on the research agenda.

Sections 2 and 3 describe the model and its calibration, respectively. Section 4 discusses the results, and section 5 concludes.

2 The Model

The model can be described as KM augmented with both nominal and real frictions. The economic actors in the model are households, whose members are entrepreneurs and workers, the government, intermediate and final goods firms, labor agencies, and capital producers.

2.1 Households

The economy is populated by a continuum of identical households of measure one. Each household consists of a continuum of members indexed by $j \in [0,1]$. In every period, household members receive an i.i.d. draw that determines whether they are entrepreneurs or workers. The probability of being an entrepreneur is $\varkappa$, which, by the law of large numbers, is also the fraction of entrepreneurs in the household. Each entrepreneur

[5] Kurlat (2010) shows that this constraint arises endogenously in a model in which entrepreneurs have asymmetric information about the quality of existing assets.

$j \in [0, \varkappa)$ has an opportunity to invest but does not work. Each worker member $j \in [\varkappa, 1]$ supplies differentiated labor of type j but does not invest.[6]

Let $C_t(j)$ denote the amount of the consumption good each member of the household purchases in the market place in period t. A key assumption of the representative household structure is that, at the end of the period, all members bring the consumption purchases back to the household, and these goods get distributed equally among all members. Utility thus depends upon the sum of all the consumption goods bought by the different household members

$$C_t \equiv \int_0^1 C_t(j)\, dj. \tag{1}$$

Let $H_t(j)$ be hours worked by worker member j. The household's objective is

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{\omega}{1+\nu} \int_{\varkappa}^1 H_s(j)^{1+\nu}\, dj \right], \tag{2}$$

where $\beta \in (0, 1)$ is the subjective discount factor, $\sigma > 0$ is the coefficient of relative risk aversion, $\nu > 0$ is the inverse Frisch elasticity of labor supply, and $\omega > 0$ is a parameter that pins down the steady-state level of hours. This construction of the representative household permits us to study a situation in which people face idiosyncratic investment opportunities, while at the same time retaining the tractability of the representative household structure, thus abstracting from consumption heterogeneity across different types of agents.[7]

[6] Although each member randomly becomes an entrepreneur or a worker, we re-number household members every period so that a member $j \in [0, \chi)$ is an entrepreneur and a member $j \in [\chi, 1]$ is a worker who supplies type-j labor.

The original KM model features heterogeneity. Each entrepreneur occasionally receives an opportunity to invest while workers never do. Aggregation is obtained by imposing a few additional restrictive assumptions. In this paper, we adopt a modified version of the KM model based on Shi (2011), who assumes that both entrepreneurs and workers are members of the same household. This variation of the KM model is more amenable to modifications, allowing us to perform a more extensive sensitivity analysis.

[7] Lucas (1990) presents a similar representative household structure where only some agents face cash-in-advance constraints. This construct is also very similar to the perfect consumption insurance assumption common in the literature (e.g., Rogerson (1988)).

At the end of each period, the household also shares all the assets accumulated during the period among members. Entering the next period, therefore, each member of the household holds an equal share of the household's assets. A key assumption is that, after the idiosyncratic shock is realized and each member knows its type, the household cannot reshuffle the allocation of resources among its members. Instead, those household members who would like to obtain more funds need to seek the money from other sources. The assets available to household members are described in the table below, which summarizes the household's balance sheet at the beginning of period t (before interest payments), expressed in terms of the consumption goods. Households own government-issued nominal bonds B_t, where P_t is the price level, K_t is physical capital, and N_t^O represents claims on other households' capital. Households' liabilities consist of claims on own capital sold to other households N_t^I, and net equity N_t is defined as

$$N_t = N_t^O + K_t - N_t^I. \tag{3}$$

Capital is homogeneous, yields a per-unit dividend stream r_t^k, and has a unit value of q_t. A fraction δ of capital depreciates in each period. Bonds pay a gross nominal interest rate R_t. Note that all households liabilities – all claims to the assets of the private sector in the model – are in the form of equity.

Household's Balance Sheet (Tradable Assets)

Assets		Liabilities	
nominal bonds	B_t/P_t	equity issued	$q_t N_t^I$
others' equity	$q_t N_t^O$		
capital stock	$q_t K_t$	net worth	$q_t N_t + B_t/P_t$

Households also own a fully-diversified, non-tradable portfolio of intermediate-goods-producing firms and capital-producing firms, which pay the per-period profits $D_t = \int_0^1 D_t(i)\, di$ and D_t^I, respectively. Finally, households pay lump-sum taxes τ_t to the government. All of these income flows are obtained by the household at the beginning of the period and are evenly distributed across members.

During the operation of the market, members decide how to allocate their resources between purchases of the non-storable consumption good, savings in the different assets, and, if entrepreneurs, investment in new capital. Those members who are workers also supply the hours demanded by firms at the wage contracted by the labor unions (as we shall see, workers have some monopolistic power and wages are sticky) and can therefore include their salaries among the available resources. Specifically, each household member's flow of funds is

$$\begin{aligned} C_t(j) + p_t^I I_t(j) + q_t[N_{t+1}(j) - I_t(j)] + \frac{B_{t+1}(j)}{P_t} \\ = \left[r_t^k + (1-\delta) q_t\right] N_t + \frac{R_{t-1} B_t}{P_t} + \frac{W_t(j)}{P_t} H_t(j) + D_t + D_t^I - \tau_t, \quad (4) \end{aligned}$$

where $H_t(j) = 0$ for entrepreneurs ($j \in [0, \varkappa)$) and $I_t(j) = 0$ for workers ($j \in [\varkappa, 1]$), $W_t(j)$ is the nominal wage for type-j labor and p_t^I is the cost of a unit of new capital in terms of the consumption good, which differs from 1 due to capital adjustment costs.

Most of the action in the model is a consequence of the financial frictions, which translate into constraints on the financing of new investment projects by entrepreneurs and on the evolution of the balance sheet.[8] The key frictions proposed by KM that we adopt here are of two forms. First, a *borrowing constraint* implies that any entrepreneur can only issue new equity up to a fraction θ of her investment. Second, a *resaleability constraint* implies that in any given period a household member can sell only a fraction ϕ_t of her existing equity holdings. An important simplification in KM is that the equity issued by the other households is a perfect substitute for the equity position in the household's own business (capital stock minus equity issued) and thus subject to exactly the same resaleability constraint.[9] As a consequence, the two resaleability constraints (on

[8]These frictions are also front and center in the original KM formulation. We assume a slightly different asset market structure in which government-issued paper, rather than money (effectively a "bubble asset"), serves as the liquid asset and pays a nominal interest rate R_t. We make this assumption because we characterize conventional monetary policy in terms of nominal interest rate setting, as standard in the New Keynesian literature (e.g., Woodford (2003)) and we study issues related to the zero lower bound.

[9]Thus, in addition to selling a fraction ϕ_t of the equity holdings of the other households, each household can remortgage a fraction ϕ_t of capital stock that has not been borrowed against previously. This simplification is essential for aggregation in KM. While not indispensable in our model with a representative household, we continue to use this assumption in order to simplify the algebra.

claims on capital of other households and on claims on own capital) can be consolidated (see the appendix for the explicit derivation) and written in terms of net equity N_t

$$N_{t+1}(j) \geq (1-\theta)I_t(j) + (1-\phi_t)(1-\delta)N_t. \tag{5}$$

The first part of the right-hand side of the inequality in (5), $(1-\theta_t)I_t(j)$, represents a constraint on borrowing to finance new investment for those agents who have an investment opportunity. If θ were equal to 1, the entrepreneur would be able to finance the entire investment by selling equity in financial markets. When $\theta < 1$, the entrepreneur is forced to retain $1-\theta$ fraction of investment as own equity and use her own fund to partly finance the investment cost. The second part of the right-hand side, $(1-\phi_t)(1-\delta)N_t$, represents the resaleability constraint. In period t, household members can sell only a fraction ϕ_t of their existing equity. While literally ϕ_t represents a transaction cost, we follow KM in interpreting changes in ϕ_t as "liquidity shocks". These shocks capture, in reduced form, changes in market liquidity. Alternatively, ϕ_t can also be thought of as a haircut in the repo market – a measure of how much liquidity entrepreneurs can obtain for 100 dollars of collateral. Under this interpretation, shocks to ϕ_t capture changes in funding conditions in the repo market.[10] The purpose of this paper is to investigate whether this shock alone can be responsible for the bulk of the Great Recession, and the extent to which unconventional policy was successful in mitigating the impact of this shock.

Another important feature of the model is that the asset B_t is not subject to any resaleability constraint and is therefore "liquid." Obviously, household members for whom constraint (5) is binding would like to acquire resources from the market by issuing liquid assets. We rule out this possibility by assuming that only the government can issue the liquid asset while households can only take a long position in it:

$$B_{t+1}(j) \geq 0. \tag{6}$$

Broadly speaking, we think of equity in the model as comprising all claims on private assets, which in reality take the form of equity or debt, while B_t represents any form of

[10] Gorton and Metrick (2011) argue that a run on the repo market is at the origin of the collapse of financial markets in the fall of 2008.

government paper. The two constraints (5) and (6) are central to the analysis. The next section argues that, in equilibrium, both constraints are binding for entrepreneurs and studies the consequences for the household decision problem as a whole.

At the end of the period, household equity, bond holdings, and capital are given, respectively, by

$$
\begin{aligned}
N_{t+1} &= \int N_{t+1}(j)\,dj, && (7)\\
B_{t+1} &= \int B_{t+1}(j)\,dj, && (8)\\
K_{t+1} &= (1-\delta)K_t + \int I_t(j)\,dj. && (9)
\end{aligned}
$$

We now move to the actual decisions of each type of household member. An important assumption is that each member of the household acts in the interest of the whole family.

2.1.1 Entrepreneurs

The flow of funds for entrepreneur $j \in [0, \varkappa)$ is given by expression (4), with $H_t(j) = 0$. That constraint clarifies that, as long as the market price of equity q_t is greater than the price of newly produced capital p_t^I, entrepreneurs trying to maximize the household's utility will use all available resources to create new capital. In the rest of the paper, we focus on constrained equilibria in which the condition $q_t > p_t^I$ is satisfied.[11] In these equilibria, entrepreneurs sell all holdings of government bonds because the expected return on new investment dominates the return on the liquid asset. Furthermore, the entrepreneur also sells as much existing equity as possible and issues the maximum amount of new equity to take full advantage of the investment opportunity. As a consequence, the constraints arising from financial frictions (5) and (6) are both binding, and entrepreneurs spend no resources on consumption goods:

$$
\begin{aligned}
N_{t+1}(j) &= (1-\theta)I_t(j) + (1-\phi_t)(1-\delta)N_t(j). && (10)\\
B_{t+1}(j) &= 0, && (11)\\
C_t(j) &= 0, && (12)
\end{aligned}
$$

[11] We first ensure that the condition $q_t > p_t^I$ holds at steady state, and then check that it is satisfied in our numerical experiments.

for $j \in [0, \varkappa]$.[12]

Substituting (10) through (12) into the flow of funds (4) and setting $H_t(j) = 0$, we obtain the amount of investment by each entrepreneur:

$$I_t(j) = \frac{\left[r_t^k + (1-\delta) q_t \phi_t\right] N_t + \frac{R_{t-1}B_t}{P_t} + D_t + D_t^I - \tau_t}{p_t^I - \theta q_t}. \tag{13}$$

Therefore, aggregate investment in the economy equals

$$I_t = \int_0^{\varkappa} I_t(j)\, dj = \varkappa \frac{\left[r_t^k + (1-\delta) q_t \phi_t\right] N_t + \frac{R_{t-1}B_t}{P_t} + D_t + D_t^I - \tau_t}{p_t^I - \theta q_t}. \tag{14}$$

The denominator represents the liquidity needs for one unit of investment – the gap between the investment goods price and the amount the entrepreneur can finance by issuing equity (θq_t). The numerator measures the amount of liquidity available to entrepreneurs. Clearly, a drop in ϕ_t reduces the amount of liquidity available to finance investment.[13]

2.1.2 Workers

The flow of funds for worker $j \in [\varkappa, 1]$ is given by expression (4), with $I_t(j) = 0$. Workers do not choose hours directly. Rather, the union who represents each type of worker member sets wages on a staggered basis, as explained in the next section. As a consequence, the household supplies labor as demanded by firms at the posted wages.

In order to find the workers' decisions in terms of asset and consumption choices, we derive the household's decisions for N_{t+1}, B_{t+1}, and C_t as a whole, taking wages and hours as given. Since we know the solution for entrepreneurs from the last section (that

[12]The fact that entrepreneurs allocate no resources to consumption is a consequence of our assumption on the household. Both entrepreneurs and workers can buy the consumption good that is jointly consumed at the end of the period. Since entrepreneurs are constrained, it is optimal for workers to buy all the consumption goods, directing all of the liquidity of entrepreneurs to investment.

[13]The entrepreneurs should not be thought of as the same characters populating the entrepreneurship literature in macroeconomics (see Quadrini (2009) for an extensive review). Instead, entrepreneurs here are best thought as capturing the broad functions of financial markets – funneling resources from savers to the production sector of the economy. The key friction in the model consists of an impediment to this "funneling", which intensifies in the event of a financial crisis.

is, $N_{t+1}(j)$, $B_{t+1}(j)$ and $C_t(j)$ for $j \in [0, \varkappa]$), constraints (1), (7), and (8) determine $C_t(j)$, $N_{t+1}(j)$, and $B_{t+1}(j)$ for workers. We then check that these choices satisfy the financing constraints (5) and (6) for workers.[14]

The aggregation of workers' and entrepreneurs' budget constraints yields

$$C_t + p_t^I I_t + q_t(N_{t+1} - I_t) + \frac{B_{t+1}}{P_t}$$
$$= \left[r_t^k + (1-\delta)\, q_t\right] N_t + \frac{R_{t-1}B_t}{P_t} + \int_{\varkappa}^{1} \frac{W_t(j)\, H_t(j)}{P_t} dj + D_t + D_t^I - \tau_t. \quad (15)$$

Households choose C_t, N_{t+1}, and B_{t+1} in order to maximize (2) subject to (15) and (13). As long as $q_t > p_t^I$, the first-order conditions for bonds and equity are, respectively,

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \left[\frac{R_t}{\pi_{t+1}} + \frac{\varkappa(q_{t+1} - p_{t+1}^I)}{p_{t+1}^I - \theta q_{t+1}} \frac{R_t}{\pi_{t+1}} \right] \right\}, \quad (16)$$

where π_t is the gross inflation rate, and

$$C_t^{-\sigma} = \beta \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \left[\frac{r_{t+1}^k + (1-\delta) q_{t+1}}{q_t} + \frac{\varkappa(q_{t+1} - p_{t+1}^I)}{p_{t+1}^I - \theta q_{t+1}} \frac{r_{t+1}^k + (1-\delta)\phi_{t+1} q_{t+1}}{q_t} \right] \right\}. \quad (17)$$

Equations (14), (16), and (17) describe the household's choice of investment, consumption and portfolio for a given price process.

The payoff from holding paper, either bonds or equity, consists of two parts. The first is the standard return: $\frac{R_t}{\pi_{t+1}}$ for bonds and $\frac{r_{t+1}^k + (1-\delta) q_{t+1}}{q_t}$ for equity. The second is the premium associated with the fact that this paper, when in the hand of entrepreneurs, relaxes their investment constraint. The value of this premium is $\frac{\varkappa(q_t - p_t^I)}{p_t^I - \theta q_t}$. The quantity $\frac{\varkappa}{p_t^I - \theta q_t}$ measures the increase in investment afforded by an extra dollar of liquidity (where $\varkappa$ and $\frac{1}{p_t^I - \theta q_t}$ capture the fraction of liquidity going to entrepreneurs and the extent to which the investment constraint is relaxed per extra unit of liquidity, respectively). The magnitude $q_t - p_t^I$ measures the value to the household of relaxing the constraint. The larger the difference between q_t and p_t^I, the more valuable for the household to acquire

[14]Different workers will buy different amounts of consumption goods and assets because of different labor incomes as a result of wage stickiness. But this heterogeneity does not affect the household level of consumption and portfolio choices due to end-of-period aggregation.

capital by investing and pay p_t^I per unit, rather than pay q_t on the market. This premium applies to the entirety of bond returns, but only to the liquid part of the equity return $\frac{r_{t+1}^k+(1-\delta)\phi_{t+1}q_{t+1}}{q_t}$, if ϕ_{t+1} is less than 1. Hence, equity pays a premium relative to bonds because of its lower "liquidity."

2.1.3 Labor Agencies and Wage Setting

Competitive labor agencies combine j-specific labor inputs into a homogeneous composite H_t according to

$$H_t = \left[\left(\frac{1}{1-\varkappa}\right)^{\frac{\lambda_w}{1+\lambda_w}} \int_{\varkappa}^{1} H_t\left(j\right)^{\frac{1}{1+\lambda_w}} dj\right]^{1+\lambda_w}, \tag{18}$$

where $\lambda_w \geq 0$.[15] Firms hire the labor input from the labor agencies at the wage W_t, which in turn remunerate the household for the labor actually provided. The zero profit condition for labor agencies implies that

$$W_t H_t = \int_{\varkappa}^{1} W_t(j) H_t(j) dj. \tag{19}$$

The demand for the j^{th} labor input is

$$H_t\left(j\right) = \frac{1}{1-\varkappa}\left[\frac{W_t\left(j\right)}{W_t}\right]^{-\frac{1+\lambda_w}{\lambda_w}} H_t, \tag{20}$$

where $W_t\left(j\right)$ is the wage specific to type j and W_t is the aggregate wage index that comes out of the zero profit condition for labor agencies

$$W_t = \left[\frac{1}{1-\varkappa}\int_{\varkappa}^{1} W_t(j)^{-\frac{1}{\lambda_w}} dj\right]^{-\lambda_w}. \tag{21}$$

Labor unions representing workers of type j set wages on a staggered basis, taking as given the demand for their specific labor input (Erceg et al. (2000)). In each period, with probability $1-\zeta_w$, a union is able to reset the wage $W_t(j)$, while with the complementary probability the wage remains fixed. Workers are committed to supply whatever amount of labor is demanded at that wage. In the event of a wage change at time t, unions

[15] We add constant $(1-\varkappa)^{-1}$ to the labor composite (18) so that it is equal to the average labor used under symmetry. Because there is no entry of new types of labor, it only simplifies the notation without changing the substance.

choose the wage $\tilde{W}_t(j)$ to minimize the present discounted value of the disutility from work conditional on not changing the wage in the future subject to (20).[16] The optimality condition for this problem, which is standard, is fully spelled out in the appendix.

2.2 Final and Intermediate Goods Producers

Competitive final-goods producers combine intermediate goods Y_{it}, where $i \in [0,1]$ indexes intermediate-goods-producing firms, to sell a homogeneous final good Y_t according to the technology

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{1}{1+\lambda_f}} di\right]^{1+\lambda_f}, \tag{22}$$

where $\lambda_f \geq 0$. Their demand for the generic i^{th} intermediate good is

$$Y_{it} = \left[\frac{P_{it}}{P_t}\right]^{-\frac{1+\lambda_f}{\lambda_f}} Y_t, \tag{23}$$

where P_{it} is the nominal price of good i. The zero profit condition for competitive final goods producers implies that the aggregate price level is

$$P_t = \left[\int_0^1 P_{it}^{-\frac{1}{\lambda_f}} di\right]^{-\lambda_f}. \tag{24}$$

The intermediate goods firm i uses K_{it} units of capital and H_{it} units of composite labor to produce output Y_{it} according to the production technology

$$Y_{it} = A_t K_{it}{}^{\gamma} H_{it}{}^{1-\gamma}, \tag{25}$$

where $\gamma \in (0,1)$ and A_t is an aggregate productivity shock. Intermediate-goods firms operate in monopolistic competition and set prices on a staggered basis (Calvo (1983)) taking the real wage $\frac{W_t}{P_t}$ and the rental rate of capital r_t^k as given. With probability $1-\zeta_p$, the firm resets its price, while with the complementary probability the price remains fixed. In the event of a price change at time t, the firm chooses the price $\tilde{P}_{it}$ to maximize the present discounted value of profits ($D_{is} = P_{is}Y_{is} - w_s H_{is} - r_s^K K_{is}$,

[16] Although each household supplies many types of labor, it is difficult for unions (which represent many households) to cooperate. Thus, each union is monopolistically competitive, taking the wages of the other unions as given.

$s \geq t$) conditional on not changing prices in the future subject to the demand for its own good (23). The intermediate firms' problem yields the standard New Keynesian Phillips curve as shown in the appendix.

2.3 Capital-Goods Producers

Capital-goods producers are perfectly competitive. These firms transform consumption goods into investment goods. Their problem consists of choosing the amount of investment goods produced I_t to maximize the profits

$$D_t^I = \left\{ p_t^I - \left[1 + S\left(\frac{I_t}{I} \right) \right] \right\} I_t, \tag{26}$$

taking the price of investment goods p_t^I as given. The price of investment goods differs from the price of consumption goods because of the adjustment cost function, which depends on the deviations of actual investment from its steady-state value I. We assume that, when evaluated in steady state, the adjustment cost function and its first derivative are zero ($S(1) = S'(1) = 0$), while its second derivative is positive ($S''(1) > 0$).

2.4 The Government

The government conducts conventional monetary policy, unconventional credit policy, and fiscal policy. Conventional monetary policy consists of the central bank setting the nominal interest rate following a standard feedback rule

$$R_t = \max\{R\pi_t^{\psi_\pi}, 0\}, \tag{27}$$

where $\psi_\pi > 1$. Unconventional credit policy corresponds to government purchases of private paper (denoted by N_{t+1}^g) as a function of its liquidity

$$\frac{N_{t+1}^g}{K} = \psi_k \left(\frac{\phi_t}{\phi} - 1 \right), \tag{28}$$

where $\psi_k < 0$.[17] Rule (28) captures the behavior of the Federal Reserve in terms of the liquidity facilities, as shown in Figure 1. According to this rule, the government

[17]Rule (28) is expressed in terms of the steady-state capital stock K to allow for the possibility that the steady-state government holdings of private assets are zero ($N^g = 0$).

intervenes when the liquidity of private paper is abnormally low. When the liquidity returns to normal, the facilities are discontinued. Since we assume that the liquidity shock follows a Markov process with two states (the steady state and the crisis state), we believe that this description of the intervention captures quite accurately the behavior of the Fed during the financial crisis of 2008. We calibrate the parameter ψ_k to deliver a balance-sheet increase in line with the data.

We stress that the government intervenes in the open market. Therefore, the intervention does not directly relax any agents' resaleability constraint (5). The intervention affects macroeconomic outcomes by changing the *aggregate* portfolio composition of the private sector, skewing it toward liquid assets. Therefore, even if the economy is subject to a liquidity shock, entrepreneurs can muster resources to finance investments (see expression (14)). In the first period, the portfolio composition of the private sector is predetermined, however. Hence, on impact, the intervention is effective only via its impact on expectations and prices.

The government budget constraint is

$$q_t N_{t+1}^g + \frac{R_{t-1} B_t}{P_t} = \tau_t + \left[r_t^k + (1-\delta) q_t \right] N_t^g + \frac{B_{t+1}}{P_t}. \tag{29}$$

The government purchase of equity and debt repayment is financed by a net tax (primary surplus), returns on equity holdings, and the new debt issuances. We assume that the government ensures intertemporal solvency by following a fiscal rule, written in deviations from steady state, according to which taxes are proportional to the beginning-of-period government net debt position

$$\tau_t - \tau = \psi_\tau \left[\left(\frac{R_{t-1} B_t}{P_t} - \frac{RB}{P} \right) - q_t N_t^g \right], \tag{30}$$

where $\psi_\tau > 0$, and where τ and $\frac{RB}{P}$ are steady-state taxes and beginning-of-period government debt, respectively (the steady-state value of N_t^g is zero by assumption). Because the adjustment of taxes to debt is gradual (to the extent that ψ_τ is small), the government has to finance emergency private paper purchases almost entirely by issuing debt.

2.5 Equilibrium and Solution Strategy

In equilibrium, households and firms maximize their objectives subject to their constraints. The market-clearing conditions for composite labor and capital use are

$$H_t = \int_0^1 H_{it} di$$

and

$$K_t = \int_0^1 K_{it} di.$$

The aggregate capital stock evolves according to

$$K_{t+1} = (1 - \delta) K_t + I_t, \tag{31}$$

and capital stock is owned by either households or government as

$$K_{t+1} = N_{t+1} + N_{t+1}^g. \tag{32}$$

Finally, the aggregate resource constraint requires that

$$Y_t = C_t + \left[1 + S\left(\frac{I_t}{I}\right)\right] I_t. \tag{33}$$

We approximate the model's equilibrium conditions up to the first order around a steady state in which the liquidity frictions are binding. The appendix shows the full set of equilibrium conditions.[18] The only non-linearity arises from the zero lower bound constraint on the nominal interest rate, which we account for by using the solution method described in Eggertsson (2008). Specifically, in order to solve the model, we assume that the log-deviation of the shock variable ϕ_t from its steady-state value follows a two-state Markov process. In the crisis/low state, the resaleability constraint is below its steady state value, i.e., $\hat{\phi}_L < 0$, while in the normal/high state, ϕ_t reverts back to steady state, i.e., $\hat{\phi}_H = 0$. The economy moves from the steady state in period 0 to the crisis state in period 1. Thereafter, the crisis persists with probability $1 - \zeta_{ZB}$ while the shock goes back to its average value with probability ζ_{ZB}. Once the crisis is over (at some stochastic date $\mathcal{T}$), we assume no other shock can occur.

[18]In what follows, a "hat" denotes the log of a variable in deviations from its steady-state value (i.e., for a generic variable x_t, $\hat{x}_t \equiv \ln(x_t/x)$, where x is the steady-state value of x_t).

3 Calibration

We calibrate the model at quarterly frequency. Table 1 shows the calibrated values of the parameters. The main challenge is choosing ϕ and θ, the steady-state values of the parameters capturing the financial frictions, as most other parameters are relatively standard. First, we tie our hands by setting $\phi = \theta$. Next, we use data from the U.S. Flow of Funds between 1952Q1 and 2008Q4 to construct a variable called "liquidity share,"defined as

$$LS_t = \frac{B_{t+1}}{B_{t+1} + P_t q_t K_{t+1}}. \tag{34}$$

We use this variable for two different purposes. First, the steady-state value of the liquidity share provides an indication of the average importance of financial frictions in the economy. Second, we use the change in this variable as a way to assess the magnitude of the liquidity shock in the aftermath of the Lehman default. Figure 2 shows the evolution of the liquidity share. Appendix 5 describes the details of how we construct this variable.

The sample average of the liquidity share is 12.64%. The top-left panel of Figure 3 shows the monotonic relationship between ϕ and the steady state of the liquidity share for a given steady-state level of the government-debt-to-GDP ratio ($\frac{B}{PY}$), which we fix at 40% consistent with the flow-of-funds measure of the average real amount of government paper in the economy relative to GDP.[19] The intuition for this relation can be gained from the top-right panel of Figure 3, which shows that the steady-state value of q increases as ϕ decreases. The more stringent the financial frictions, the scarcer the amount of capital in the economy and the higher its value in terms of consumption goods. A higher value of q translates into a lower value of the liquidity share. Given the observed level of government-debt-to-GDP ratio, the value of ϕ that delivers a steady-state liquidity share roughly in line with the data (13%) is 0.185.[20]

[19]We also computed this figure using Congressional Budget Office estimates for the overall amount of government liabilities and found the same value

[20]A value of $\phi = 0.185$ means the entrepreneur can sell up to 18.5% of the equity holdings within a quarter and $1 - (0.815)^4 = 56\%$ within a year. A measure of steady-state q would also provide direct evidence on ϕ. Such a measure is hard to obtain, however, especially given our broad definition of capital.

The steady-state returns on the two assets in the economy, government paper and private capital, also provide information on the average importance of credit frictions. Equations (16) and (17) show that equity pays a premium relative to liquid assets, which increases as ϕ drops. The relationship between this "liquidity premium" and ϕ is depicted in the bottom-right panel of Figure 3. The figure shows that for $\phi = 0.185$ we obtain a premium of roughly 1.5% percent. An empirical counterpart for the liquidity premium is hard to obtain because of our broad definition of capital. Even if we were to use the standard equity premium as measured from the stock market, it is not clear which fraction of the premium can be attributed to the notion of "illiquidity" used in this paper. As ϕ drops, the required return on the liquid assets also drops, as shown in the bottom-left panel of Figure 3. This relation occurs because q rises as financial frictions become more stringent, and so does the shadow value of relaxing the investment constraint for entrepreneurs. This effect makes liquid assets more valuable (see equation (16)). Our chosen value for ϕ implies a return on liquid assets of 2.2%. This value falls right in between the average ex-post real returns (nominal yield minus realized CPI inflation rate) over the period 1953Q2-2009Q4 on one-year Treasury bills (1.72%) and ten-year Treasuries (2.57%).

We calibrate the probability of receiving an investment opportunity in each quarter $\varkappa$ to 5%, consistent with U.S. evidence in Doms and Dunne (1998) and Gourio and Kashyap (2007).[21] The remaining parameters correspond to standard values in the business cycle literature. We set the subjective discount factor β to 0.99 and the inverse Frisch elasticity of labor supply ν to 1. We choose a capital share γ of 0.4, an annual depreciation rate of 10% ($\delta = 0.025$) and $S''(1) = 1$ so that the price elasticity of investment is consistent with instrumental variable estimates in Eberly (1997). We calibrate symmetrically the degree of monopolistic competition in labor and product markets, assuming a steady-

Measuring the liquidity share is therefore an indirect way of measuring q.

[21] Cooper et al. (1999) find that the fraction of U.S. manufacturing plants adjusting their capital each year is 40%. This value would translate into a 10% probability of receiving an investment opportunity in each quarter. However, the fraction of plants that make sizable investments in a year probably represents an upper bound to calibrate the quarterly average frequency of investment to the extent that, in the data, very few plants adjust their capital stock more than once a year. For this reason, we adopt the more conservative number in the literature.

state markup of 10% ($\lambda_p = \lambda_w = 0.1$). The average duration of price and wage contracts is 4 quarters ($\zeta_p = \zeta_w = 0.75$), in line with the recent estimates in Nakamura and Steinsson (2008).[22] Finally, we set the feedback coefficient on inflation ψ_π in the interest rate rule (27) to 1.5, the value commonly used in the literature. Transfers slowly adjust to the government net wealth position after intervention ($\psi_\tau = 0.1$) so that government debt finances most of the intervention in the short run and transfers follow a smooth path.[23] In section 4.5 we study the robustness of our results to alternative values for some of the parameters.

We present simulations in which the economy is subject only to a liquidity shock ϕ_t. The purpose of the paper is to investigate the effect of the market freeze that followed the default of Lehman Brothers in September 2008 and the concurrent Fed's response. Although it is certainly simplistic to assume that only one shock captures the events in late 2008 and 2009, we pursue this line to assess how much mileage we can get from that one shock and to evaluate the associated policy intervention.

Three key parameters are the magnitude and the expected duration of the shock, i.e., $\hat{\phi}_L$ and ζ_{ZB}, and the intensity of the government response ψ_k. The increase in the liquidity share right after Lehman's bankruptcy represents a target that we use to quantify the magnitude of the shock during the crisis. At the apex of the financial crisis (right after Lehman's collapse), the liquidity share jumped by 26.6%, from a value of 12.23% in 2008Q3 consistent with its historical average to 15.48% in 2008Q4. As the liquidity shock hits, liquid assets become relatively more valuable than illiquid ones since entrepreneurs can resell the former but not the latter, thereby raising the liquidity share. The movement in the liquidity share is of course not independent of the government intervention that increases the amount of liquid assets. Since we know the size of this intervention, we use this second target to jointly calibrate the size of the shock and the intensity of the government response to the liquidity crisis. Specifically, we simultane-

[22]A lower degree of price rigidities (more in line with the evidence in Bils and Klenow (2004)) would deliver the same value for the reduced-form slope of the Phillips curve if we were to incorporate real rigidities in the model.

[23]The results are not very sensitive to this choice as long as the intervention reflects a temporary increase in aggregate debt.

ously pick $\hat{\phi}_L$ and ψ_k to match: (i) an increase in the liquidity share on impact of about 26.6%; and (ii) a government intervention of about $1.4 trillion (10% of GDP), consistent with the increase in the asset side of the Fed's balance sheet after the collapse of Lehman Brothers, as displayed in Figure 1. The resulting size of the shock is $\hat{\phi}_L = -0.6$, i.e. the resaleability of equity in the secondary market ϕ_t falls 60% from 0.185 to 0.074. As mentioned earlier, in our model $1 - \phi_t$ can be interpreted as the haircut on equity in secondary markets. The size of the shock is comparable to the average increase of haircuts on structured products in the repo market during 2008 (see Figure 2 in Gorton and Metrick (2010)).

We choose the expected duration of the shock, which coincides with the agents' expectation that the zero lower bound will be binding, to be six quarters ($\zeta_{ZB} = 0.167$). This value falls close to the midpoint between survey evidence of market participants (Moore (2008)) and the predictions of an estimated interest rate rule (Rudebusch (2009)). Later, we present results based on expectations of more severe financial disruption.

4 Results

4.1 Simulating the Financial Crisis: The Impact on Macroeconomic and Financial Variables

Figure 4 shows the response of output, inflation and the nominal interest rate to a liquidity shock ϕ_t in the model and compares it to the dynamics in the data during the Great Recession. Specifically, the top panel plots the expected path of variables conditional on the shock hitting at the beginning of the first period – that is, averages of all possible state contingent paths induced by the Markov process, using the associated probabilities as weights. The bottom panel shows the changes in the data relative to the end of 2008Q3, when the Lehman bankruptcy occurred, until the end of the recession in 2009Q2. Output is measured as the sum of consumption and investment from the NIPA tables and is normalized so that its log-level in 2008Q3 is zero. For inflation, we use the annualized percentage change in the GDP deflator. The nominal interest rate is the federal funds rate.

The model broadly explains the simultaneous drop in output, inflation, and interest rates observed during the financial crisis. If we compare the changes in macroeconomic variables in the model for the period in which the shock hits with the maximum peak effect in the data relative to 2008Q3, the drop in output and inflation in the model and in the data are essentially the same (7.39% versus 7.27% for output and −5.07% versus −4.62% for inflation). The interest rate reaches the zero lower bound, which is indeed what happened by the end of 2008Q4.[24]

Figure 5 shows the decomposition of the output drop in the relative contribution of consumption and investment in the model (top panel) and in the data (bottom panel). Our empirical counterpart of consumption excludes durable goods, which instead we treat as part of investment, consistently with much of the literature (e.g., Justiniano et al. (2010)). Both consumption and investment are normalized so that their log-level in 2008Q3 is zero. In the model, investment does not drop quite as much as in the data (−14.57% versus −23.95%) while consumption falls somewhat more (−5.03% versus −1.39%).The model may underpredict the fall investment because of the absence of an explicit residential sector. The disparity between model and data in terms of consumption may be due to the the fact that the model ignores some of the other demand-boosting policies enacted during this period, most notably the American Recovery Act passed in January 2009. Nevertheless, the broad empirical patterns are correct, in that investment drops substantially more than consumption in percentage terms both in the model and the data.[25]

[24] Recall that impulse responses show the expected path of a given variable from an ex ante perspective, not a particular realized path in the model. This observation is particularly important to keep in mind when interpreting the impulse response for the nominal interest rate. Since for all realized paths the interest rate is bounded below at zero, the *ex ante* average of these paths is necessarily greater than zero after the initial period. Still, for each realized path, the zero bound remains binding as long as the shock is in its crisis state, i.e., ϕ_L.

[25] Figure 5 also shows that the response of investment to the liquidity shocks features a "kink" in the first period. Recall from section 2.4 that entrepreneurs sell all the liquid assets on their balance sheet and the fraction ϕ_t of private paper due to the resaleability constraint. The intervention occurs in the open market and does not relax the individual entrepreneur's resaleability constraint. What changes is the total amount of liquid assets in the hands of the private sector, so that after the intervention the private sector holds more liquid assets that are not subject to the resaleability constraint. This

The liquidity shock also generates a non-negligible response of financial variables in the model, although generally less than in the data. The upper-left panel of Figure 6 displays the spread of equity versus the liquid asset, defined as the difference in expected returns between the two types of assets:

$$\mathbb{E}_t \left[\frac{r^k_{t+1} + (1-\delta)\, q_{t+1}}{q_t} - \frac{R_t}{\pi_{t+1}} \right].$$

The liquidity shock causes this spread to increase by 124 annualized basis points. The spread in the model has no immediate empirical counterpart, given our broad definition of capital. However, the bottom-left panel of Figure 6 shows for reference the first principal component extracted from a number of spreads between U.S. Treasury securities at various maturities and corporate bonds of different ratings (normalized to zero in 2008Q3).[26] The model explains more than 60% of the increase in this measure of spreads. Finally, the upper-right panel of Figure 6 shows that the model produces a drop in the value of capital ($P_t q_t K_t$) of about 6%. This change explains between a quarter and a third of the actual total decrease in the value of capital since Lehman's bankruptcy, as measured from the flow of funds (bottom-right panel of Figure 6, log-level normalized to zero in 2008Q3).

In short, our simulated crisis generates movements in macroeconomic variables following a liquidity shock that are broadly in line with their empirical counterparts following the Lehman's failure. The response of financial variables to a liquidity shock is also substantial, albeit not as strong as in the data.

change in balance sheet composition affects the amount of liquidity available to entrepreneurs to finance investment, as shown in expression (14). In the first period, however, the amount of liquid assets the entrepreneur can sell is given by last period's holdings. Policy then has an effect only via changes in prices during the first period, which is nevertheless far from negligible, as shown in the next section. This discontinuity generates a kink in the response of investment to a liquidity shock in the presence of unconventional policy, which can also be seen in the response of output.

[26]See Adrian et al. (2010). This series is essentially a weighted average of spreads, where the weights are proportional to the degree of comovement. We thank Tobias Adrian for providing the data.

4.2 The Great Escape? What Would Have Happened in the Absence of the Intervention?

What would have happened after the liquidity shock in the absence of unconventional policy? This is the central question of the paper, which we can address using our DSGE model with liquidity frictions. Figure 7 shows the counterfactual responses of output and inflation to a liquidity shock in the absence of intervention (dashed lines), along with the responses with intervention (solid lines) already shown in Figure 4. The model suggests that, without the facilities, the drop in output would have increased from 7 to 10.5%, that is, the output contraction during the Great Recession would have been 50% larger. The impact on inflation is even stronger. Inflation would have declined by almost 8%, compared to 5% with intervention.

The intervention also affects spreads, but interestingly the impact is fairly small (34 basis points) relative to the impact on other macro variables. Much of the early literature on the effect of the facilities focused on the reduction in spreads as the main metric to interpret their success (McAndrews et al. (2008); and Taylor and Williams (2009)). Our model suggests that this metric may not be entirely appropriate. Even if the reduction in spreads is limited, the macroeconomic impact is substantial. Regardless, the magnitude of the model-implied effect of unconventional policy on spreads is broadly consistent with some of the existing estimates of the effects of the Term Auction Facility on the Libor-OIS spread (McAndrews et al. (2008)).

Looking at the first period understates the importance of unconventional policy, given our assumption that, on impact, the facilities do not relax the entrepreneurs' resaleability constraint and only affect the macroeconomy thanks to their impact on prices. From the second period onward, the effect is larger, as the policy changes the aggregate amount of liquidity in the economy. Table 2 summarizes the overall effect of the intervention on output by reporting the "balance-sheet multiplier", defined as

$$M_{B,0} = \frac{\mathbb{E}_0 \sum_{t=0}^{\infty} (\hat{Y}_t^I - \hat{Y}_t^N)}{\mathbb{E}_0 \sum_{t=0}^{\infty} \hat{N}_{t+1}^g} \left(\frac{Y}{qK} \right),$$

where $\hat{Y}_t^I$ is the baseline value of output and $\hat{Y}_t^N$ is the counterfactual value of output in the absence of intervention. This statistic measures how much the expected value of output increases given a one dollar expected increase in liquidity. The numerator is the area between the impulse response for output with and without intervention, while the denominator is the area under the impulse response of the Fed's balance sheet in the model. In the baseline case, this multiplier is equal to 0.45.

The natural comparison for the balance-sheet multiplier is the fiscal multiplier of Keynesian fame. In the case of government spending, the multiplier measures the effect on aggregate output of the government consuming some real resources – that is, by how many dollars output increases for every dollar in spending. In our case, no real resources are effectively consumed. The government increases liquidity in exchange for privately issued assets. The balance-sheet multiplier measures by how much output increases for every dollar of increased liquidity. In the model, not only is the private sector better off because of the liquidity injection, but the government actually ends up making money off the transaction (about \$57.6 billion in expected terms, in our example).[27]

The second column of Table 2 reports the multiplier from a more extreme calibration than in the baseline case in terms of the expected duration of the shock.[28] Like the recent financial crisis, the Great Depression in the US and the Lost Decade in Japan were also characterized by zero interest rates, deflationary pressures and weakness in the financial sector.Both the Great Depression and the Lost Decade, however, lasted much longer than two years. To capture the possibility that the public had expected a Depression-like crisis, we consider a shock with an expected lifetime of two and a half years (implying a persistence of the low state ζ_{ZB} equal to 0.1). We correspondingly adjust (reduce) the size of the shock so that we hit the same targets (a 26.6% increase in the liquidity share and a \$1.4 trillion increase in the government holdings of privately issued assets) as in the baseline experiment. In this case, the balance-sheet multiplier is 3.22, more than seven times bigger than in the baseline calibration.

Figure 8 shows output and inflation in this extreme scenario. Without intervention,

[27]Federal Reserve transfers to the U.S. Treasury (profits minus operating expenses) reached two consecutive records in 2009 (\$47.4 billion) and 2010 (\$78.4 billion), largely as a result of the increased interest income on security holdings.

[28]Recall that in our simulation the duration of the zero bound corresponds exactly to the duration of the shock.

the equilibrium is a disaster. Output collapses by more than 40% and prices tumble as well. In short, the main macroeconomic variables take values observed during the Great Depression (and possibly worse). As the multiplier hints, the unconventional credit policy becomes much more effective. As shown by the solid line, the policy response essentially creates a similar outcome as before, i.e., a recession of similar order as seen in the data. This "divine coincidence" (Christiano et al. (2009a)), according to which the policy intervention becomes more effective as the economy approaches the "disaster area" (where the equilibrium becomes indeterminate as the probability of exiting the crisis state goes to zero), represents another element of commonality with the literature on the multiplier of government spending (see also Eggertsson (2009)). We do not think that the exact numbers of this experiment necessarily reflect a realistic prediction of how much output and inflation would collapse in such an extreme scenario because our linear approximation may not be too reliable in this case. We do think, however, that the experiment illustrates a basic quantitative point that is likely to be independent of the solution technique. The "disaster area" is also the region in which the unconventional credit policy becomes extremely effective and the balance-sheet multiplier very big. Figure 9 illustrates this property of the model. The multiplier increases more than proportionally as the duration of the crisis lengthens.

4.3 The Role of Nominal Frictions

The previous two sections showed that the KM liquidity shocks can rationalize the behavior of macroeconomic and financial variables during the Great Recession, and that unconventional credit policy might have prevented an even larger downturn. The next two sections shed some light on the key ingredients behind our main results. We start from the role of nominal rigidities. Absent this friction, liquidity shocks would only affect the composition of output, decreasing investment and increasing consumption, but would have virtually no effect on aggregate activity.

The four panels of Figure 10 show the response of output, investment, consumption, and the real interest rates with (solid line) and without (dashed line) nominal rigidities under the baseline calibration. For simplicity, but also to magnify the differences, we show the responses without policy intervention. The upper-left panel of Figure 10 shows that, with flexible prices and wages, the response of output is indeed very small, even though the liquidity shock has a large impact on investment also in the absence of nominal rigidities (upper-right panel). The equilibrium condition for investment (equation (14)) shows that when ϕ_t falls the amount of resources available to entrepreneurs for investment drops, regardless of price and wage stickiness. Clearly, the financial frictions are driving the fall in investment. Nominal frictions exacerbate the effect of the shock on prices and in particular on the value of capital q_t. This observation explains why quantitatively the response of investment is larger in this case, but qualitatively the two impulse responses are quite similar.

Conversely, consumption moves in opposite directions, depending on whether prices and wages are flexible or not (bottom-left panel). Consumption rises under flexible prices and wages, instead of falling as in our simulation with nominal rigidities. Intuitively, consumption needs to make up for the drop in investment since, in that case, output does not drop as much without nominal frictions. The reason for small response of output absent nominal rigidities is that the liquidity shock only affects the accumulation of the capital to be used for production in the future, but has no effect on either productivity or the existing capital stock. Output would drop substantially only if, for some reason, labor were to be used much less intensively for production. If prices are flexible and the elasticity of labor supply is not too extreme, however, the effect on hours is not very pronounced. Hence, aggregate output remains more or less unchanged.

The mechanism of adjustment hinges upon the behavior of the real interest rate. To get people to spend more, the real interest rate needs to decline. The bottom-right panel of Figure 10 shows that the real interest rate becomes negative in response to the liquidity shock absent nominal frictions, so that consumption rises. This fall in the real interest rate is hard to achieve, however, when prices are rigid. With some (but not full) price flexibility, people start expecting some deflation in future periods when the shock is still perturbing the economy, while the Taylor rule implies zero inflation as soon as the shock is over. The interaction of the zero bound and price frictions leads to higher real interest rates owing to expected deflation, which causes consumption to fall with investment. The longer the private sector expects the shock to last, the stronger is deflation and hence the rise in real rates.

The fact that the liquidity shock cannot generate much effect if all prices are flexible is a main quantitative findings of this paper. recession in response to liquidity shock is a combination of the financial frictions and some nominal rigidities that prevents the real interest rate from falling.

4.4 The Zero Lower Bound

Given the relevance of nominal rigidities stressed in the previous section, not surprisingly conventional monetary policy also plays an important role in our results. But full monetary policy stabilization is impaired by the presence of the zero lower bound (henceforth, ZLB) on nominal interest rates.

Figure 11 shows six panels. The top three panels show the response of output, the nominal interest rate, and the real interest rate under the ZLB constraint, with (solid lines) and without (dashed lines) policy intervention. The bottom three panels show the response of the same variables ignoring the ZLB constraint, again with (solid lines) and without (dashed lines) policy intervention. In order to show that the ZLB works as an amplification mechanism for the liquidity shock, let us focus on the case without intervention – that is, the dashed lines. As we have seen in Figure 7, output drops by more than 10 percent without intervention when the ZLB is binding (top-left panel). In the absence of the ZLB, even without intervention, output would have fallen only by half that amount (bottom-left panel). The reason is that, in this case, a monetary authority following the Taylor-type rule (27) would have lowered the nominal interest rates below -6% (bottom-middle panel), thereby inducing a fall in the real interest rate from 2.1% percent (the steady state) into negative territory (bottom-right panel). In contrast, when we explicitly consider the ZLB, the nominal interest rate is stuck at zero (top-middle panel). The zero bound amplifies the effect of the liquidity shock not only because the constraint is binding in a given period, but especially because agents expect it to be binding in the future. This belief lowers expected future income and generates deflationary expectations. Such expectations lead to a rise in real rates (top-right panel) and a decline in demand.

In this situation, the intervention achieves a substantial reduction in real rates relative to the no-intervention case (top-right panel of Figure 11), even though the nominal rate is still constrained at zero. Unconventional policy stimulates demand by changing the portfolio composition of the private sector, thereby enabling entrepreneurs to pursue more investment opportunities. The impact of the policy on investment supports demand in all periods when the economy is in the crisis state, indirectly boosting consumption via its effect on inflation expectations and the real rate. In this sense, unconventional policy can substitute for the conventional one when the latter is hindered by the ZLB.

If the ZLB is not binding, unconventional policy is much less effective, simply because conventional policy can do its job in boosting demand. Indeed, the bottom-left panel of Figure 11 shows that the paths of output with and without unconventional policy (solid and dashed lines) are similar when the ZLB is not binding. The "great escape" calibration forcefully illustrates the role of expectations in determining the effectiveness of unconventional policy. When agents expect the shock to last for a long period, unconventional policy becomes very effective only when expectations are of persistent deflation in the presence of ZLB. The second column of Table 2 shows that, indeed, the effect of the unconventional policy becomes an order of magnitude smaller when the ZLB is not considered.

4.5 Robustness

In this section, we discuss the robustness of our quantitative results to a selected number of key parameters: (i) the degree of price and wage rigidities ζ_p and ζ_w; (ii) the level of relative risk aversion σ; and (iii) the adjustment cost parameter $S''(1)$. In each case, we adjust the size of the shock $\hat{\phi}_L$ and the feedback coefficient in the policy rule for private asset purchases ψ_k to hit the same targets (a 26.6% increase in the liquidity share and a \$1.4 trillion expansion of the Fed's balance sheet) as in the baseline case. Table 3 summarizes the results.

The first column reports the multiplier and the impact response to the liquidity shock of output and its components, the inflation rate, the nominal value of the capital stock (all in percentage deviations from steady state), and the spreads (in annualized basis points) in the baseline case. In the next five columns, we vary one parameter at a time and report the same statistics.

Column (2) shows the implications of having a higher degree of price and wage rigidity relative to the baseline calibration, equal to 0.85, implying that firms (unions) reset prices (wages) every six and a half quarters. A lower frequency of price changes induces a smaller drop in inflation relative to the baseline and a larger drop in output. The multiplier is slightly smaller because higher rigidities imply smaller deflation and hence a lower real interest rate given that the nominal rate is zero. The composition of output is closer to the data in that investment drops by more than 20% (6% more than in the baseline) while consumption decreases by 5.6% (compared to 5% in the baseline). The price of capital is comparable to the baseline case while spreads almost double. Column (3) considers a lower degree of price and wage rigidity relative to the baseline calibration, equal to 0.66, implying that firms (unions) reset prices (wages) every three quarters. While this parameterization implies that the model is further away (albeit not dramatically) from the data on a number of dimensions, the multiplier is larger. More flexible prices make the intervention more powerful when the ZLB is binding, because the real interest rate increases by more when prices are more flexible.

Column (4) of Table 3 reports the results of increasing the coefficient of relative risk aversion to 2. The fall in output and inflation is a bit smaller than in the baseline calibration, but the relative contribution of consumption and investment is more in line with the empirical evidence. Spreads move more than in the baseline case, while the decrease in the price of capital is not significantly different from that in the baseline case. The multiplier is a bit smaller.

The last two columns of Table 3 show the results of abstracting from adjustment costs (column (7)) or of a higher level of adjustment costs (column (8)). Here, the differences with the baseline calibration are more evident. Absent adjustment costs, the drop in output is more than 50% larger than in the baseline. The difference is entirely explained by the collapse in investment, which overshoots the movement in the data by almost 10%. Inflation also drops more than in the data. The main problem with this parameterization is that spreads also fall, albeit not by much. This finding is not surprising, perhaps, since the real rate increases substantially more than in the baseline calibration. At the opposite extreme, a larger adjustment cost allows us to obtain a substantial movement in spreads. However, the composition of output becomes counterfactual, as consumption and investment drop almost by the same magnitude.

5 Conclusions

In this paper, we have proposed an analysis of the economic and financial crisis of 2008 based on shocks to liquidity of private paper. We have incorporated a set of financial frictions into a standard DSGE model to show that unconventional credit policy may have been effective in preventing the Great Recession from becoming a second Great Depression when nominal prices and wages are sticky and the nominal interest rate is near zero.

Our results rely on the crucial distinction that the government can issue perfectly liquid papers while the private sector cannot. The ability of governments to issue fiat currency and raise taxes provides a rationale for this assumption. If government bonds become subject to default risk and sensitive to information on a possible default, also their liquidity would become much less than perfect, as in the recent cases of Greece, Portugal, and Ireland. In this case, the government has only limited ability to conduct unconventional credit policy and the expectations about future fiscal policy would affect both the valuation and the liquidity of government bonds. We leave this topic for future research.[29]

[29] We have also abstracted from the wealth distribution across heterogenous agents by assuming complete sharing of consumption and assets among family members at the end of every period. Absent this pooling of resources, the distribution of net worth across heterogeneous producers and consumers affects aggregate production and asset prices as in Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). Because the balance sheet takes time to adjust, the recovery of aggregate production may be slow after a large financial crisis. In such an economy, the government purchase of illiquid private paper

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Appendix: Construction of the Liquidity Share

The liquidity share in the model is defined as $LS_t = \frac{B_{t+1}}{B_{t+1}+P_t q_t K_{t+1}}$ (equation (34)). The two quantities in the definition of the liquidity share are the dollar value of the amount of U.S. government liabilities B_{t+1} (by assumption, the empirical counterpart of the liquid assets in the model) and of net claims on private assets (capital) $P_t q_t K_{t+1}$, respectively.

Recall that in the model, as in the actual economy, households hold claims on the capital held by other households (the N_{t+1}^{O} and N_{t+1}^{I} terms mentioned in the discussion of the household's balance sheet). The term $P_t q_t K_{t+1}$, however, measures the net amount of these claims – that is, the value of capital in the economy. We therefore consolidate the balance sheet of households, the non-corporate and the corporate sectors to obtain the market value of aggregate capital. For households, we sum real estate (B.100 line 3), equipment and software of non-profit organizations (B.100 line 6), and consumer durables (B.100 line 7). For the non-corporate sector, we sum real estate (B.103 line 3), equipment and software (B.103 line 6) and inventories (B.103 line 9). For the corporate sector, we obtain the market value of the capital stock by summing the market value of equity (B.102 line 35) and liabilities (B.102 line 21) net of financial assets (B.102 line 6). We then subtract from the market value of capital for the private sector the government credit market instruments (B.106 line 5), TARP (B.106 line 10), and trade receivables (B.106 line 11).

Our measure of liquid assets B_{t+1} consists of all liabilities of the federal government – that is, Treasury securities (L.106 line 17) net of holdings by the monetary authority (L.106 line 12) and the budget agency (L.209 line 20) plus reserves (L.108 line 26), vault cash (L.108 line 27) and currency (L.108 line 28) net of remittances to the federal government (L.108 line 29).

Three qualifications are in order. First, no data are available for the physical capital stock of the financial sector. Second, not all of the assets in the flow of funds are evaluated at market value. Specifically, the capital stock of households (consumer durable goods) and non-corporate firms (equipment and software owned by non-profit organizations) are measured at replacement cost. Last, in our calculations we do not net out liquid and illiquid assets held by the rest of the world. Even if we do, however, the numbers are not very different, since the rest of the world, on net, holds both liquid (government liabilities) and illiquid (private sector liabilities) assets in roughly the same proportion. The liquidity share calculated excluding the foreign sector averages 10.56%, as opposed to 12.64%, over the sample period and exhibits very similar dynamics.

Table 1: Calibration

β	$=$	0.99		Discount factor
σ	$=$	1		Relative risk aversion
δ	$=$	0.025		Depreciation rate
γ	$=$	0.40		Capital share
$S''(1)$	$=$	1		Adjustment cost parameter
ν	$=$	1		Inverse Frisch elasticity
ψ	$=$	1.5		Taylor rule coefficient
$\zeta_p = \zeta_w$	$=$	0.75		Price/wage Calvo probability
$\lambda_p = \lambda_p$	$=$	0.1		Price/wage steady-state markup
$\varkappa$	$=$	0.05		Probability of investment opportunity
ϕ	$=$	0.19		Resaleability constraint
θ	$=$	0.19		Borrowing constraint
L	$=$	0.40		Steady-state liquidity/GDP
ξ_τ	$=$	0.1		Transfer rule coefficient
		Baseline	Great Escape	
$\hat{\phi}_L$	$=$	-0.600	-0.295	Size of the liquidity shock
ζ_{ZB}	$=$	0.167	0.100	Probability of exiting the crisis state
ξ_k	$=$	-0.063	-0.127	Government intervention coefficient

Notes: The table shows the parameter values of the model for the baseline calibration. The last three rows also report the size of the shock, the probability of exiting the crisis state, and the coefficient in the government rule for purchases of private assets in the Great Escape calibration.

Table 2: Multipliers

	Baseline	Great Escape
Full model	0.45	3.22
No zero bound constraint	0.25	0.25
No nominal rigidities	0.09	0.08

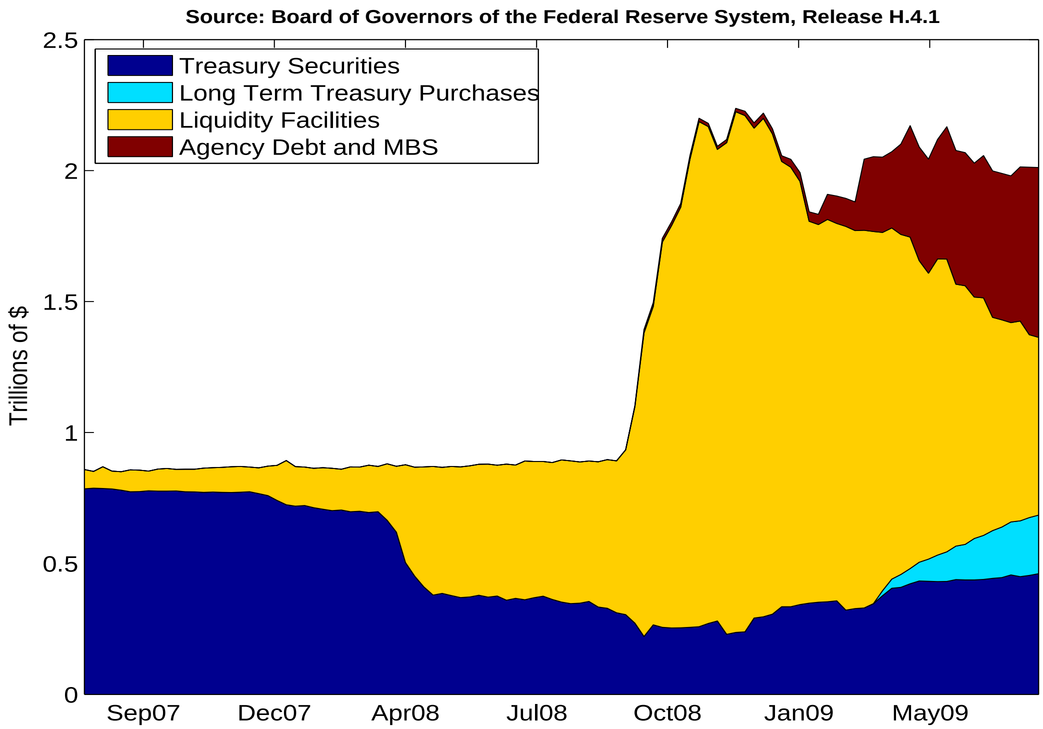
Notes: The table shows the balance-sheet multiplier for the baseline calibration and Great Escape scenario for the full model, for the case of flexible prices and wages, and for the case in which we ignore the zero lower bound constraint on the nominal interest rate (third row).

Table 3: Robustness

	Baseline	*Alternative*	*Parameterizations*			
	(1)	(2)	(3)	(4)	(5)	(6)
Nominal Rigidities (ζ_p, ζ_w)	0.75	0.85	0.66			
Risk Aversion (σ)	1			2		
Adjustment Costs $(S''(1))$	1				0	3
Multiplier $M_{B,0}$	0.45	0.36	0.86	0.26	0.65	0.35
Period 1 *response*						
Output	-7.39	-9.25	-5.43	-6.43	-10.8	-5.44
Consumption	-5.02	-5.58	-4.08	-2.89	-4.14	-4.97
Investment	-14.6	-20.4	-9.54	-17.2	-31.1	-6.88
Inflation	-5.07	-2.56	-6.58	-4.31	-6.64	-4.02
Value of Capital	-5.56	-4.92	-5.49	-5.04	-4.95	-5.29
Spreads	124	236	34	179	-38	196

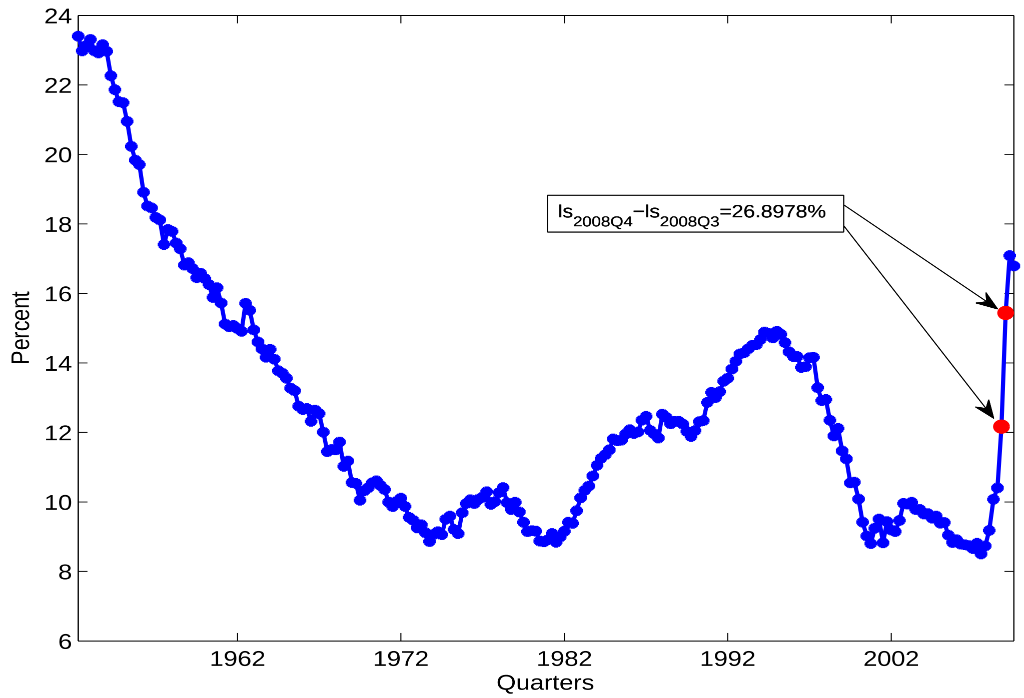
Notes: The table shows the balance-sheet multiplier and the first-period response of output, consumption, investment, inflation, value of capital, and spreads for the baseline calibration and for alternative parameterizations of nominal rigidities, risk aversion, and adjustment costs.

Figure 1: Federal Reserve's Assets: August 2007 – July 2009



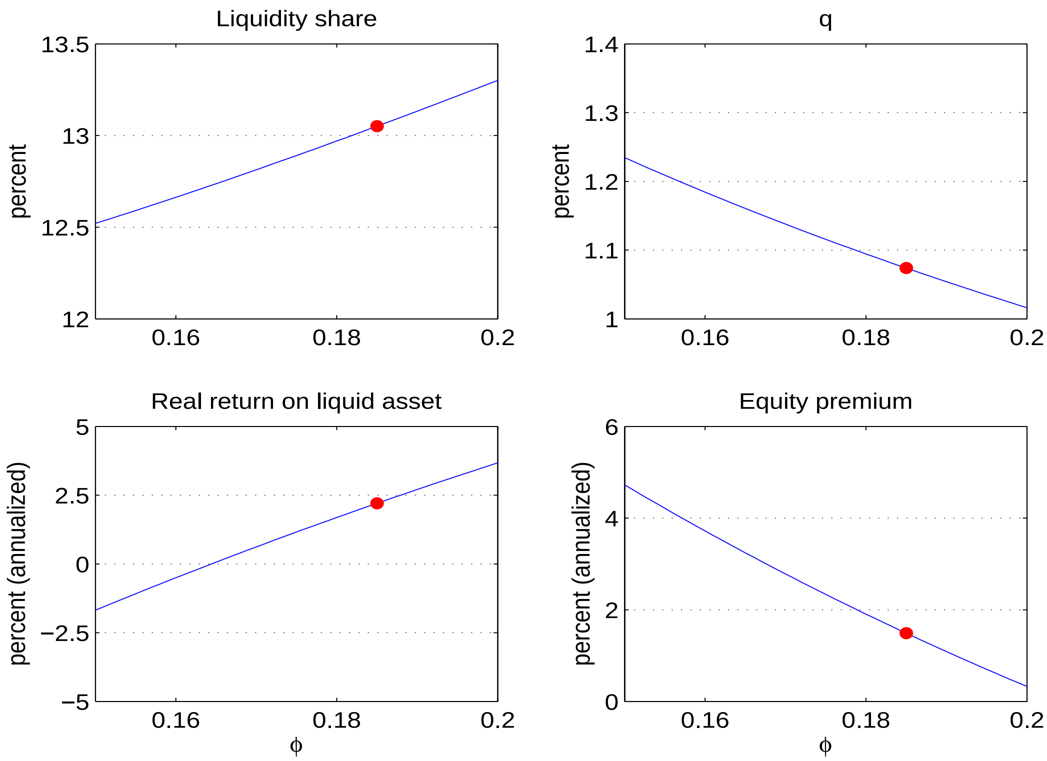
Notes: The figure plots the evolution of the asset side of the Federal Reserve balance sheet from August 2007 through July 2009.

Figure 2: The Liquidity Share in the Data



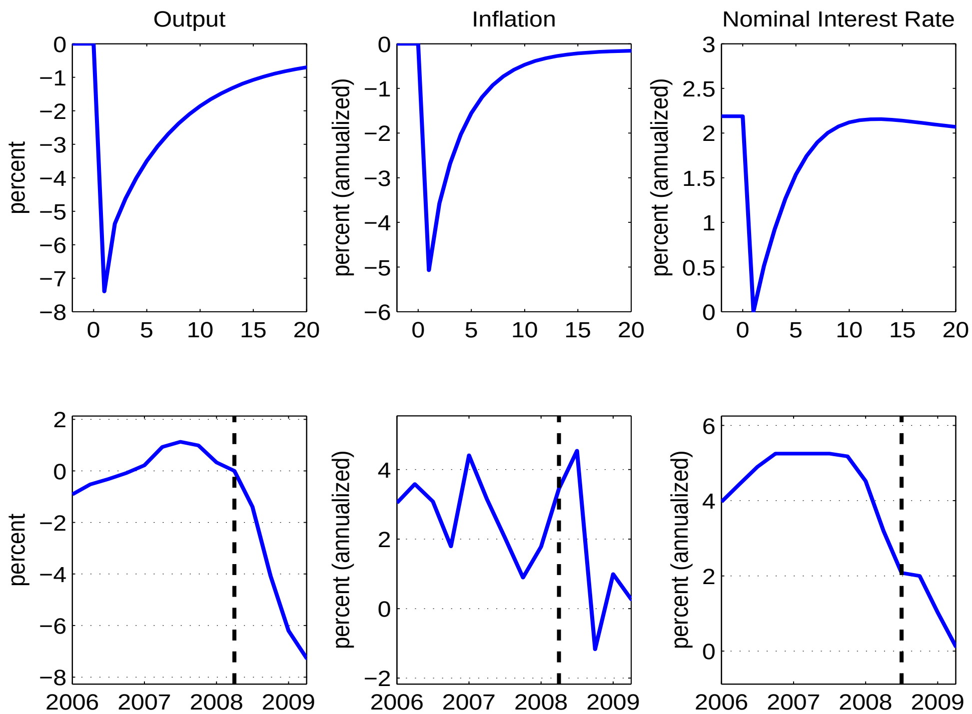
Notes: The figure plots the evolution of the liquidity share, defined as the ratio of government liabilities (liquid assets) to total assets in the U.S. economy, over the sample period 1952Q1:2009Q2.

Figure 3: Steady State as a Function of ϕ_*



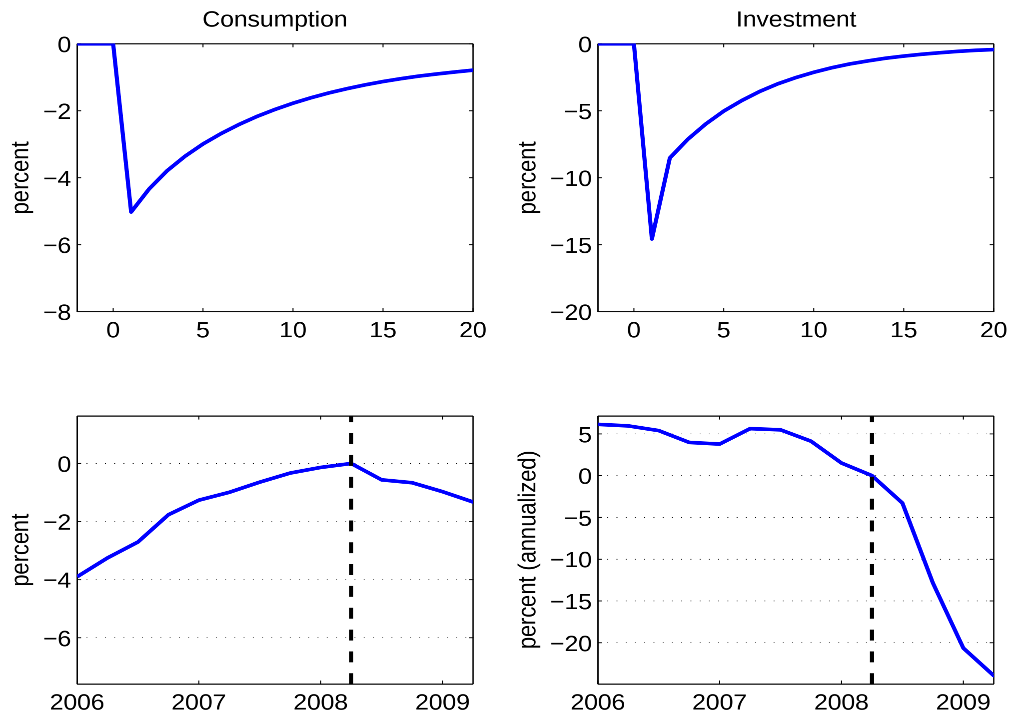
Notes: The figure plots the steady-state liquidity share (top-left panel), price of equity (top-right panel), real return on the liquid asset (bottom-left panel), and equity premium (bottom-right panel) as a function of the steady-state value of the resaleability parameter ϕ.

Figure 4: Response of Key Macro Variables: Model and Data



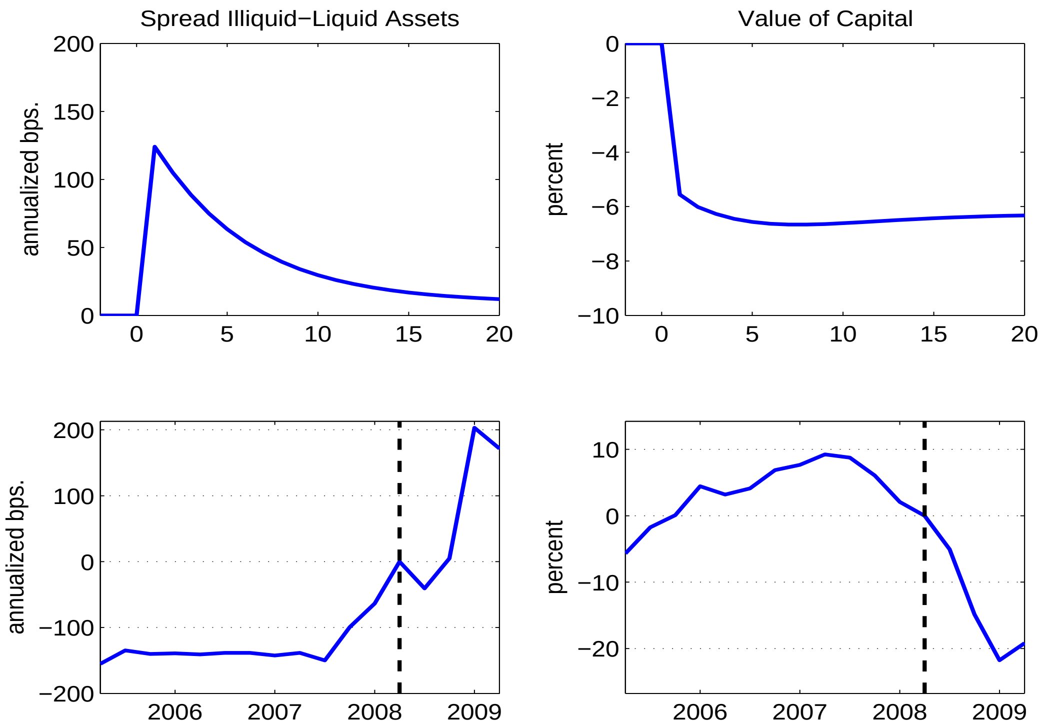
Notes: The figure compares the impulse response function to the liquidity shock of output (top-left panel), inflation (top-center panel) and nominal interest rate (top-right panel) in the model with the evolution of real GDP (bottom-left panel), GDP deflator inflation rate (bottom-center panel) and federal funds rate (bottom-right panel) in the data during the financial crisis. The vertical dashed line corresponds to 2008Q3. The log of real GDP is normalized to zero in 2008Q3.

Figure 5: Response of Consumption and Investment: Model and Data



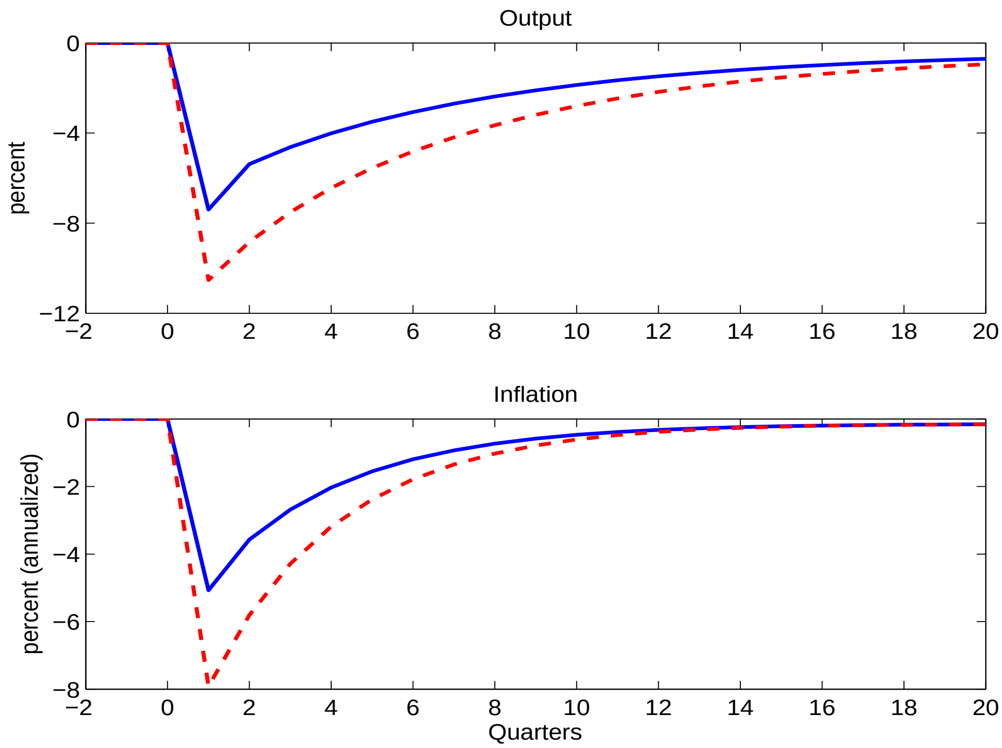
Notes: The figure compares the impulse response function to the liquidity shock of consumption (top-left panel), and investment (top-right panel) in the model with the evolution of non-durable consumption (bottom-left), and investment plus durable consumption (bottom-right) in the data during the financial crisis. The vertical dashed line corresponds to 2008Q3. The log of consumption and investment in the data are normalized to zero in 2008Q3.

Figure 6: Response of Spreads and the Value of Capital: Model and Data



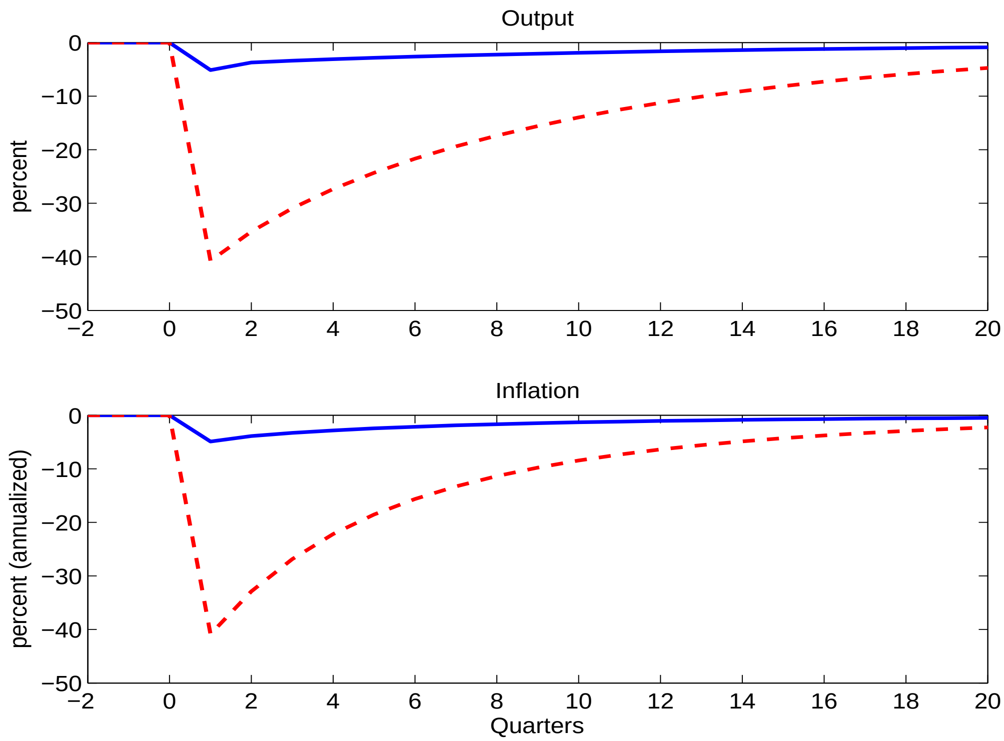
Notes: The figure compares the impulse response function to the liquidity shock of the spread between the return on the illiquid and the liquid assets (top-left panel) and the nominal value of the capital stock (top-right panel) in the model with the evolution of the first principal component of various empirical spreads from Adrian et al. (2010) (bottom-left panel), and the nominal value of the capital stock from the Flow of Funds (bottom-right panel) during the financial crisis. The vertical dashed line corresponds to 2008Q3. Empirical spreads and the log of the nominal value of the capital stock in the data are normalized to zero in 2008Q3.

Figure 7: The Effect of Policy Intervention



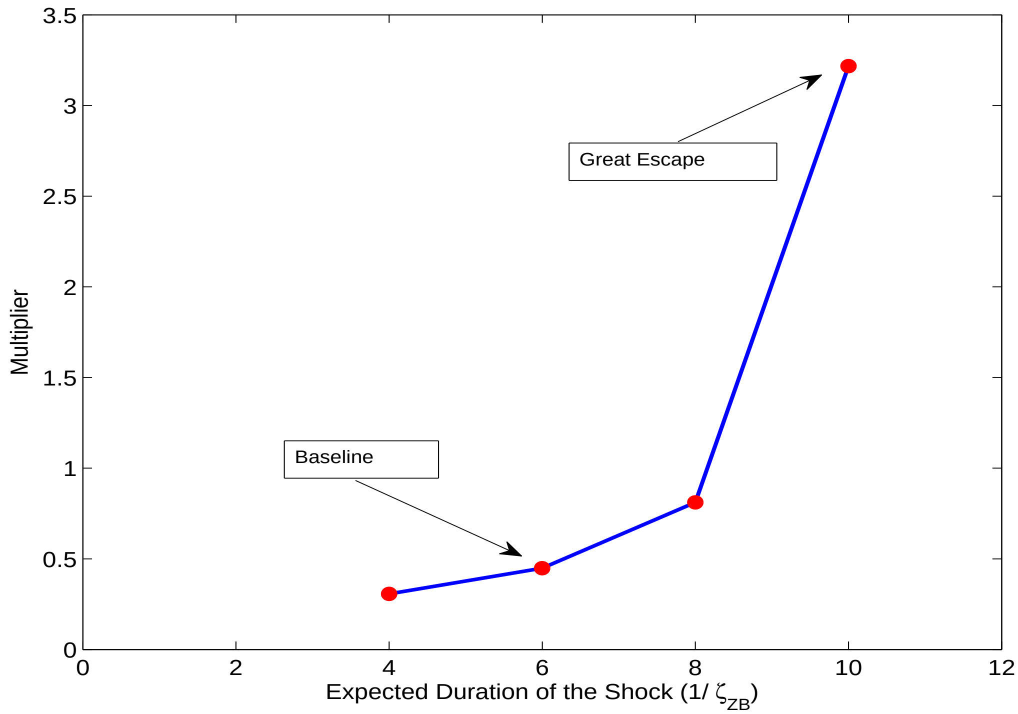
Notes: The figure compares the impulse response function to the liquidity shock of output (top) and inflation (bottom) in the model under the baseline calibration with (continuous blue line) and without (dashed red line) intervention.

Figure 8: The Great Escape?



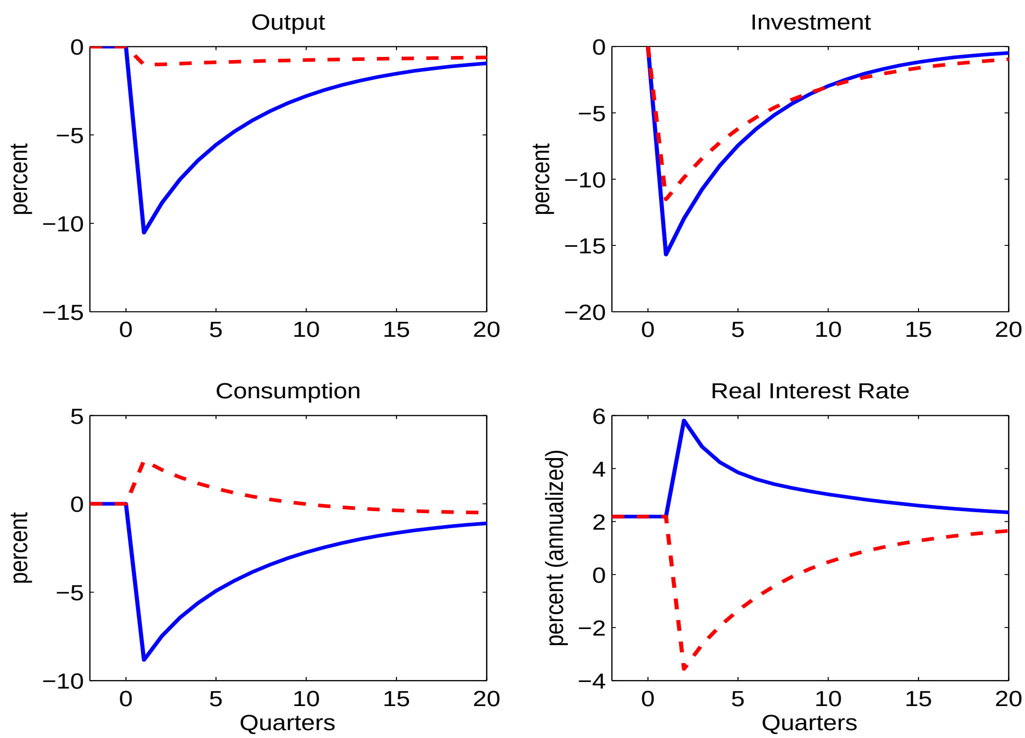
Notes: The figure compares the impulse response function to the liquidity shock of output (top panel) and inflation (bottom panel) in the model under the Great Escape calibration with (continuous blue line) and without (dashed red line) intervention.

Figure 9: The Balance-Sheet Multiplier and the Expected Duration of the Crisis



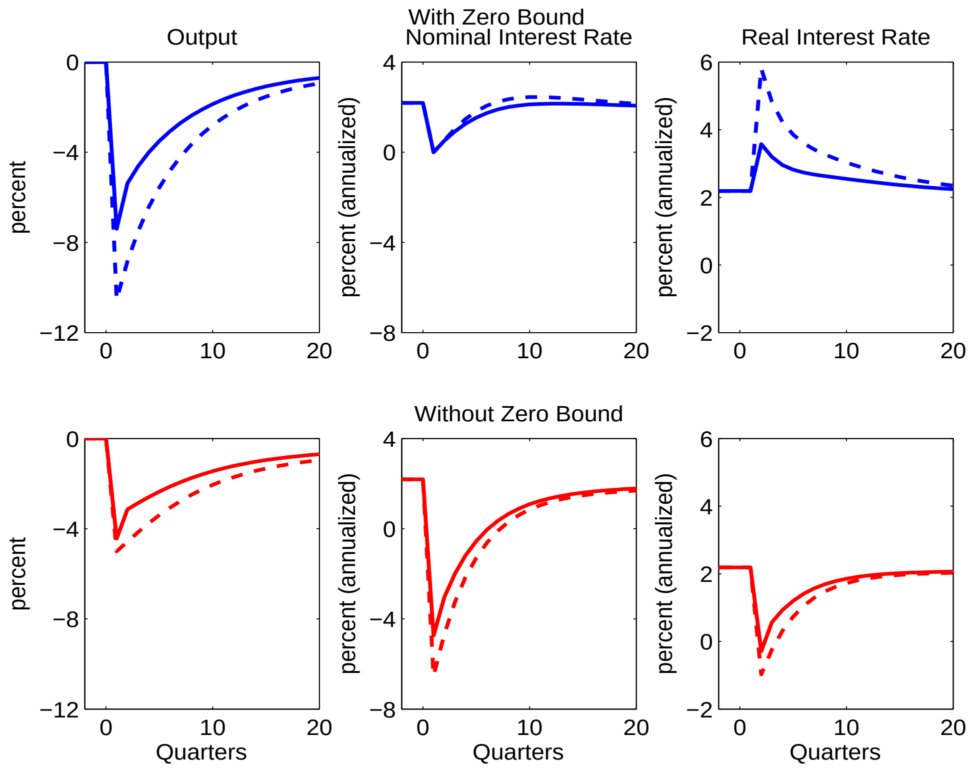
Notes: The figure plots the balance-sheet multiplier as a function of the expected duration of the crisis (the inverse of the probability of exiting the crisis state $\zeta_Z B$).

Figure 10: The Role of the Nominal Rigidities



Notes: The figure compares the impulse response function to the liquidity shock of output (top-left panel), investment (top-right panel), consumption (bottom-left panel), and real interest rate (bottom-right panel) in the model under the baseline calibration with (continuous blue line) and without (dashed red line) nominal rigidities.

Figure 11: The Role of the Zero Bound



Notes: The figure compares the impulse response function to the liquidity shock of output (left), nominal interest rate (center) and real interest rate (right) with (continuous line) and without (dashed line) intervention. We impose the zero lower bound constraint on the nominal interest rate in the top row, but not in the bottom row.

A Appendix

A.1 Derivation of Liquidity Constraint

The household's balance sheet (excluding human capital) is given in Table in Section 2.1 in the text. The existence of two financial frictions constrains the evolution of both equity issued and others' equity. The entrepreneur cannot issue new equity more than a fraction θ of the investment undertaken in the current period plus a fraction $\phi_t^I \in (0,1)$ of the undepreciated capital stock previously not mortgaged $(K_t - N_t^I)$. Therefore, equity issued evolves according to

$$N_{t+1}^I(j) \leq (1-\delta)N_t^I + \theta I_t(j) + (1-\delta)\phi_t^I(K_t - N_t^I). \tag{A.1}$$

Similarly, the entrepreneur cannot sell more than a fraction ϕ_t^O of holdings of the others' equity remained. Therefore, others' equity evolves according to

$$N_{t+1}^O(j) \geq (1-\delta)N_t^O - (1-\delta)\phi_t^O N_t^O. \tag{A.2}$$

The key assumption that allows us to derive a single constraint on the evolution of net equity ($N_t \equiv N_t^O + K_t - N_t^I$) is that the "resaleability" parameters are the same, that is $\phi_t^I = \phi_t^O = \phi_t$. Then two constraints (A.1) and (A.2) yield (5) in the text.

A.2 Optimality conditions

A.2.1 Household's Optimality Conditions

Because each entrepreneur must satisfy the financing constraints on equity holdings (5) , bond holdings (6) and non-negativity constraint of consumption, the aggregate investment of the representative household must satisfy

$$I_t \equiv \int_0^{\varkappa} I_t(j)dj \leq \varkappa \frac{\left[r_t^k + (1-\delta)\, q_t\phi_t\right] N_t + \frac{R_{t-1}B_t}{P_t} + D_t + D_t^I - \tau_t}{p_t^I - \theta q_t}. \tag{A.3}$$

As explained in the text, we separate the wage setting from the consumption, investment and portfolio decision. The household chooses C_t, I_t, N_{t+1} and B_{t+1} to maximize the utility (2) subject to the budget constraint (15) and the financing constraint of investment (A.3). Let ξ_t and η_t be the Lagrange multipliers attached to (15) and (A.3). The first order conditions for consumption, investment, equity and government bond are respectively

$$C_t^{-\sigma} = \xi_t, \tag{A.4}$$

$$\xi_t(q_t - p_t^I) = \eta_t, \tag{A.5}$$

$$q_t\xi_t = \beta\mathbb{E}_t\left\{\xi_{t+1}[r_{t+1}^k + (1-\delta)q_{t+1}] + \eta_{t+1}\frac{\varkappa[r_{t+1}^k + (1-\delta)\phi_{t+1}q_{t+1}]}{p_{t+1}^I - \theta q_{t+1}}\right\}, \tag{A.6}$$

$$\xi_t = \beta\mathbb{E}_t\left[\frac{R_t}{\pi_{t+1}}\left(\xi_{t+1} + \eta_{t+1}\frac{\varkappa}{p_{t+1}^I - \theta_{t+1}q_{t+1}}\right)\right]. \tag{A.7}$$

We focus on equilibria in which the financing constraint on investment is sufficiently tight so that the price of equity is bigger than its installation cost, i.e. $q_t > p_t^I$ in the neighborhood of the steady state equilibrium. This condition is also always satisfied in our simulations outside the steady state. Therefore, the Lagrange multiplier η_t on the financing constraint on investment equation (A.3) is always positive. This result implies that each entrepreneur satisfies the financing constraints on equity holdings (5) and bond holdings (6) with equality and his/her consumption is zero, i.e. $C_t(j) = 0$ for $j \in [0, \varkappa)$. Also (A.3) holds with equality ((14) in the text). Substituting the Lagrange multipliers from (A.4) and (A.5) into (A.6) and (A.7) gives the Euler equations for bond and equity that characterize the household portfolio decisions (16) and (17) .

Let L_{t+1} be the real value of liquid assets at the end of period

$$L_{t+1} \equiv \frac{B_{t+1}}{P_t}. \tag{A.8}$$

Together with the expression for dividends, aggregate investment (14) can be rewritten as

$$I_t = \varkappa\frac{\left[r_t^k + (1-\delta)\,q_t\phi_t\right]N_t + \frac{R_{t-1}L_t}{\pi_t} + Y_t - w_tH_t - r_t^kK_t + p_t^II_t - I_t\left[1 + S\left(\frac{I_t}{I}\right)\right] - \tau_t}{p_t^I - \theta q_t} \tag{A.9}$$

A.2.2 Wage Setting Decision

Competitive labor agencies chooses $H_t(j)$ to maximize their profits

$$W_t H_t - \int_{\varkappa}^{1} W_t(j) H_t(j) dj$$

subject to (18), taking wages $W_t(j)$ as given. The first order condition determines the demand for the j^{th} labor input (20), where $W_t(j)$ is the wage specific to type j and W_t is the aggregate wage index that comes out of the zero profit condition for labor agencies (21).

Labor unions representing suppliers of type-j labor set wages on a staggered basis, taking as given the demand for their specific labor input. In each period, with probability $1-\zeta_w$, a union is able to reset the wage $W_t(j)$, while with the complementary probability the wage remains fixed. Household are committed to supply whatever labor is demanded at that wage. In the event of a wage change at time t, unions choose the wage $\tilde{W}_t(j)$ to maximize

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\zeta_w)^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{\omega}{1+\nu} \int_{\varkappa}^{1} H_s(j)^{1+\nu} dj \right]$$

subject to (15) and (20) with $W_{t+s}(j) = \tilde{W}_t(j), \forall s \geq 0$.

The first order condition for this problem is

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\zeta_w)^{s-t} C_s^{-\sigma} \left[\frac{\tilde{W}_t(j)}{P_s} - (1+\lambda_w) \frac{\omega H_s(j)^{\nu}}{C_s^{-\sigma}} \right] H_s(j) = 0.$$

All unions face an identical problem. We focus on a symmetric equilibrium in which all unions choose the same wage $\tilde{W}_t(j) = \tilde{W}_t$. Let $w_t \equiv W_t / P_t$ denote the real wage. The first order condition for optimal wage setting becomes

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\zeta_w)^{s-t} C_s^{-\sigma} \left\{ \frac{\tilde{w}_t}{\pi_{t,s}} - (1+\lambda_w) \frac{\omega \left[\frac{1}{1-\varkappa} \left(\frac{\tilde{w}_t}{\pi_{t,s} w_s} \right)^{-\frac{1+\lambda_w}{\lambda_w}} H_s \right]^{\nu}}{C_s^{-\sigma}} \right\} \left(\frac{\tilde{w}_t}{\pi_{t,s} w_s} \right)^{-\frac{1+\lambda_w}{\lambda_w}} H_s = 0, \tag{A.10}$$

where

$$\pi_{t,s} = \begin{cases} 1, \text{ for } s = t \\ \pi_{t+1} \cdot \pi_{t+2} \cdots \pi_s, \text{ for } s \geq t+1 \end{cases}.$$

By the law of large numbers, the probability of changing the wage corresponds to the fraction of types who actually do change their wage. Consequently, from expression (21), the real wage evolves according to

$$w_t^{-\frac{1}{\lambda_w}} = (1-\zeta_w)\tilde{w}_t^{-\frac{1}{\lambda_w}} + \zeta_w \left(\frac{w_{t-1}}{\pi_t}\right)^{-\frac{1}{\lambda_w}}. \tag{A.11}$$

A.2.3 Final and Intermediate Goods Producers

Competitive final goods producers choose Y_{it} to maximize profits

$$P_t Y_t - \int_0^1 P_{it} Y_{it} di,$$

where P_{it} is the price of the i^{th} variety, subject to (22). The solution to the profit maximization problem yields the demand for the generic i^{th} intermediate good (23). The zero profit condition for competitive final goods producers implies that the aggregate price level is (24).

Monopolistically competitive intermediate goods producers hire labor from households and rent capital from entrepreneurs to produce intermediate goods according to the production technology (25) and subject to the demand condition (23). We solve the problem for intermediate goods producers in two steps. First, we solve for the optimal amount of inputs (capital and labor) demanded. For this purpose, intermediate goods producers minimize costs

$$r_t^k K_{it} + w_t H_{it}$$

subject to (25). Let mc_{it} be the Lagrange multiplier on the constraint, the real marginal cost. The first order condition implies that the capital-labor ratio at the firm level is independent of firm-specific variables as

$$\frac{K_{it}}{H_{it}} = \frac{K_t}{H_t} = \frac{\gamma}{1-\gamma}\frac{w_t}{r_t^k}. \tag{A.12}$$

Then the marginal cost is independent of firm-specific variables as

$$mc_{it} = mc_t = \frac{1}{A_t}\left(\frac{r_t^k}{\gamma}\right)^{\gamma}\left(\frac{w_t}{1-\gamma}\right)^{1-\gamma}. \tag{A.13}$$

The second step consists of characterizing the optimal price setting decision in the event that firm i can adjust its price. Recall that this adjustment occurs in each period with probability $1-\zeta_p$, independent of previous history. If a firm can reset its price, it chooses $\tilde{P}_{it}$ to maximize

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\zeta_p)^{s-t} C_s^{-\sigma} \left[\frac{\tilde{P}_{it}}{P_s} - mc_s \right] Y_s(i),$$

subject to (23). The first order condition for this problem is

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\zeta_p)^{s-t} C_s^{-\sigma} \left[\frac{\tilde{P}_{it}}{P_s} - (1+\lambda_f) mc_s \right] Y_s(i) = 0.$$

All intermediate goods producers face an identical problems. As for the wage setting decision, we focus on a symmetric equilibrium in which all firms choose the same price $\tilde{P}_{it} = \tilde{P}_t$. Let $\tilde{p}_t \equiv \tilde{P}_t / P_t$ denote the optimal relative price. The first order condition for optimal price setting becomes

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta\zeta_p)^{s-t} C_s^{-\sigma} \left[\frac{\tilde{p}_t}{\pi_{t,s}} - (1+\lambda_f) mc_s \right] \left(\frac{\tilde{p}_t}{\pi_{t,s}} \right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_s = 0. \tag{A.14}$$

By the law of large numbers, the probability of changing the price coincides with the fraction of firms who actually do change the price in equilibrium. Therefore, from expression (24), inflation depends on the optimal reset price according to

$$1 = (1-\zeta_p)\tilde{p}_t^{-\frac{1}{\lambda_f}} + \zeta_p \left(\frac{1}{\pi_t} \right)^{-\frac{1}{\lambda_f}} \tag{A.15}$$

The fact that the capital-output ratio is independent of firm-specific factors implies that we can obtain an aggregate production function

$$A_t K_t^{\gamma} H_t^{1-\gamma} = \int_0^1 Y_{it} di = \sum_{s=0}^{\infty} \zeta_p \left(1-\zeta_p\right)^{t-s} \left(\frac{\widetilde{p}_{t-s}}{\pi_{t-s,t}} \right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_t, \tag{A.16}$$

where $K_t \equiv \int_0^1 K_{it} di$ and $H_t \equiv \int_0^1 H_{it} di$.

A.2.4 Capital Producers

Capital producers transform consumption into investment goods and operate in a competitive national market. Their problem consists of choosing the amount of investment goods produced I_t to maximize (26) taking the price of investment goods p_t^I as given. The first order condition for this problem is

$$p_t^I = 1 + S\left(\frac{I_t}{I}\right) + S'\left(\frac{I_t}{I}\right)\frac{I_t}{I}. \tag{A.17}$$

A.2.5 Government Budget

Using the expression of real value of liquidity, the government budget constraint (29) and the tax rule (30)can be written as

$$q_t N_{t+1}^g + \frac{R_{t-1}L_t}{\pi_t} = \tau_t + [r_t^k + (1-\delta)q_t]N_t^g + L_{t+1}. \tag{A.18}$$

and

$$\tau_t - \tau = \psi_\tau\left(\frac{R_{t-1}L_t}{\pi_t} - \frac{RL}{\pi} - q_t N_t^g\right) \tag{A.19}$$

A.3 Equilibrium Conditions

The total factor productivity and resaleability constraint (A_t, ϕ_t) follow an exogenous Markov process. In addition to these, we have four endogenous state variables of $(K_t, N_t^g, R_{t-1}L_t, w_{t-1})$ - aggregate capital stock, government ownership of capital, and a real liquidity measure and the real wage rate from the previous period. The recursive competitive equilibrium is defined as nine endogenous quantities $\left(I_t, C_t, Y_t, H_t, K_{t+1}, N_{t+1}, N_{t+1}^g, L_{t+1}, \tau_t\right)$ and nine prices $\left(q_t, p_t^I, \widetilde{w}_t, w_t, \widetilde{p}_t, \pi_t, r_t^K, mc_t, R_t\right)$ as a function of the state variables $(K_t, N_t^g, R_{t-1}L_t, w_{t-1}, A_t, \phi_t)$ which satisfies the eighteen equilibrium conditions (16, 17, 27, 28, 31, 32, 33, A.9, A.10, A.11, A.12, A.13, A.14, A.15, A.16, A.17, A.18, A.19). Once all the market clearing condition and the government budget constraints are satisfied, the household budget constraint (15) is satisfied by Walras' Law.

Additionally, we define the following variables:

$$R_t^q = \mathbb{E}_t\left[\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{q_t}\right] : \text{Expected rete of return on equity} \tag{A.20}$$

$$R_t^k = \mathbb{E}_t[r_{t+1}^k + (1-\delta)] : \text{Expected return to capital} \tag{A.21}$$

A.3.1 Steady state

We consider a steady-state economy in which there is no change in the total factor productivity, resaleability, the nominal price level, and the endogenous quantities and prices. Condition (A.12) at steady state implies

$$\frac{K}{H} = \frac{\gamma}{1-\gamma}\frac{w}{r^k}. \tag{A.22}$$

In steady state all firms charge the same price, hence $\tilde{p} = 1$ and the real marginal cost is equal to the inverse of the markup

$$mc = \frac{1}{A}\left(\frac{r^k}{\gamma}\right)^{\gamma}\left(\frac{w}{1-\gamma}\right)^{1-\gamma} = \frac{1}{1+\lambda_f}. \tag{A.23}$$

Incorporating these two equations into the steady-state version of the production function (A.16) yields a relation between the capital-output ratio and the rental rate of capital

$$\frac{Y}{K} = \frac{(1+\lambda_f)r^k}{\gamma}. \tag{A.24}$$

Because the ratio between capital and hours is a function of the capital-output ratio (from the production function), equation (A.23) also yields an expression for the real wage as a function of the rental rate

$$w = (1-\gamma)\left(\frac{A}{1+\lambda_f}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{r^k}\right)^{\frac{\gamma}{1-\gamma}}. \tag{A.25}$$

In steady state, the real wage is equal to a markup over the marginal rate of substitution between labor and consumption

$$w = (1+\lambda_w)\frac{[H/(1-\varkappa)]^{\nu}}{C^{-\sigma}}. \tag{A.26}$$

From the steady-state version of (16), we can solve for the steady-state real interest rate ($r \equiv R/\pi = R$) as a function of q

$$\beta^{-1} = r\left(1+\varkappa\frac{q-1}{1-\theta q}\right) \tag{A.27}$$

where we used the fact that in steady state $p^I = 1$ because ($S(1) = S'(1) = 0$ (from A.17). This condition implies that the liquid asset has a return that is less that β^{-1} as long as $1 < q < 1/\theta$.

Steady-state tax obtain from (A.18)

$$\tau = (r-1)L. \tag{A.28}$$

In steady state, condition (A.9) implies

$$I = \varkappa \frac{[r^k + (1-\delta)\phi q]K + rL + \frac{\lambda_f}{1+\lambda_f}Y - \tau}{1-\theta q} = \varkappa \frac{[r^k + (1-\delta)\phi q]K + L + \frac{\lambda_f}{1+\lambda_f}Y}{1-\theta q}, \tag{A.29}$$

where we used (A.28) to eliminate transfers and the fact that in steady state $K = N$ since by assumption $N^g = 0$. Steady-state investment is simply equal to depreciated steady-state capital

$$\frac{I}{K} = \delta. \tag{A.30}$$

Combining (A.29) with (A.30), we obtain

$$\delta(1-\theta q) = \varkappa \left[r^k + (1-\delta)\phi q + \frac{L}{K} + \frac{\lambda_f}{1+\lambda_f}\frac{Y}{K} \right]. \tag{A.31}$$

Using the steady-state capital output ratio (A.24), we obtain a relationship between r^k and q

$$\delta - [\delta\theta + \varkappa(1-\delta)\phi]q = \varkappa \left(1 + \frac{1+\lambda_f}{\gamma}\frac{L}{Y} + \frac{\lambda_f}{\gamma} \right) r^k, \tag{A.32}$$

where L/Y is ratio of liquid assets to GDP that we take as exogenous in our calibration.

Another relationship between r^k and q obtains from the steady-state version of (17)

$$\beta^{-1} = \frac{r^k + (1-\delta)q}{q}\left(1 + \varkappa\frac{q-1}{1-\theta q}\right) - \frac{\varkappa(1-\delta)(1-\phi)(q-1)}{1-\theta q} \tag{A.33}$$

where $(r^k + \lambda q)/q$ is the steady-state return on equity. As long as $\phi < 1$, the return on equity is larger than the steady-state return on the liquid assets by

$$\frac{\varkappa(1-\delta)(1-\phi)(q-1)}{(1-\theta q)\left(1 + \varkappa\frac{q-1}{1-\theta q}\right)}. \tag{A.34}$$

We can insert the solution for r^k from (A.32) into (A.33) and solve for q. Once we have q and r^k, r can be obtained from (A.27), w from (A.25), K/Y from (A.24), K/H from (A.22), I/K from (A.30) and C/Y from the resource constraint. Finally, the size of the economy Y is determined to satisfy (A.26).

A.3.2 Log-linear Approximation

Define $\hat{x}_t \equiv \log(x_t/x)$ where x is the steady-state value of x_t. From investment function (A.9), we have:

$$\begin{aligned}
&\delta(1-\varkappa)\hat{p}_t^I + (1-\theta q)\delta \hat{I}_t - [\theta\delta + \varkappa(1-\delta)\phi]q\hat{q}_t \\
&-\varkappa(1-\delta)\phi q\hat{\phi}_t - \varkappa[r^k + (1-\delta)\phi q]\hat{N}_t - \varkappa r\tfrac{L}{K}\left(\hat{R}_{t-1} + \hat{L}_t - \hat{\pi}_t\right) \\
&+\varkappa\tfrac{\tau}{K}\hat{\tau}_t - \varkappa\tfrac{Y}{K}\hat{Y}_t + \varkappa\tfrac{(1-\gamma)r^k}{\gamma}(\hat{w}_t + \hat{H}_t) + \varkappa r^k\hat{K}_t = 0.
\end{aligned} \tag{A.35}$$

From the Euler equation for equity (17), we get:

$$\begin{aligned}
&\hat{q}_t - \sigma\hat{C}_t = -\sigma\mathbb{E}_t[\hat{C}_{t+1}] + \beta\tfrac{r^k}{q}\left(1 + \varkappa\tfrac{q-1}{1-\theta q}\right)\mathbb{E}_t[\hat{r}^k_{t+1}] + \beta(1-\delta)\varkappa\tfrac{q-1}{1-\theta q}\phi\mathbb{E}_t[\hat{\phi}_{t+1}] \\
&+\beta\left[(1-\delta) + \varkappa(1-\delta)\tfrac{q-1}{1-\theta q}\phi + \varkappa(r^k + (1-\delta)\phi q)\tfrac{1-\theta}{(1-\theta q)^2}\right]\mathbb{E}_t[\hat{q}_{t+1}] \\
&-\beta\varkappa\left[\tfrac{r^k}{q} + (1-\delta)\phi\right]\tfrac{q(1-\theta)}{(1-\theta q)^2}\mathbb{E}_t[\hat{p}^I_{t+1}].
\end{aligned} \tag{A.36}$$

The Euler equation for bonds (16) becomes:

$$\begin{aligned}
&-\sigma\hat{C}_t = -\sigma\mathbb{E}_t[\hat{C}_{t+1}] + \hat{R}_t - \mathbb{E}_t[\hat{\pi}_{t+1}] + \beta\varkappa\tfrac{1-\theta}{(1-\theta q)^2}rq\mathbb{E}_t[\hat{q}_{t+1}] \\
&-\beta\varkappa\tfrac{1-\theta}{(1-\theta q)^2}rq\mathbb{E}_t[\hat{p}^I_{t+1}].
\end{aligned} \tag{A.37}$$

The resource constraint (33) is written as:

$$\hat{Y}_t = \frac{I}{Y}\hat{I}_t + \frac{C}{Y}\hat{C}_t. \tag{A.38}$$

The marginal costs (A.13) is:

$$\widehat{mc}_t = (1-\gamma)\hat{w}_t + \gamma\hat{r}^k_t - \hat{A}_t. \tag{A.39}$$

The price setting decision for the firm (A.14) combined with the relation between inflation and optimal relative price (A.15) leads to:

$$\hat{\pi}_t = \frac{(1-\zeta_f\beta)(1-\zeta_f)}{\zeta_f}\widehat{mc}_t + \beta\mathbb{E}_t[\hat{\pi}_{t+1}]. \tag{A.40}$$

The relationship determining the capital/labor ratio (A.12) becomes:

$$\hat{K}_t = \hat{w}_t - \hat{r}^k_t + \hat{H}_t. \tag{A.41}$$

The law of motion for aggregate wages (A.11) is now:

$$\hat{w}_t = (1-\zeta_w)\widehat{\widetilde{w}}_t + \zeta_w(\hat{w}_{t-1} - \hat{\pi}_t). \tag{A.42}$$

The wage-setting decision of the household (A.10) leads to:

$$\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\hat{\tilde{w}}_t-(1-\zeta_w\beta)\nu\frac{1+\lambda_w}{\lambda_w}\hat{w}_t$$
$$=(1-\zeta_w\beta)\left(\nu\hat{H}_t+\sigma\hat{C}_t\right)+\zeta_w\beta\left(1+\nu\frac{1+\lambda_w}{\lambda_w}\right)\mathbb{E}_t\left(\hat{\tilde{w}}_{t+1}+\hat{\pi}_{t+1}\right). \quad \text{(A.43)}$$

The aggregate production function (A.16) is written as:

$$\hat{Y}_t=\hat{A}_t+\gamma\hat{K}_t+(1-\gamma)\hat{H}_t. \quad \text{(A.44)}$$

The first order condition for capital producers (A.17) becomes:

$$\hat{p}_t^I=S''(1)\hat{I}_t. \quad \text{(A.45)}$$

The law of motion of capital (31) is now:

$$\hat{K}_{t+1}=\delta\hat{I}_t+(1-\delta)\hat{K}_t. \quad \text{(A.46)}$$

The market clearing condition for capital implies:

$$\hat{K}_{t+1}=\hat{N}_{t+1}+\hat{N}^g_{t+1}, \quad \text{(A.47)}$$

where $\hat{N}^g_{t+1}\equiv N^g_{t+1}/K$. The interest rate rule (27) is:

$$\hat{R}_t=\psi_\pi\hat{\pi}_t. \quad \text{(A.48)}$$

When the ZLB is binding, by definition $R_t=1$ and hence, in terms of log-linearized variables, we have

$$\hat{R}_t=-\log(R) \quad \text{(A.49)}$$

The government rule for purchasing capital (28) is now:

$$\hat{N}^g_{t+1}=\psi_k\hat{\phi}_t. \quad \text{(A.50)}$$

The government budget constraint (A.18) becomes:

$$\frac{\tau}{K}\hat{\tau}_t=q\hat{N}^g_{t+1}+\frac{rL}{K}(\hat{R}_{t-1}-\hat{\pi}_t+\hat{L}_t)-[r^k+(1-\delta)q]\hat{N}^g_t-\frac{L}{K}\hat{L}_{t+1}. \quad \text{(A.51)}$$

The rule for transfers/taxes (A.19) is written as:

$$\frac{\tau}{K}\hat{\tau}_t=\psi_\tau\left[\frac{rL}{K}(\hat{R}_{t-1}-\hat{\pi}_t+\hat{L}_t)-q\hat{N}^g_t\right]. \quad \text{(A.52)}$$

In order to avoid rewriting different version of the equilibrium conditions depending on whether the ZLB binds, in the code we make use of the Fisher relationship:

$$\hat{R}_t = \hat{r}_t - \mathbb{E}_t[\hat{\pi}_{t+1}], \tag{A.53}$$

where r_t is the real rate.

Other definitions included in the codes are:

$$\hat{R}^q_t = \frac{\frac{r^k}{q}}{\frac{r^k}{q} + (1-\delta)} \mathbb{E}_t \hat{r}^k_{t+1} + \frac{(1-\delta)}{\frac{r^k}{q} + (1-\delta)} \mathbb{E}_t \hat{q}_{t+1} - \hat{q}_t, \tag{A.54}$$

$$\hat{R}^k_t = \frac{r^k}{r^k + (1-\delta)} \mathbb{E}_t \hat{r}^k_{t+1}. \tag{A.55}$$

$$\widehat{LS}_t = \frac{1}{1 + \frac{L}{qK}} [\hat{L}_{t+1} - (\hat{q}_t + \hat{K}_{t+1})]. \tag{A.56}$$