Urban Population and Amenities

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Abstract

We use a frictionless neoclassical general-equilibrium model to explain cross-metro variation in population density based on three broad amenity types: quality of life, productivity in tradables, and productivity in non-tradables. Analytically, we demonstrate the dependence of quantities on amenities through substitution possibilities in consumption and production. Our calibrated model predicts large elasticities, consistent with variation in U.S. data, and estimates of local labor supply and demand. From only differences in wages and housing costs, we explain half of the variation in density, especially through quality-of-life amenities. We also show density information can provide or refine measures of land value and local productivity. We show how our approach can be used to study a wide variety of urban quantities.

Keywords: Population density, productivity, quality of life.

JEL Numbers: H2, H4, J3, Q5, R1

1 Introduction

Population densities vary across space considerably more than the prices of labor and housing. At the metropolitan level, the average residential density of New York is almost 50 times that of Texarkana. Meanwhile, wage levels in the highest-paying metro are not even double that of the lowest, and housing costs in the most expensive metro average only four times that of the lowest. Below, we examine how small differences in prices are compatible with large differences in quantities, like population density, in the neo-classical model of Rosen (1979) and Roback (1982) with mobile households and firms, and both tradable and non-tradable sectors.

In the neo-classcial model, differences in prices and quantities across metro areas stem from local amenities, which work through three different channels: quality of life, tradable-sector productivity, and non-tradable-sector productivity. The first two channels determine the extent to which people follow jobs or jobs follow people, a topic long debated (Blanco 1963, Borts and Stein 1964).[1] The third, determines whether both jobs and people follow availabile housing, a subject that has received more recent attention (Glaeser and Gyourko 2006, Glaeser, Gyourko, and Saks 2006, Saks 2008). Although researchers use the neo-classical model heavily to examine the relationship between prices and amenities, they rarely do so to examine the relationship between quantities and amenities. When they have, the models has been examined numerically, putting strong restrictions on the model (e.g. Haughwout and Inman 2001, Rappaport 2008a, 2008b), or altering its structure, (e.g. Desmet and Rossi Hansberg 2012, Moretti 2011), particularly in the non-tradable sector.[2]

Here we consider the relationships between amenities and quantities analytically, using the canonical neo-classical model with few restrictions. This allows us to analyze how quantity dif-

[1] See Hoogstra, Florax, and Dijk (2005) for an interesting meta-analysis of this literature.

[2] Haughwout and Inman (2001) reduce the non-tradable sector to a fixed land market. Rappaport (2008a, 2008b) constrains productivity in the non-tradable sector to be the same as in the tradable sector, and assumes the elasticity of substitution between factors in tradable production is one. Glaeser, Gyourko and Saks (2006) and Moretti (2011) use an ad-hoc partial equilibrium supply function, thereby excluding labor from the non-tradable sector, and force households to consume a fixed amount of housing. Desmet and Rossi-Hansberg (2012) constrain elasticities of substitution in consumption and tradable production to be one, and model the non-tradable sector using a monocentric city, where households consume a single unit of housing. Only Rappaports work is useful for studying population density, although his work is done numerically, and is not linked to data in a close manner.

ferences depend on cost and expenditure shares, tax rates, and separate margins of substitution in consumption and in both types of production. The substitution margins reflect three separate behavioral responses that lead to higher densities, including the construction of housing at greater heights, the willingness of households to crowd into existing housing, and shifts in production away from land-intensive goods. It turns out that urban quantities depend on these substitution possibilities in a first-order manner, while for prices, they do not. Using a pre-set calibration of the United States economy from Albouy (2009), our results suggest that substitution possibilities in the non-tradable sector, including housing, are particularly important.

The analytical exercise maps reduced-form elasticities, estimated in the literature, e.g., of local labor or housing supply, to more elementary structural parameters. This mapping reframes partial-equilibrium shifts in supply and demand as general-equilibrium responses to amenity changes, e.g. an increase in labor demand is mapped to an increase in traded-sector productivity. The calibrated model implies that quantities are much more responsive than prices to differences in amenities over the long run. The model produces large (positive) labor-supply elasticities that are remarkably consistent with estimates found in Bartik (1991) and Notowidigo (2012), and even larger (negative) labor-demand elasticities consistent with estimates in Card (2001). Moreover, our numbers are consistent with the stylized fact that population density varies by an order of magnitude more than wages and housing costs across metro areas.

Our research complements that on agglomeration, which examines the reverse relationship of how population affects amenities, especially productivity. For example, we can model how areas with higher quality of life become denser, thereby making them more productive through agglomeration. Agglomeration then creates a multiplier effect through feedback, whereby higher density increases productivity, bringing forth even higher density and productivity. We also consider the possibility of greater density reducing quality of life through congestion. Under our calibration, we find that these multiplier effects are potentially important, magnifying or dampening long-run behavioral responses up to 25 percent.

We apply the model empirically by using it to relate observable prices to population densities in 276 American metropolitan areas using Census data. The pre-set calibration does remarkably well, explaining half of the variation in population densities through quality-of-life and trade-productivity predicted by two simple measures of wages and housing costs. Our calibration fits the data better than those that ignore substitution possibilities, e.g. in consumption or non-tradable production, or assume that they are all unit elastic, as in a Cobb-Douglas economy.

If the calibration produces accurate elasticity values, variation in population density not explained by quality of life, may substitute for missing data on land prices, and help to identify productivity in the traded sector and in the non-traded sector. From this method, metro areas such as New York, Chicago, and Houston appear to have rather productive non-tradable sectors, at least historically. On the other hand, metros such as San Francisco and Seattle have far less productive non-tradable sectors despite having very productive tradable sectors.

Our last exercise determines the relative importance of different amenities in explaining where people live. A variance decomposition suggests that quality of life explains a greater fraction of population density than does trade-productivity, even though the latter varies more in value, and affects wage and housing costs more. This conclusion is reinforced if agglomeration increases trade-productivity or reduces quality of life. Productivity in non-tradables explains density more than the other types of amenities, although this may have much to do with how it is measured. We also simulate how population density might change if federal taxes were made geographically neutral. This tends to make trade-productivity a stronger determinant and causes multipliers from agglomeration feedback to be larger.

The general-equilibrium model of location, with homogenous agents, provides a different point of view than partial equilibrium models of location with heterogenous agents, often with dynamics (e.g. Kennan and Walker 2011). These approaches typically do not consider how wages and housing costs depend on population sizes. Moreover, the focus of such work is to explain migration decisions over short and medium-run horizons, while our intention is to examine population differences over the very long run.

The rest of the paper is organized as follows: Section 2 introduces the model. Section 3 calibrates the model, provides numerical results, and discusses identification. Section 5.3 provides new estimates of trade and home-productivity. Section 4 estimates long-run elasticities of labor and housing demand and supply. Section 6 concludes.

2 Locational Equilibrium of Quantities, Prices, and Amenities

2.1 Elements of the Neo-Classical Location Model for Metro Areas

To explain how prices and quantities vary with amenity levels across cities, we use the model of Albouy (2009a), which adds federal taxes to the general-equalibrium three-equation Roback (1982) model. The national economy contains many cities, indexed by j, which trade with each other and share a homogenous population of mobile households. Households supply a single unit of labor in the city they live in, and consume a numeraire traded good x and a non-traded "home" good y with local price p^j.[3] All input and output markets are perfectly competitive, and all prices and quantities are homogenous within cities, though they vary across cities.

Cities differ exogenously in three general attributes, each of which is an index meant to summarize the value of amenities to households and firms: (i) quality of life, Q^j, raises household utility; (ii) trade-productivity, A_X^j, lowers costs in the traded-good sector, and (iii) home-productivity, A_Y^j, lowers costs in the home-good sector.[4]

Firms produce traded and home goods out of land, capital, and labor. Within a city, factors receive the same payment in either sector. Land L is heterogenous across cities, immobile, and receives a city-specific price r^j. Each city's land supply $L^j(r)$ may depend positively on r^j.

Capital K is fully mobile across cities and receives the price $\bar{\imath}$ everywhere. The supply of capital in each city, K^j, is perfectly elastic at this price. The national level of capital may be

[3]In application, the price of the home good is equated with the cost of housing services. Non-housing goods are considered to be a composite commodity of traded goods and non-housing home goods.

[4]All of these attributes depend on a vector of natural and artificial city amenities, $\mathbf{Z}^j = (Z_1^j, ..., Z_K^j)$, through functional relationships $Q^j = \widetilde{Q}(\mathbf{Z}^j)$, $A_X^j = \widetilde{A_X}(\mathbf{Z}^j)$, and $A_Y^j = \widetilde{A_Y}(\mathbf{Z}^j)$. For a consumption amenity, e.g. clement weather, $\partial\widetilde{Q}/\partial Z_k > 0$; for a trade-production amenity, e.g. navigable water, $\partial\widetilde{A_X}/\partial Z_k > 0$; for a home-production amenity, e.g. flat geography, $\partial\widetilde{A_Y}/\partial Z_k > 0$. It is possible that a single amenity affects more than one attribute or affects an attribute negatively.

fixed or depend on $\bar{\imath}$. Households N are fully mobile, have identical tastes and endowments, and each supplies a single unit of labor. Household size is fixed. Wages w^j vary across cities because households care about local prices and quality of life. The total number of households is $N^{TOT} = \sum_j N^j$, which may be fixed or determined by international migration.

Households own identical diversified portfolios of land and capital, which pay an income $R = \frac{1}{N_{TOT}} \sum_j r^j L^j$ from land and $I = \frac{1}{N_{TOT}} \sum_j \bar{\imath} K^j$ from capital. Total income $m^j = R + I + w^j$ varies across cities only as wages vary. Out of this income households pay a linear federal income tax τm^j which is redistributed in uniform lump-sum payments, T. Household preferences are modeled by a utility function $U(x, y; Q^j)$ which is quasi-concave over x, y, and Q^j. The expenditure function for a household in city j is $e(p^j, u; Q^j) \equiv \min_{x,y}\{x + p^j y : U(x, y; Q^j) \geq u\}$. Assume Q enters neutrally into the utility function and is normalized so that $e(p^j, u; Q^j) = e(p^j, u)/Q^j$, where $e(p^j, u) \equiv e(p^j, u; 1)$.

Operating under perfect competition, firms produce traded and home goods according to the functions $X^j = A_X^j F_X^j(L_X^j, N_X^j, K_X^j)$ and $Y^j = A_Y^j F_Y^j(L_Y^j, N_Y^j, K_Y^j)$, where F_X and F_Y are concave and exhibit constant returns to scale, and A_X^j and A_Y^j are assumed to be Hicks-Neutral. Unit cost in the traded-good sector is $c_X(r^j, w^j, \bar{\imath}; A_X^j) \equiv \min_{L,N,K}\{r^j L + w^j N + \bar{\imath} K : A_X^j F(L, N, K) = 1\}$. Similar to the relationship between quality of life and the expenditure function, let $c_X(r^j, w^j, \bar{\imath}; A_X^j) = c_X(r^j, w^j, \bar{\imath})/A_X^j$, where $c_X(r^j, w^j, \bar{\imath}) \equiv c_X(r^j, w^j, \bar{\imath}; 1)$. A symmetric definition holds for the unit cost in the home-good sector c_Y.[5]

2.2 Equilibrium Conditions

Each city can be described by a system of sixteen equations in sixteen endogenous variables: three prices p^j, w^j, r^j, and thirteen quantities $x^j, y^j, X^j, Y^j, N^j, N_X^j, N_Y^j, L^j, L_X^j, L_Y^j, K^j, K_X^j, K_Y^j$. We begin by having these depend on three exogenous attributes Q^j, A_X^j, A_Y^j and a land supply function $L(r)$. In this scenario, the system of equations has a block-recursive structure, allowing us to first

[5]The model generalizes to a case with heterogenous workers that supply different fixed amounts of labor if these workers are perfect substitutes in production, have identical homothetic preferences, and earn equal shares of income from labor.

determine prices - where most researchers stop - second, per-capita consumption quantities, and last, production quantities, including total population. This block-recursive structure is broken if amenities are made endogenous to quantities, e.g. if $A_X^j = A_{X0}^j (N^j)^\alpha$ where A_{X0}^j is due to fixed natural advantages, and $(N^j)^\alpha$ is due to agglomeration economies. This is more important when doing comparative statics, e.g. by changing A_{X0}^j, which then changes N^j, than when doing measurement, where N^j may be treated as fixed so long as we are satisfied in measuring the composition $A_{X0}^j (N^j)^\alpha$. Throughout, we adopt a "small open city" assumption and take nationally determined variables $\bar{u}, \bar{\imath}, I, R, T$ as given for any individual city.[6]

2.2.1 Price Conditions

Since households are fully mobile, they must receive the same utility across all inhabited cities. Higher prices or lower quality of life are compensated with greater after-tax income,

$$e(p^j, \bar{u})/Q^j = (1-\tau)(w^j + R + I) + T, \tag{1}$$

where $\bar{u}$ is the level of utility attained nationally by all households. Firms earn zero profits in equilibrium. For given output prices, firms in more productive cities must pay higher rents and wages,

$$c_X(r^j, w^j, \bar{\imath})/A_X^j = 1 \tag{2}$$

$$c_Y(r^j, w^j, \bar{\imath})/A_Y^j = p^j. \tag{3}$$

Equations (1), (2), and (3) simultaneously determine the city-level prices p^j, r^j, and w^j for each city as implicit functions of the three attributes Q^j, A_X^j, and A_Y^j. In equilibrium, these conditions provide a one-to-one mapping between unobserved city attributes and potentially observable prices, obviating the need to examine quantities.

[6] In a closed city, we could instead take N^j or K^j as given, and endogenize factor incomes R^j or I^j. In the open city we assume that the federal government's budget is given by, $\tau \sum_j N^j m^j + T \sum_j N^j = 0$, so a city with average income receives a transfer which exactly offsets its taxes.

2.2.2 Consumption Conditions

In deciding their consumption quantities x^j, y^j, households face the budget constraint

$$x^j + p^j y^j = (1 - \tau)(w^j + R + I)^j + T \tag{4}$$

where p^j and w^j are determined by the price conditions. Optimal consumption is determined in conjunction with the tangency condition

$$(\partial U/\partial y) \,/\, (\partial U/\partial x) = p^j. \tag{5}$$

As we assume preferences are homothetic, Q^j does not affect the marginal rate of substitution. Thus, in areas where Q^j is higher, but p^j is the same, households consume less of x and y in equal proportions, holding the ratio y/x constant, similar to an income effect. Holding Q^j constant, increases in p^j are compensated by increases in w^j so that households reduce their relative consumption of y to x due to a pure substitution effect.

2.2.3 Production Conditions

With prices and per-capita consumption levels accounted for, Levels of output X^j, Y^j, employment N^j, N_X^j, N_Y^j, capital K^j, K_X^j, K_Y^j, and land L^j, L_X^j, L_Y^j are determined by eleven equations describing production sector and market clearing. The first six express conditional factor demands using Shepard's Lemma. Because of constant returns to scale, and Hicks neutrality, the derivative of the uniform unit-cost function equals the ratio of the relevant input, augmented by city-specific productivity, to output:

$$\partial c_X/\partial w = A_X^j N_X^j / X^j \tag{6}$$

$$\partial c_X/\partial r = A_X^j L_X^j / X^j \tag{7}$$

$$\partial c_X/\partial i = A_X^j K_X^j / X^j \tag{8}$$

$$\partial c_Y/\partial w = A_Y^j N_Y^j / Y^j \tag{9}$$

$$\partial c_Y/\partial r = A_Y^j L_Y^j / Y^j \tag{10}$$

$$\partial c_Y/\partial i = A_Y^j K_Y^j / Y^j \tag{11}$$

The next three conditions express the local resource constraints for labor, land, and capital, under the assumption that factors are fully employed.

$$N^j = N_X^j + N_Y^j \tag{12}$$

$$L^j = L_X^j + L_Y^j \tag{13}$$

$$K^j = K_X^j + K_Y^j \tag{14}$$

Equation (13) differs from the others as local land is determined by the supply function,

$$L^j = L(r^j), \tag{15}$$

Together, the assumptions of an internally homogenous open city, with exogenous amenities, and cost and expenditure functions that are homogenous of degree one, imply that all of the production quantity predictions increase proportionally with the quantity of land. If land in a city doubles, labor and capital will migrate in to also double, so that all prices and per-capita quantities remain the same. The open city and constant returns to scale assumptions imply that all of the model's quantity predictions increase one-for-one with the quantity of land. If the available land in city j doubled, then labor and capital would migrate inwards such that, in the new equilibrium, all of the prices and per-capita quantities would return to the initial equilibrium while the aggregate quantities would increase by the same amount as the increase in land supply. By focusing on density, we can essentially normalize land supply to a single unit $L(r^j) = 1$.[7]

The last condition is a market clearing condition for the local market in home-goods.

$$Y^j = N^j y^j \tag{16}$$

Walras' Law makes redundant the market clearing equation for tradable output, which include net tranfers from the federal government $T^j - \tau m^j$.

2.3 Log-Linearization around National Averages

The system described by conditions (1) to (16) is generically non-linear.[8] To make the system yield close-form solutions, enabling analytical interpretation , we log-linearize these conditions. Hence, we express each city's price and quantity differentials in terms of its amenity differentials, relative to the national average. These differentials are expressed in logarithms so that for any variable z, $\hat{z}^j \equiv \ln z^j - \ln \bar{z} \cong (z^j - \bar{z})/\bar{z}$ approximates the percent difference in city j of z relative to the average $\bar{z}$.

To express the log-linearization, we define several economic parameters, which take values at the national average. For households, denote the share of gross expenditures spent on the traded and home good as $s_x \equiv x/m$ and $s_y \equiv py/m$; denote the share of income received from land, labor, and capital income as $s_R \equiv R/m$, $s_w \equiv w/m$, and $s_I \equiv I/m$. For firms, denote the cost share of land, labor, and capital in the traded-good sector as $\theta_L \equiv rL_X/X$, $\theta_N \equiv wN_X/X$, and $\theta_K \equiv \bar{\imath}K_X/X$; denote equivalent cost shares in the home-good sector as ϕ_L, ϕ_N, and ϕ_K. Finally, denote the share of land, labor, and capital used to produce traded goods as $\lambda_L \equiv L_X/L$,

[7]Land supply can vary on two different margins. At the extensive margin, an increase in land supply corresponds to a growing city boundary. At the intensive margin, an increase in land supply takes the form of employing previously unused land within a city's border. By focusing on density we are ruling out the extensive case.

[8]Except when the economy is fully Cobb-Douglas, and there is no income received from land, capital, or government. In Appendix A, we provide results from a nonlinear simulation of the model.

$\lambda_N \equiv N_X/N$, and $\lambda_K \equiv K_X/K$. Assume the home-good is more cost-intensive in land relative to labor than the traded-good, both absolutely, $\phi_L \geq \theta_L$, and relatively, $\phi_L/\phi_N \geq \theta_L/\theta_N$, implying $\lambda_L \leq \lambda_N$.

The first three price conditions are log-linearized as

$$-s_w(1-\tau)\hat{w}^j + s_y\hat{p}^j = \hat{Q}^j \tag{1*}$$

$$\theta_L\hat{r}^j + \theta_N\hat{w}^j = \hat{A}_X^j \tag{2*}$$

$$\phi_L\hat{r}^j + \phi_N\hat{w}^j - \hat{p}^j = \hat{A}_Y^j \tag{3*}$$

These conditions are examined in depth in Albouy (2009b), and so here we just note how these expressions involve only cost and expenditure shares, and the marginal tax rate τ.

The log-linearized conditions describing consumption introduce the elasticity of substitution in consumption, $\sigma_D \equiv -e \cdot (\partial^2 e/\partial^2 p)/[\partial e/\partial p \cdot (e - p \cdot \partial e/\partial p)]$,

$$s_x\hat{x}^j + s_y\left(\hat{p}^j + \hat{y}^j\right) = (1-\tau)s_w\hat{w}^j \tag{4*}$$

$$\hat{x}^j - \hat{y}^j = \sigma_D\hat{p}^j \tag{5*}$$

Substituting in equation (1*) into this system produces the solutions $\hat{x}^j = s_y\sigma_D\hat{p}^j - \hat{Q}^j$ and $\hat{y}^j = -s_x\sigma_D\hat{p}^j - \hat{Q}^j$. These describe the substitution and quality-of-life effects described earlier.

Even though our model contains homogenous households, one can think of higher values of σ_D as approximating households with heterogeneous preferences who sort across cities. Households with stronger tastes for y will choose to live in areas with lower prices p. At the equilibrium levels of utility, an envelope of the mobilitiy conditions for each type forms that of a representative household, with greater preference heterogeneity reflected as more flexible substitution. [9]

The next six log-linearizations, of the conditional factor demands, describe how input demands

[9] Roback (1980) provides discussion along these lines.

depend on output, productivity, and relative input prices

$$\hat{N}_X^j = \hat{X}^j - \hat{A}_X^j + \theta_L \sigma_X^{LN} \left(\hat{r}^j - \hat{w}^j\right) - \theta_K \sigma_X^{NK} \hat{w}^j \tag{6*}$$

$$\hat{L}_X^j = \hat{X}^j - \hat{A}_X^j + \theta_N \sigma_X^{LN} (\hat{w}^j - \hat{r}^j) - \theta_K \sigma_X^{KL} \hat{r}^j \tag{7*}$$

$$\hat{K}_X^j = \hat{X}^j - \hat{A}_X^j + \theta_L \sigma_X^{KL} \hat{r}^j + \theta_N \sigma_X^{NK} \hat{w}^j \tag{8*}$$

$$\hat{N}_Y^j = \hat{Y}^j - \hat{A}_Y^j + \phi_L \sigma_Y^{LN} (\hat{r}^j - \hat{w}^j) - \phi_K \sigma_Y^{NK} \hat{w}^j \tag{9*}$$

$$\hat{L}_Y^j = \hat{Y}^j - \hat{A}_Y^j + \phi_N \sigma_Y^{LN} (\hat{w}^j - \hat{r}^j) - \phi_K \sigma_Y^{KL} \hat{r}^j \tag{10*}$$

$$\hat{K}_Y^j = \hat{Y}^j - \hat{A}_Y^j + \phi_L \sigma_Y^{KL} \hat{r}^j + \phi_N \sigma_Y^{NK} \hat{w}^j \tag{11*}$$

The dependence on input prices is determined by three partial (Allen-Uzawa) elasticities of substitution in each sector. These are defined for each pair of factors, where $\sigma_X^{LN} \equiv c_X \cdot (\partial^2 c_X / \partial w \partial r) \, / \, (\partial c_X / \partial w \cdot \partial c_X /$ is for labor and land in the production of X, etc. These values are taken at the national average, although they could vary across locations. To simplify matters we also assume that the partial elasticities within sectors are the same, i.e., $\sigma_X^{NK} = \sigma_X^{KL} = \sigma_X^{LN} \equiv \sigma_X$, and similarly for σ_Y, as with a constant elasticity of substitution production function.

A higher value of σ_X corresponds to more flexible production of the traded-good. With a single traded good, firms can vary their production by changing inputs. In a generalization with multiple traded goods sold at fixed prices, firms could adjust their product mix to specialize in producing goods where their input costs are relatively low. For example, an areas with high land costs, but low labor costs will produce goods that are use labor intensely but not land. A representative zero-profit condition can be drawn as an envelope of the zero-profit conditions for each good, with a greater variety of goods reflected as greater substituion possibilities, i.e., a larger σ_X.

A related argument may be made for home goods, as firms may produce them using different factor proportions. For instance, a high value of σ_Y means that housing producers can use labor and capital to build taller buildings in areas where the price of land is high. Residential units can also be subdivided to produce more effective livable space. If these goods are perfect substitutes, then an envelope of zero-profit conditions may be used as a represenative zero-profit condition.

Because housing is durable, this process may take a long time. For other home goods, one can imagine retailers using taller shelves, and restaurants hiring more labor to move clients through faster.

Log-linearizing the resource constraints for labor, land, and capital.

$$\hat{N}^j = \lambda_N \hat{N}_X^j + (1-\lambda_N)\hat{N}_Y^j \tag{12*}$$

$$\hat{L}^j = \lambda_L \hat{L}_X^j + (1-\lambda_L)\hat{L}_Y^j \tag{13*}$$

$$\hat{K}^j = \lambda_K \hat{K}_X^j + (1-\lambda_K)\hat{K}_Y^j \tag{14*}$$

$$\tag{17}$$

These imply that the sector-specific changes in factors affect overall changes in proportion to the factor share. The condition for land supply uses the elasticity $\varepsilon_{L,r} \equiv \partial L/\partial r \cdot (r/L)$.

$$\hat{L}^j = \varepsilon_{L,r}\hat{r}^j \tag{15*}$$

As new land is assumed identical to old land, then the impact of amenities on population and other production quantities will involve the term $\varepsilon_{L,r}\hat{r}^j$, with whatever impact the amenities have on $\hat{r}^j$, since quantities are proportional to the amount of land. By focusing on density, we rule this part out assuming $\varepsilon_{L,r} = 0$. Wrapping up, the market clearing condition for home-goods is simply

$$\hat{N}^j + \hat{y}^j = \hat{Y}^j. \tag{16*}$$

2.4 Solving the Model

The solutions for the endogenous variables are expressed in terms of the amenity differentials $\hat{Q}^j$, $\hat{A}_X^j$, and $\hat{A}_X^j$. Because of the block-recursive structure, only equations (1*) to (3*) are needed for the price differentials, discussed in Albouy (2009b).

$$\hat{r}^j = \frac{1}{s_R}\frac{\lambda_N}{\lambda_N - \tau\lambda_L}\left[\hat{Q}^j + \left(1 - \frac{1}{\lambda_N}\tau\right) s_x\hat{A}^j_X + s_y\hat{A}^j_Y\right] \tag{18}$$

$$\hat{w}^j = \frac{1}{s_w}\frac{1}{\lambda_N - \tau\lambda_L}\left[-\lambda_L\hat{Q}^j + (1-\lambda_L)s_x\hat{A}^j_X - \lambda_L s_y\hat{A}^j_Y\right] \tag{19}$$

$$\hat{p}^j = \frac{1}{s_y}\frac{1}{\lambda_N - \tau\lambda_L}\left[(\lambda_N - \lambda_L)\hat{Q}^j + (1-\tau)(1-\lambda_L)s_x\hat{A}^j_X - (1-\tau)\lambda_L s_y\hat{A}^j_Y\right] \tag{20}$$

Higher quality of life leads to higher land and home-good prices but lower wages. Higher trade-productivity increases all three prices, while higher home-productivity increases land prices but decreases wages and the home-good price.

Putting solution (20) in equations (4*) and (5*), yields the per-capita consumption differentials

$$\hat{x}^j = \frac{\sigma_D(1-\tau)}{\lambda_N - \tau\lambda_L}\left[\frac{(\sigma_D(\lambda_N - \lambda_L) - \lambda_N + \tau\lambda_L}{\sigma_D(1-\tau)}\hat{Q}^j + (1-\lambda_L)s_x\hat{A}^j_X - \lambda_L s_y\hat{A}^j_Y\right] \tag{21}$$

$$\hat{y}^j = \frac{s_x}{s_y}\frac{\sigma_D(1-\tau)}{\lambda_N - \tau\lambda_L}\left[-\frac{\sigma_D(\lambda_N - \lambda_L) + \lambda_N - \tau\lambda_L}{\sigma_D(1-\tau)}\hat{Q}^j - (1-\lambda_L)s_x\hat{A}^j_X + \lambda_L s_y\hat{A}^j_Y\right] \tag{22}$$

Households in trade-productive areas substitute towards tradable consumption away from non-tradable consumption, while households in home-productive areas do the opposite. In nicer areas, firms unambiguously consume fewer home goods; their consumption of tradable goods is more ambiguous as the substitution effect is positive, while the income effect is negative.

Unfortunately, solutions for the other quantities, which also rely on production equations (6*) through (16*) are more complicated and harder to intuit. As a notational short-cut, we express the change in each quantity with respect to amenities using three reduced-form elasticities, each composed of structural parameters. For example, the solution for population is expressed by

$$\hat{N}^j = \varepsilon_{N,Q}\hat{Q}^j + \varepsilon_{N,A_X}\hat{A}^j_X + \varepsilon_{N,A_Y}\hat{A}^j_Y, \tag{23}$$

where $\varepsilon_{N,Q}$ is the elasticity of population with respect to quality of life, ε_{N,A_X} is the population elasticity for trade-productivity, etc. The first reduced-form elatsicity is given structurally by

$$\begin{aligned}\varepsilon_{N,Q} = &\left[\frac{\lambda_N - \lambda_L}{\lambda_N}\right] + \sigma_D\left[\frac{s_x(\lambda_N - \lambda_L)^2}{s_y\lambda_N(\lambda_N - \lambda_L\tau)}\right] + \sigma_X\left[\frac{\lambda_L^2}{s_w(\lambda_N - \lambda_L\tau)} + \frac{\lambda_L\lambda_N}{s_R(\lambda_N - \lambda_L\tau)}\right]\\ &+ \sigma_Y\left[\frac{\lambda_L^2(1-\lambda_N)}{s_w\lambda_N(\lambda_N - \lambda_L\tau)} + \frac{\lambda_N(1-\lambda_L)}{s_R(\lambda_N - \lambda_L\tau)} - \frac{(\lambda_N - \lambda_L)^2}{s_y\lambda_N(\lambda_N - \lambda_L\tau)}\right]\\ &+ \varepsilon_{L,r}\left[\frac{\lambda_N}{s_R(\lambda_N - \lambda_L\tau)}\right] \end{aligned} \tag{24}$$

We provide similar expressions for ε_{N,A_X} and ε_{N,A_Y} in the appendix.

The components of these reduced-form elasticities may be interpreted. By collecting terms by their corresponding structural elasticity, as in (24, it is possible to see how higher quality of life raises population through five behavioral responses. The first term expresses how households accept to consume fewer goods, especially housing, in nicer areas through the income effect. The second term, with σ_D, captures how households increase density by substituting away from land-intensive goods, e.g., by crowding into existing housing. The third, with σ_X, gets at the ability of firms in the traded-sector to substitute away from land towards labor and capital. The fourth, with σ_D, reflects how home-goods become less land intensive, e.g., buildings get taller. The fifth, with $\varepsilon_{L,r}$ provides the population gain on the extensive margin, from more land being used, rather than on the intensive margin, from greater population density.

Each of the reduced-form elasticities between a quantity and a type of amenity may have up to five similar structural effects. The key differences between the price and quanity solutions is that the latter depend heavily on the substitution elasticities.

2.5 Agglomeration Economy Feedback and Multiplier Effects

The notationally compact formulation above make it straightforward to model simple forms of endogenous amenities. We consider two types we believe to be the most common: positive economies of scale in tradable production, and negative economies of scale in quality of life. For simplicity, both are assumed to depend on total population, with $A_X^j = A_{X0}^j(N^j)^\alpha$ and

$Q^j = Q_0^j (N^j)^{-\gamma}$, where A^j_{X0} and Q^j_0 represent city j's "natural advantages" and $\alpha \geq 0$ and $\gamma \geq 0$ are the reduced-form agglomeration elasticities. These natural advantages could be determined by local geographic features, local policies, or be the result of historical path dependence (Bleakley and Lin 2012). The agglomeration processes for productivity may be due to non-rival input sharing or knowledge spillovers, while the diseconomies in quality of life may be due to congestion or pollution. The main assumption here is that these processes follow a power law.

These agglomeration feedback effects result in the population solution now being

$$\hat{N}^j = \left[\varepsilon_{N,Q}(\hat{Q}^j_0 + \alpha \hat{N}^j) + \varepsilon_{N,A_X}(\hat{A}^j_{X0} - \gamma \hat{N}^j) + \varepsilon_{N,A_Y}\hat{A}^j_{Y0}\right] \tag{25}$$

$$= \frac{1}{1 - \alpha\varepsilon_{N,A_X} + \gamma\varepsilon_{N,Q}} \left(\varepsilon_{N,Q}\hat{Q}^j + \varepsilon_{N,A_X}\hat{A}^j_{X0} + \varepsilon_{N,A_Y}\hat{A}^j_Y\right) \tag{26}$$

$$= \left(\varepsilon^0_{N,Q}\hat{Q}^j + \varepsilon^0_{N,A_X}\hat{A}^j_{X0} + \varepsilon^0_{N,A_Y}\hat{A}^j_{Y0}\right) \tag{27}$$

taking A^j_{Y0} as fixed. The second expression begins with the multiplier which reflects how impact of natural advantages is magnified through positive economies of scale and dampened by negative ones. The multiplier effect depends as much on the population elasticities $\varepsilon_{N,A_X} and \varepsilon_{N,Q}$ as on the agglomeration parameters α and γ. The third equation simply re-expresses the reduced-form elasticities in terms of only the natural advantages. These elasticities may be smaller or larger than the originals, depending on the agglomeration effects and are appropriate to use in comparative static exercises, when the level of a natural advantage changes.

This framework could also be used to study a variety of more complicated endogenous feedback effects, although these would require more complicated solutions.

2.6 Identification of Production Amenities and Land Values

With accurate data on all price differentials $\hat{r}^j, \hat{w}^j, \hat{p}^j$ and knowledge of national economic parameters, we can estimate amenity differentials $\hat{Q}^j, \hat{A}^j_X, \hat{A}^j_Y$ with equations (1*), (2*), (3*). Reliable land value data comparable across metropolitan areas is not readily available, making it hard to identify trade- and home-productivity using equations (2*) and (3*).[10] Combining these equations to eliminate $\hat{r}^j$ we are left with

$$\frac{\theta_L}{\phi_L}\hat{p}^j + \left(\theta_N - \phi_N \frac{\theta_L}{\phi_L}\right)\hat{w}^j = \hat{A}_X^j - \frac{\theta_L}{\phi_L}\hat{A}_Y^j. \tag{28}$$

As Albouy (2009b) discusses, trade-productivity estimates may be estimated using the inferred cost formula on the left-hand side if we give up on estimating home-productivity, and assume it is constant across cities $\hat{A}_Y^j = 0$. These estimates are biased downwards, albeit slightly, in home-productive areas.[11]

Without such a restriction, home and trade productivity cannot be separately identified, since higher trade-productivity pushes wages and housing costs upwards in the same proportions that home-productivity pushes them downwards. [12] Assuming constant home-productivity $\hat{A}_Y^j = 0$, we can construct an initial estimate of trade-productivity as in equation (28) using parameters and data on wages and housing prices.

To solve this identification problem, we use additional information from population density not predicted by quality of life. This comes from combining equations (1*) and (23), yielding

$$\hat{N}^j - \varepsilon_{N,Q}[\underbrace{s_y\hat{p}^j - s_w(1-\tau)\hat{w}^j}_{\hat{Q}^j}] = \varepsilon_{N,A_X}\hat{A}_X^j + \varepsilon_{N,A_Y}\hat{A}_Y^j. \tag{29}$$

On the right-hand side we see that this excess density measure should be explained by either trade- or home-productivity. Because we are exactly identified, our amenity estimates will *perfectly predict* population densities given our parameter choices. Solving this system of two equations we

[10]Albouy and Ehrlich (2012) estimates $\hat{r}^j$ using recent transaction purchase data, although this is limited size and available only for recent years. Their analysis also discusses several conceptual and empirical challenge from this approach.

[11]This point is seen directly in equation (28) after noting that $\theta_L << \phi_L$.

[12]From equation (2*), note that $\hat{A}_X^j$ equals the costs faced by traded-good firms. We define $\hat{A}_X^j - \frac{\theta_L}{\phi_L}\hat{A}_Y^j$ as the costs of traded-good firms relative to home-good firms. The adjustment factor θ_L/ϕ_L arises because we eliminate $\hat{r}^j$.

obtain measures of productivity based on the differentials $\hat{N}^j, \hat{w}^j, \hat{p}^j$

$$\hat{A}_X^j = \frac{\theta_L[N^j - \varepsilon_{N,Q}(s_y p^j - s_w(1-\tau)w^j)] + \phi_L \varepsilon_{N,A_Y}[\frac{\theta_L}{\phi_L}p^j + (\theta_N - \phi_N \frac{\theta_L}{\phi_L}w^j]}{\theta_L \varepsilon_{N,A_X} + \phi_L \varepsilon_{N,AY}} \tag{30}$$

$$\hat{A}_Y^j = \frac{\phi_L[N^j - \varepsilon_{N,Q}(s_y p^j - s_w(1-\tau)w^j)] - \phi_L \varepsilon_{N,A_Y}[\frac{\theta_L}{\phi_L}p^j + (\theta_N - \phi_N \frac{\theta_L}{\phi_L}w^j]}{\theta_L \varepsilon_{N,A_X} + \phi_L \varepsilon_{N,AY}} \tag{31}$$

Trade-productiivty is measured by higher excess density and inferred costs. Home-productivity is measured more strongly by higher excess density, and by lower inferred costs.

This strategy identifies land value differences by substituting the solutions into (2*) or (3*):

$$\hat{r}^j = \frac{[\hat{N}^j - \varepsilon_{N,Q}(s_y \hat{p}^j - s_w(1-\tau)\hat{w}^j)] - \varepsilon_{N,A_X}\theta_N \hat{w}^j - \varepsilon_{N,A_Y}[\hat{p}^j - \phi_N \hat{w}^j]}{\theta_L \varepsilon_{N,A_X} + \phi_L \varepsilon_{N,AY}}$$

This rent measure is based off of excess density not predicted by the restricted productivity differentials we would estimate if land values were equal, $\hat{r}^j = 0$, i.e. $\hat{A}_X^j = \theta_N \hat{w}^j$ and $\hat{A}_Y^j = \phi_N \hat{w}^j - \hat{p}^j$. Excess density beyond that amount indicates higher land values.

3 Calibrating the Model and Calculating Elasticities

3.1 Parameter Choices

Calibrating the model to the U.S. economy has varying degrees of difficulty. Cost and expenditure shares require information on the first moments of data (i.e. means) and may be ascertained with some accuracy. Elasticities of substitution require credible identification involving second meoments (i.e. covariances of quantities with prices), and thus are subject to less confidence.

The main calibration we use is shown in Table 1. It follows that of Albouy (2009a). The expenditure and cost shares are already discussed in former work, so we leave these to the appendix, and keep them as fixed. Here we focus on the elasticities of substitution and consider ways of calibrating them differently.

We also must determine three elasticities of substitution. Following the literature review and discussion in Albouy (2009a), we initially use his values of $\sigma_D = \sigma_X = \sigma_Y = 0.667$. We provide sensitivity analysis surrounding our elasticities of substitution below.

For illustrative purposes, we consider fairly large values for the agglomeration elasticities. Thus, we use $\alpha = 0.06$ for the positive effect of population on trade-productivity, $\gamma = 0.015$ for the negative effect on quality of life. [13]

A few potential complications deserve special attention. First, incorrect parameter values might bias our estimates. As mentioned above, the parameters come from a variety of sources and are generally estimated across different years, geographies, and industries. Second, the log-linearized model is most accurate for small deviations from the national average. Population density varies significantly, which could bias our results. We present a non-linear simulation in Appendix A which suggests that our main conclusions are not affected by the linear approximation.

Furthermore, the elasticity of traded-good production σ_X might vary at different levels of aggregation. Specifically, the national elasticity might be larger than the city-level elasticity because of greater flexibility at the national level of production. On the other hand, if the national output mix is effectively fixed but cities can specialize in production, then this suggest that σ_X might be larger at the city-level.

We demonstrate how elasticities of substitution affect reduced form elasticities in Table 2. Our estimates also might contain error due to certain modeling assumptions, e.g. frictionless household relocation. We do not adjust for misspecification error. Our model most appropriately describes a long-run equilibrium, where moving costs or other frictions likely have little impact. Finally, the elasticity of home-good production may vary across cities, as Saiz (2010) demonstrates. For example, home-producers in coastal cities might find it more difficult to substitute away from capital or labor towards land. We do not incorporate city-specific production elasticities into our model. If σ_Y varies among cities, the misspecification error will appear in our productivity estimates.

[13] 0.06 is the estimated elasticity of wages on productivity seen in Ciccone and Hall (1996). Rosenthal and Strange (2004) argue that a one-percent increase in population leads to no more than a 0.03-0.08 percent increase in productivity. The relevant empirical elasticity is actually how a one-percent increase in population density affects productivity. Glaeser and Gottlieb 2008 estimate increases in commute times, pollution, and crime with population, possibly justifiying such a value.

3.2 Reduced-Form Elasticities

In Table 2, we demonstrate how the reduced-form population elasticities depend on the structural elasticiities of substitution by substituting in the values of all of the other parameters. Thus the five effects seen in (24) are calibrated as

$$\varepsilon_{N,Q} \approx 0.77 + 1.14\sigma_D + 1.95\sigma_X + 8.02\sigma_Y + 11.85\varepsilon_{L,r}$$

Calculating all of the elasticiities using the main calibration where $\sigma_D = \sigma_X = \sigma_Y = 0.667$ and $\varepsilon_{L,r} = 0$ yields the population differential in terms of the three amenity types:[14]

$$\hat{N}^j \approx 8.18\hat{Q}^j + 2.16\hat{A}_X^j + 2.88\hat{A}_Y^j. \tag{32}$$

This expression is potentially misleading since a one-point increase in $\hat{Q}^j$ has the value of a one-point increase in income, while one-point increases in $\hat{A}_X^j$ and $\hat{A}_X^j$ have values of s_x and s_y of income due to the size of their respective sectors. Normalizing the effects so that they are of equal value increases the coefficients on the productivity effects

$$\begin{aligned} \hat{N}^j \approx& \varepsilon_{N,Q}\hat{Q}^j + \frac{\varepsilon_{N,A_X}}{s_x} s_x\hat{A}_X^j + \frac{\varepsilon_{N,A_Y}}{s_y} s_y\hat{A}_Y^j \\ =& \, 8.18\hat{Q}^j + 3.37 s_x\hat{A}_X^j + 8.01 s_y\hat{A}_Y^j \end{aligned} \tag{33}$$

Thus we see that both quality of life and home-productivity have large impacts on local population, with an increase equal to one-percent of income increasing population by more than 8 log points. The effect of trade productivity is less than half of that. Much of these differences are related to taxes, which discourage workers from being in trade-productive areas and push them towards high quality-of-life and home-productive areas (Albouy 2009a). Making taxes neutral results in

[14] Note that when we allow $\varepsilon_{L,r} > 0$ we can no longer interpret $\hat{N}^j$ as the population density differential, but instead only as the population differential.

amenities having more simiilar effects:

$$\hat{N}^j \approx 6.32\hat{Q}^j + 5.81 s_x \hat{A}_X^j + 7.55 s_y \hat{A}_Y^j.$$

The effects are still not equal: when quality of life increases, income effects imply households will pack themselves more into housing, while substitution effects will cause producers to substitute away from land towards labor, making consumption less land intensive. When trade-productivity rises these substitution effects are weaker, and households still demand compensation in terms of land-intensive goods.

The numbers in Table 2 imply that the most important substitution elasticity affecting location decisions is σ_Y. Without it, additional home-good production comes only from increases in home-productivity or land released from the traded-good sector. Letting σ_Y remain a free parameter,

$$\hat{N}^j \approx (2.84 + 5.34\sigma_Y)\hat{Q}^j + (1.23 + 2.14\sigma_Y) s_x \hat{A}_X^j + (3.18 + 4.83\sigma_Y) s_y \hat{A}_Y^j. \quad (34)$$

Setting $\sigma_Y = 0$ yields much lower elasticities: population densities cannot increase much when the housing stock cannot be made denser. [15] On the other hand, setting $\sigma_D = 0$ eliminates substitution effects in consumption, but allows for income effects. As can be seen from Table 2, lowering $\sigma_D = 0$ means households respond less to quality of life and trade-productivity, but more to home-productivity. In this case, a city's productivity in building housing is more important than the consumption amenities it offers households.

The overall dependence of density on substitution possibilities may be gauged by restricting the elasticities to be equal $\sigma_D = \sigma_X = \sigma_Y = \sigma$, revealing relatively small constants:

$$\hat{N}^j \approx (0.77 + 11.11\sigma)\hat{Q}^j + (5.06\sigma) s_x \hat{A}_X^j + (0.77 + 10.17\sigma) s_y \hat{A}_Y^j. \quad (35)$$

In a Cobb-Douglas economy $\sigma = 1$, the implied elasticities are almost 50-percent higher than in

[15] These estimates might be more accurate in predicting population flows to negative shocks in the spirit of Glaeser and Gyourko (2005), who highlight the asymmetric impact of durable housing on population flows.

the base calibration $\sigma = 0.667$. Assuming a Cobb-Douglas economy seems innocuouos when predicting prices, when substitution elasticities have no first-order effect, but it is not innocuous when modeling quantities.

The multiplier effect for agglomeration feed back can also be calibrated. Calibrating our fairly large values for positive and negative economies together, we get that the two could possibly offset.

$$\frac{1}{1-\alpha\varepsilon_{N,A_X}+\gamma\varepsilon_{N,Q}} \approx \frac{1}{1-(0.06)(2.16)+(0.015)(8.18)} \approx 1.01$$

Separately, the multiplier for positive feedback is 1.12 while for negative feedback it is 0.89. These calibrated values suggests that the bias from ignoring agglomeration feedaback is probably not tremendous. Basic agglomeration economies or diseconomies do not seem to dominate other location forces due to natural advantages, historical path dependence, or other local idiosyncracies.

We have discussed results for only one urban quantity, population density. In Table 3 we list the reduced-form elasticities for all endogenous prices and quantities. Panel A presents results for the baseline tax treatment, while Panel B presents results for a geographically neutral federal taxes.

4 General Equilibrium Elasticities and Empirical Estimates

Our model sheds light on commonly estimated elasticities of local labor demand or housing supply, predicated on partial equilibrium models that consider labor and housing markets separately. Our general-equilibrium model considers housing and labor markets simultaneously. The adjustments underlying these elasticities might take place over the course of decades, if not generations. For example, our model may account for changes in the durable housing stock or shifts in labor across exportable sectors. The source of the change in supply or demand may matter a great deal.

4.1 Local Labor Supply and Demand

Conventionally, labor economists think of the workforce of a city being determined by supply and demand. In the general equilibrium context here, an increase in labor demand is brought about by an increase in trade-productivity A_X. If tradable goods are heterogenous and the number of cities is large, this could be due to an increase in the world price of the output produced in the city. Holding productivity (and agglomeration) constant, a greater work force psuhes down wages, as firms must complement it with ever scarcer and more expensive land.[16]

An increase in labor supply can be brought about by an increase in quality of life, Q, as workers willing to accept a lower wage. With homogenous workers the supply wage increases with the wage, as workers need to be compensated for rising home-good prices. [17]

According to the calibration, an increase in trade-productivity produces the ratio

$$\frac{\partial \hat{N}}{\partial \hat{w}} = \frac{\partial \hat{N}/\partial \hat{A}_X}{\partial \hat{w}/\partial \hat{A}_X} \approx \frac{2.159}{1.090} \approx 1.98$$

This may be interpreted as an elasticity of local labor supply. Researchers have frequently tried to estimate this elasticity using a method from Bartik (1991). It predicts changes in local labor demand based on national changes in industrial composition. This instrumental variable can identify this elasticity so long as it correlated with changes in trade-productivity, and uncorrelated with changes in quality of life or home-productivity. Estimates seen in Bartik and Nowtowidigo (2012) are generally in the range of 2 to 4. These fairly large values are remarkably close to that predicted by the calibration. In addition, we can use the model to interpret possible issues with the estimates. If increases in demand (i.e., increases in A_X) are positively correlated with increases in supply (i.e., increases in Q), then the elasticity of labor supply will be be biased upwards.[18]

[16]Some models simply assume a fixed factor in produciton, such as from land only for the tradable sector. Here land in the tradable sector must compete with land in the non-trdable sector, esp. residential, causing the price to rise as more households enter.

[17]If workers have heterogeneous tastes, then the supply curve would rise as higher wages will attract those with weaker tastes for living in a location.

[18]The estimates in Notowdigdo (2012) reveal an increase in housing costs along with higher wages that are consistent with a small increase in quality of life, enough to produce a small upward bias, but it is not highly significant.

The slope of the labor demand curve may be identified from an exogenous change in quality of life. According to our calibration the elasticity of labor demand is large:

$$\frac{\partial \hat{N}}{\partial \hat{w}} = \frac{\partial \hat{N}/\partial \hat{Q}}{\partial \hat{w}/\partial \hat{Q}} \approxeq \frac{8.185}{-0.359} \approx -22.79,$$

Empriical strategies that attempt to estimate this elasticity are surprisingly few. The area where this appears to be the most active is in the area of immigration. The empirical strategies here (e.g. Card, 2001) consider places with enclaves of immigrants to be more attractive to new immigrants from similar source countries. When workers are sufficiently substitutable, then cities whose existing populations resemble that of new immigrants become relatively more desirable to the typical worker, as in an increase in Q^j. In general this literature has found wages at the city level to be fairly unresponsive to increases in labor supply, consistent with the large elasticity above.

The model also highlights an atypical supply and demand increase that could be brought forth through higher housing productivity. In this case, firms may demand more labor to produce more home-goods, while the supply of workers increases because of lowers local costs-of-living. The net result is many more workers paid slightly lower wages (assuming away agglomeration).

$$\frac{\partial \hat{N}}{\partial \hat{w}} = \frac{\partial \hat{N}/\partial \hat{A}_Y}{\partial \hat{w}/\partial \hat{A}_Y} \approxeq \frac{2.885}{-0.117} \approx -24.66.$$

We are not aware of any estimates of this elasticity, altough works by Saks (2008) and others has highlighted the importance of housing supply in accomodating workers. This may be interpreted through variation in σ_Y seen in (34).

4.2 Local Housing Supply and Demand

As labor and housing markets both clear in the neo-classical model, the population is closely tied to the amount of housing, which we interpret as home goods in (16). The difference between population and houisng is due to substitution and income effects in consumption, calibrated as

$$\hat{Y}^j = \hat{N}^j - 0.43\hat{p}^j - \hat{Q}^j \tag{36}$$

$$= 6.20\hat{Q}^j + 2.40 s_x \hat{A}_X^j + 8.20 s_y \hat{A}_Y^j. \tag{37}$$

This makes housing somewhat less responsive to quality-of-life and trade-productivity differences and more responsive to home-productivity differences. Most empriical studies have generally ignored changes in per-capita housing consumption, and equated housing with population.

Housing elasticities are, of course, examined with respect to changes in housing prices. It is worth repeating that the source of the shift in housing supply is important, with trade-productucity increases having a smaller effect than quality-of-life shifts.[19]

$$\frac{\partial \hat{Y}}{\partial \hat{p}} = \frac{\partial \hat{Y}/\partial \hat{A}_X}{\partial \hat{p}/\partial \hat{A}_X} \approxeq \frac{1.539}{1.607} \approx 0.96$$
$$\frac{\partial \hat{Y}}{\partial \hat{p}} = \frac{\partial \hat{Y}/\partial \hat{Q}}{\partial \hat{p}/\partial \hat{Q}} \approxeq \frac{6.197}{2.543} \approx 2.44.$$

These calibrated values are within the range seen in Saiz (2010) from 0.80 to 5.45 for different cities. He uses both shifts in industrial composition, immigrant enclaves, as well as sunshine as sources of exogenous variation to identify these elasticities. Thus, he appears to estimating a hybrid of the two elasticities above: places deemed to have a greater housing supply elasticity may have instead experienced a greater quality-of-life chaneg than trade-productivity change. Of course part of the variation also stems from local variation in σ_Y and possibly $\varepsilon_{L,r}$.

Our log-linearization predicts that housing prices will not be affected by increasing housing supply through the production elasticity σ_Y, since there is no first-order dependence. An increase in housing-productivity, interpretable as an increase in housing supply, does lower prices, but by

[19]When land supply is fixed, the total home-good differential represents a housing density differential.

much less than it increases the amount of housing: $\hat{A}_Y$,

$$\frac{\partial \hat{Y}}{\partial \hat{p}} = \frac{\partial \hat{Y}/\partial \hat{A}_Y}{\partial \hat{p}/\partial \hat{A}_Y} \approx \frac{2.951}{-0.172} \approx -17.16.$$

Thus, with homogenous tastes and mobile factors, measures that increase housing productivity, such as reducing regulations, will be seen much more in quantities than prices.

The calculations above show that the frictionless neoclassical model generates own-price demand elasticities which are roughly an order of magnitude larger than supply elasticities.

5 Empriical Relationship between Density, Prices, and Amenities

5.1 Data

We define cities at the Metropolitan Statistical Area (MSA) level using 1999 OMB definitions of consolidated MSAs (e.g. San Francisco is combined with Oakland and San Jose), of which there are 276. We use the 5-percent sample of 2000 United States Census from Ruggles et. al (2004) to calculate wage and housing price differentials, controlling for relevant covariates.[20] Population density also comes from the 2000 Census: density is calculated at the census tract level then averaged according to population to form an MSA density value. All of our empirical results below use MSA population weights.

5.2 Population Density and Calibrated Substitution Elasticities

Here, we consider how well the model can predict population densities using price information. We also exa as well as examine the potential accuracy of our calibrated substitution elasticities. We do this, following the discussion above, by assuming $\hat{A}_X^j = 0$ and using $\hat{w}^j, \hat{p}^j$ to identify

[20] See Appendix C for more details on the calculation of wage and price differentials.

$\hat{Q}^j, \hat{A}_X^j$ from (1*) and (28). This predicts $\hat{N}^j$ for each city without home-productivity differences and compare this with actual population density differences.

The overall variance in log population density differences across MSAs, weighted by population is 0.770. The variance of the prediction error, equal to the difference between the actual density minus predicted by the model using wages and housing costs, is only 0.398. This means that a remarkable 48 percent of density variation is explained by this model, based on a calibration pre-set in Albouy (2009a). It is worth repeating that the values here were taken from the literature, and were not estimated from population density.

To assess whether we could explain more variation by choosing different elasticities of substitution, we consider how well different combinations of $\sigma_D, \sigma_X, \sigma_Y$ predict densities. [21] In figure 3, we graph the variance of the prediction error as a function of the elasticities of substitution. If, for simplicity's sake, we restrict $\sigma_D = \sigma_X = \sigma_Y = \sigma$, as in equation (35), prediction error is minimized at roughly $\sigma = 0.667$, our initial specification. Other values increase prediction error, including the Cobb-Douglas case $\sigma = 1$. The next curve fixes $\sigma_X = 0.667$, which reduces the prediction error but only by a small amount. Also fixing $\sigma_D = 0.667$, as in the last curve, reduces the specifcation by roughly the same amount. The greatest reduction comes from setting $\sigma_Y = 0.667$, furhter emphasizing its importance. If σ_Y is set to zero, the model reduces the variance by only half as much. The takeaway from this exercise is that our the preset calibration does quite well relative to other potential calibrations. [22]

[21] It is worth noting that one potentially could estimate the model's parameters with sixteen moment conditions (1*)-(16*). But this approach requires data on all endogenous prices and quantities for each city, which are not available.

[22] An unrestricted regression of log density on wages and housing costs natually produces a higher R-squared of 0.72 ¿ 0.48, with $\hat{N}^j = 4.40\hat{w}^j + 0.90\hat{p}^j + e^j = 0.63\hat{Q}^j + 6.26\hat{A}_X^j + e^j$. Relative to the calibration, this produces an estimate of $\varepsilon_(N, Q)$ that is far too low, and $\varepsilon_(N, A_X)$ that is far too high. The two estimated reduced-form elasticities are insufficient for identifying the three elasticity parameters. Furthermore, since the estimated coefficient on $\hat{Q}^j$ of 0.63 is less than the constant 0.77 in equation (**??**), then at least one of the substitution elasticiites would have to be negative (e.g. constraining the two production elasticities to equal, $\sigma_D = 14.66$ and $\sigma_X = \sigma_Y = -1.70$), which is untenable. The calibrated model suggests that ϵ^j, which includes $\hat{A}_Y^j$, is positively correlated with A_X^j or negatively correlated with $\hat{Q}^j$. If instead, we constrain the estimates to fit the restriction $\sigma_D = \sigma_X = \sigma_Y = \sigma$, as in equation (35), then we obtain $\hat{N}^j = 8.59\hat{Q}^j + 2.28\hat{A}_X^j + e^j$ implying a $\sigma = 0.663$, very close to the calibration.

5.3 Trade and Home-Productivity Estimates

We now use density information to identify trade- and home- productivity separately, using the method proposed in 2.6. Figure 3 displays the estimated measures of relative cost and excess density for different MSAs from the left-hand sides of equations (28) and (29). The figure includes iso-productivity lines for both tradable and home sectors. To understand how trade-productivity is inferred, consider the downward-sloping iso trade-productivity line, where cities all have the average trade-productivity. Above and to the right of this line, cities have higher excess density or relative costs, indicating above average trade-productivity. Above and to the left of the iso home-productivity line, cities have high excess density, or low relative costs, indicating high home-productivity. For example, San Francisco, has high trade-productivity and low home-productivity; San Antonio has the opposite; New York and Chicago are productive in both sectors, while Santa Fe and Myrtle Beach are unproductive in both.

Figure 4 uses the same data as Figure 3, but instead graphs trade- and home-productivity directly. Overall, New York is the most productive city. San Francisco, which is the second most valuable city, is not a leader in productivity due to its relatively low home-productivity. Figure 4 also includes isoclines for excess density and relative costs. Holding quality of life constant, trade-productivity and home-productivity must move in opposite directions to keep population density constant. Holding quality of life constant, home-productivity must rise faster than trade-productivity to keep relative costs constant.

Two important points should be made about about the home-productivity estimates. First, they strongly reflect the residual measure of population density.[23] Second, the measure is highly indicative of the accumulated housing stock of a city. Older cities, like New York, Chicago, and Philadelphia, all have high home-productivity. We can explain part of this by noting that these cities have been built up over the past century, when building and land use regulations were less restrictive.[24]

[23] Recall that we can estimate $\hat{Q}^j$ perfectly and $\hat{A}_X^j$ quite well with only wage and housing price data.

[24] Some of these findings appear to conflict with recent work by Albouy and Ehrlich (2012), who use data on land values to infer productivity in the housing sector, which comprises most of the non-tradable sector. While the two

The estimation procedure outlined above also refines estimates of trade-productivity over those provided in Albouy (2009). Cities with high relative costs and high levels of excess density are inferred to have high levels of trade-productivity. However, equation (28) shows that home-productivity reduces trade-productivity estimates via cost reductions. Appendix Table 2 compares the trade-productivity estimates between the constant home-productivity case and the procedure used here. In addition, this table lists the population density, quality of life, and home-productivity for all metropolitan and non-metropolitan areas.

5.4 Variance Decomposition of Population Density

In this last section we attempt to answer the ambitious question of why the population lives where it does using straightforward variance decomposition, which we present in Tables 6 and 7. The first relies on the simpler estimates of Q adn A_X based only on price data, while 7 uses density information to identify A_Y, providing a fuller, if more tautological, decomposition. In the first, quality of life explains more than half of the total variance in predicted population density, even though the variance of trade-productivity is an order of magnitude larger than the variance of quality of life. Relatively small differences in quality of life explain a large amount of the population distribution. In other words, the constant home-productivity frictionless neoclassical model predicts that "jobs follow people" much more than "people follow jobs." The other key takeaway from Table 6 is that wage and housing prices explain nearly half of the observed variance in population density. Specifically, the variance of the predicted population density divided by the variance of observed population density equals approximately 0.48.

In Table 7A, we decompose the variance of observed (which now equals predicted) population

approaches largely agree on which large areas have high housing productivity, the land values approach suggests that larger, denser cities generally have lower, rather than higher housing productivity. This apparent contradiction actually highlights what the two methodologies infer differently. Productivity measures based on land values provide a better insight into the marginal cost of increasing the housing supply, by essentially inferring the replacement cost. Productivity measures based on density are more strongly related to the average cost of the housing supply, thereby reflecting the whole history of building in a city. The distinction matters particularly for cities with older housing built on the easiest terrain in the decades prior to the diffusion of residential land-use regulations when factor prices were relatively low.

density. In comparing quality of life and trade productivity, we note a similar outcome as in Table 6. In fact, the ratio of variance explaned by quality of life to variance explained by trade-productivity is larger in Table 7A than in Table 6. The relatively large fraction of variance explained by home-productivity suggests that there remains some portion of household location decisions which our simple model does not explain. Nevertheless, quality of life and trade productivity explain nearly half of the total variation in population density.

Table 7B explores What would happen to population density if federal taxes were made geographically neutral. We can use our estimates of quality of life, trade- and home-productivity, along with the calibrated model, to predict prices and quantities (including population density) for each city in the absence of distortionary federal income taxes.[25] Table 7B presents the variance decomposition of the geographically netural tax counterfactual. Trade-productivity now explains a larger fraction of population density than does quality of life. As described above, federal taxes introduce a wedge between trade-productivity and the benefits that households receive by locating in productive cities. Eliminating the geographic distortion in the tax code would allow households to benefit more from highly productive cities.

6 Conclusion

Under plausible specifications of substitution elasticities, matching a neoclassical general equilibrium model with reasonable parameter estimates generates exceptionally large elasticities of population density with respect to amenities. The model also generates extremely large elasticities of local labor demand, while the elasticity of labor supply closely matches existing empirical estimates. Our model reflects the interrelationship between urban quantities and prices and connects both to amenities in consumption and production. Urban quantities depend particularly on substitution elasticities and the complementarity of amenities.

[25] Because we estimate amenities using observed density, wage, and housing price data, we cannot estimate amenities in the absence of distortionary federal taxes.

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Table 1: Calibrated Parameters

Parameter Name	Notation	Calibrated Value
Cost and Expenditure Shares		
Home-good expenditure share	s_y	0.36
Income share to land	s_R	0.10
Income share to labor	s_w	0.75
Traded-good cost share of land	θ_L	0.025
Traded-good cost share of labor	θ_N	0.825
Home-good cost share of land	ϕ_L	0.233
Home-good cost share of labor	ϕ_N	0.617
Share of land used in traded good	λ_L	0.17
Share of labor used in traded good	λ_N	0.70
Tax Parameters		
Average marginal tax rate	τ	0.361
Average deduction level	δ	0.291
Structural Elasticities		
Elasticity of substitution in consumption	σ_D	0.667
Elasticity of traded-good production	σ_X	0.667
Elasticity of home-good production	σ_Y	0.667
Elasticity of land supply	$\varepsilon_{L,r}$	0.0

Table 2: Sensitivity Analysis

	$\varepsilon_{N,Q}$	ε_{N,A_X}	ε_{N,A_Y}
σ_D	1.143	0.718	-0.076
σ_X	1.953	0.466	0.635
σ_Y	8.015	2.051	2.607
$\varepsilon_{L,r}$	11.853	4.009	3.856
Constant	0.772	0.000	0.772

Table 2 describes the effect on reduced-form elasticities of increasing each structural elasticity by one. For example, increasing σ_D by 1 increases $\varepsilon_{N,Q}$ by 1.14.

TABLE 3: Base Elasticities

A: With Taxes			
	$\hat{Q}$	$\hat{A}_X$	$\hat{A}_Y$
$\hat{r}$	11.853	4.009	3.856
$\hat{w}$	-0.359	1.090	-0.116
$\hat{p}$	2.543	1.607	-0.172
$\hat{x}$	-0.444	0.348	-0.037
$\hat{y}$	-1.987	-0.620	0.066
$\hat{N}$	8.185	2.159	2.884
$\hat{L}$	0.000	0.000	0.000
$\hat{K}$	7.940	2.861	2.779
$\hat{X}$	7.966	3.335	2.934
$\hat{Y}$	6.197	1.539	2.951
$\hat{N}_X$	8.206	2.275	3.012
$\hat{N}_Y$	8.134	1.884	2.581
$\hat{L}_X$	0.060	0.328	0.362
$\hat{L}_Y$	-0.011	-0.062	-0.069
$\hat{K}_X$	7.966	3.002	2.934
$\hat{K}_Y$	7.894	2.611	2.503
B: Neutral Taxes			
	$\hat{Q}$	$\hat{A}_X$	$\hat{A}_Y$
$\hat{r}$	10.001	6.400	3.600
$\hat{w}$	-0.303	1.018	-0.109
$\hat{p}$	2.146	2.121	-0.227
$\hat{x}$	-0.555	0.794	-0.085
$\hat{y}$	-0.916	-0.905	0.097
$\hat{N}$	6.318	3.721	2.717
$\hat{L}$	0.000	0.000	0.000
$\hat{K}$	6.181	4.384	2.616
$\hat{X}$	5.814	4.804	2.777
$\hat{Y}$	5.402	2.815	2.814
$\hat{N}_X$	6.017	3.792	2.849
$\hat{N}_Y$	7.036	3.551	2.402
$\hat{L}_X$	-0.856	0.202	0.375
$\hat{L}_Y$	0.163	-0.038	-0.0715
$\hat{K}_X$	5.814	4.471	2.777
$\hat{K}_Y$	6.834	4.230	2.330

Each value in Table 3 represents the partial effect that a one-percent increase in each amenity has on each price or quantity, i.e. $\partial\hat{r}/\partial\hat{Q} = 11.845$. The values in panel A are derived using the parameters in Table 1. The values in panel B are derived using $\tau = 0$.

Table 4: Agglomeration Elasticities

	A: With Taxes		
	$\hat{Q}$	$\hat{A}_{X0}$	$\hat{A}_Y$
$\hat{r}$	13.692	4.494	4.505
$\hat{w}$	0.141	1.222	0.059
$\hat{p}$	3.281	1.802	0.087
$\hat{x}$	-0.284	0.391	0.019
$\hat{y}$	-2.271	-0.695	-0.033
$\hat{N}$	9.175	2.420	3.234
$\hat{L}$	0.000	0.000	0.000
$\hat{K}$	9.253	3.208	3.242
$\hat{X}$	9.497	3.739	3.473
$\hat{Y}$	6.903	1.725	3.200
$\hat{N}_X$	9.250	2.550	3.380
$\hat{N}_Y$	8.998	2.112	2.886
$\hat{K}_X$	9.344	3.365	3.420
$\hat{K}_Y$	9.092	2.927	2.925
$\hat{L}_X$	0.211	0.368	0.415
$\hat{L}_Y$	-0.040	-0.070	-0.079
	B: Neutral Taxes		
	$\hat{Q}$	$\hat{A}_{X0}$	$\hat{A}_Y$
$\hat{r}$	12.485	7.863	4.668
$\hat{w}$	0.092	1.250	0.060
$\hat{p}$	2.969	2.605	0.126
$\hat{x}$	-0.247	0.976	0.047
$\hat{y}$	-1.267	-1.112	-0.054
$\hat{N}$	7.763	4.571	3.338
$\hat{L}$	0.000	0.000	0.000
$\hat{K}$	7.883	5.387	3.348
$\hat{X}$	7.679	5.902	3.579
$\hat{Y}$	6.495	3.459	3.284
$\hat{N}_X$	7.489	4.659	3.483
$\hat{N}_Y$	8.414	4.363	2.995
$\hat{K}_X$	7.550	5.493	3.523
$\hat{K}_Y$	8.476	5.197	3.036
$\hat{L}_X$	-0.777	0.248	0.409
$\hat{L}_Y$	0.148	-0.047	-0.077

Endogenous productivity: $A^j_X = A^j_{X0}(N^j)^\alpha$, $\alpha = 0.05$

Table 5: Congestion Cost Elasticities

	$\hat{Q}_0$	$\hat{A}_X$	$\hat{A}_Y$
A: With Taxes			
$\hat{r}$	10.185	3.569	3.269
$\hat{w}$	-0.308	1.103	-0.099
$\hat{p}$	2.185	1.513	-0.298
$\hat{x}$	-0.382	0.365	-0.015
$\hat{y}$	-1.410	-0.467	0.269
$\hat{N}$	7.033	1.855	2.479
$\hat{L}$	0.000	0.000	0.000
$\hat{K}$	6.823	2.567	2.385
$\hat{X}$	6.846	3.039	2.539
$\hat{Y}$	5.325	1.309	2.644
$\hat{N}_X$	7.052	1.970	2.605
$\hat{N}_Y$	6.989	1.582	2.178
$\hat{K}_X$	6.846	2.706	2.539
$\hat{K}_Y$	6.783	2.318	2.112
$\hat{L}_X$	0.052	0.326	0.359
$\hat{L}_Y$	-0.009	-0.062	-0.068
B: Neutral Taxes			
	$\hat{Q}_0$	$\hat{A}_X$	$\hat{A}_Y$
$\hat{r}$	8.879	5.739	3.117
$\hat{w}$	-0.269	1.038	-0.094
$\hat{p}$	1.905	1.979	-0.330
$\hat{x}$	-0.493	0.831	-0.058
$\hat{y}$	-0.813	-0.844	0.141
$\hat{N}$	5.609	3.303	2.412
$\hat{L}$	0.000	0.000	0.000
$\hat{K}$	5.488	3.976	2.317
$\hat{X}$	5.162	4.420	2.496
$\hat{Y}$	4.796	2.458	2.553
$\hat{N}_X$	5.341	3.394	2.559
$\hat{N}_Y$	6.246	3.086	2.063
$\hat{K}_X$	5.162	4.087	2.496
$\hat{K}_Y$	6.067	3.778	2.000
$\hat{L}_X$	-0.760	0.259	0.416
$\hat{L}_Y$	0.144	-0.049	-0.079

Congestion costs: $Q^j = Q_0^j(N^j)^{-\gamma}$, $\gamma = 0.02$

Table 6: Variance Decomposition, Two Amenity

	Variance	Fraction of variance explained by		
		Quality of Life	Trade Productivity	Covariance
	(1)	(2)	(3)	(4)
Predicted	0.348	0.502	0.180	0.316

Table 6 presents the variance decomposition of predicted population density using data on wages and house prices only.

Table 7: Variance Decomposition, Three Amenity

A: Observed Population Density and Prices. Var = 0.770

Fraction of variance explained by

	$Cov(\varepsilon_{N,Q}\hat{Q},\cdot)$	$Cov(\varepsilon_{N,A_X}\hat{A}_X,\cdot)$	$Cov(\varepsilon_{N,A_Y}\hat{A}_Y,\cdot)$
$Cov(\cdot,\varepsilon_{N,Q}\hat{Q})$	0.227	·	·
$Cov(\cdot,\varepsilon_{N,A_X}\hat{A}_X)$	0.132	0.097	·
$Cov(\cdot,\varepsilon_{N,A_Y}\hat{A}_Y)$	-0.136	0.235	0.443

B: Counterfactual Density and Prices. Var=1.357

Fraction of variance explained by

	$Cov(\varepsilon_{N,Q}\hat{Q},\cdot)$	$Cov(\varepsilon_{N,A_X}\hat{A}_X,\cdot)$	$Cov(\varepsilon_{N,A_Y}\hat{A}_Y,\cdot)$
$Cov(\cdot,\varepsilon_{N,Q}\hat{Q})$	0.106	·	·
$Cov(\cdot,\varepsilon_{N,A_X}\hat{A}_X)$	0.137	0.227	·
$Cov(\cdot,\varepsilon_{N,A_Y}\hat{A}_Y)$	-0.077	0.299	0.307

Panel A presents the variance decomposition using data on population density, wages, and house prices. Panel B presents the variance decomposition under geographically neutral income taxes. Both panels use the same amenity estimates.

Figure 1: Distribution, 2000

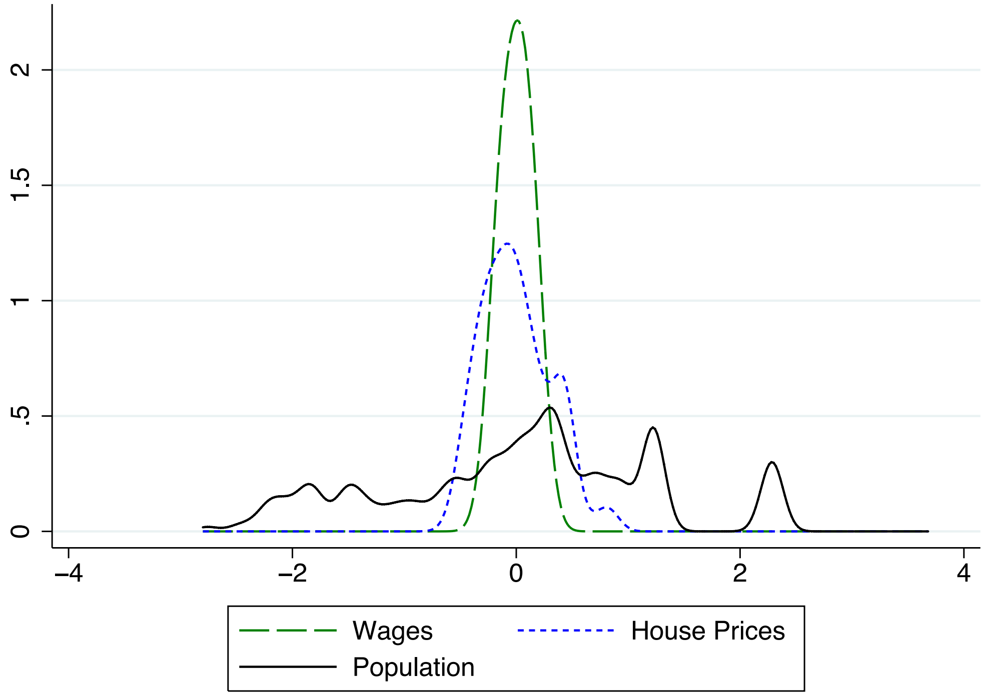

Figure 1 is smoothed with a Gaussian kernel, bandwidth=0.05.

Figure 2: Error in Fitting Pop. Density using $\hat{Q}$ and $\hat{A}_X$ Only

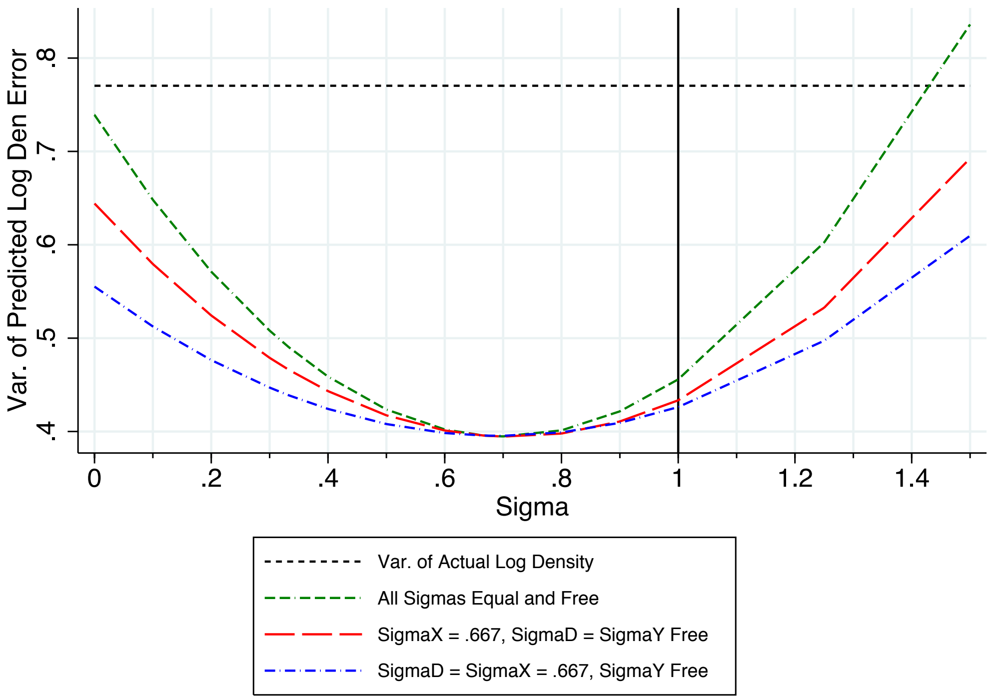

Figure 3: Excess Density and Relative Cost Estimates, 2000

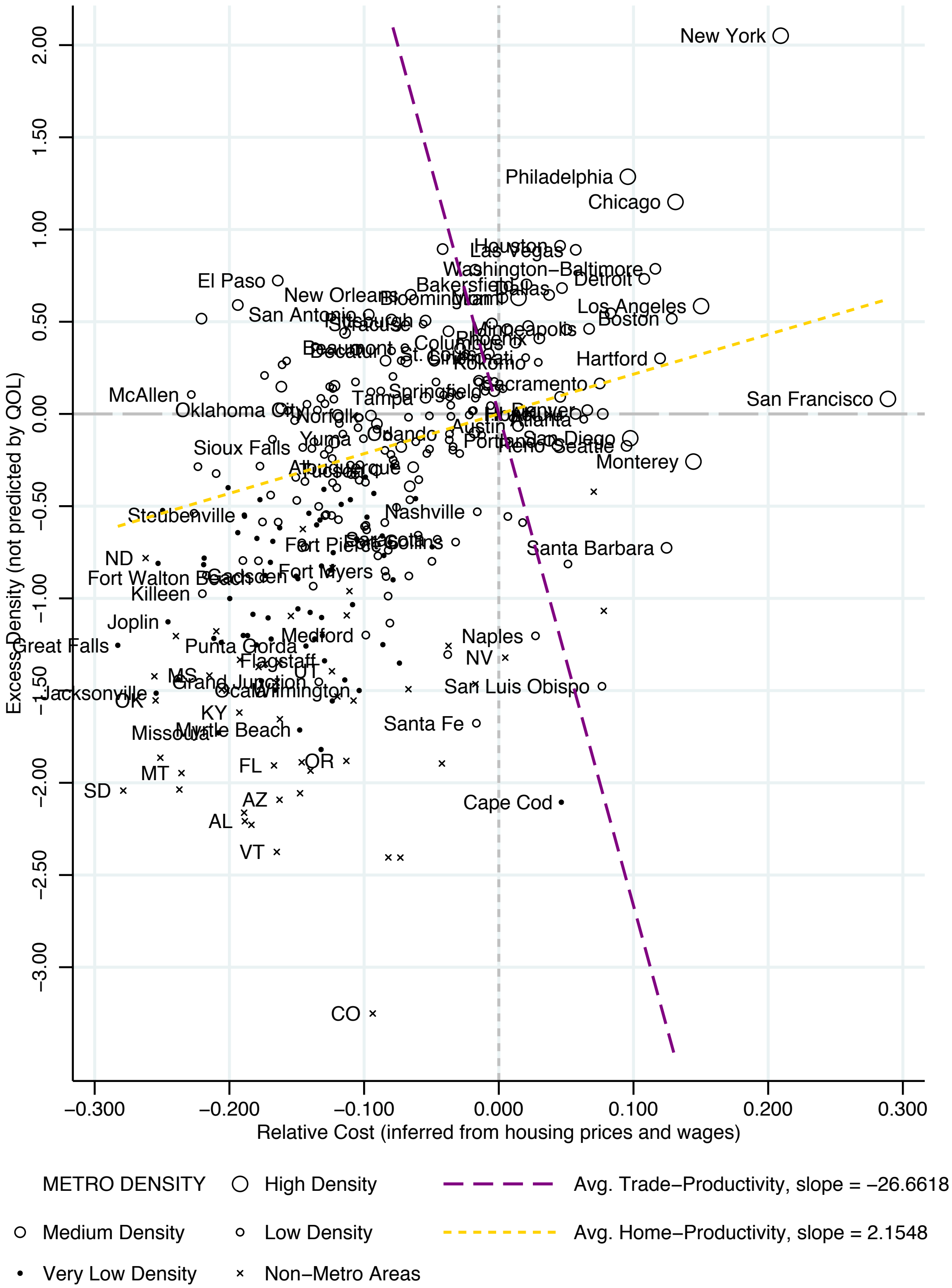

Figure 4: Trade- and Home-Productivity Estimates, 2000

Appendix - Not for Publication

A Nonlinear Simulation

As in Rappaport (2008a, 2008b), we employ a two-step simulation method to solve the model without log-linearization. Let the utility function be $U(x, y; Q) = Qx^{1-s_y}y^{s_y}$, which implies that $\sigma_D = 1$. Define the production function in the traded-good sector to be $X = A_X L_X^{\theta_L} N_X^{\theta_N} K_X^{1-\theta_L-\theta_N}$, which implies that $\sigma_X = 1$. The production function for the home-good sector is defined similarly. This is a Cobb-Douglas economy.

We first consider a "large" city with amenity values normalized so that $Q = A_X = A_Y = 1$. We fix the amount of land and population. We additionally normalize the value of $\bar{\iota}$ and solve a system of fifteen equations, corresponding to equations (1)-(14) and (16), for fifteen unknown variables, $\bar{u}, w, r, p, x, y, X, Y, N_X, N_Y, L_X, L_Y, K_X, K_Y, K$. After solving for these variables, we verify that the values of $s_R, s_w, \lambda_L, \lambda_N$ match those in Table 1 given our values for $s_y, \theta_L, \theta_N, \phi_L, \phi_K$. We also obtain values for $\bar{u}$, R, and I.

We then consider a "small" city, which we endow with land equal to $1/1,000,000$ of the large city's land.[26] Unlike the large city, the population for the small city is endogenous. We solve the same system as for the large city, but we now solve for $w, r, p, x, y, X, Y, N_X, N_Y, L_X, L_Y, K_X, K_Y, N, K$. The average amenity values are $Q = A_X = A_Y = 1$. Varying the values of Q, A_X, A_Y for the small city and then solving the system yields elasticities of quantities and prices with respect to amenities.

For the log-linearized model, we estimate $\hat{N}^j = \varepsilon_{NQ}\hat{Q}^j + \varepsilon_{NA_X}\hat{A}_X^j + \varepsilon_{NA_Y}\hat{A}_Y^j$. Importantly, the reduced form elasticities are estimated using parameters taken at the national average. In general, each elasticity is itself a nonlinear function which we compute using our simulation. The primary goal of this exercise is to ensure that the reduced form elasticities which we obtain via the log-linearized model are reasonable.

When $\sigma_D = \sigma_X = \sigma_Y = 1$ in the log-linearized model, we obtain $\hat{N} = 11.88\hat{Q} + 3.24\hat{A}_X + 3.94\hat{A}_Y$. For the nonlinear simulation, we obtain $\hat{N} = 10.82\hat{Q} + 2.68\hat{A}_X + 4.02\hat{A}_Y$. Reassuringly, the results are quite similar. Appendix Figure 1 graphs the elasticity of population density with respect to amenities, where the bottom-right panel collects the other three graphs.

Appendix Figure 2 graphs the elasticity with respect to amenities, where we now change multiple amenities, in equal amounts, at the same time. The top right panel is particularly interesting. Increasing quality of life and home-productivity both decrease wages; higher quality of life increases the cost of housing, while home-productivity reduces the cost. One explanation for the graph is that, for $Q = A_Y > 1.7$, quality of life dominates home-productivity, driving home prices up and reducing the increase in population density. The bottom-right panel shows that population density responds very strongly, and on a similar order of magnitude as our log-linearized model predicts, to a change in all three amenities.

[26] We do this to avoid any feedback effect from the small city to the large one.

B System of Equations

After log-linearizing, we can represent our system of equations in matrix form as:

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ \lambda_N & 0 & 0 & 0 & 1-\lambda_N & 0 & 0 & 0 & -1 \\ 0 & \lambda_L & 0 & 0 & 0 & 1-\lambda_L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \hat{N}_X \\ \hat{L}_X \\ \hat{K}_X \\ \hat{X} \\ \hat{N}_Y \\ \hat{L}_Y \\ \hat{K}_Y \\ \hat{Y} \\ \hat{N} \end{bmatrix} = \begin{bmatrix} (\sigma_X - 1)\hat{A}_X - \sigma_X \hat{w} \\ (\sigma_X - 1)\hat{A}_X - \sigma_X \hat{r} \\ (\sigma_X - 1)\hat{A}_X \\ (\sigma_Y - 1)\hat{A}_Y + \sigma_Y(\hat{p} - \hat{w}) \\ (\sigma_Y - 1)\hat{A}_Y + \sigma_Y(\hat{p} - \hat{r}) \\ (\sigma_Y - 1)\hat{A}_Y + \sigma_Y \hat{p} \\ 0 \\ \varepsilon_{L,r}\hat{r} \\ \hat{y} \end{bmatrix}$$

Solving the matrix above gives us $\hat{N}$ as a function of $\hat{Q}, \hat{A}_X, \hat{A}_Y, \hat{p}, \hat{r}, \hat{w}$, and parameters. Using equations (18), (19), and (20), we obtain a solution for $\hat{N}$ in terms of the three amenities, $\hat{N} = \varepsilon_{N,Q}\hat{Q} + \varepsilon_{N,A_X}\hat{A}_X + \varepsilon_{N,A_Y}\hat{A}_Y$. For simplicity, we present below solutions which do not account for deductions or tax differences across states. As described in Section D, our empirical results do account for these issues.

$$\begin{aligned} \varepsilon_{N,Q} = & \left[\frac{\lambda_N - \lambda_L}{\lambda_N}\right] + \sigma_D\left[\frac{s_x(\lambda_N - \lambda_L)^2}{s_y\lambda_N(\lambda_N - \lambda_L\tau)}\right] + \sigma_X\left[\frac{\lambda_L^2}{s_w(\lambda_N - \lambda_L\tau)} + \frac{\lambda_L\lambda_N}{s_R(\lambda_N - \lambda_L\tau)}\right] \\ & + \sigma_Y\left[\frac{\lambda_L^2(1-\lambda_N)}{s_w\lambda_N(\lambda_N - \lambda_L\tau)} + \frac{\lambda_N(1-\lambda_L)}{s_R(\lambda_N - \lambda_L\tau)} - \frac{(\lambda_N - \lambda_L)^2}{s_y\lambda_N(\lambda_N - \lambda_L\tau)}\right] \\ & + \varepsilon_{L,r}\left[\frac{\lambda_N}{s_R(\lambda_N - \lambda_L\tau)}\right] \end{aligned}$$

$$\begin{aligned} \varepsilon_{N,A_X} = & \sigma_D\left[\frac{s_x^2(\lambda_N - \lambda_L)(1-\lambda_L)(1-\tau)}{s_y\lambda_N(\lambda_N - \lambda_L\tau)}\right] + \sigma_X\left[\frac{s_x\lambda_L(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L\tau)} - \frac{s_x\lambda_L(1-\lambda_L)}{s_w(\lambda_N - \lambda_L\tau)}\right] + \\ & \sigma_Y\left[\frac{s_x(1-\lambda_L)(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L\tau)} - \frac{s_x\lambda_L(1-\lambda_L)(1-\lambda_N)}{s_w\lambda_N(\lambda_N - \lambda_L\tau)} - \frac{s_x(1-\lambda_L)(\lambda_N - \lambda_L)(1-\tau)}{s_y\lambda_N(\lambda_N - \lambda_L\tau)}\right] \\ & + \varepsilon_{L,r}\left[\frac{s_x(\lambda_N - \tau)}{s_R(\lambda_N - \lambda_L\tau)}\right] \end{aligned}$$

$$\begin{aligned} \varepsilon_{N,A_Y} = & \left[\frac{\lambda_N - \lambda_L}{\lambda_N}\right] + \sigma_D\left[\frac{-s_x\lambda_L(\lambda_N - \lambda_L)(1-\tau)}{\lambda_N(\lambda_N - \lambda_L\tau)}\right] + \sigma_X\left[\frac{s_y\lambda_N\lambda_L}{s_R(\lambda_N - \lambda_L\tau)} + \frac{s_y\lambda_L^2}{s_w(\lambda_N - \lambda_L\tau)}\right] \\ & + \sigma_Y\left[-\left(\frac{\lambda_N - \lambda_L}{\lambda_N}\right) + \frac{s_y\lambda_L^2(1-\lambda_N)}{s_w\lambda_N(\lambda_N - \lambda_L\tau)} + \frac{s_y\lambda_N(1-\lambda_L)}{s_R(\lambda_N - \lambda_L\tau)} + \frac{\lambda_L(\lambda_n - \lambda_L)(1-\tau)}{\lambda_N(\lambda_N - \lambda_L\tau)}\right] \\ & + \varepsilon_{L,r}\left[\frac{s_y\lambda_N}{s_R(\lambda_N - \lambda_L\tau)}\right] \end{aligned}$$

We can set home-productivity constant across ciites, $\hat{A}_Y^j = 0$, and use population density to estimate the elasticity of non-tradable good production σ_Y^j for each city. In particular, we have

$$\hat{N}^j = \varepsilon_{N,Q}\hat{Q}^j + \varepsilon_{N,A_X}\hat{A}_X^j,$$

where $\varepsilon_{N,Q}$ and ε_{N,A_X} are as defined above. When home-productivity is constant, we can identify trade-productivity using information on wages on housing prices,

$$\hat{A}_X^j = \frac{\theta_L}{\phi_L}\hat{p}^j + \left(\theta_N - \phi_N\frac{\theta_L}{\phi_L}\right)\hat{w}^j$$

and so we have a single equation in one unknown variable, σ_Y^j.

When we set $\hat{y}^j = 0$, we can solve for $\hat{N}^j = \tilde{\varepsilon}_{N,Q}\hat{Q}^j + \tilde{\varepsilon}_{N,A_X}\hat{A}_X^j + \tilde{\varepsilon}_{N,A_Y}\hat{A}_Y^j$, where the coefficients are defined as:

$$\begin{aligned}
\tilde{\varepsilon}_{N,Q} &= \sigma_X\left[\frac{\lambda_L^2}{s_w(\lambda_N-\lambda_L\tau)} + \frac{\lambda_L\lambda_N}{s_R(\lambda_N-\lambda_L\tau)}\right] + \varepsilon_{L,r}\left[\frac{\lambda_N}{s_R(\lambda_N-\lambda_L\tau)}\right] \\
&\quad + \sigma_Y\left[\frac{\lambda_L^2(1-\lambda_N)}{s_w\lambda_N(\lambda_N-\lambda_L\tau)} + \frac{\lambda_N(1-\lambda_L)}{s_R(\lambda_N-\lambda_L\tau)} - \frac{(\lambda_N-\lambda_L)^2}{s_y\lambda_N(\lambda_N-\lambda_L\tau)}\right] \\
\tilde{\varepsilon}_{N,A_X} &= \sigma_X\left[\frac{s_x\lambda_L(\lambda_N-\tau)}{s_R(\lambda_N-\lambda_L\tau)} - \frac{s_x\lambda_L(1-\lambda_L)}{s_w(\lambda_N-\lambda_L\tau)}\right] + \varepsilon_{L,r}\left[\frac{s_x(\lambda_N-\tau)}{s_R(\lambda_N-\lambda_L\tau)}\right] \\
&\quad + \sigma_Y\left[\frac{s_x(1-\lambda_L)(\lambda_N-\tau)}{s_R(\lambda_N-\lambda_L\tau)} - \frac{s_x\lambda_L(1-\lambda_L)(1-\lambda_N)}{s_w\lambda_N(\lambda_N-\lambda_L\tau)} - \frac{s_x(1-\lambda_L)(\lambda_N-\lambda_L)(1-\tau)}{s_y\lambda_N(\lambda_N-\lambda_L\tau)}\right] \\
\tilde{\varepsilon}_{N,A_Y} &= \sigma_X\left[\frac{s_y\lambda_N\lambda_L}{s_R(\lambda_N-\lambda_L\tau)} + \frac{s_y\lambda_L^2}{s_w(\lambda_N-\lambda_L\tau)}\right] + \varepsilon_{L,r}\left[\frac{s_y\lambda_N}{s_R(\lambda_N-\lambda_L\tau)}\right] \\
&\quad + \sigma_Y\left[\frac{s_y\lambda_L^2(1-\lambda_N)}{s_w\lambda_N(\lambda_N-\lambda_L\tau)} + \frac{s_y\lambda_N(1-\lambda_L)}{s_R(\lambda_N-\lambda_L\tau)} + \frac{\lambda_L(\lambda_n-\lambda_L)(1-\tau)}{\lambda_N(\lambda_N-\lambda_L\tau)}\right]
\end{aligned}$$

These reduced-form elasticities no longer depend on the elasticity of substitution in consumption, σ_D. Quality of life and home-productivity no longer lead to population density independently of the substitution elasticities, i.e. the term $(\lambda_N - \lambda_L)/\lambda_N$ drops out of the elasticities.

C Data and Estimation

We use United States Census data from the 2000 Integrated Public-Use Microdata Series (IPUMS), from Ruggles et al. (2004), to calculate wage and housing price differentials. The wage differentials are calculated for workers ages 25 to 55, who report working at least 30 hours a week, 26 weeks a year. The MSA assigned to a worker is determined by their place of residence, rather than their place of work. The wage differential of an MSA is found by regressing log hourly wages on individual covariates and indicators for which MSA a worker lives in, using the coefficients on these MSA indicators. The covariates consist of

- 12 indicators of educational attainment;
- a quartic in potential experience, and potential experience interacted with years of education;

- 9 indicators of industry at the one-digit level (1950 classification);
- 9 indicators of employment at the one-digit level (1950 classification);
- 4 indicators of marital status (married, divorced, widowed, separated);
- an indicator for veteran status, and veteran status interacted with age;
- 5 indicators of minority status (Black, Hispanic, Asian, Native American, and other);
- an indicator of immigrant status, years since immigration, and immigrant status interacted with black, Hispanic, Asian, and other;
- 2 indicators for English proficiency (none or poor).

All covariates are interacted with gender.

This regression is first run using census-person weights. From the regressions a predicted wage is calculated using individual characteristics alone, controlling for MSA, to form a new weight equal to the predicted wage times the census-person weight. These new income-adjusted weights are needed since workers need to be weighted by their income share. The new weights are then used in a second regression, which is used to calculate the city-wage differentials from the MSA indicator variables. In practice, this weighting procedure has only a small effect on the estimated wage differentials.

Housing price differentials are calculated using the logarithm reported gross rents and housing values. Only housing units moved into within the last 10 years are included in the sample to ensure that the price data are fairly accurate. The differential housing price of an MSA is calculated in a manner similar to wages, except using a regression of the actual or imputed rent on a set of covariates at the unit level. The covariates for the adjusted differential are

- 9 indicators of building size;
- 9 indicators for the number of rooms, 5 indicators for the number of bedrooms, number of rooms interacted with number of bedrooms, and the number of household members per room;
- 2 indicators for lot size;
- 7 indicators for when the building was built;
- 2 indicators for complete plumbing and kitchen facilities;
- an indicator for commercial use;
- an indicator for condominium status (owned units only).

A regression of housing values on housing characteristics and MSA indicator variables is first run using only owner-occupied units, weighting by census-housing weights. A new value-adjusted weight is calculated by multiplying the census-housing weights by the predicted value from this first regression using housing characteristics alone, controlling for MSA. A second regression is run using these new weights for all units, rented and owner-occupied, on the housing characteristics fully interacted with tenure, along with the MSA indicators, which are not interacted. The house-price differentials are taken from the MSA indicator variables in this second regression. As with the wage differentials, this adjusted weighting method has only a small impact on the measured price differentials.

D Additional Tax Issues

D.1 Deduction

Tax deductions are applied to the consumption of home goods at the rate $\delta \in [0, 1]$, so that the tax payment is given by $\tau(m - \delta py)$. With the deduction, the mobility condition becomes

$$\begin{aligned} \hat{Q}^j &= (1 - \delta\tau')s_y\hat{p}^j - (1 - \tau')s_w\hat{w}^j \\ &= s_y\hat{p}^j - s_w\hat{w}^j + \frac{d\tau^j}{m} \end{aligned}$$

where the tax differential is given by $d\tau^j/m = \tau'(s_w\hat{w}^j - \delta s_y p^j)$. This differential can be solved by noting

$$\begin{aligned} s_w\hat{w}^j &= s_w\hat{w}_0^j + \frac{\lambda_L}{\lambda_N}\frac{d\tau^j}{m} \\ s_y\hat{p}^j &= s_y\hat{p}_0^j - \left(1 - \frac{\lambda_L}{\lambda_N}\right)\frac{d\tau^j}{m} \end{aligned}$$

and substituting them into the tax differential formula, and solving recursively,

$$\begin{aligned} \frac{d\tau^j}{m} &= \tau' s_w\hat{w}_0^j - \delta\tau' s_y\hat{p}_0^j + \tau'\left[\delta + (1-\delta)\frac{\lambda_L}{\lambda_N}\right] \\ &= \tau'\frac{s_w\hat{w}_0^j - \delta s_y\hat{p}_0^j}{1 - \tau'\left[\delta + (1-\delta)\lambda_L/\lambda_N\right]} \end{aligned}$$

We can then solve for the tax differential in terms of amenities:

$$\frac{d\tau^j}{m} = \tau'\frac{1}{1 - \tau'\left[\delta + (1-\delta)\lambda_L/\lambda_N\right]}\left[(1-\delta)\left(\frac{1-\lambda_L}{\lambda_N}s_x\hat{A}_X^j - \frac{\lambda_L}{\lambda_N}s_y A_Y^j\right) - \frac{(1-\delta)\lambda_L + \delta\lambda_N}{\lambda_N}\hat{Q}^j\right]$$

This equation demonstrates that the deduction reduces the dependence of taxes on productivity and increases the implicit subsidy for quality-of-life.

D.2 State Taxes

The tax differential with state taxes is computed by including an additional component based on wages and prices relative to the state average, as if state tax revenues are redistributed lump-sum to households within the state. This produces the augmented formula

$$\frac{d\tau^j}{m} = \tau' \left(s_w \hat{w}^j - \delta\tau' s_y \hat{p}^j \right) + \tau'_S [s_w(\hat{w}^j - \hat{w}^S) - \delta_S s_y (\hat{p}^j - \hat{p}^S)] \tag{A.1}$$

where τ'_S and δ_S are are marginal tax and deduction rates at the state-level, net of federal deductions, and $\hat{w}^S$ and $\hat{p}^S$ are the differentials for state S as a whole relative to the entire country.

D.3 Cost and Expenditure Shares

We calibrate the model using the data described below and national-level parameters. Starting with income shares, Krueger (1999) argues that s_w is close to 75 percent. Poterba (1998) estimates that the share of income from corporate capital is 12 percent, so s_I should be higher and is taken as 15 percent. This leaves 10 percent for s_R, which is roughly consistent with estimates in Keiper et al. (1961) and Case (2007).[27]

Turning to expenditure shares, Albouy (2008), Moretti (2008), and Shapiro (2006) find that housing costs approximate non-housing cost differences across cities. The cost-of-living differential is $s_y \hat{p}^j$, where $\hat{p}^j$ equals the housing-cost differential and s_y equals the expenditure share on housing plus an additional term which captures how a one percent increase in housing costs predicts a $b = 0.26$ percent increase in non-housing costs.[28] In the Consumer Expenditure Survey (CEX), the share of income spent on shelter and utilities, s_{hous}, is 0.22, while the share of income spent on other goods, s_{oth}, is 0.56, leaving 0.22 spent on taxes or saved (Bureau of Labor Statistics 2002).[29] Thus, our coefficient on the housing cost differential is $s_y = s_{hous} + s_{oth} b = 0.22 + 0.56 \times 0.26 = 36$ percent. This leaves s_x at 64 percent.

We choose the cost shares to be consistent with the expenditure and income shares above. θ_L appears small: Beeson and Eberts (1986) use a value of 0.027, while Rappaport (2008a, 2008b) uses a value of 0.016. Valentinyi and Herrendorff (2008) estimate the land share of tradables at 4 percent, although their definition of tradables differs from the one here. We use a value of 2.5 percent for θ_L here. Following Carliner (2003) and Case (2007), the cost-share of land in home-goods, ϕ_L, is taken at 23.3 percent; this is slightly above values from McDonald (1981), Roback (1982), and Thorsnes (1997) to account for the increase in land cost shares over time described by Davis and Palumbo (2007). Together the cost and expenditure shares imply λ_L is 17 percent, which appears reasonable since the remaining 83 percent of land for home goods includes all residential land and much commercial land; the cost and expenditure shares also agree with s_R at 10 percent.[30] Finally, we choose the cost shares of labor and capital in both production sectors. As

[27] The values Keiper reports were at a historical low. Keiper et al. (1961) find that total land value was found to be about 1.1 times GDP. A rate of return of 9 percent would justify using $s_R = 0.10$. Case (2007), ignoring agriculture, estimates the value of land to be $5.6 trillion in 2000 when personal income was $8.35 trillion.

[28] See Albouy (2008) for details.

[29] Utility costs account for one fifth of s_{hous}, which means that without them this parameter would be roughly 0.18.

[30] These proportions are roughly consistent with other studies. In the base calibration of the model, 51 percent of land is devoted to actual housing, 32 percent is for non-housing home goods, and 17 percent is for traded goods, including those purchased by the federal government. Keiper et al. (1961) find that about 52.5 of land value is in residential uses, a 22.9 percent in industry, 20.9 percent in agriculture. Case (2007), ignoring agriculture, finds that in 2000 residential real estate accounted for 76.6 percent of land value, while commercial real estate accounted for the remaining 23.4 percent.

separate information on ϕ_K and θ_K does not exist, we set both cost shares of capital at 15 percent to be consistent with s_I. Accounting identities then determine that θ_N is 82.5 percent, ϕ_N is 62 percent, and λ_N is 70.4 percent.

The federal tax rate, when combined with relevant variation in wages with state tax rates, produces an approximate marginal tax rate, τ, of 36.1 percent. Details on this tax rate, as well as housing deductions, are discussed in Appendix D.4.

D.4 Calibration of Tax Parameters

The federal marginal tax rate on wage income is determined by adding together federal marginal income tax rate and the effective marginal payroll tax rate. TAXSIM gives an average marginal federal income tax rate of 25.1 percent in 2000. In 2000, Social Security (OASDI) and Medicare (HI) tax rates were 12.4 and 2.9 percent on employer and employee combined. Estimates from Boskin et al. (1987, Table 4) show that the marginal benefit from future returns from OASDI taxes is fairly low, generally no more than 50 percent, although only 85 percent of wage earnings are subject to the OASDI cap. HI taxes emulate a pure tax (Congressional Budget Office 2005). These facts suggest adding 37.5 percent of the Social Security tax and all of the Medicare tax to the federal income tax rate, adding 8.2 percent. The employer half of the payroll tax (4.1 percent) has to be added to observed wage levels to produce gross wage levels. Overall, this puts an overall federal tax rate, τ' , of 33.3 percent tax rate on gross wages, although only a 29.2 percent rate on observed wages.

Determining the federal deduction level requires taking into account the fact that many households do not itemize deductions. According to the Statistics on Income, although only 33 percent of tax returns itemize, they account for 67 percent of reported Adjusted Gross Income (AGI). Since the income-weighted share is what matters, 67 percent is multiplied by the effective tax reduction given in TAXSIM, in 2000 of 21.6 percent. Thus, on average these deductions reduce the effective price of eligible goods by 14.5 percent. Since eligible goods only include housing, this deduction applies to only 59 percent of home goods. Multiplying 14.5 percent times 59 percent gives an effective price reduction of 8.6 percent for home goods. Divided by a federal tax rate of 33.3 percent, this produces a federal deduction level of 25.7 percent.

State income tax rates from 2000 are taken from TAXSIM, which, per dollar, fall at an average marginal rate of 4.5 percent. State sales tax data in 2000 are taken from the Tax Policy Center, originally supplied by the Federation of Tax Administrators. The average state sales tax rate is 5.2 percent. Sales tax rates are reduced by 10 percent to accommodate untaxed goods and services other than food or housing (Feenberg et al. 1997), and by another 8 percent in states that exempt food. Overall state taxes raise the marginal tax rate on wage differences within state by an average of 5.9 percentage points, from zero points in Alaska to 8.8 points in Minnesota.

State-level deductions for housing expenditures, explicit in income taxes, and implicit in sales taxes, should also be included. At the state level, deductions for income taxes are calculated in an equivalent way using TAXSIM data. Furthermore, all housing expenditures are deducted from the sales tax. Overall this produces an average effective deduction level of $\delta = 0.291$.

Appendix Table 1: Relationship between Observed and Predicted Variables

	$\hat{Q}$	$\hat{A}_X$	$\hat{A}_Y$	$\hat{r}$	$\hat{x}$	$\hat{y}$	$\hat{L}$	$\hat{K}$	$\hat{X}$	$\hat{Y}$	$\hat{N}_X$	$\hat{N}_Y$	$\hat{K}_X$	$\hat{K}_Y$	$\hat{L}_X$	$\hat{L}_Y$
$\hat{w}$	-0.478	0.837	0.731	0.488	0.476	0.482	0.000	0.617	1.115	0.471	0.169	-0.439	0.836	0.228	0.511	-0.097
	(0.002)	(0.001)	(0.006)	(0.026)	(0.003)	(0.002)	(0.000)	(0.018)	(0.018)	(0.019)	(0.018)	(0.017)	(0.018)	(0.017)	(0.002)	(0.000)
$\hat{p}$	0.324	0.008	-0.925	0.320	-0.106	-0.712	-0.000	0.034	-0.098	-0.702	-0.101	0.273	-0.101	0.273	-0.314	0.060
	(0.001)	(0.000)	(0.002)	(0.010)	(0.001)	(0.001)	(0.000)	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)	(0.001)	(0.000)
$\hat{N}$	0.000	0.034	0.320	1.373	-0.000	-0.000	0.000	0.989	1.055	0.998	1.044	0.892	1.044	0.892	0.128	-0.024
	(0.000)	(0.000)	(0.001)	(0.002)	(0.000)	(0.000)	(0.000)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.000)	(0.000)

Each column presents coefficients (standard errors) from an OLS regression of (unobserved) estimated amenity, price, or quantity on observed prices and population density ($\hat{w}, \hat{p}, \hat{N}$).

Appendix Table 2: List of Metropolitan and Non-Metropolitan Areas Ranked by Inferred Land Value

Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	Restricted $\hat{A}_X^j$	$\hat{A}_Y^j$	$\hat{r}^j$
New York, Northern New Jersey, Long Island, NY-NJ-CT-PA	2.285	0.029	0.209	0.264	0.513	3.384
San Francisco-Oakland-San Jose, CA	1.209	0.138	0.289	0.270	-0.174	2.035
Los Angeles-Riverside-Orange County, CA	1.250	0.081	0.150	0.159	0.084	1.932
Chicago-Gary-Kenosha, IL-IN-WI	1.191	0.005	0.131	0.161	0.278	1.777
Salinas (Monterey-Carmel), CA	0.863	0.137	0.144	0.125	-0.183	1.460
San Diego, CA	0.872	0.123	0.098	0.086	-0.110	1.423
Philadelphia-Wilmington-Atlantic City, PA-NJ-DE-MD	0.958	-0.040	0.096	0.133	0.346	1.395
Miami-Fort Lauderdale, FL	0.964	0.041	0.015	0.035	0.192	1.357
Santa Barbara-Santa Maria-Lompoc, CA	0.713	0.176	0.125	0.090	-0.319	1.282
Boston-Worcester-Lawrence, MA-NH-ME-CT	0.797	0.034	0.128	0.137	0.077	1.254
Washington-Baltimore, DC-MD-VA-WV	0.683	-0.013	0.116	0.135	0.172	1.051
Las Vegas, NV-AZ	0.684	-0.025	0.057	0.083	0.246	0.989
New Orleans, LA	0.686	0.005	-0.065	-0.038	0.253	0.860
Providence-Fall River-Warwick, RI-MA	0.591	0.014	0.022	0.037	0.138	0.853
Stockton-Lodi, CA	0.529	-0.002	0.083	0.096	0.118	0.800
Milwaukee-Racine, WI	0.573	-0.009	0.037	0.057	0.181	0.794
Denver-Boulder-Greeley, CO	0.467	0.054	0.066	0.062	-0.039	0.721
Phoenix-Mesa, AZ	0.507	0.012	0.030	0.042	0.111	0.717
Sacramento-Yolo, CA	0.434	0.033	0.075	0.075	0.001	0.703
Buffalo-Niagara Falls, NY	0.448	-0.054	-0.042	-0.008	0.316	0.612
Seattle-Tacoma-Bremerton, WA	0.324	0.061	0.095	0.082	-0.121	0.583
Modesto, CA	0.389	-0.008	0.050	0.062	0.112	0.578
Provo-Orem, UT	0.447	0.019	-0.048	-0.034	0.126	0.564
Detroit-Ann Arbor-Flint, MI	0.346	-0.047	0.108	0.125	0.160	0.557
Champaign-Urbana, IL	0.435	-0.009	-0.080	-0.056	0.219	0.555
Laredo, TX	0.524	-0.008	-0.194	-0.159	0.324	0.518
Salt Lake City-Ogden, UT	0.394	0.026	-0.015	-0.007	0.068	0.517
Reading, PA	0.403	-0.046	-0.017	0.011	0.263	0.509
Madison, WI	0.333	0.053	-0.018	-0.020	-0.020	0.487
Reno, NV	0.263	0.053	0.043	0.034	-0.085	0.462
Dallas-Fort Worth, TX	0.318	-0.044	0.047	0.067	0.186	0.461
Houston-Galveston-Brazoria, TX	0.323	-0.072	0.045	0.073	0.261	0.444
Cleveland-Akron, OH	0.329	-0.016	0.006	0.022	0.143	0.442
Allentown-Bethlehem-Easton, PA	0.308	-0.022	-0.005	0.012	0.161	0.410
West Palm Beach-Boca Raton, FL	0.235	0.017	0.046	0.045	-0.002	0.370
Portland-Salem, OR-WA	0.241	0.047	0.037	0.030	-0.073	0.368
State College, PA	0.292	0.036	-0.120	-0.111	0.081	0.337
El Paso, TX	0.386	-0.041	-0.164	-0.127	0.346	0.333
Fresno, CA	0.232	-0.008	-0.014	-0.002	0.106	0.320
Lincoln, NE	0.330	0.022	-0.122	-0.108	0.134	0.301
Minneapolis-St. Paul, MN-WI	0.203	-0.032	0.067	0.078	0.102	0.294
Hartford, CT	0.087	-0.026	0.120	0.121	0.014	0.260
Lafayette, IN	0.236	-0.006	-0.069	-0.054	0.139	0.260
Springfield, MA	0.143	0.002	-0.003	0.001	0.042	0.235
Norfolk-Virginia Beach-Newport News, VA-	0.210	0.027	-0.095	-0.088	0.063	0.224
Bakersfield, CA	0.189	-0.063	0.020	0.043	0.211	0.221
Columbus, OH	0.156	-0.028	0.013	0.025	0.115	0.198
San Antonio, TX	0.222	-0.039	-0.097	-0.071	0.240	0.182

Appendix Table 2: List of Metropolitan and Non-Metropolitan Areas Ranked by Inferred Land Value

Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	Restricted $\hat{A}_X^j$	$\hat{A}_Y^j$	$\hat{r}^j$
Bloomington-Normal, IL	0.126	-0.061	0.003	0.024	0.200	0.153
Austin-San Marcos, TX	0.069	0.016	0.014	0.011	-0.031	0.131
Tucson, AZ	0.122	0.052	-0.091	-0.095	-0.035	0.118
Toledo, OH	0.113	-0.041	-0.037	-0.019	0.170	0.092
Erie, PA	0.153	-0.035	-0.114	-0.090	0.220	0.090
Pittsburgh, PA	0.119	-0.047	-0.054	-0.033	0.200	0.085
Tampa-St. Petersburg-Clearwater, FL	0.109	0.003	-0.054	-0.047	0.066	0.078
Iowa City, IA	0.103	0.034	-0.072	-0.073	-0.007	0.074
Albuquerque, NM	0.113	0.049	-0.064	-0.069	-0.049	0.073
Colorado Springs, CO	0.060	0.055	-0.066	-0.075	-0.080	0.042
Rochester, NY	0.019	-0.041	-0.029	-0.014	0.134	0.041
Omaha, NE-IA	0.136	-0.019	-0.084	-0.068	0.151	0.029
St. Louis, MO-IL	0.052	-0.034	-0.007	0.004	0.110	0.012
Non-metro, AK	0.000	0.012	0.037	0.000	0.000	0.000
Honolulu, HI	0.000	0.204	0.057	0.000	0.000	0.000
Anchorage, AK	0.000	0.023	0.077	0.000	0.000	0.000
Non-metro, HI	0.000	0.126	0.013	0.000	0.000	0.000
Bryan-College Station, TX	0.063	0.027	-0.122	-0.118	0.035	-0.013
Lancaster, PA	-0.002	-0.011	-0.017	-0.013	0.040	-0.018
Albany-Schenectady-Troy, NY	-0.026	-0.041	-0.026	-0.013	0.118	-0.019
Corpus Christi, TX	0.074	-0.034	-0.106	-0.086	0.185	-0.028
Non-metro, RI	-0.094	0.040	0.071	0.051	-0.184	-0.032
Cincinnati-Hamilton, OH-KY-IN	-0.007	-0.038	0.020	0.029	0.084	-0.034
Fort Collins-Loveland, CO	-0.045	0.079	-0.032	-0.054	-0.201	-0.052
Lubbock, TX	0.075	-0.009	-0.161	-0.144	0.159	-0.067
Spokane, WA	0.011	0.008	-0.090	-0.086	0.045	-0.086
Bloomington, IN	-0.015	0.032	-0.110	-0.112	-0.013	-0.092
Louisville, KY-IN	-0.016	-0.023	-0.047	-0.037	0.088	-0.103
Orlando, FL	-0.057	0.006	-0.037	-0.038	-0.009	-0.128
Syracuse, NY	-0.081	-0.069	-0.056	-0.035	0.195	-0.137
Memphis, TN-AR-MS	-0.044	-0.060	-0.013	0.003	0.151	-0.137
Visalia-Tulare-Porterville, CA	-0.087	-0.016	-0.036	-0.031	0.040	-0.144
Green Bay, WI	-0.093	-0.011	-0.022	-0.021	0.014	-0.165
Pueblo, CO	0.006	-0.003	-0.162	-0.149	0.122	-0.167
Scranton–Wilkes-Barre–Hazleton, PA	-0.048	-0.027	-0.106	-0.093	0.129	-0.173
Amarillo, TX	-0.028	-0.010	-0.142	-0.130	0.116	-0.189
Sarasota-Bradenton, FL	-0.138	0.066	-0.046	-0.066	-0.186	-0.204
Brownsville-Harlingen-San Benito, TX	0.048	-0.057	-0.221	-0.187	0.319	-0.204
Des Moines, IA	-0.085	-0.022	-0.037	-0.031	0.055	-0.208
South Bend, IN	-0.094	-0.047	-0.072	-0.057	0.143	-0.229
Eugene-Springfield, OR	-0.159	0.088	-0.084	-0.108	-0.225	-0.247
Dayton-Springfield, OH	-0.146	-0.030	-0.030	-0.024	0.053	-0.249
Kansas City, MO-KS	-0.125	-0.037	-0.015	-0.008	0.066	-0.264
Yuma, AZ	-0.121	0.002	-0.100	-0.098	0.027	-0.273
Altoona, PA	-0.082	-0.045	-0.158	-0.136	0.202	-0.279
Indianapolis, IN	-0.183	-0.039	0.003	0.008	0.042	-0.281
Merced, CA	-0.215	-0.012	-0.013	-0.016	-0.028	-0.300
Lansing-East Lansing, MI	-0.218	-0.046	-0.008	-0.002	0.058	-0.310

Appendix Table 2: List of Metropolitan and Non-Metropolitan Areas Ranked by Inferred Land Value

Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	Restricted $\hat{A}_X^j$	$\hat{A}_Y^j$	$\hat{r}^j$
Appleton-Oshkosh-Neenah, WI	-0.185	-0.021	-0.052	-0.048	0.032	-0.324
Grand Rapids-Muskegon-Holland, MI	-0.236	-0.044	-0.010	-0.005	0.046	-0.333
Harrisburg-Lebanon-Carlisle, PA	-0.222	-0.029	-0.020	-0.018	0.019	-0.338
Lexington, KY	-0.150	-0.033	-0.095	-0.084	0.104	-0.344
Waterloo-Cedar Falls, IA	-0.132	-0.023	-0.129	-0.118	0.107	-0.354
Richmond-Petersburg, VA	-0.228	-0.033	-0.006	-0.004	0.019	-0.358
Fargo-Moorhead, ND-MN	-0.115	-0.039	-0.174	-0.154	0.187	-0.371
Rockford, IL	-0.237	-0.069	-0.024	-0.011	0.122	-0.376
Boise City, ID	-0.185	0.010	-0.077	-0.081	-0.033	-0.380
Atlanta, GA	-0.291	-0.032	0.063	0.057	-0.053	-0.381
Oklahoma City, OK	-0.146	-0.020	-0.135	-0.124	0.100	-0.397
Odessa-Midland, TX	-0.154	-0.063	-0.136	-0.114	0.211	-0.398
Davenport-Moline-Rock Island, IA-IL	-0.215	-0.041	-0.088	-0.077	0.101	-0.407
Wichita, KS	-0.187	-0.048	-0.079	-0.066	0.120	-0.412
San Luis Obispo-Atascadero-Paso Robles, CA	-0.458	0.124	0.077	0.020	-0.527	-0.418
Portland, ME	-0.239	0.051	-0.060	-0.078	-0.170	-0.428
York, PA	-0.277	-0.032	-0.036	-0.033	0.020	-0.431
Jacksonville, FL	-0.265	-0.009	-0.051	-0.054	-0.026	-0.442
Yakima, WA	-0.287	-0.009	-0.029	-0.034	-0.049	-0.445
Lawrence, KS	-0.240	0.038	-0.129	-0.138	-0.088	-0.445
Binghamton, NY	-0.302	-0.054	-0.123	-0.109	0.130	-0.472
Cedar Rapids, IA	-0.266	-0.002	-0.078	-0.081	-0.026	-0.475
Sheboygan, WI	-0.302	-0.019	-0.062	-0.062	-0.006	-0.489
Savannah, GA	-0.307	-0.011	-0.080	-0.082	-0.015	-0.491
Charlottesville, VA	-0.329	0.054	-0.090	-0.109	-0.185	-0.492
Naples, FL	-0.424	0.095	0.027	-0.016	-0.405	-0.497
Rochester, MN	-0.325	-0.061	-0.003	0.003	0.059	-0.500
Sioux Falls, SD	-0.230	-0.006	-0.146	-0.141	0.043	-0.512
Muncie, IN	-0.272	-0.043	-0.122	-0.111	0.111	-0.515
Canton-Massillon, OH	-0.322	-0.024	-0.083	-0.081	0.018	-0.526
Sioux City, IA-NE	-0.220	-0.060	-0.161	-0.140	0.197	-0.540
Gainesville, FL	-0.301	0.024	-0.134	-0.141	-0.068	-0.554
Yuba City, CA	-0.394	0.009	-0.066	-0.077	-0.104	-0.559
Tulsa, OK	-0.280	-0.032	-0.104	-0.097	0.065	-0.570
Abilene, TX	-0.256	0.004	-0.223	-0.217	0.063	-0.577
Chico-Paradise, CA	-0.444	0.053	-0.067	-0.092	-0.236	-0.578
Utica-Rome, NY	-0.375	-0.064	-0.125	-0.111	0.134	-0.583
La Crosse, WI-MN	-0.355	-0.020	-0.126	-0.123	0.026	-0.603
Peoria-Pekin, IL	-0.399	-0.061	-0.041	-0.034	0.061	-0.606
Janesville-Beloit, WI	-0.395	-0.050	-0.019	-0.017	0.019	-0.619
Medford-Ashland, OR	-0.425	0.095	-0.099	-0.133	-0.317	-0.620
Melbourne-Titusville-Palm Bay, FL	-0.360	-0.000	-0.104	-0.108	-0.044	-0.621
Elmira, NY	-0.415	-0.061	-0.132	-0.119	0.119	-0.640
Decatur, IL	-0.385	-0.089	-0.080	-0.062	0.165	-0.642
Richland-Kennewick-Pasco, WA	-0.440	-0.051	0.011	0.009	-0.015	-0.646
Topeka, KS	-0.349	-0.024	-0.137	-0.132	0.047	-0.654
St. Joseph, MO	-0.347	-0.026	-0.168	-0.160	0.072	-0.666
Springfield, IL	-0.435	-0.039	-0.082	-0.080	0.019	-0.669

Appendix Table 2: List of Metropolitan and Non-Metropolitan Areas Ranked by Inferred Land Value

Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}^j_X$	Restricted $\hat{A}^j_X$	$\hat{A}^j_Y$	$\hat{r}^j$
Fort Walton Beach, FL	-0.383	0.062	-0.174	-0.192	-0.165	-0.681
Billings, MT	-0.329	0.013	-0.169	-0.172	-0.024	-0.684
Corvalis, OR	-0.474	0.081	-0.081	-0.114	-0.308	-0.686
Fort Myers-Cape Coral, FL	-0.454	0.049	-0.084	-0.107	-0.215	-0.693
Columbia, MO	-0.401	0.023	-0.164	-0.172	-0.075	-0.700
Raleigh-Durham-Chapel Hill, NC	-0.503	0.011	0.018	-0.004	-0.201	-0.720
Tallahassee, FL	-0.451	0.022	-0.098	-0.112	-0.134	-0.724
Saginaw-Bay City-Midland, MI	-0.489	-0.074	-0.035	-0.028	0.062	-0.726
Burlington, VT	-0.453	0.065	-0.082	-0.110	-0.260	-0.729
Baton Rouge, LA	-0.465	-0.031	-0.053	-0.057	-0.032	-0.737
Roanoke, VA	-0.470	-0.017	-0.107	-0.110	-0.032	-0.754
Waco, TX	-0.439	-0.047	-0.118	-0.111	0.065	-0.758
Evansville-Henderson, IN-KY	-0.458	-0.047	-0.104	-0.099	0.049	-0.762
Nashville, TN	-0.539	-0.001	-0.016	-0.033	-0.159	-0.768
Grand Junction, CO	-0.518	0.114	-0.134	-0.174	-0.374	-0.768
Williamsport, PA	-0.471	-0.031	-0.130	-0.128	0.019	-0.778
Lewiston-Auburn, ME	-0.435	-0.008	-0.123	-0.127	-0.034	-0.785
Cheyenne, WY	-0.414	0.056	-0.217	-0.231	-0.130	-0.799
Charleston-North Charleston, SC	-0.534	0.025	-0.082	-0.101	-0.180	-0.808
Youngstown-Warren, OH	-0.512	-0.052	-0.090	-0.087	0.036	-0.816
Columbus, GA-AL	-0.488	-0.055	-0.152	-0.142	0.094	-0.836
Fort Wayne, IN	-0.533	-0.063	-0.067	-0.063	0.041	-0.840
Santa Fe, NM	-0.641	0.127	-0.017	-0.073	-0.527	-0.849
Birmingham, AL	-0.561	-0.047	-0.034	-0.038	-0.033	-0.860
Kalamazoo-Battle Creek, MI	-0.602	-0.056	-0.037	-0.039	-0.020	-0.868
San Angelo, TX	-0.487	-0.025	-0.177	-0.174	0.032	-0.868
Beaumont-Port Arthur, TX	-0.526	-0.108	-0.070	-0.052	0.164	-0.873
Columbia, SC	-0.566	-0.007	-0.076	-0.088	-0.110	-0.881
Kokomo, IN	-0.618	-0.110	0.029	0.037	0.069	-0.910
Fayetteville, NC	-0.566	0.028	-0.178	-0.193	-0.132	-0.916
Daytona Beach, FL	-0.562	0.019	-0.144	-0.158	-0.132	-0.927
Montgomery, AL	-0.578	-0.003	-0.124	-0.134	-0.091	-0.930
Pittsfield, MA	-0.689	0.014	-0.050	-0.073	-0.222	-0.931
New London-Norwich, CT-RI	-0.765	0.006	0.051	0.020	-0.297	-0.940
Springfield, MO	-0.559	0.003	-0.175	-0.183	-0.066	-0.942
Fort Pierce-Port St. Lucie, FL	-0.620	0.011	-0.078	-0.097	-0.174	-0.943
Bellingham, WA	-0.701	0.074	-0.038	-0.080	-0.392	-0.956
Charlotte-Gastonia-Rock Hill, NC-SC	-0.660	-0.013	0.007	-0.013	-0.183	-0.961
Eau Claire, WI	-0.613	-0.026	-0.120	-0.125	-0.045	-0.965
Shreveport-Bossier City, LA	-0.583	-0.042	-0.124	-0.123	0.008	-0.969
Jamestown, NY	-0.633	-0.079	-0.157	-0.144	0.114	-0.970
Sharon, PA	-0.610	-0.033	-0.151	-0.151	-0.006	-0.989
Victoria, TX	-0.623	-0.074	-0.104	-0.097	0.066	-1.017
Jackson, MS	-0.627	-0.031	-0.099	-0.104	-0.050	-1.017
Jackson, MI	-0.722	-0.064	-0.034	-0.038	-0.039	-1.038
McAllen-Edinburg-Mission, TX	-0.541	-0.079	-0.228	-0.208	0.192	-1.039
Elkhart-Goshen, IN	-0.707	-0.043	-0.059	-0.067	-0.073	-1.058
Duluth-Superior, MN-WI	-0.676	-0.069	-0.116	-0.111	0.045	-1.060

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Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	Restricted $\hat{A}_X^j$	$\hat{A}_Y^j$	$\hat{r}^j$
Wichita Falls, TX	-0.601	-0.008	-0.226	-0.228	-0.016	-1.064
Mobile, AL	-0.676	-0.016	-0.128	-0.137	-0.086	-1.079
Killeen-Temple, TX	-0.645	0.040	-0.220	-0.237	-0.160	-1.080
Las Cruces, NM	-0.638	0.019	-0.190	-0.203	-0.123	-1.083
Athens, GA	-0.729	0.016	-0.125	-0.146	-0.189	-1.084
Terre Haute, IN	-0.676	-0.060	-0.139	-0.135	0.036	-1.097
Pensacola, FL	-0.676	0.003	-0.146	-0.159	-0.123	-1.100
Tuscaloosa, AL	-0.714	-0.013	-0.099	-0.112	-0.125	-1.101
Dubuque, IA	-0.667	-0.024	-0.150	-0.155	-0.047	-1.109
Mansfield, OH	-0.722	-0.048	-0.110	-0.113	-0.030	-1.115
Little Rock-North Little Rock, AR	-0.699	-0.011	-0.100	-0.113	-0.127	-1.116
Panama City, FL	-0.723	0.026	-0.138	-0.160	-0.204	-1.138
Lake Charles, LA	-0.721	-0.064	-0.085	-0.085	-0.005	-1.139
Owensboro, KY	-0.702	-0.041	-0.144	-0.146	-0.018	-1.150
Greenville, NC	-0.766	-0.022	-0.085	-0.099	-0.130	-1.172
Lakeland-Winter Haven, FL	-0.759	-0.023	-0.119	-0.130	-0.101	-1.204
Grand Forks, ND-MN	-0.700	-0.046	-0.210	-0.205	0.042	-1.208
Lima, OH	-0.787	-0.062	-0.103	-0.105	-0.017	-1.211
Greensboro–Winston Salem–High Point, NC	-0.848	-0.016	-0.049	-0.071	-0.197	-1.258
Macon, GA	-0.844	-0.068	-0.079	-0.083	-0.036	-1.273
Non-metro, CA	-0.982	0.059	-0.017	-0.066	-0.458	-1.277
St. Cloud, MN	-0.859	-0.048	-0.110	-0.118	-0.073	-1.281
Biloxi-Gulfport-Pascagoula, MS	-0.818	-0.026	-0.135	-0.146	-0.099	-1.301
Albany, GA	-0.860	-0.063	-0.099	-0.103	-0.041	-1.306
Punta Gorda, FL	-0.859	0.049	-0.143	-0.176	-0.305	-1.312
Wilmington, NC	-0.914	0.071	-0.104	-0.148	-0.409	-1.313
Tyler, TX	-0.884	-0.025	-0.106	-0.121	-0.144	-1.340
Benton Harbor, MI	-0.938	-0.029	-0.081	-0.099	-0.167	-1.341
Barnstable-Yarmouth (Cape Cod), MA	-1.111	0.121	0.046	-0.029	-0.708	-1.355
Monroe, LA	-0.867	-0.036	-0.133	-0.143	-0.093	-1.363
Augusta-Aiken, GA-SC	-0.895	-0.057	-0.093	-0.101	-0.074	-1.364
Huntsville, AL	-0.908	-0.055	-0.062	-0.073	-0.105	-1.370
Redding, CA	-1.011	0.041	-0.074	-0.115	-0.382	-1.376
Lafayette, LA	-0.874	-0.057	-0.130	-0.134	-0.041	-1.384
Casper, WY	-0.833	-0.002	-0.219	-0.231	-0.110	-1.415
Non-metro, CT	-1.122	-0.007	0.078	0.035	-0.397	-1.415
Knoxville, TN	-0.937	-0.011	-0.125	-0.145	-0.185	-1.436
Missoula, MT	-0.905	0.101	-0.208	-0.252	-0.411	-1.454
Auburn-Opelika, AL	-0.950	-0.015	-0.132	-0.150	-0.173	-1.459
Charleston, WV	-0.917	-0.052	-0.117	-0.125	-0.076	-1.460
Hattiesburg, MS	-0.910	-0.029	-0.180	-0.189	-0.092	-1.465
Chattanooga, TN-GA	-0.970	-0.035	-0.105	-0.121	-0.147	-1.468
Johnstown, PA	-0.911	-0.062	-0.201	-0.200	0.011	-1.472
Rapid City, SD	-0.948	0.033	-0.212	-0.238	-0.244	-1.547
Lawton, OK	-0.942	-0.016	-0.253	-0.262	-0.085	-1.567
Wausau, WI	-1.066	-0.049	-0.086	-0.103	-0.152	-1.588
Bismarck, ND	-0.918	-0.048	-0.250	-0.249	0.005	-1.589
Flagstaff, AZ-UT	-1.089	0.030	-0.129	-0.166	-0.340	-1.601

Appendix Table 2: List of Metropolitan and Non-Metropolitan Areas Ranked by Inferred Land Value

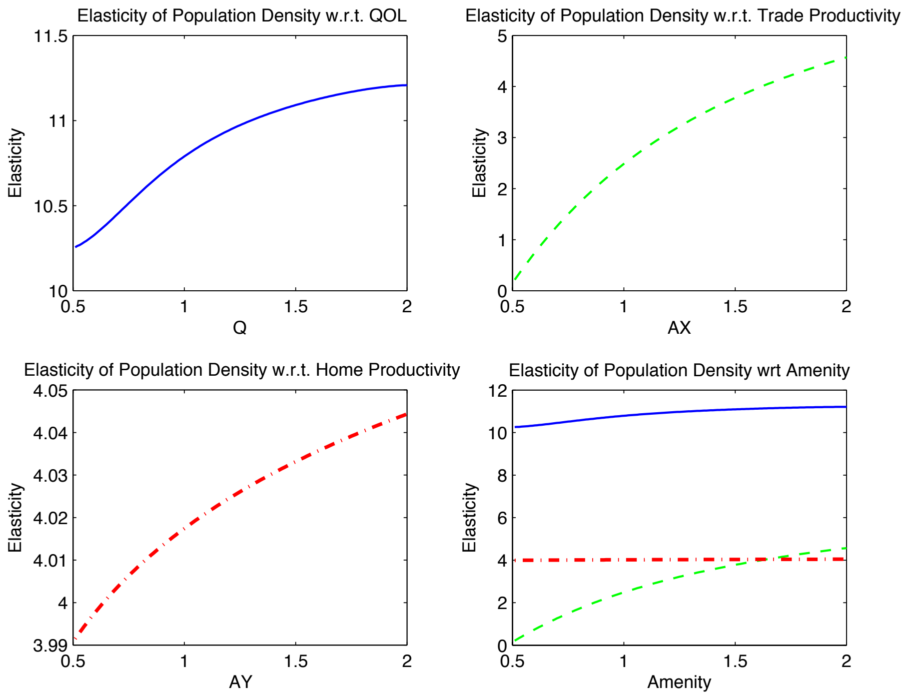
Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	Restricted $\hat{A}_X^j$	$\hat{A}_Y^j$	$\hat{r}^j$
Non-metro, PA	-1.057	-0.053	-0.145	-0.156	-0.100	-1.617
Steubenville-Weirton, OH-WV	-1.023	-0.058	-0.189	-0.194	-0.045	-1.622
Fayetteville-Springdale-Rogers, AR	-1.066	0.005	-0.132	-0.160	-0.263	-1.630
Wheeling, WV-OH	-1.026	-0.058	-0.189	-0.194	-0.047	-1.632
Jackson, TN	-1.073	-0.063	-0.098	-0.110	-0.112	-1.636
Great Falls, MT	-0.957	0.036	-0.283	-0.305	-0.207	-1.640
Fort Smith, AR-OK	-1.010	-0.045	-0.194	-0.202	-0.072	-1.653
Jacksonville, NC	-1.094	0.051	-0.254	-0.288	-0.310	-1.671
Glens Falls, NY	-1.201	-0.020	-0.109	-0.136	-0.257	-1.673
Pocatello, ID	-1.042	-0.061	-0.141	-0.149	-0.075	-1.676
Danville, VA	-1.088	-0.057	-0.163	-0.172	-0.085	-1.678
Non-metro, WA	-1.186	0.037	-0.067	-0.113	-0.432	-1.683
Enid, OK	-1.041	-0.032	-0.219	-0.229	-0.099	-1.693
Greenville-Spartanburg-Anderson, SC	-1.149	-0.031	-0.078	-0.104	-0.234	-1.709
Huntington-Ashland, WV-KY-OH	-1.072	-0.074	-0.177	-0.180	-0.026	-1.715
Alexandria, LA	-1.127	-0.031	-0.173	-0.191	-0.161	-1.753
Non-metro, NY	-1.246	-0.050	-0.123	-0.143	-0.183	-1.769
Pine Bluff, AR	-1.128	-0.053	-0.168	-0.179	-0.105	-1.805
Houma, LA	-1.194	-0.054	-0.123	-0.140	-0.156	-1.816
Non-metro, MA	-1.376	0.063	-0.042	-0.104	-0.579	-1.829
Non-metro, ND	-1.113	-0.041	-0.262	-0.270	-0.069	-1.861
Dover, DE	-1.327	-0.009	-0.086	-0.123	-0.342	-1.903
Joplin, MO	-1.216	-0.011	-0.246	-0.266	-0.192	-1.908
Non-metro, ID	-1.256	0.012	-0.174	-0.207	-0.315	-1.913
Non-metro, UT	-1.315	0.010	-0.124	-0.163	-0.362	-1.917
Non-metro, NV	-1.409	-0.011	0.005	-0.041	-0.427	-1.929
Asheville, NC	-1.343	0.058	-0.132	-0.185	-0.493	-1.930
Non-metro, OR	-1.377	0.062	-0.113	-0.169	-0.525	-1.967
Clarksville-Hopkinsville, TN-KY	-1.275	-0.004	-0.206	-0.233	-0.255	-1.970
Lynchburg, VA	-1.331	-0.031	-0.140	-0.167	-0.248	-1.975
Non-metro, MD	-1.441	-0.022	-0.037	-0.078	-0.378	-1.978
Decatur, AL	-1.356	-0.072	-0.085	-0.105	-0.187	-2.022
Non-metro, OH	-1.387	-0.052	-0.111	-0.136	-0.232	-2.033
Myrtle Beach, SC	-1.402	0.038	-0.148	-0.196	-0.447	-2.034
Longview-Marshall, TX	-1.360	-0.057	-0.149	-0.169	-0.183	-2.065
Bangor, ME	-1.365	-0.018	-0.169	-0.198	-0.274	-2.097
Florence, AL	-1.398	-0.042	-0.149	-0.174	-0.236	-2.112
Cumberland, MD-WV	-1.434	-0.040	-0.171	-0.197	-0.236	-2.115
Sumter, SC	-1.391	-0.037	-0.182	-0.206	-0.222	-2.123
Non-metro, WY	-1.402	0.007	-0.165	-0.203	-0.353	-2.132
Parkersburg-Marietta, WV-OH	-1.394	-0.072	-0.170	-0.185	-0.141	-2.140
Sherman-Denison, TX	-1.449	-0.028	-0.137	-0.168	-0.297	-2.151
Non-metro, IN	-1.500	-0.050	-0.113	-0.142	-0.272	-2.200
Gadsden, AL	-1.446	-0.069	-0.150	-0.170	-0.178	-2.203
Non-metro, IL	-1.519	-0.052	-0.154	-0.181	-0.245	-2.230
Goldsboro, NC	-1.509	-0.007	-0.176	-0.213	-0.343	-2.240
Johnson City-Kingsport-Bristol, TN-VA	-1.485	-0.028	-0.180	-0.210	-0.277	-2.251
Non-metro, NM	-1.482	0.002	-0.202	-0.239	-0.341	-2.269

Appendix Table 2: List of Metropolitan and Non-Metropolitan Areas Ranked by Inferred Land Value

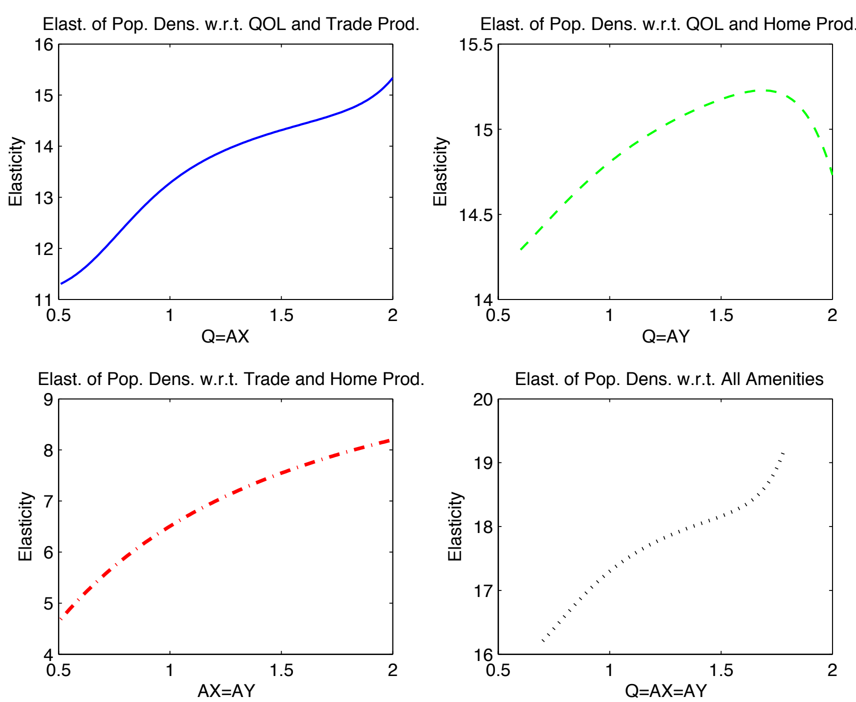
Full Name of Metropolitan Area	$\hat{N}^j$	$\hat{Q}^j$	$\hat{A}_X^j$	Restricted $\hat{A}_X^j$	$\hat{A}_Y^j$	$\hat{r}^j$
Non-metro, MT	-1.461	0.059	-0.236	-0.285	-0.462	-2.272
Non-metro, KS	-1.488	-0.035	-0.240	-0.263	-0.220	-2.296
Dothan, AL	-1.533	-0.040	-0.186	-0.214	-0.257	-2.331
Non-metro, WV	-1.523	-0.042	-0.210	-0.235	-0.233	-2.343
Hickory-Morganton-Lenoir, NC	-1.624	-0.008	-0.124	-0.168	-0.414	-2.366
Non-metro, IA	-1.554	-0.027	-0.192	-0.224	-0.295	-2.367
Ocala, FL	-1.582	-0.010	-0.166	-0.205	-0.365	-2.376
Rocky Mount, NC	-1.640	-0.024	-0.114	-0.155	-0.384	-2.395
Florence, SC	-1.606	-0.049	-0.131	-0.163	-0.295	-2.395
Anniston, AL	-1.579	-0.046	-0.190	-0.217	-0.254	-2.403
Texarkana, TX-Texarkana, AR	-1.556	-0.068	-0.200	-0.219	-0.183	-2.409
Non-metro, NE	-1.592	-0.021	-0.256	-0.286	-0.279	-2.468
Non-metro, MN	-1.735	-0.047	-0.163	-0.197	-0.320	-2.514
Non-metro, WI	-1.761	-0.028	-0.120	-0.164	-0.409	-2.546
Jonesboro, AR	-1.651	-0.026	-0.238	-0.270	-0.298	-2.552
Non-metro, AZ	-1.789	0.037	-0.163	-0.223	-0.559	-2.588
Non-metro, VT	-1.775	0.073	-0.165	-0.234	-0.648	-2.603
Non-metro, MI	-1.864	-0.038	-0.108	-0.153	-0.424	-2.644
Non-metro, FL	-1.823	0.010	-0.167	-0.220	-0.496	-2.694
Non-metro, LA	-1.846	-0.058	-0.178	-0.212	-0.317	-2.764
Non-metro, TX	-1.848	-0.043	-0.206	-0.242	-0.337	-2.786
Non-metro, VA	-1.908	-0.031	-0.163	-0.207	-0.419	-2.786
Non-metro, OK	-1.830	-0.034	-0.255	-0.289	-0.322	-2.804
Non-metro, NH	-2.059	0.042	-0.082	-0.159	-0.715	-2.916
Non-metro, ME	-2.004	0.027	-0.184	-0.247	-0.588	-2.944
Non-metro, MS	-1.956	-0.066	-0.215	-0.248	-0.306	-2.972
Non-metro, MO	-2.048	-0.023	-0.251	-0.297	-0.424	-3.065
Non-metro, KY	-2.086	-0.057	-0.193	-0.234	-0.386	-3.111
Non-metro, NC	-2.164	-0.013	-0.148	-0.207	-0.558	-3.127
Non-metro, SD	-2.036	0.001	-0.279	-0.328	-0.462	-3.138
Non-metro, GA	-2.219	-0.040	-0.146	-0.200	-0.505	-3.192
Non-metro, SC	-2.203	-0.033	-0.140	-0.196	-0.524	-3.205
Non-metro, CO	-2.333	0.112	-0.094	-0.198	-0.978	-3.231
Non-metro, DE	-2.322	0.010	-0.073	-0.150	-0.721	-3.246
Non-metro, AR	-2.267	-0.028	-0.237	-0.290	-0.489	-3.402
Non-metro, TN	-2.470	-0.038	-0.189	-0.249	-0.563	-3.631
Non-metro, AL	-2.761	-0.067	-0.189	-0.250	-0.578	-4.045

See text for estimation procedure. $\hat{A}_X^j$ corresponds to the trade-productivity estimates obtained using wage, housing price, and density data, while Restricted $\hat{A}_X^j$ corresponds to trade-productivity estimates obtained using wage and housing price data plus the constant home-productivity assumption.

Appendix Figure 1: Nonlinear Reduced-Form Elasticities, Single Amenity



Appendix Figure 2: Nonlinear Reduced-Form Elasticities, Multiple Amenities



Appendix Figure 3: Estimated Amenity Distribution, 2000

Appendix Figure 3 is smoothed with a Gaussian kernel, bandwidth=0.05.