Spatial frictions

Kristian Behrens* Giordano Mion† Yasusada Murata‡ Jens Südekum§

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ABSTRACT: The world is replete with spatial frictions. Shipping goods across cities entails trade frictions. Commuting within cities causes urban frictions. How important are these frictions in shaping the spatial economy? We develop and quantify a novel framework to address this question at three different levels: Do spatial frictions matter for the city-size distribution? Do they affect individual city sizes? Do they contribute to the productivity advantage of large cities and the toughness of competition in cities? The short answers are: no; yes; and it depends.

Keywords: trade frictions; urban frictions; city-size distribution; productivity; markups

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*Canada Research Chair in Regional Impacts of Globalization, Department of Economics, Université du Québec à Montréal (UQAM), Canada; CIRPÉE, Canada; and CEPR, UK. E-mail: behrens.kristian@uqam.ca.

†Department of Geography and Environment, London School of Economics (LSE), UK; CEP, UK; and CEPR, UK. E-mail: g.mion@lse.ac.uk

‡Advanced Research Institute for the Sciences and Humanities (ARISH), Nihon University, Tokyo, Japan. E-mail: murata.yasusada@nihon-u.ac.jp

§Mercator School of Management, Universität Duisburg-Essen, Germany; Ruhr Graduate School of Economics, Germany; and IZA, Germany. E-mail: jens.suedekum@uni-due.de

1. Introduction

The world is replete with spatial frictions. Trade frictions for shipping goods across cities induce consumers and firms to spatially concentrate to take advantage of large local markets. Yet, such a concentration generates urban frictions within cities – people spend a lot of time commuting and pay high land rents. Economists have studied this fundamental trade-off between agglomeration and dispersion forces for decades, analyzing how firms and workers choose their locations depending on the magnitudes of – and changes in – spatial frictions (Fujita *et al.*, 1999; Fujita and Thisse, 2002). However, little is known about the quantitative importance of urban and trade frictions in shaping the spatial economy. To what extent do spatial frictions matter for the city-size distribution? By how much do they affect individual city sizes? To what degree do they contribute to the productivity advantage of large cities and the toughness of competition in cities?

Answering these questions is difficult for at least two reasons. First, one needs a spatial model with costly trade and commuting, featuring endogenous location decisions. To investigate the productivity advantage of large cities and the toughness of competition in cities, productivity and markups also need to be endogenous and responsive to changes in spatial frictions. Second, to perform counterfactual analysis aimed at quantifying the importance of those frictions, one must keep track of all general equilibrium interactions when taking the model structurally to the data. To the best of our knowledge, there exist to date no spatial models dealing jointly with these difficulties.

Our aim in this paper is to develop and quantify a novel multi-city general equilibrium model that can fill this gap. Most closely related to our framework is the model by Desmet and Rossi-Hansberg (2013). These authors develop a system-of-cities model with perfect competition to quantify the contribution of efficiency, amenities, and local distortions to the observed size distribution of cities. They do, however, assume that trade between cities is costless, and their perfectly competitive setup does not allow them to investigate endogenous productivity and markup responses due to changes in spatial frictions.

In our model, city sizes, their distribution, productivity, and markups are all endogenously determined and react to changes in urban and trade frictions. Given the population distribution, changes in spatial frictions affect productivity and markups, as well as wages, in cities. These changes, in turn, generate utility differences across cities, thereby affecting individual location decisions. In a nutshell, shocks to spatial frictions affect productivity and competition, as emphasized in the recent trade literature, and trigger population movements, as highlighted in urban economics and the 'new economic geography' (NEG). We quantify our framework using data for 356 US metropolitan statistical areas (MSAs) in 2007. The model performs well in replicating several empirical facts that are not used in the quantification stage, both at the MSA and firm levels. The model can also be extended to encompass external agglomeration economies, which is important as Combes *et al.* (2012) argue that the productivity advantage of large cities is largely due to such externalities. The key qualitative and quantitative properties of our model are robust to that and a number of other extensions, however.

We conduct two counterfactual experiments. First, we consider a scenario where commuting within cities is costless. Second, we analyze a scenario where consumers face the same trade costs for local and non-local products. In both cases, we compare the actual and the counterfactual equilibria to assess the quantitative importance of spatial frictions for the city-size distribution, individual city sizes, as well as productivity and markups in cities. Those counterfactuals are meaningful as they provide *bounds* that suggest to what extent the US economic geography is affected by urban and trade costs.

What are our main quantitative findings? First, neither type of frictions significantly affects the US city-size distribution. Even in a world where urban or trade frictions are eliminated for all cities, that distribution would still follow the rank-size rule also known as Zipf's law. Second, eliminating spatial frictions would change individual city sizes within the stable distribution. Without urban frictions, large congested cities would gain, while small isolated cities would lose population. For example, the size of New York would increase by 8.5%, i.e., its size is limited by 8.5% by the presence of urban frictions. By contrast, in a world without trade frictions, large cities would shrink compared to small cities as local market access no longer matters. For example, the size of New York would decrease by 10.8%, i.e., its size is boosted by 10.8% by the presence of trade frictions. Turning to productivity and competition, eliminating trade frictions would lead to aggregate productivity gains of 68% and markup reductions of 40%, both of which are highly unevenly distributed across MSAs. Eliminating urban frictions generates smaller productivity gains up to 1.4%. Still it leads to a notable markup reduction of about 10% in the aggregate, but again with a lot of variation across MSAs. Summing up, our counterfactual analysis suggests that spatial frictions do not matter for the city-size distribution, they do matter for individual city sizes, and they matter differently for productivity and competition across cities.

Our analysis contributes to both the recent empirical NEG and urban economics literatures. Although these literatures have made some important progress recently (e.g., Redding and Sturm, 2008; Combes and Lafourcade, 2011), it is fair to say that spatial models have so far been confronted with data mostly in a reduced-form manner. Two notable exceptions are Desmet and Rossi-Hansberg (2013) and Ahlfeldt *et al.* (2012), although the latter deal only with a single city. Our framework is also related to the structural international trade literature that, since Eaton and Kortum (2002), has been flourishing (see, among others, Holmes and Stevens, 2010; Eaton *et al.*, 2011; Corcos *et al.*, 2012; Behrens *et al.*, 2012). Yet, those models abstract from population movements across locations. Our contribution brings those various strands of literature closer together and provides the first structural estimation of an urban system model with costly trade across cities and costly commuting within cities.

The rest of the paper is organized as follows. In Section 2 we describe the basic setup of our model, and then analyze the equilibrium in Section 3. Section 4 describes our quantification procedure and discusses the model fit. We then turn to our counterfactual experiments in Section 5. Section 6 provides some extensions and discusses the robustness of our main results. Section 7 concludes. Several proofs and details about our model and quantification procedure are relegated to a supplementary online appendix.

2. The model

We consider an economy that consists of K cities, with L_r identical workers/consumers in city $r = 1,...,K$. Labor is the only factor of production.

2.1 *Preferences and demands*

There is a final consumption good, provided as a continuum of horizontally differentiated varieties. Consumers have identical preferences that display 'love of variety' and give rise to demands with variable elasticity. Let $p_{sr}(i)$ and $q_{sr}(i)$ denote the price and the per capita consumption of variety i when it is produced in city s and consumed in city r. Following Behrens and Murata (2007, 2012a,b) the utility maximization problem of a representative consumer in city r is given by:

$$\max_{q_{sr}(j),\, j\in\Omega_{sr}} \quad U_r \equiv \sum_s \int_{\Omega_{sr}} \left[1 - \mathrm{e}^{-\alpha q_{sr}(j)}\right] \mathrm{d}j \qquad \text{s.t.} \quad \sum_s \int_{\Omega_{sr}} p_{sr}(j) q_{sr}(j) \mathrm{d}j = E_r, \tag{1}$$

where Ω_{sr} denotes the endogenously determined set of varieties produced in s and consumed in r, and where E_r denotes consumption expenditure. Solving (1) yields the following demand functions:

$$q_{sr}(i) = \frac{E_r}{N_r^c \overline{p}_r} - \frac{1}{\alpha}\left\{\ln\left[\frac{p_{sr}(i)}{N_r^c \overline{p}_r}\right] + \eta_r\right\}, \quad \forall i \in \Omega_{sr}, \tag{2}$$

where N_r^c is the mass of varieties consumed in city r, and

$$\overline{p}_r \equiv \frac{1}{N_r^c}\sum_s \int_{\Omega_{sr}} p_{sr}(j)\mathrm{d}j \quad \text{and} \quad \eta_r \equiv -\sum_s \int_{\Omega_{sr}} \ln\left[\frac{p_{sr}(j)}{N_r^c \overline{p}_r}\right] \frac{p_{sr}(j)}{N_r^c \overline{p}_r}\mathrm{d}j$$

denote the average price and the differential entropy of the price distribution, respectively.[1] Since marginal utility at zero consumption is bounded, the demand for a variety need not be positive. Indeed, as can be seen from (2), the demand for a local variety i (respectively, a non-local variety j) is positive if and only if the price of variety i (variety j) is lower than the reservation price p_r^d. Formally,

$$q_{rr}(i) > 0 \iff p_{rr}(i) < p_r^d \quad \text{and} \quad q_{sr}(j) > 0 \iff p_{sr}(j) < p_r^d,$$

where $p_r^d \equiv N_r^c \overline{p}_r \mathrm{e}^{\alpha E_r/(N_r^c \overline{p}_r) - \eta_r}$ depends on the price aggregates $\overline{p}_r$ and η_r. The definition of the reservation price allows us to express the demands for local and non-local varieties concisely as follows:

$$q_{rr}(i) = \frac{1}{\alpha}\ln\left[\frac{p_r^d}{p_{rr}(i)}\right] \quad \text{and} \quad q_{sr}(j) = \frac{1}{\alpha}\ln\left[\frac{p_r^d}{p_{sr}(j)}\right]. \tag{3}$$

Observe that the price elasticity of demand is given by $1/[\alpha q_{rr}(i)]$ for variety i, and respectively, by $1/[\alpha q_{sr}(j)]$ for variety j. Thus, if individuals consume more of those varieties, which is for instance the case when their expenditure increases, they become less price sensitive. Last, since $\mathrm{e}^{-\alpha q_{sr}(j)} = p_{sr}(j)/p_r^d$, the indirect utility in city r is given by

$$U_r = N_r^c - \sum_s \int_{\Omega_{sr}} \frac{p_{sr}(j)}{p_r^d}\mathrm{d}j = N_r^c\left(1 - \frac{\overline{p}_r}{p_r^d}\right), \tag{4}$$

which we use to compute the equilibrium utility in the subsequent analysis.

2.2 *Technology and market structure*

Prior to production, firms decide in which city they enter and engage in research and development. The labor market in each city is perfectly competitive, so that all firms take

[1] As shown in Reza (1994, pp.278-279), the differential entropy takes its maximum value when there is no dispersion, i.e., $p_{sr}(i) = \overline{p}_r$ for all $i \in \Omega_{sr}$ for all s. In that case, we would observe $\eta_r = -\ln(1/N_r^c)$ and thus $q_{sr}(i) = E_r/(N_r^c \overline{p}_r)$ by (2). Behrens and Murata (2007, 2012a,b) focus on such a symmetric case. In contrast, this paper considers firm heterogeneity, so that not only the average price $\overline{p}_r$ but the entire price distribution matter for the demand $q_{sr}(i)$. The differential entropy η_r captures the latter price dispersion.

the wage rate w_r as given. Entry in city r requires a fixed amount F of labor paid at the market wage. Each firm i that enters in city r discovers its marginal labor requirement $m_r(i) \geq 0$ only after making this irreversible entry decision. We assume that $m_r(i)$ is drawn from a known, continuously differentiable distribution G_r.[2] We introduce *trade frictions* into our model by assuming that shipments from city r to city s are subject to trade costs $\tau_{rs} > 1$ for all r and s, which firms incur in terms of labor. Since entry costs are sunk, firms will survive (i.e., operate) provided they can charge prices $p_{rs}(i)$ above marginal costs $\tau_{rs} m_r(i) w_r$ in at least one city. The surviving firms operate in the same city where they enter.

We assume that product markets are segmented, i.e., resale or third-party arbitrage is sufficiently costly, so that firms are free to price discriminate between cities. The operating profit of a firm i located in city r is then as follows:

$$\pi_r(i) = \sum_s \pi_{rs}(i) = \sum_s L_s q_{rs}(i) \left[p_{rs}(i) - \tau_{rs} m_r(i) w_r \right], \tag{5}$$

where $q_{rs}(i)$ is given by (3). Each surviving firm maximizes (5) with respect to its prices $p_{rs}(i)$ separately. Since there is a continuum of firms, no individual firm has any impact on p_r^d, so that the first-order conditions for (operating) profit maximization are given by:

$$\ln \left[\frac{p_s^d}{p_{rs}(i)} \right] = \frac{p_{rs}(i) - \tau_{rs} m_r(i) w_r}{p_{rs}(i)}, \quad \forall i \in \Omega_{rs}. \tag{6}$$

A price distribution satisfying (6) is called a *price equilibrium*. Equations (3) and (6) imply that $q_{rs}(i) = (1/\alpha)[1 - \tau_{rs} m_r(i) w_r / p_{rs}(i)]$. Thus, the minimum output that a firm in market r may sell in market s is given by $q_{rs}(i) = 0$ at $p_{rs}(i) = \tau_{rs} m_r(i) w_r$. This, by (6), implies that $p_{rs}(i) = p_s^d$. Hence, a firm located in r with draw $m_{rs}^x \equiv p_s^d / (\tau_{rs} w_r)$ is just indifferent between selling and not selling to s, whereas all firms in r with draws below m_{rs}^x are productive enough to sell to s. In what follows, we refer to $m_{ss}^x \equiv m_s^d$ as the *internal cutoff* in city s, whereas m_{rs}^x with $r \neq s$ is the *external cutoff*. External and internal cutoffs are linked as follows:[3]

$$m_{rs}^x = \frac{\tau_{ss}}{\tau_{rs}} \frac{w_s}{w_r} m_s^d. \tag{7}$$

[2]Differences in G_r across cities thus reflect production amenities such as local knowledge that are not transferable across space. Firms take those differences into account when making their entry decisions.

[3]Expression (7) reveals an interesting relationship of how trade costs and wage differences affect firms' abilities to break into different markets. In particular, when wages are equalized across cities ($w_r = w_s$) and internal trade is costless ($\tau_{ss} = 1$), all external cutoffs must fall short of the internal cutoffs since $\tau_{rs} > 1$. Breaking into market s is then always harder for firms in $r \neq s$ than for local firms in s, which is the standard case in the firm heterogeneity literature (e.g., Melitz, 2003; Melitz and Ottaviano, 2008). However, in the presence of wage differences and internal trade costs, the internal cutoff need not be larger than the external cutoff in equilibrium. The usual ranking $m_s^d > m_{rs}^x$ prevails only when $\tau_{ss} w_s < \tau_{rs} w_r$.

Given those cutoffs, and a mass of entrants N_r^E in city r, only $N_r^p = N_r^E\, G_r\left(\max_s \{m_{rs}^x\}\right)$ firms survive, namely those which are productive enough to sell at least in one market (which need not be their local market). The mass of varieties consumed in city r is then

$$N_r^c = \sum_s N_s^E G_s(m_{sr}^x), \tag{8}$$

which is the sum of all firms that are productive enough to sell to market r.

Since all firms in each city differ only by their marginal labor requirements, we can express all firm-level variables in terms of m. Specifically, solving (6) by using the Lambert W function, defined as $\varphi = W(\varphi)\mathrm{e}^{W(\varphi)}$, the profit-maximizing prices and quantities, as well as operating profits, are given by:[4]

$$p_{rs}(m) = \frac{\tau_{rs} m w_r}{W}, \quad q_{rs}(m) = \frac{1}{\alpha}\left(1 - W\right), \quad \pi_{rs}(m) = \frac{L_s \tau_{rs} m w_r}{\alpha}(W^{-1} + W - 2), \tag{9}$$

where W denotes the Lambert W function with argument $\mathrm{e}m/m_{rs}^x$, which we suppress to alleviate notation. Since $W(0) = 0$, $W(\mathrm{e}) = 1$ and $W' > 0$ for all non-negative arguments, we have $0 \leq W \leq 1$ if $0 \leq m \leq m_{rs}^x$. The expressions in (9) show that a firm in r with a draw m_{rs}^x charges a price equal to marginal cost, faces zero demand, and earns zero operating profits in market s. Furthermore, using the properties of W', we readily obtain $\partial p_{rs}(m)/\partial m > 0$, $\partial q_{rs}(m)/\partial m < 0$, and $\partial \pi_{rs}(m)/\partial m < 0$. In words, firms with higher productivity (lower m) charge lower prices, sell larger quantities, and earn higher operating profits. These properties are similar to those of the Melitz (2003) model with constant elasticity of subtitution (CES) preferences. Yet, our specification with variable demand elasticity also features higher markups for more productive firms. Indeed, the markup for a firm located in city r and a consumer located in city s,

$$\Lambda_{rs}(m) \equiv \frac{p_{rs}(m)}{\tau_{rs} m w_r} = \frac{1}{W} \tag{10}$$

implies that $\partial \Lambda_{rs}(m)/\partial m < 0$. Unlike Melitz and Ottaviano (2008), who use quasi-linear preferences, we incorporate this feature into a full-fledged general equilibrium model with income effects for varieties.

2.3 *Urban structure*

Each city consists of a large amount of land that stretches out on a two-dimensional featureless plane. Land is used for housing only. Each agent consumes inelastically one

[4]Further details about the Lambert W function, the technical properties of which are key to making our model tractable, can be found in Behrens *et al.* (2012) and in the supplementary online appendix.

unit of land, and the amount of land available at each location is set to one. All firms in city r are located at a dimensionless Central Business District (CBD). A monocentric city of size L_r then covers the surface of a disk with radius $\overline{x}_r \equiv \sqrt{L_r/\pi}$, with the CBD located in the middle of that disk and the workers evenly distributed within it.

We introduce *urban frictions* in a standard way into our model by assuming that agents have to commute to the CBD for work. In particular, we assume that each individual in city r is endowed with $\overline{h}_r$ hours of time, which is the gross labor supply per capita in hours, including commuting time. Commuting costs are of the 'iceberg' type: the *effective* labor supply of a worker living at a distance $x_r \leq \overline{x}_r$ from the CBD is given by

$$s_r(x_r) = \overline{h}_r \mathrm{e}^{-\theta_r x_r}, \tag{11}$$

where $\theta_r \geq 0$ captures the time loss due to commuting and thus measures the commuting technology of city r.[5] The *total effective labor supply* at the CBD is then given by

$$S_r = \int_0^{\overline{x}_r} 2\pi x_r s_r(x_r) \mathrm{d}x_r = \frac{2\pi \overline{h}_r}{\theta_r^2} \left[1 - \left(1 + \theta_r \sqrt{L_r/\pi} \right) \mathrm{e}^{-\theta_r \sqrt{L_r/\pi}} \right]. \tag{12}$$

Define the *effective labor supply per capita* as $h_r \equiv S_r/L_r$, which is the average number of hours worked in city r. It directly follows from (12) that S_r is positive and increasing in L_r, while h_r is decreasing in L_r: given gross labor supply per capita $\overline{h}_r$ and commuting technology $\theta_r > 0$, the effective labor supply per capita is lower in a larger city.[6] We can further show that $\partial h_r / \partial \theta_r < 0$. The effective labor supply per capita is lower, ceteris paribus, the more severe the urban frictions are in city r, that is, the worse the commuting technology is. Notice that with $\theta_r = 0$ we would have $h_r = \overline{h}_r$ for all L_r workers.

Since firms locate at the CBD, the wage income net of commuting costs earned by a worker residing at the city edge is $w_r s_r(\overline{x}_r) = w_r \overline{h}_r \mathrm{e}^{-\theta_r \overline{x}_r}$. Because workers are identical, the wages net of commuting costs and land rents are equalized across all locations in the city: $w_r s_r(x_r) - R_r(x_r) = w_r s_r(\overline{x}_r) - R_r(\overline{x}_r)$, where $R_r(x_r)$ is the land rent at a distance x_r from the CBD. We normalize the opportunity cost of land at the urban fringe to zero, i.e., $R_r(\overline{x}_r) = 0$. The equilibrium land rent schedule is then given by $R_r^*(x_r) = w_r(\mathrm{e}^{-\theta_r x_r} -$

[5] We use an exponential commuting cost since a linear specification, as in, e.g., Murata and Thisse (2005), is subject to a boundary condition to ensure positive effective labor supply at each location in the city. Keeping track of this condition becomes tedious with multiple cities and intercity movements of people. The exponential specification has been used extensively in the literature (e.g., Lucas and Rossi-Hansberg, 2002), and the convexity of the time loss with respect to distance from the CBD can also be justified in a modal choice framework of intra-city transportation (e.g., Glaeser, 2008, pp.24–25).

[6] Here we abstract from an "urban rat race" in larger cities. However, when quantifying the model in Section 4, we use data on $\overline{h}_r$ across MSAs, which shows that $\overline{h}_r$ is higher in big cities like New York.

$\mathrm{e}^{-\theta_r \overline{x}_r})\overline{h}_r$, which yields the following aggregate land rents:

$$ALR_r = \int_0^{\overline{x}_r} 2\pi x_r R_r^*(x_r)\mathrm{d}x_r = \frac{2\pi w_r \overline{h}_r}{\theta_r^2}\left[1 - \left(1 + \theta_r\sqrt{L_r/\pi} + \frac{\theta_r^2 L_r}{2\pi}\right)\mathrm{e}^{-\theta_r\sqrt{L_r/\pi}}\right]. \quad (13)$$

We assume that each worker in city r owns an equal share of the land in that city, and thus receives an equal share of aggregate land rents. Furthermore, each worker has an equal claim to aggregate profits Π_r in the respective city. Accordingly, the per capita expenditure which consists of the wage net of commuting costs, land rent and profit income, is then given by $E_r = w_r\overline{h}_r \mathrm{e}^{-\theta_r\sqrt{L_r/\pi}} + ALR_r/L_r + \Pi_r/L_r = w_r h_r + \Pi_r/L_r$.

3. Equilibrium

3.1 *Single city case*

To illustrate how our model works, we first consider the case of a single city. There are two equilibrium conditions in that case: zero expected profits, and labor market clearing. These two conditions can be solved for the internal cutoff m^d and the mass of entrants N^E, which completely characterize the market equilibrium. For notational convenience, we drop the subscript r and normalize the internal trade costs to one.

Using (5) and (9), the zero expected profit (ZEP) condition $\int_0^{m^d} \pi(m)\mathrm{d}G(m) = Fw$ can be rewritten as:

$$\frac{L}{\alpha}\int_0^{m^d} m(W^{-1} + W - 2)\mathrm{d}G(m) = F, \quad (14)$$

which is a function of m^d only and yields a unique equilibrium cutoff because the left-hand side of (14) is shown to be strictly increasing in m^d from 0 to ∞. Furthermore, using (9), the labor market clearing (LMC) condition, $N^E[L\int_0^{m^d} mq(m)\mathrm{d}G(m) + F] = S$, can be expressed as follows:

$$N^E\left[\frac{L}{\alpha}\int_0^{m^d} m\,(1 - W)\,\mathrm{d}G(m) + F\right] = S, \quad (15)$$

which can be uniquely solved for N^E given the cutoff m^d obtained from (14).[7]

As in Melitz and Ottaviano (2008) and many other existing studies, we choose a particular distribution function for firms' productivity draws, $1/m$, namely a Pareto

[7] From the ZEP condition $L\int_0^{m^d}[p(m) - mw]\,q(m)\mathrm{d}G(m) = Fw$, and from the budget constraint $N^E\int_0^{m^d} p(m)q(m)\mathrm{d}G(m) = E$, we get $EL/(wN^E) = L\int_0^{m^d} mq(m)\mathrm{d}G(m) + F$ which, together with LMC, yields $E = (S/L)w = hw$. The per capita expenditure thus depends only on effective labor supply per capita and the wage in equilibrium, whereas profits per capita, Π/L, are zero.

distribution: $G(m) = (m/m^{\max})^k$, where $m^{\max} > 0$ and $k \geq 1$ are the upper bound and the shape parameter, respectively. This distribution is useful for deriving analytical results and taking the model to data. In particular, we obtain the following closed-form solutions for the equilibrium cutoff and the mass of entrants in the single city case:

$$m^d = \left(\frac{\mu^{\max}}{L}\right)^{\frac{1}{k+1}} \quad \text{and} \quad N^E = \frac{\kappa_2}{\kappa_1 + \kappa_2}\frac{S}{F}, \tag{16}$$

where $\mu^{\max} \equiv \left[\alpha F (m^{\max})^k\right] / \kappa_2$ and κ_1 and κ_2 are positive constants that solely depend on k. The term $\mu^{\max}$ can be interpreted as an inverse measure of *technological possibilities*: the lower is the fixed labor requirement for entry, F, or the lower is the upper bound, $m^{\max}$, the lower is $\mu^{\max}$ and, hence, the better are the city's technological possibilities.

How do population size and technological possibilities affect entry and selection? Recall from (12) that S is increasing in L. The second expression in (16) then shows that there are more entrants N^E in a larger city. The first expression in (16), in turn, shows that a larger L or a smaller $\mu^{\max}$ entail a smaller cutoff m^d and, thus, a lower survival probability $G(m^d)$ of entrants. This tougher selection maps into higher average productivity, $1/\overline{m}$, since $\overline{m} \equiv (1/N)\int_\Omega m(i)\mathrm{d}i = [k/(k+1)]m^d$ under a Pareto distribution. The mass of surviving firms $N^p = N^E G(m^d)$, which is equivalent to consumption diversity N^c in the single city case, is then equal to

$$N = \frac{\alpha}{\kappa_1 + \kappa_2}\frac{h}{m^d} = \frac{\alpha h}{\kappa_1 + \kappa_2}\left(\frac{L}{\mu^{\max}}\right)^{\frac{1}{k+1}}. \tag{17}$$

Since firms are heterogeneous and have different markups and market shares, the simple (unweighted) average of markups is not an adequate measure of consumers' exposure to market power. Using (9) and (10), we hence define the (expenditure share) weighted average of firm-level markups as follows:

$$\overline{\Lambda} \equiv \frac{1}{G(m^d)}\int_0^{m^d} \frac{p(m)q(m)}{E}\Lambda(m)\mathrm{d}G(m) = \frac{\kappa_3}{\alpha}\frac{m^d}{h}, \tag{18}$$

where κ_3 is a positive constant that solely depends on k.[8] Note that the average markup is proportional to the cutoff. It thus follows from (17) and (18) that our model displays pro-competitive effects, since $\overline{\Lambda} = [\kappa_3/(\kappa_1+\kappa_2)]\,(1/N)$ decreases with the mass of competing firms. Finally, indirect utility in the single city case can be expressed as

$$U = \alpha\left[\frac{1}{(k+1)(\kappa_1+\kappa_2)} - 1\right]\frac{h}{m^d} = \left[\frac{1}{(k+1)(\kappa_1+\kappa_2)} - 1\right]\frac{\kappa_3}{\overline{\Lambda}}, \tag{19}$$

[8] Recent empirical work by Feenstra and Weinstein (2010) uses a similar (expenditure share) weighted average of markups in a translog framework.

where the term in square brackets is, by construction, positive for all $k \geq 1$. Alternatively, indirect utility can be written as $U = [1/(k+1) - (\kappa_1 + \kappa_2)]N$. Hence, as can be seen from expressions (16)–(19), a city with better technological possibilities allows for higher utility because of tougher selection, tougher competition, and greater consumption diversity.

The impact of city size on consumption diversity, the average markup, and indirect utility can be established as follows. Using (12) and (16), we can rewrite indirect utility as

$$U = \alpha \left[\frac{1}{(k+1)(\kappa_1 + \kappa_2)} - 1 \right] \left\{ \frac{2\pi \overline{h}}{\theta^2 L} \left[1 - \left(1 + \theta\sqrt{L/\pi} \right) \mathrm{e}^{-\theta\sqrt{L/\pi}} \right] \right\} \left(\frac{L}{\mu^{\max}} \right)^{\frac{1}{k+1}}. \quad (20)$$

The term in braces in (20) equals the effective labor supply per capita, h, which decreases with L. The last term in expression (20) captures the cutoff productivity level, $1/m^d$, which increases with L. The net effect of an increase in L on the indirect utility U is thus ambiguous, highlighting the trade-off between a dispersion force (urban frictions) and an agglomeration force (tougher firm selection) inherent in our model. Yet, it can be shown that U is single-peaked with respect to L as in Henderson (1974). Since the indirect utility is proportional to N, it immediately follows that consumption diversity also exhibits a $\cap$-shaped pattern, while the average markup $\overline{\Lambda}$ is $\cup$-shaped with respect to population size L.

Observe that for now in our model, larger cities are more productive because of tougher selection, but not because of technological externalities associated with agglomeration. In line with an abundant empirical literature (e.g., Rosenthal and Strange, 2004), we extend our framework to allow for such agglomeration economies in Section 6.

3.2 Urban system: Multiple cities

We now turn to the urban system with multiple cities. The timing of events is as follows. First, workers/consumers choose their locations. Second, given the population distribution across cities, firm entry, selection and production take place.[9] We start the analysis by deriving the market equilibrium conditions for given city sizes, and then define the spatial equilibrium where individuals endogenously choose their locations.

3.2.1 Market equilibrium

There are three sets of market equilibrium conditions in the urban system. For each city, LMC and ZEP can be written analogously as in the single city setup. In addition, trade

[9] This timing simplifies our model because we need not specify which types of firms relocate as workers move across cities. The spatial sorting of firms or workers is not the topic of the present paper.

must be balanced for each city, which requires that the total value of exports equals the total value of imports.

As in the single city case, we assume Pareto distributions for productivity draws. The shape parameter $k \geq 1$ is assumed to be identical, but the upper bounds are allowed to vary across cities, i.e., $G_r(m) = (m/m_r^{\max})^k$. Under this assumption, the market equilibrium conditions – LMC, ZEP, and the trade balance – can be written as follows:

$$N_r^E \left[\frac{\kappa_1}{\alpha\left(m_r^{\max}\right)^k} \sum_s L_s \tau_{rs} \left(\frac{\tau_{ss}}{\tau_{rs}} \frac{w_s}{w_r} m_s^d \right)^{k+1} + F \right] = S_r. \tag{21}$$

$$\mu_r^{\max} = \sum_s L_s \tau_{rs} \left(\frac{\tau_{ss}}{\tau_{rs}} \frac{w_s}{w_r} m_s^d \right)^{k+1}, \tag{22}$$

$$\frac{N_r^E w_r}{\left(m_r^{\max}\right)^k} \sum_{s \neq r} L_s \tau_{rs} \left(\frac{\tau_{ss}}{\tau_{rs}} \frac{w_s}{w_r} m_s^d \right)^{k+1} = L_r \sum_{s \neq r} \tau_{sr} \frac{N_s^E w_s}{\left(m_s^{\max}\right)^k} \left(\frac{\tau_{rr}}{\tau_{sr}} \frac{w_r}{w_s} m_r^d \right)^{k+1}. \tag{23}$$

where $\mu_r^{\max} \equiv \left[\alpha F \left(m_r^{\max}\right)^k\right]/\kappa_2$ denotes technological possibilities. Note that $\mu_r^{\max}$ is city-specific, and captures the local production amenities that are not transferable across space.

The $3 \times K$ conditions (21)–(23) depend on $3 \times K$ unknowns: the wages w_r, the masses of entrants N_r^E, and the internal cutoffs m_r^d. The external cutoffs m_{rs}^x can be recovered from (7). Combining (21) and (22), we can immediately show that

$$N_r^E = \frac{\kappa_2}{\kappa_1 + \kappa_2} \frac{S_r}{F}, \tag{24}$$

which implies that more firms choose to enter in larger cities in equilibrium. Adding the term in r that is missing on both sides of (23), and using (22) and (24), we obtain the following equilibrium relationship:

$$\frac{h_r}{\left(m_r^d\right)^{k+1}} = \sum_s S_s \tau_{rr} \left(\frac{\tau_{rr}}{\tau_{sr}} \frac{w_r}{w_s} \right)^k \frac{1}{\mu_s^{\max}}. \tag{25}$$

The $2 \times K$ conditions (22) and (25) summarize how wages, cutoffs, technological possibilites, trade costs, population sizes, and effective labor supplies are related in the market equilibrium. Using those expressions, it can be shown that the mass of varieties consumed in city r is inversely proportional to the internal cutoff, and proportional to the effective labor supply per capita in that city:

$$N_r^c = \frac{\alpha}{(\kappa_1 + \kappa_2)\tau_{rr}} \frac{h_r}{m_r^d}. \tag{26}$$

Furthermore, the (expenditure share) weighted average of markups that consumers face in city r can be expressed as follows:

$$\overline{\Lambda}_r \equiv \frac{\sum_s N_s^E \int_0^{m_{sr}^x} \dfrac{p_{sr}(m)q_{sr}(m)}{E_r} \Lambda_{sr}(m) \mathrm{d}G_s(m)}{\sum_s N_s^E G_s(m_{sr}^x)} = \frac{\kappa_3 \tau_{rr}}{\alpha} \frac{m_r^d}{h_r}. \tag{27}$$

It follows from (26) and (27) that there are pro-competitive effects, since $\overline{\Lambda}_r$ decreases with the mass N_r^c of competing firms in city r as $\overline{\Lambda}_r = [\kappa_3/(\kappa_1+\kappa_2)](1/N_r^c)$. Last, the indirect utility is given by

$$U_r = \frac{\alpha}{\tau_{rr}} \left[\frac{1}{(k+1)(\kappa_1+\kappa_2)} - 1 \right] \frac{h_r}{m_r^d} = \left[\frac{1}{(k+1)(\kappa_1+\kappa_2)} - 1 \right] \frac{\kappa_3}{\overline{\Lambda}_r}, \tag{28}$$

which implies that greater effective labor supply per capita, $h_r = S_r/L_r$, tougher selection, and a lower average markup in city r translate into higher indirect utility. Alternatively, the indirect utility can be rewritten as $U_r = [1/(k+1) - (\kappa_1+\kappa_2)]N_r^c$, i.e., it is proportional to the mass of varieties consumed in city r.

3.2.2 *Spatial equilibrium*

We now move to the spatial equilibrium where individuals endogenously choose their locations. We introduce city-specific amenities and taste heterogeneity in residential location into our model. This is done for two reasons. First, individuals in reality choose their location not only based on wages, prices, and productivities that result from market interactions, but also based on non-market features such as amenities (e.g., climate or landscape). Second, individuals do not necessarily react in the same way to regional gaps in wages and cost-of-living (Tabuchi and Thisse, 2002; Murata, 2003). Such taste heterogeneity tends to offset the extreme outcome that often arises in typical NEG models, namely that *all* mobile economic activity concentrates in a single city. When we take our model to data, taste heterogeneity is thus useful for capturing an observed non-degenerate equilibrium distribution of city sizes.

We assume that the location choice of an individual ℓ is based on linear random utility $V_r^\ell = U_r + A_r + \xi_r^\ell$, where U_r is given by (28) and A_r subsumes city-specific amenities that are equally valued by all individuals. For the empirical implementation, we assume that $A_r \equiv A(A_r^o, A_r^u)$, where A_r^o refers to observed amenities such as costal location and A_r^u to the unobserved part. The random variable ξ_r^ℓ then captures idiosyncratic taste differences in residential location. Following McFadden (1974), we assume that the ξ_r^ℓ are i.i.d. across individuals and cities according to a double exponential distribution with zero mean and variance equal to $\pi^2\beta^2/6$, where β is a positive constant. Since β has a positive relationship with variance, the larger the value of β, the more heterogeneous are the consumers' attachments to each city. Given the population distribution, an individual's probability of choosing city r can then be expressed as a logit form:

$$\mathbb{P}_r = \Pr\left(V_r^\ell > \max_{s \neq r} V_s^\ell\right) = \frac{\exp((U_r + A_r)/\beta)}{\sum_{s=1}^K \exp((U_s + A_s)/\beta)}. \tag{29}$$

If $\beta \to 0$, which corresponds to the case without taste heterogeneity, people choose their location based only on $U_r + A_r$, i.e., they choose a city with the highest $U_r + A_r$ with probability one. By contrast, if $\beta \to \infty$, individuals choose their location with equal probability $1/K$. In that case, taste for residential location is extremely heterogeneous, so that $U_r + A_r$ does not affect location decisions at all.

A *spatial equilibrium* is defined as a city-size distribution satisfying

$$\mathbb{P}_r = \frac{L_r}{\sum_{s=1}^K L_s}, \quad \forall r. \tag{30}$$

In words, a spatial equilibrium is a fixed point where the choice probability of each city is equal to that city's share of the economy's total population.[10]

3.3 *The impact of spatial frictions: An example with two cities*

To build intuition for our counterfactual experiments, we consider an example with two cities, as is standard in the literature. The formal analysis is in the supplementary online appendix, whereas the main text focuses on the intuition of how spatial frictions affect the fundamental trade-off between agglomeration and dispersion forces.

We assume that trade costs are symmetric ($\tau_{12} = \tau_{21} = \tau$ and $\tau_{11} = \tau_{22} = t$), and that intra-city trade is less costly than inter-city trade ($t \leq \tau$). The market equilibrium for any given city sizes L_1 and L_2 is then uniquely determined, and yields the relative wage $\omega \equiv w_1/w_2$ and the two internal cutoffs m_1^d and m_2^d.

Now suppose that city 1 is larger than city 2 ($L_1 > L_2$) while the two cities are identical with respect to their gross labor supplies per capita ($\overline{h}_1 = \overline{h}_2 = \overline{h}$), commuting technologies ($\theta_1 = \theta_2 = \theta$), and technological possibilities ($\mu_1^{\max} = \mu_2^{\max} = \mu^{\max}$). Then, the market equilibrium is such that *the larger city has the higher wage* ($\omega > 1$) *and the lower cutoff* ($m_1^d < m_2^d$). The intuition is that – due to trade frictions – firms in the larger city

[10] In theory, there can of course be multiple city-size distributions satisfying (30). However, this is not an issue given the aim of our paper. Indeed, in Section 4, where we take our model to data, we use the observed US city sizes for the spatial equilibrium to be uniquely determined.

have an advantage in terms of local market size, and this advantage must be offset by the higher wage and the tougher selection in equilibrium.

Turning to choice probabilities, for any given city sizes L_1 and L_2, (29) can be written as

$$\mathbb{P}_1 = \frac{\exp(\Upsilon/\beta)}{\exp(\Upsilon/\beta)+1} \quad \text{and} \quad \mathbb{P}_2 = \frac{1}{\exp(\Upsilon/\beta)+1},$$

where $\Upsilon \equiv (U_1 - U_2) + (A_1 - A_2)$. Hence, $\mathbb{P}_1$ is increasing and $\mathbb{P}_2$ is decreasing in Υ. Plugging (28) into the definition of Υ, we readily obtain

$$\Upsilon = \left(\frac{\alpha}{t}\right)\left[\frac{1}{(k+1)(\kappa_1+\kappa_2)} - 1\right]\left(\frac{h_1}{m_1^d} - \frac{h_2}{m_2^d}\right), \tag{31}$$

where we set $A_1 = A_2$ for simplicity. Recalling that $L_1 > L_2$, the lower cutoff in city 1 ($m_1^d < m_2^d$) constitutes an agglomeration force as it raises the indirect utility difference Υ. Yet, due to urban frictions, the larger city also has lower effective labor supply per capita ($h_1 < h_2$), which negatively affects Υ, thus representing a dispersion force.

For the population distribution $L_1 > L_2$ to be a spatial equilibrium, condition (30) requires that $\mathbb{P}_1 > \mathbb{P}_2$, which in turn implies $\Upsilon > 0$ and $h_1/m_1^d > h_2/m_2^d$ by (31). The larger city then has greater consumption diversity ($N_1^c > N_2^c$) according to (26) and a lower average markup ($\overline{\Lambda}_1 < \overline{\Lambda}_2$) according to (27) than the smaller city. Taking such a spatial equilibrium as the starting point, we now consider what happens if either urban frictions or trade frictions are eliminated.

No urban frictions. Our first counterfactual experiment will be to eliminate urban frictions while leaving trade frictions unchanged. This is equivalent to setting $\theta = 0$, holding τ and t constant. In what follows, we consider how Υ is affected by such a change. This allows us to study if eliminating urban frictions involves more agglomeration (larger $\mathbb{P}_1$) or more dispersion (smaller $\mathbb{P}_1$). Let $\widetilde{\Upsilon}$ be the value of Υ in the counterfactual scenario, keeping city sizes fixed at their initial levels. Other counterfactual values are also denoted with a tilde. Observing that $\widetilde{h}_1 = \widetilde{h}_2 = \overline{h}$ when $\theta = 0$, we have

$$\text{sign}\left\{\widetilde{\Upsilon} - \Upsilon\right\} = \text{sign}\left\{\frac{1}{\widetilde{m}_1^d}(\overline{h} - h_1) - \frac{1}{\widetilde{m}_2^d}(\overline{h} - h_2) + h_1\left(\frac{1}{\widetilde{m}_1^d} - \frac{1}{m_1^d}\right) - h_2\left(\frac{1}{\widetilde{m}_2^d} - \frac{1}{m_2^d}\right)\right\}. \tag{32}$$

The first two terms in (32) stand for the direct effects of eliminating urban frictions. In the initial situation where $\theta > 0$, we know that $h_1 < h_2 < \overline{h}$ as urban frictions are greater in the larger city. We also know that $m_1^d < m_2^d$ holds even without urban frictions as $L_1 > L_2$, so that $\widetilde{m}_1^d < \widetilde{m}_2^d$. Hence, the first positive term always dominates the second negative term, thus showing that the direct effects favor the large city by increasing the probability $\mathbb{P}_1$ of choosing city 1. However, eliminating urban frictions also induces indirect effects through the cutoffs, which are captured by the second two terms in (32). Both of these terms are negative and thus work in the opposite direction than the direct effects. Specifically, it can be shown that setting $\theta = 0$ implies $m_1^d < \widetilde{m}_1^d < \widetilde{m}_2^d < m_2^d$. That is, average productivity goes down in the larger city when the population distribution is held fixed, while it goes up in the smaller city.[11]

If the direct effects dominate the indirect effects, we have $\widetilde{\Upsilon} > \Upsilon$ so that $\mathbb{P}_1$ increases and the large city becomes even larger as urban frictions are eliminated. The increase in population then leads to a productivity gain, which may offset the productivity drop at a given population size. As we show below, such a pattern indeed emerges in the quantified multi-city model (see Figures 1, 2, and 4): big cities like New York become even larger. Holding the initial population fixed, productivity goes down in New York, while it goes up once we take population changes into account, as shown in Figure 4. By the same argument, small cities may end up with a lower productivity due to their loss in population. Hence, eliminating urban frictions makes the productivity change in the economy as a whole ambiguous.

No trade frictions. Our second counterfactual experiment will be to eliminate trade frictions while leaving urban frictions unchanged. More specifically, we consider a scenario where consumers face the same trade costs for local and non-local varieties. This is equivalent to setting $\tau = t$, holding θ constant. As before, let $\widetilde{\Upsilon}$ be the value of Υ in the counterfactual scenario, while keeping city sizes fixed at the initial level. Noting that h_1 and h_2 remain constant, the change in Υ can now be written as

$$\text{sign}\left\{\widetilde{\Upsilon} - \Upsilon\right\} = \text{sign}\left\{h_1\left(\frac{1}{\widetilde{m}_1^d} - \frac{1}{m_1^d}\right) - h_2\left(\frac{1}{\widetilde{m}_2^d} - \frac{1}{m_2^d}\right)\right\}. \tag{33}$$

It can be shown that now *both* cutoffs decrease for given population sizes, i.e., $\widetilde{m}_1^d < m_1^d$ and $\widetilde{m}_2^d < m_2^d$. Both cities, therefore, experience a productivity gain. The first term in brackets in (33) is thus positive and the second term is negative. Yet it can be shown that $\widetilde{\Upsilon} < \Upsilon$ holds if $\mu_2^{\max}/\mu_1^{\max} \leq (h_2/h_1)^{k+1}$. In other words, the large city becomes smaller

[11] The reason is the following: the reduction of θ from any given positive value to zero raises aggregate labor supply S_r in both cities. The increase is relatively stronger in the larger city (S_1/S_2 goes up), so that the relative wage ω increases. To offset this, the equilibrium cutoff must thus *increase* in the larger city and *decrease* in the smaller city.

if the two cities are not too different in terms of their technological possibilities. In the simple case where $\mu_2^{\max}/\mu_1^{\max} = 1$, the large city always becomes smaller as $h_2/h_1 > 1$. In contrast, the small city becomes larger and, consequently, experiences a stronger productivity gain than the large city. We show below that such a pattern also emerges in our quantified multi-city model (see Figures 5 and 6).[12]

4. Quantification

We now take our multi-city model to the data by estimating or calibrating its parameters. This procedure can be divided into two broad stages, namely the quantification of the *market equilibrium* and that of the *spatial equilibrium*, which we now explain in turn.

4.1 Market equilibrium

The quantification of the market equilibrium consists of the following five steps:

1. Using the definition of total effective labor supply and data on commuting time, hours worked, and city size at the MSA level, we obtain the city-specific commuting technology parameters $\widehat{\theta}_r$ that constitute *urban frictions*.

2. Using the specification $\tau_{rs} \equiv d_{rs}^{\gamma}$, where d_{rs} is the distance from r to s, we estimate a gravity equation that relates the value of bilateral trade flows to distance. For a given value of the Pareto shape parameter k, we obtain the distance elasticity $\widehat{\gamma}$ that constitutes *trade frictions*.

3. The estimated distance elasticity, together with data on labor supply, value added per worker, and city size, allows us to back out the set of city-specific technological possibilities $\widehat{\mu}_r^{\max}$ and wages $\widehat{w}_r$ that are consistent with the market equilibrium conditions.

4. Using the set of city-specific technological possibilities thus obtained, we draw a large sample of firms from within the model to compute the difference between the simulated and observed establishment size distributions.

[12]Other two-region NEG models with commuting costs (Tabuchi, 1998; Murata and Thisse, 2005) would come to qualitatively similar conclusions about how falling transport or commuting costs affect the spatial equilibrium. Helpman (1998) considers a fixed supply of land instead of commuting, but his model would also display a similar pattern as falling transport costs are dispersive, while greater abundance of land is agglomerative. Though useful for illustrative purposes, such two-region examples do not convey a sense of magnitude about the quantitative importance of spatial frictions in practice, however. Sections 4–6 of this paper deals precisely with this issue.

5. Iterating through steps 2 to 4, we search over the parameter space to find the value of the Pareto shape parameter k that minimizes the sum of squared differences between the simulated and observed establishment size distributions.

Several details about this procedure and the data are relegated to the supplementary online appendix. As for the quantification results, our iterative procedure yields the Pareto shape parameter $\widehat{k} = 6.4$. Columns 1 and 2 of Table 1 below show that, despite having only a single degree of freedom, the fit of the simulated establishment size distribution to the observed establishment size distribution is quite good.

Turning to spatial frictions, we obtain an estimate for the commuting technology parameter that constitutes *urban frictions* for each MSA. As shown in Table 4 in the supplementary online appendix, the value of $\widehat{\theta}_r$ ranges from 0.0708 (Los Angeles-Long Beach-Santa Ana and New York-Northern New Jersey-Long Island) and 0.0867 (Chicago-Naperville-Joliet) to 0.9995 (Yuba City, CA) and 1.4824 (Hinesville-Fort Stewart, GA). Thus, big cities tend to have better commuting technologies per unit of distance.[13] For *trade frictions*, our fixed effects estimation of the gravity equation yields $\widehat{\gamma k} = 1.2918$ (with standard error 0.0271) which, given $\widehat{k} = 6.4$, implies $\widehat{\gamma} = 0.2018$.

We then obtain the values of the technological possibilities $\widehat{\mu}_r^{\max}$, which may be viewed as a measure for MSA-level production amenities. Table 4 in the supplementary online appendix reports those values, along with the observed MSA populations scaled by their mean (i.e., $L_r/\overline{L}$) and average productivities ($1/\overline{m}_r$). From the quantification procedure we also obtain the wages $\widehat{w}_r$ that are consistent with the market equilibrium conditions, which we compare to the observed wages at the MSA-level in Section 4.3. Ultimately, the quantification of the market equilibrium allows us to measure the indirect utility $\widehat{U}_r$ from (28) by using data on $h_r = S_r/L_r$ and m_r^d, as well as the estimate of $\widehat{\tau}_{rr}$.

4.2 Spatial equilibrium

Using the spatial equilibrium conditions (30), the expression of indirect utility $\widehat{U}_r$, and data on observed amenities A_r^o, we obtain a measure for unobserved amenities A_r^u and the relative weight of indirect utility and amenities for individual location decisions that are consistent with the observed city-size distribution.

[13]For any given distance x from the CBD, a smaller θ implies that people spend less time to commute to the CBD. However, this does not necessarily mean that average commuting time is shorter in larger cities because of longer average commuting distances. Our finding that big cities tend to have better commuting technologies also holds when assuming a linear commuting technology as in Murata and Thisse (2005).

Setting $U_1 + A_1 \equiv 0$ as a normalization, and using the observed L_r for the 356 MSAs, the spatial equilibrium conditions $\mathbb{P}_r = L_r/L$ for $r = 2,3,\ldots,K$ can be *uniquely* solved for $(U_r + A_r)/\beta$.[14] We thus obtain the values of $(U_r + A_r)/\beta$ that replicate the observed city-size distribution as a spatial equilibrium. Let $\widehat{D}_r$ denote this solution satisfying

$$\mathbb{P}_r = \frac{\exp(\widehat{D}_r)}{\sum_{s=1}^{K} \exp(\widehat{D}_s)} = \frac{L_r}{L}, \quad \widehat{D}_1 = 0. \tag{34}$$

Having solved (34) for $\widehat{D}_r$, we then gauge the relative importance of indirect utility $\widehat{U}_r$ and observed amenities A_r^o in consumers' location choices by estimating a simple OLS regression as follows,

$$\widehat{D}_r = \alpha_0 + \alpha_1 \widehat{U}_r + \alpha_2 A_r^o + \varepsilon_r, \tag{35}$$

which yields

$$\widehat{D}_r = \underset{(0.2644)}{-0.2194} + \underset{(0.5289)}{1.7481^{***}} \widehat{U}_r + \underset{(0.0199)}{0.0652^{***}} A_r^o + \widehat{\varepsilon}_r. \tag{36}$$

Consistent with theory, both indirect utility and observed amenities significantly influence the spatial distribution of population across MSAs, both coefficients being positive. The fitted residuals $\widehat{\varepsilon}_r$ can be interpreted as a measure of the unobserved part of the MSA amenities. We hence let $\widehat{A}_r^u \equiv \widehat{\varepsilon}_r$ which by construction is uncorrelated with A_r^o. In Section 6.2, we discuss the robustness of our results with respect to the value of α_1.

Table 4 in the supplementary online appendix reports the observed and unobserved consumption amenities, as well as the production amenities. Several points are worth emphasizing. First, in contrast to Roback (1982) type approaches, spatial patterns of MSA-level consumption and production amenities ($\widehat{A}_r^u$ and $\widehat{\mu}_r^{\max}$) are derived from a quantified spatial equilibrium framework where trade frictions are explicitly taken into account. Second, both observed and unobserved consumption amenities are positively correlated with city size, the correlation being stronger for the latter (0.7023) than for the former (0.1334). Third, while the correlation between A_r^o and $\widehat{A}_r^u$ is zero by construction, there is also little correlation between technological possibilities and each type of consumption amenities (-0.0867 and 0.0713 for A_r^o and $\widehat{A}_r^u$, respectively). This is consistent with the results by Chen and Rosenthal (2008) who find that good business locations in the US need not have good consumption amenities.

[14] See the supplementary online appendix for the proof of uniqueness.

4.3 MSA- and firm-level model fit

Before turning to the counterfactual analysis, it is important to point out that our model can replicate several empirical facts, both at the MSA and firm levels, that have not been used in the quantification procedure. We briefly summarize some of those dimensions and again relegate several details of this model fit analysis to the supplementary online appendix.

First, since our key objective is to investigate the importance of urban and trade frictions, having an idea of how well our model captures empirical facts about these dimensions is particularly important.

Urban frictions. We first consider urban frictions by comparing the 'model-based' and observed aggregate land rents. The former can be obtained by making use of (13). The latter is, in turn, obtained by $\text{ALR}_r = \text{GMR}_r/(1 - \text{ownershare}_r)$, where GMR is the (aggregate) gross monthly rent.[15] The simple correlation between the model-based and observed aggregate land rents across MSAs is 0.9805, while the Spearman rank correlation is 0.9379. Alternatively, we can use $\text{ALR}_r = \text{ERV}_r/(\text{ownershare}_r)$, where ERV_r is the equivalent rent value for houses that are owned. Under this alternative formula, the correlation between the model-based and observed aggregate land rents becomes 0.9624, while the Spearman rank correlation is 0.9129. In all cases, the correlations are high, thus suggesting that our model does a good job in capturing urban frictions across MSAs.[16]

Trade frictions. We next turn to trade frictions. Note that our estimate of the distance elasticity $\widehat{\gamma k}$ for the year 2007 closely matches the value of 1.348 reported by Hillberry and Hummels (2008) from estimation of a gravity equation at the 3-digit zip code level using the confidential CFS microdata. We can further assess to what extent our model can

[15]The formula can be obtained as follows. First, the total amount of expenditure in housing services (ALR) is given by the sum of the gross monthly rent (GMR) and the equivalent rent value for houses that are owned (ERV). Data on GMR, which can be decomposed as (average rent) $\times$ (number of houses that are rented), is available. Now assume that GMR/(number of houses rented) $=$ ERV/(number of houses owned) holds in each city at equilibrium by arbitrage. We then obtain ALR $=$ GMR/$(1 -$ share of houses that are owned).

[16]One might argue that our simple monocentric city model is not the most appropriate specification as large MSAs are usually polycentric. To see how urban frictions relate to polycentricity, we compute a simple correlation between $\widehat{\theta}_r$ and the number of employment centers in each MSA for the year 2000 as identified by Arribas-Bel and Sanz Gracia (2010). The correlation is -0.4282, while the Spearman rank correlation is -0.5643, thus suggesting that our monocentric model with city-specific commuting technology captures the tendency that larger cities are more efficient for commuting as they allow for more employment centers, thereby reducing the average commuting distance through employment decentralization.

cope with existing micro evidence on the spatial structure of shipping patterns. As shown in the supplementary online appendix, both aggregate shipment values and the number of shipments predicted by our model fall off very quickly with distance – becoming very small beyond a threshold of about 200 miles – whereas price per unit first rises with distance and average shipment values do not display a clear pattern. These results are nicely in line with those in Hillberry and Hummels (2008). Furthermore, we can also compare shipping shares and shipping distances by establishment size class predicted by our model, and the observed counterparts as reported by Holmes and Stevens (2012). Our model can qualitatively reproduce their observed shipment shares. It can also explain their finding that the mean distance shipped increases with establishment size.

Second, the correlation between actual relative wages and those predicted by our model is 0.7379 and thus reasonably high.

Third, the representative firm sample drawn from our quantified model can replicate the observed distribution of establishments across MSAs. Table 1 reports the mean, standard deviation, minimum, and maximum of the number of establishments (top part) and average establishment size (bottom part) at the MSA level, and the number of establishments is further broken down by employment size. The last column of Table 1 reports the correlation between the observed and our simulated data. As can be seen, the simple cross-MSA correlation for the total number of establishments is 0.7253, with a slightly larger rank correlation of 0.733. Again, these are reasonably large numbers. Furthermore, the correlations between the observed and the predicted numbers of medium-sized and large establishments across MSAs are particularly large (between 0.889 and 0.9412).

Table 1: Cross-MSA distribution of establishment numbers and average size – summary for observed and simulated data.

	Mean		St.dev.		Min		Max		Correlation
Variable	Model	Observed	Model	Observed	Model	Observed	Model	Observed	Model-Observed
# of establishments total	18067.10	18067.09	16878.09	43138.45	1738	911	109210	541255	0.7253
# of establishments size 1-19	15444.74	15461.97	12066.43	37449.79	1550	804	79181	478618	0.3824
# of establishments size 20-99	2121.56	2162.09	6320.64	4728.28	49	93	52178	51310	0.9412
# of establishments size 100-499	429.83	397.50	1729.44	922.34	14	13	24365	9951	0.8890
# of establishments size 500+	70.94	45.52	132.67	113.75	2	1	1509	1376	0.9320
Avg establishment size	11.73	15.40	11.63	2.60	0.90	6.40	131.88	23.70	0.1716

Notes: Model values are computed from a representative sample of 6,431,886 establishments. See supplementary online appendix F.2 for a detailed description.

5. Counterfactuals

Having shown that our quantified model performs well in replicating several features of the data, we now use it for counterfactual analysis. Our aim is to assess the importance of spatial frictions for the US city-size distribution, for individual city sizes, as well as for the distributions of productivity and markups across MSAs. To this end we eliminate urban frictions or trade frictions (counterfactuals CF1 and CF2, respectively).

5.1 No urban frictions

In the first counterfactual experiment (which we call 'no urban frictions'), we set all commuting-related frictions – and hence all land rents – to zero ($\widehat{\theta}_r = 0$ for all r) while keeping trade frictions $\widehat{\tau}_{rs}$, technological possibilities $\widehat{\mu}_r^{\max}$, consumption amenities (A_r^o and $\widehat{A}_r^u$), and the location choice parameters $\widehat{\alpha}_0$, $\widehat{\alpha}_1$, and $\widehat{\alpha}_2$ constant.[17] This corresponds to a hypothetical world where only goods are costly to transport while living in cities does not impose any urban costs. Comparing the counterfactual equilibrium for this scenario to the initial spatial equilibrium is then a meaningful exercise, as it provides *bounds* to what extent the actual US economic geography is shaped by urban frictions.

City sizes. Starting with city sizes, eliminating urban frictions leads to (gross) cross-MSA population movements of about 4 million people, i.e., 1.6% of the total MSA population in our sample. Figure 1 plots percentage changes in MSA population against the initial log MSA population. As can be seen, large cities would on average gain population, whereas small and medium-sized cities tend to lose. In other words, urban frictions limit the size of large cities. The size of New York, for example, would increase by about 8.5%. That is to say, urban frictions matter for the size of New York, as the city is 8.5% smaller than it would be in a hypothetical world without urban frictions. Some MSAs close to New York and Boston are affected even more by urban frictions. For example, New Haven-Milford, CT, is 12.1% smaller and Bridgeport-Stamford-Norwalk, CT, is even 15.9% smaller than it would be. The top panel of Figure 2 further indicates that the impacts of urban frictions follow a rich spatial pattern and are highly unevenly spread across MSAs.

Interestingly, although the sizes of individual cities would be substantially different in a world without urban frictions, the city-size distribution would be almost the same. This is shown in Figure 3. A standard rank-size rule regression reveals that the coefficient on log size rises slightly from -0.9249 to -0.9178, the change being statistically insignificant.[18] The hypothetical elimination of urban frictions would thus move single cities up or down

[17] Although workers are mobile in our model, we can set urban frictions to zero without having degenerate equilibria with full agglomeration. The reason is that, as explained before, consumers' location choice probabilities are expressed as a logit so that no city disappears.

[18] We follow Gabaix and Ibragimov (2011) and adjust the rank by subtracting $1/2$.

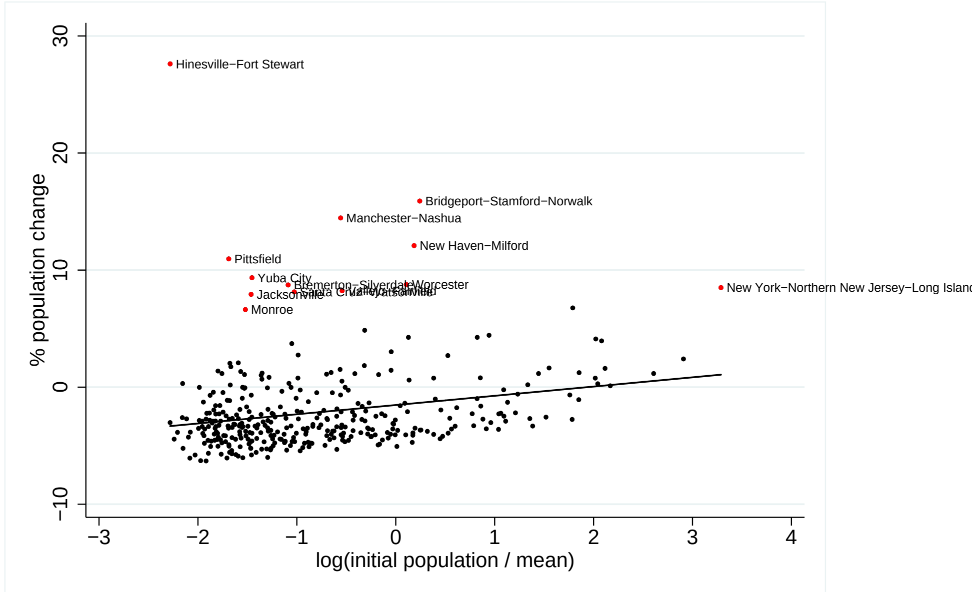

Figure 1: Changes in MSA populations and initial size (CF1)

in the urban hierarchy, but within a stable city-size distribution. We will discuss this stability in greater depth below in Section 5.3.

Productivity. Turning to average productivity, the middle panel of Figure 2 shows that the impact of urban frictions differs substantially across cities. New York's productivity is 0.76% higher in the counterfactual equilibrium. Urban frictions thus have a negative impact on productivity as they limit the size of New York. However, most MSAs would have a *lower* productivity level if urban frictions were eliminated, for example small cities like Monroe, MI, by 0.9%. This means that the presence of urban frictions in the real world leads to a higher productivity as population is retained in those cities. Computing the nation-wide productivity change, weighted by MSA population shares in the initial equilibrium, we find that eliminating urban frictions would increase average productivity by a mere 0.04%.

It is important to see that these results refer to the *long-run* impacts of eliminating urban frictions on productivity, as they include the effects of population movements. To gauge the contribution of labor mobility to these overall impacts, we disentangle the *short-run* effects, before the population reshufflings have taken place, from the long-run effects. The left panel of Figure 4 illustrates the cutoff changes across MSAs when eliminating urban frictions, holding city sizes fixed at their initial levels. It shows that the cutoffs m_r^d rise, on average, in larger cities. However, as can be seen from the right panel of Figure 4, the

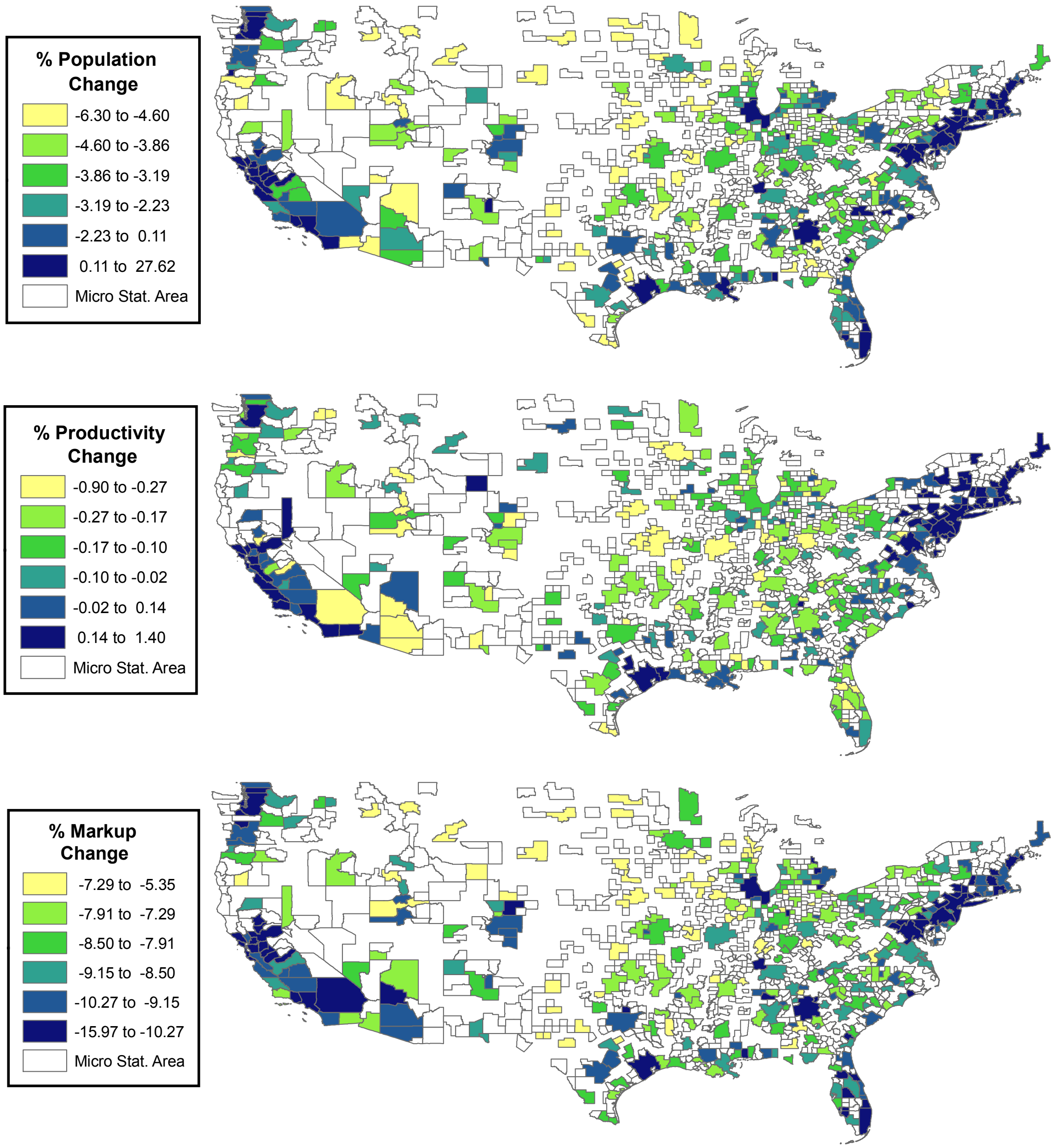

Figure 2: Spatial pattern of counterfactual changes in L_r, $1/m_r^d$ and $\overline{A}_r$ (CF1)

subsequent movements of population (which flows toward the larger cities), more than offset this initial change, thereby generating larger productivity gains in the bigger cities in the long-run equilibrium.[19] This decomposition of the short- and long-run effects can also be related to the comparative static results of Section 3.3. There, we have shown that the instantaneous impact of reducing urban frictions – keeping L_r fixed – is to raise the cutoff in the large city and to lower it in the small city. This pattern can get reversed,

[19]Some simple OLS regressions of the change in m_r^d in the short- and in the long-run on inital population yield: $\Delta m_r^d = -0.0821^{***} + 0.0127^{***} L_r$ in the short-run, and $\Delta m_r^d = \ \ 0.0817^{***} - 0.0194^{***} L_r$ in the long-run, thus showing the switch in the results depending on whether or not population is mobile.

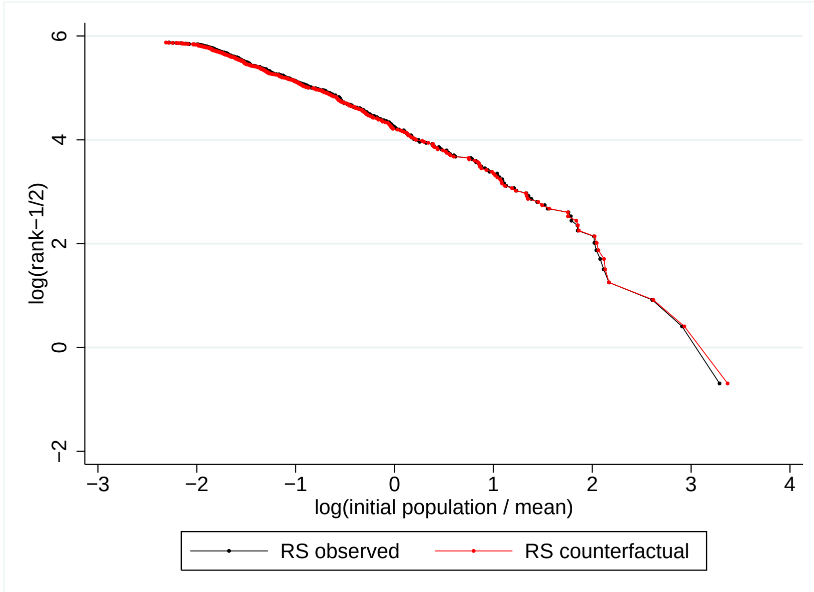

Figure 3: Rank-size rule, observed and counterfactual (CF1)

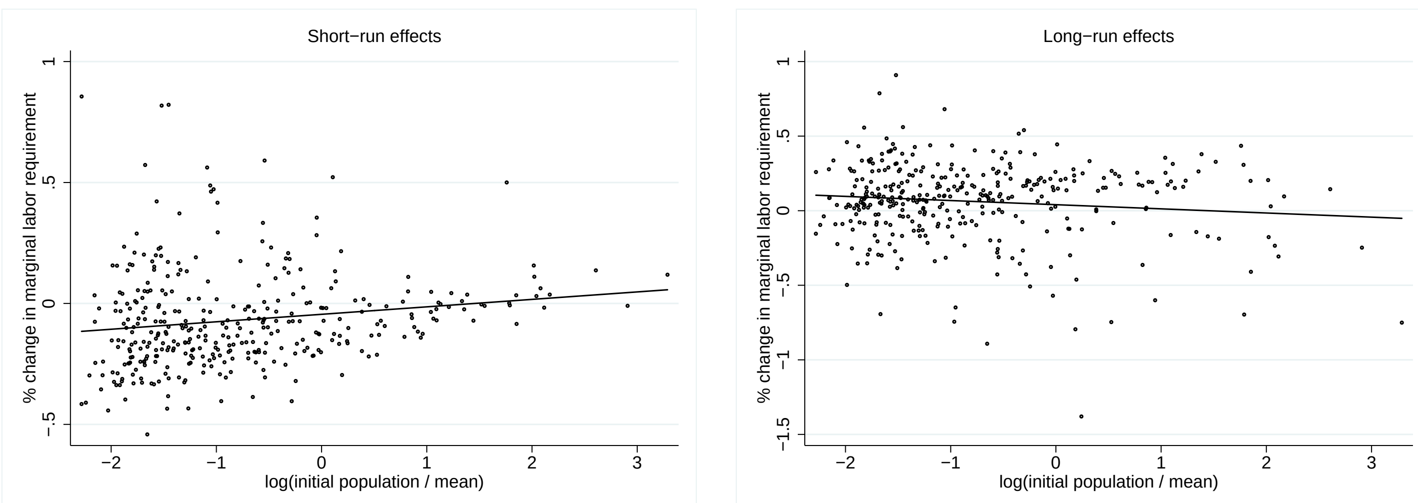

Figure 4: Difference in short- and long-run relationships between Δm_r^d and L_r (CF1)

however, once the population movements are taken into account.

Markups. Turning to the long-run impact on markups, the bottom panel of Figure 2 reveals that this is the dimension where the largest changes take place. Markups would decrease everywhere, with reductions ranging from 5.3% to about 16%, but the more so for the most populated areas of the East and West coasts. As can be seen from (27), the reason for these large changes is twofold. First, eliminating urban frictions increases the effective labor supply per capita h_r everywhere, which allows for more firms in each MSA and, therefore, for more competition. Second, there is an effect going through the cutoffs. Some places see their cutoffs fall, especially larger cities which receive population inflows, and this puts additional pressure on markups there. In contrast, cutoffs increase in cities that lose population. However, even in those cities it turns out that markups decrease, as the pro-competitive effect due to higher effective labor supply per capita dominates the anti-competitive effect of the higher cutoff.

To summarize, even without urban frictions, the city-size distribution would remain fairly stable, despite the fact that larger cities tend to grow and smaller cities tend to shrink. Furthermore, the 'no urban frictions' case supports more firms, which reduces markups and expands product diversity, though firms are not on average much more productive than in a world with urban frictions. The productivity gap between large and small cities would, however, widen.

5.2 *No trade frictions*

How do trade frictions shape the US economic geography? To address this question, we set external trade costs from s to r equal to internal trade costs in r ($\tau_{sr} = \tau_{rr}$ for all r and s) in the second counterfactual experiment (which we call 'no trade frictions'). This experiment corresponds to a hypothetical world where consumers face the same trade costs for local and non-local varieties.[20]

City sizes. Starting with city sizes, eliminating trade frictions would lead to significant (gross) cross-MSA population movements of about 10.2 million people, i.e., 4.08% of the total MSA population in our sample. Some small and remote cities would gain substantially. For example, the population of Casper, WY, would grow by about 105% and that of Hinesville-Fort Stewart, GA, by about 99.4%. That is, trade frictions limit the size of small and remote cities substantially. Figure 5 plots the percentage changes in MSA population against the initial log MSA population. Consistent with the comparative static results of Section 3.3, in a world without trade frictions larger cities lose ground and individuals move, on average, to smaller cities to relax urban costs. These changes are depicted in the top panel of Figure 6. Although individual cities would be substantially affected by the fall in trade frictions, the city-size distribution remains again quite stable,

[20]Eaton and Kortum (2002) consider a similar counterfactual scenario in the context of international trade with a fixed population distribution. We have also experimented with setting $\tau_{rs} = \tau_{rr}$ for all r and s, which corresponds to a hypothetical world where goods are as costly to trade between MSAs as within MSAs from the firms' perspective. The results are largely the same.

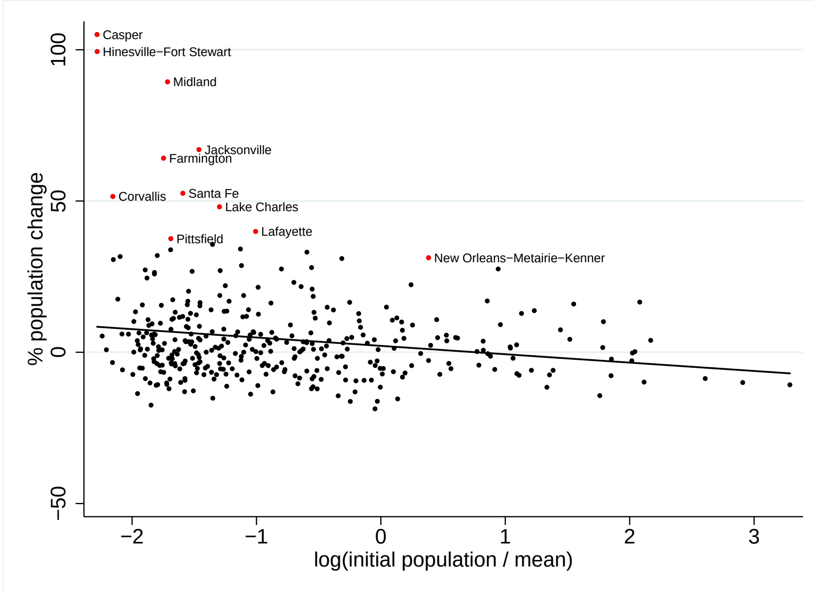

Figure 5: Changes in MSA populations and initial size (CF2)

as can be seen from Figure 7. The coefficient on log size drops from -0.9249 to -0.9392, yet this change is again statistically insignificant.

Productivity. Concerning the changes in average productivity, observe first that all MSAs gain. In other words, the existence of trade frictions in the real world causes productivity losses for the US economy. Yet, as can be seen from the middle panel of Figure 6, these impacts are unevenly spread across MSAs. If trade frictions were eliminated, some small cities would gain substantially (e.g., an increase of about 125.5% in Great Falls, MT), while large cities would gain significantly less: 41.18% in New York, 48.08% in Los Angeles, and 55.71% in Chicago. The first reason is linked to market access. Indeed, the more populated areas, e.g., those centered around California and New England, would be those gaining the least from a reduction of trade frictions, as they already provide firms with a good access to a large local market. The second reason is that, as stated above, large cities tend to lose population, thereby reducing the productivity gains brought about by the fall in trade frictions. Computing the nation-wide productivity change, weighted by MSA population shares in the initial equilibrium, we find that eliminating trade frictions would increase average productivity by 67.59%. Thus, reducing spatial frictions for shipping goods would entail substantial aggregate productivity gains.

Markups. The bottom panel of Figures 6 reveals that markups would decrease considerably in a world without trade frictions, with reductions ranging from 29% to 55%. Such

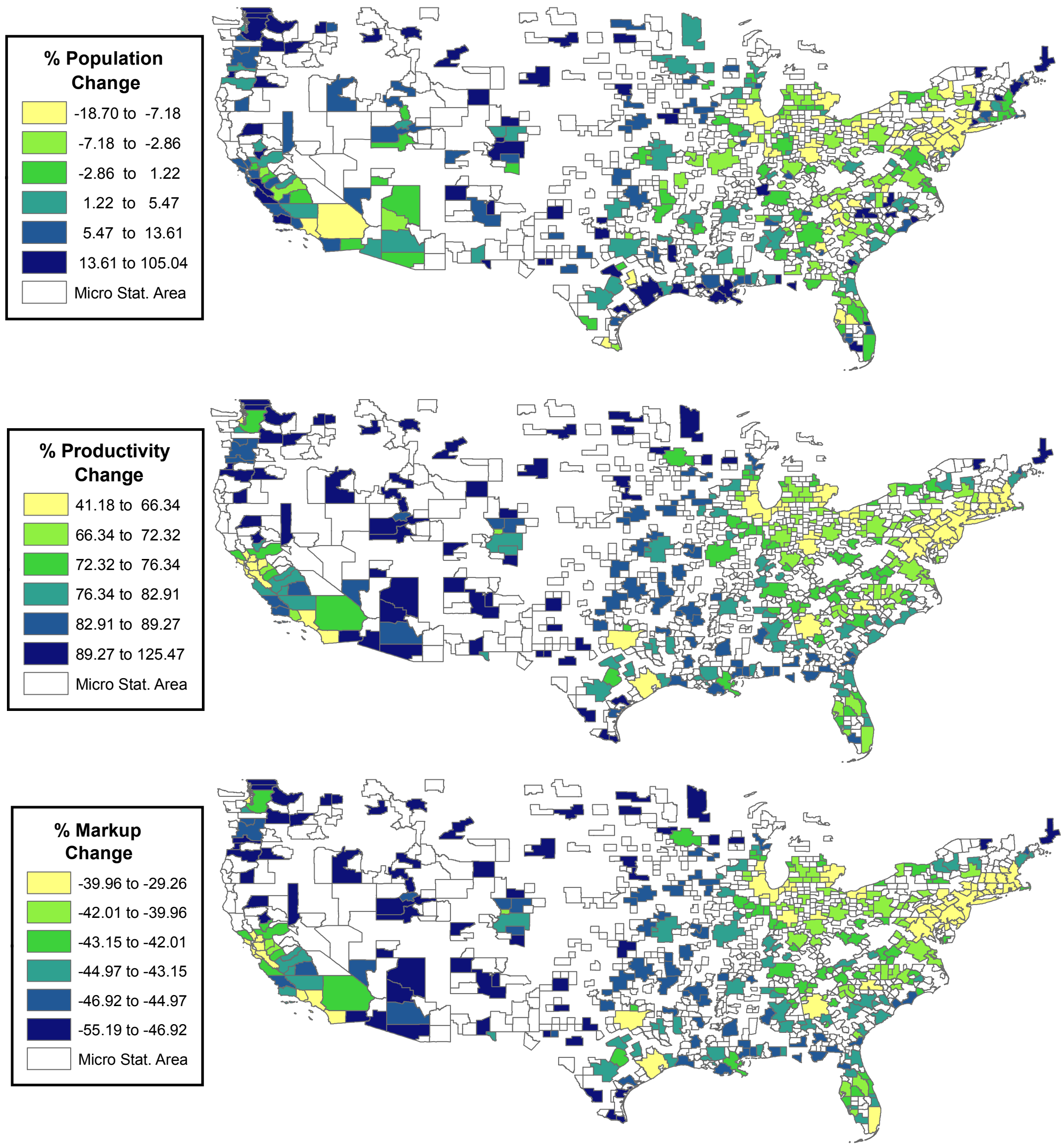

Figure 6: Spatial pattern of counterfactual changes in L_r, $1/m_r^d$ and $\overline{\Lambda}_r$ (CF2)

reductions are particularly strong in MSAs with poor market access, i.e., the center of the US and the areas close to the borders. Observe that the changes in markups – though substantial – are more compressed than the changes in productivity (the coefficient of variation for productivity changes is 0.18, while that for changes in markups is 0.09). The reason is the following. Eliminating trade frictions reduces cutoffs in all MSAs, but especially in small and remote ones. This puts downward pressure on markups. Yet, there is also an indirect effect through changes in effective labor supply h_r. An increase in h_r, which occurs in big cities that lose population, reduces markups more strongly than

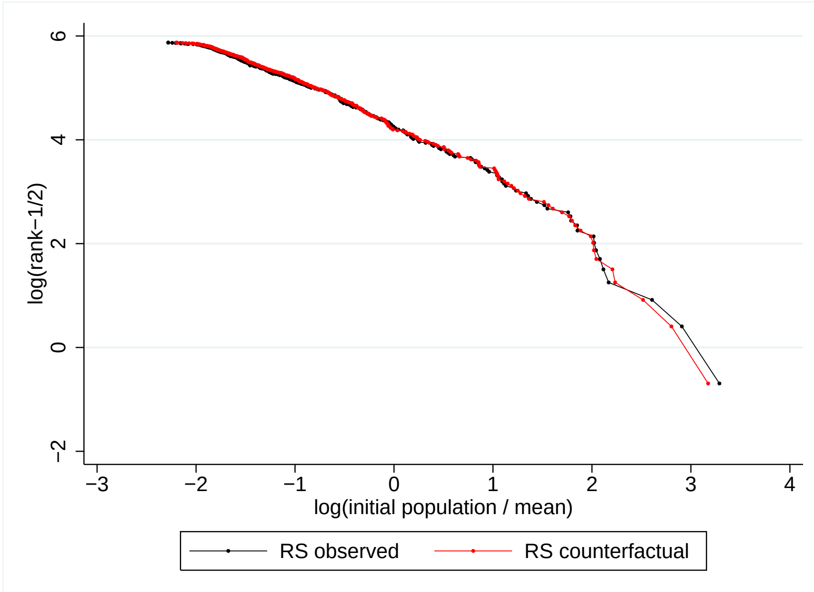

Figure 7: Rank-size rule, observed and counterfactual (CF2)

what is implied by the direct change only, while the decrease in h_r that occurs in small and remote cities gaining population works in the opposite direction and dampens the markup reductions.

To summarize, without trade frictions, the city-size distribution would remain fairly stable, despite the fact that larger cities tend to shrink and smaller cities tend to grow. Furthermore, the 'no trade frictions' case allows for higher average productivity and lower markups by intensifying competition in all MSAs, and especially in small and remote ones. The productivity gap between large and small cities would, hence, shrink.

5.3 *How important are spatial frictions?*

Our paper is, to the best of our knowledge, the first to investigate the impact of both urban and trade frictions on the size distribution of cities.[21] A key novel insight of our analysis is that spatial frictions have a quite limited impact on that distribution. The rank-size rule would still hold with a statistically identical coefficient in a world without urban or trade frictions.

[21] The influential models on the city-size distribution by Gabaix (1999), Eeckhout (2004), Duranton (2007) and Rossi-Hansberg and Wright (2007) include urban costs but assume away trade costs. None of these papers analyzes how the city-size distribution is affected by urban frictions. The most closely related paper in that respect is Desmet and Rossi-Hansberg (2013). Yet, their framework is not suited to investigate the impact of trade frictions on the city-size distribution, as it also abstracts from trade costs.

Note that our result on the stability of the city-size distribution contrasts with that of Desmet and Rossi-Hansberg (2013), who find that the size distribution tilts substantially when urban frictions are reduced. The difference in results can be understood as follows. In their analysis, the commuting friction parameter is common to all MSAs, whereas we allow commuting technologies to differ across cities. In our setting, big cities like New York or Los Angeles tend to have the best commuting technologies per unit of distance in the initial equilibrium, so that the impacts of setting $\widehat{\theta}_r = 0$ are relatively small there. By contrast, in Desmet and Rossi-Hansberg (2013), the commuting technology improves equally across all MSAs so that big cities get very large due to larger efficiency gains in commuting than in our case. Another key difference is that in Desmet and Rossi-Hansberg (2013), all consumers react in the same way to changes in utility and amenities, whereas those reactions are idiosyncratic in our model and, therefore, less extreme.

Although spatial frictions hardly affect the city-size distribution in our framework, they do matter for the sizes of individual cities within that stable distribution. Indeed, eliminating spatial frictions leads to aggregate (gross) inter-MSA reallocations of about 4–10 million people. Whether or not large or small cities gain population crucially depends on what type of spatial frictions is eliminated. Urban frictions limit the size of large cities, whereas trade frictions limit the size of small cities. As extensively discussed above, our approach is able to quantify those effects.

Notice that we have so far considered *simultaneous* reductions in spatial frictions for all cities. We can also look at a *unilateral* reduction for a single city. Specifically, let us briefly consider two additional counterfactuals. In the first one, we only eliminate urban frictions for New York. In that case, New York grows by about 19.73% (i.e., by about 3.7 million people). In the second one, we set $\tau_{sr} = \tau_{rr}$ for all s only when r is New York. That is, we improve the market access to New York for all firms that are located elsewhere, while holding the market access of firms located in New York to other MSAs constant. In that case, New York shrinks remarkably by 15.57% (i.e., about 3 million people). Hence, a unilateral change in spatial frictions for a single city has a much larger impact on the size of that city. More generally, these results show that the *relative levels across cities* of both types of frictions matter a lot to understand the sizes of individual cities.

Finally, our experiments show that urban and trade frictions matter, though to a different extent, for the distributions of productivity and markups – and ultimately welfare – across MSAs. Eliminating trade frictions would lead to significant productivity gains and substantially reduced markups. These changes are highly heterogeneous across space and tend to reduce differences in productivity and city sizes across MSAs. Concerning urban frictions, their elimination would not give rise to such significant productivity gains, but would still considerably intensify competition and generate lower markups by allowing for more firms in equilibrium.

6. Extensions and robustness

6.1 Agglomeration economies

The recent literature shows that agglomeration economies, i.e., productivity gains due to larger or denser urban areas, are a prevalent feature of the spatial economy (see Rosenthal and Strange, 2004; Melo *et al.*, 2010). We have so far focused entirely on one channel: larger cities are more productive because of tougher firm selection. Yet, larger or denser cities can become more productive for various other reasons such as sharing–matching–learning externalities (Duranton and Puga, 2004), and sorting by human capital (Combes *et al.*, 2008; Behrens *et al.*, 2010). In fact, Combes *et al.* (2012) have argued that the productivity advantage of large cities is mostly due to such agglomeration externalities.

We illustrate a simple way to extend our framework to include agglomeration economies. Specifically, we allow the upper bound in each MSA ($m_r^{\max}$) to be a function of the density of that MSA. Agglomeration economies are thus modeled as a right-shift in the *ex ante* productivity distribution: upon entry, a firm in a denser MSA has a higher probability of getting a better productivity draw.[22] Starting from the baseline model, assume that technological possibilities $\mu_r^{\max}$ can be expressed as $\mu_r^{\max} = c \cdot \text{density}_r^{-k\xi} \cdot \psi_r^{\max}$, where $\text{density}_r \equiv L_r/\text{surface}_r$, ξ is the elasticity of the *ex ante* upper bound of the marginal labor requirement with respect to density, and $\psi_r^{\max}$ is an idiosyncratic measure of technological possibilities that is purged from agglomeration effects. We can then estimate the ex ante productivity advantage of large cities by running a simple log-log regression of $\widehat{\mu}_r^{\max}$ on MSA population densities and a constant, which yields:

$$\ln(\widehat{\mu}_r^{\max}) = \underset{(0.3566)}{2.6898^{***}} - \underset{(0.0813)}{0.1889^{**}} \ln(\text{density}_r).$$

Since $\ln \mu_r^{\max} = k \ln m_r^{\max}$ plus a constant, the elasticity ξ of $m_r^{\max}$ with respect to density is given by $0.1889/\widehat{k} = 0.0295$ which is the value we use in what follows. In words, doubling MSA density reduces the upper bound (and, equivalently, the mean by the

[22] Formally, the right-shift in the *ex ante* productivity distribution implies that the distribution in a denser MSA first-order stochastically dominates that in a less dense MSA. Observe that firm selection afterwards acts as a truncation, so that the *ex post* distribution is both right-shifted and truncated.

properties of the Pareto distribution) of the *ex ante* marginal labor requirement of entrants by 2.95%. That figure, though computed for the ex ante distribution, lies within the consensus range of previous elasticity estimates for agglomeration economies measured using ex post productivity distributions (see Melo *et al*., 2010). This effect is independent of the subsequent truncation of the ex post productivity distribution, thus disentangling agglomeration from selection.

In the supplementary online appendix, we show how those agglomeration economies can be taken into account in the quantification of our model. We then run both counterfactuals ('no urban frictions' and 'no trade frictions') with the agglomeration economies specification. The results are summarized in the bottom panel of Table 2 (labeled CF3 and CF4, respectively). As can be seen, the results change little compared to our previous specification without agglomeration economies (reported in the top panel). Observe that this finding does not mean that agglomeration economies are unimportant. The reason why they do not matter much in our experiments is that not so many people move between the initial and the counterfactual equilibria. Yet, given the measured elasticities of agglomeration economies, much larger population movements would be required for them to become quantitatively more visible.

Table 2: Summary of the counterfactuals.

Baseline counterfactuals (no agglomeration economies)						
	No urban frictions (CF1)			No trade frictions (CF2)		
	Mean	Std. dev.	Weighted mean	Mean	Std. dev.	Weighted mean
% change $1/\overline{m}_r$	-0.06	0.26	0.04	78.50	14.26	67.59
% change L_r	-2.15	3.60	0	4.30	15.28	0
% change $\overline{\Lambda}_r$	-8.79	1.82	-9.85	-43.55	4.27	-39.90
% change V_r	9.69	2.24	10.98	78.17	13.79	67.62
RS coefficient	-0.9178			-0.9392		
Robustness checks (with agglomeration economies)						
	No urban frictions (CF3)			No trade frictions (CF4)		
	Mean	Std. dev.	Weighted mean	Mean	Std. dev.	Weighted mean
% change $1/\overline{m}_r$	-0.12	0.31	0.04	78.71	14.03	67.63
% change L_r	-2.21	3.74	0	4.50	16.15	0
% change $\overline{\Lambda}_r$	-8.74	1.89	-9.85	-43.60	4.33	-39.90
% change V_r	9.62	2.33	10.98	78.36	14.03	67.66
RS coefficient	-0.9176			-0.9394		

Notes: Weighted mean refers to the mean percentage change where the weights are given by the MSAs' initial population shares. The counterfactual scenarios CF3 and CF4 include the agglomeration economies specification. RS coefficient refers to the slope of the estimated rank-size relationship.

6.2 *Amenities and inter-city population reallocations*

The quantification of our model suggests that amenities and regional attachment are important for shaping the city-size distribution. One may thus wonder how important the estimated value of α_1 is for our qualitative and quantitative results. More specifically, the value of α_1 in (35) determines the relative weight of indirect utility and amenities, and any omitted variable will lead to a biased estimate of this relative weight. Hence, it could be the case that our relatively small population movements in response to shocks to spatial frictions are driven by too low an estimate of α_1. To see that our results – both qualitatively and to a large extent also quantitatively – are not very sensitive to the value of α_1, we consider the following 'no trade frictions' counterfactual.[23] We scale up the estimate $\widehat{\alpha}_1$ by either 50% or 100% and recompute the new values for the unobserved amenities, keeping $\widehat{\alpha}_0$, $\widehat{\alpha}_2$, $\widehat{D}_r$, $\widehat{U}_r$, and A_r^o constant. Using the larger values of α_1 and the new (smaller) unobserved amenities, we run the counterfactual scenario and look at how different the implied changes are. A larger value of α_1 is expected to deliver larger population movements as agents become more sensitive to differences in prices, wages, and consumption diversity across MSAs.

Whether we increase α_1 by 50% or by 100%, the city-size distribution remains fairly stable, with the Zipf coefficient going from -0.9249 to -0.9399 or to -0.9376 (see Figure 8 for the latter case). The total (gross) population movement is 14,943,005 or 19,459,006, respectively, which amounts to 5.98% or 7.78% of the urban population (recall the corresponding number in the baseline case is 4.08%). Hence, larger values of α_1 lead to greater population reallocations when trade frictions are eliminated, as people are more sensitive to indirect utility differences across cities. The changes in individual city sizes range from -26.31% to 179.78% with a 50% increase in α_1, and from -33.00% to 269.25% with a 100% increase.[24] The spatial patterns (not depicted here for the sake of brevity) look fairly similar to those in the benchmark case.

Those findings suggest that our main results are robust, both qualitatively and to a large extent quantitatively, to higher values of α_1. In particular, amenities do not matter for the city-size distribution to remain stable between the initial and counterfactual equilibria because that distribution is hardly affected even when we greatly reduce the importance of amenities relative to indirect utility in consumers' location choices.

However, amenities do matter for replicating the observed initial city-size distribution. To see this, we briefly consider a similar counterfactual exercise as in Desmet and Rossi-Hansberg (2013) and set all unobserved amenities across cities equal to their mean, holding all spatial frictions fixed. Figure 9 shows that there would be a substantial tilt of the city-size distribution. The Zipf coefficient falls from -0.9249 to -3.6715, and about a half of the US MSA population move, leading to a much less unequal city-size distribution

[23] We also considered the 'no urban frictions' counterfactual obtaining similar insights.

[24] The changes in productivity range from 25.85% to 49.18% in the former case, and from 25.80% to 49.26% in the latter case.

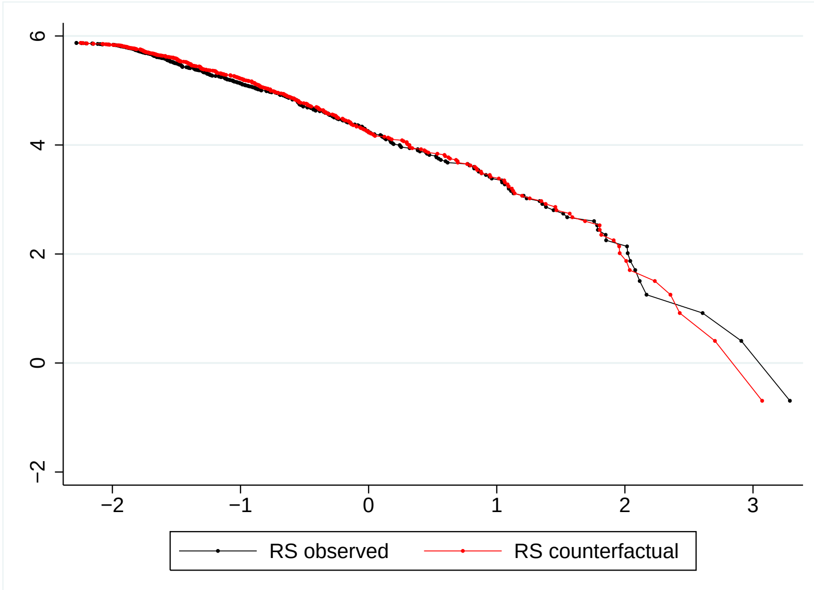

Figure 8: Changes in the city-size distribution (robustness, increasing α_1 by 100% in CF2)

– large cities shrink and small cities grow.[25]

7. Conclusions

We have developed a novel general equilibrium model of a spatial economy with multiple cities and endogenous location decisions. Using 2007 US data at the state and MSA levels, we have quantified our model using all of its market and spatial equilibrium conditions, as well as a gravity equation for trade flows and a logit model for consumers' location choice probabilities. The quantified model performs well and is able to replicate – both at the MSA and firm levels – a number of empirical features that are linked to urban and trade frictions.

To assess the importance of spatial frictions, we have used our model to study two counterfactual scenarios. Those allow us to trace out the impacts of both trade and urban frictions on the city-size distribution, the sizes of individual cities, as well as on productivity and competition across space. A first key insight is that the city-size distribution is hardly affected by the presence of either trade or urban frictions. A second key insight is that, within the stable distribution, the sizes of individual cities can be affected substantially by changes in spatial frictions. Last, our third key insight is that their presence imposes quite significant welfare costs. The reasons are too high

[25]We also experimented with setting all technological possibilities equal to the mean. In that case, 5.57% of the population moves and there is no strong impact on the city-size distribution.

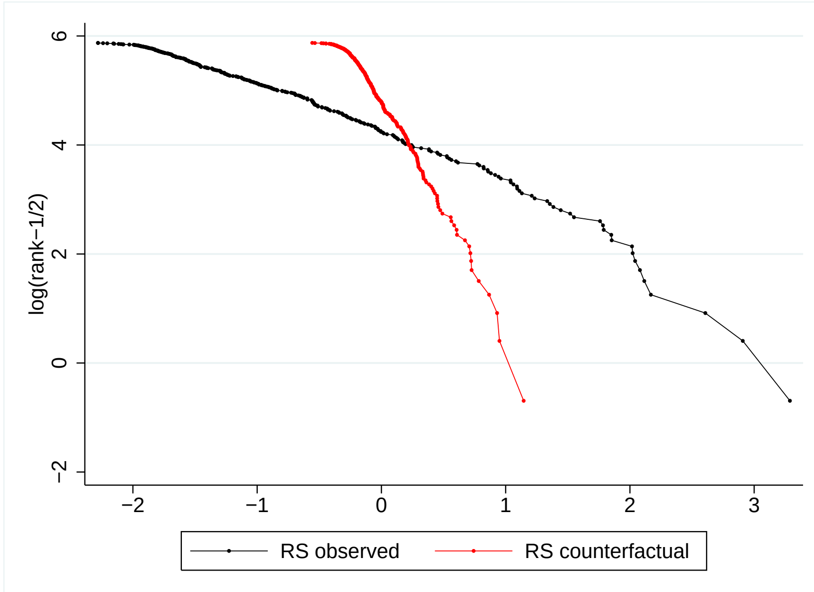

Figure 9: Changes in the city-size distribution (equal amenities case)

price-cost margins and, depending on the type of spatial frictions we consider, foregone productivity or reduced product diversity.

Our approach brings various strands of literature closer together. In particular, our model: (i) considers trade and urban frictions that are identified as being relevant by the NEG and urban economics literature; (ii) endogenizes productivity, markups, and product diversity, three aspects that loom large in the recent trade literature; (iii) allows to deal with heterogeneity along several dimensions (across space, across firms, across consumers); (iv) can be readily brought to data in very a self-contained way; and (v) fits quite nicely features of the data not used in the quantification stage. We believe that our framework provides a useful starting point for further general equilibrium counterfactual analysis.

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Supplementary Online Appendices, not intended for publication

The Appendix is structured as follows: **Appendix A** shows how to derive the demand functions (2) and the firm-level variables (9) using the Lambert W function. In **Appendix B** we provide integrals involving the Lambert W function and derive the terms $\{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$ that are used in the paper. **Appendix C** contains proofs and computations for the single city case. In **Appendix D** we derive the equilibrium conditions (21)–(23) and provide further derivations for the multi-city case. **Appendix E** deals with the example with two cities. **Appendix F** provides details about the quantification procedure, the data used, and the different elements of model fit. **Appendix G** proves that the spatial equilibrium is uniquely determined in our quantification procedure. **Appendix H** describes the procedure for conducting counterfactual analyses with our quantified framework, while **Appendix I** spells out the procedure with agglomeration economies. Finally, **Appendix J** reports some additional results tables.

Appendix A: Demand functions and firm-level variables.

A.1. Derivation of the demand functions (2). Letting λ stand for the Lagrange multiplier, the first-order condition for an interior solution to the maximization problem (1) satisfies

$$\alpha \mathrm{e}^{-\alpha q_{sr}(i)} = \lambda p_{sr}(i), \quad \forall i \in \Omega_{sr} \tag{A-1}$$

and the budget constraint $\sum_s \int_{\Omega_{sr}} p_{sr}(k) q_{sr}(k) \mathrm{d}k = E_r$. Taking the ratio of (A-1) for $i \in \Omega_{sr}$ and $j \in \Omega_{vr}$ yields

$$q_{sr}(i) = q_{vr}(j) + \frac{1}{\alpha} \ln \left[\frac{p_{vr}(j)}{p_{sr}(i)} \right] \quad \forall i \in \Omega_{sr}, \forall j \in \Omega_{vr}.$$

Multiplying this expression by $p_{vr}(j)$, integrating with respect to $j \in \Omega_{vr}$, and summing across all origins v we obtain

$$q_{sr}(i) \sum_v \int_{\Omega_{vr}} p_{vr}(j) \mathrm{d}j = \underbrace{\sum_v \int_{\Omega_{vr}} p_{vr}(j) q_{vr}(j) \mathrm{d}j}_{\equiv E_r} + \frac{1}{\alpha} \sum_v \int_{\Omega_{vr}} \ln \left[\frac{p_{vr}(j)}{p_{sr}(i)} \right] p_{vr}(j) \mathrm{d}j. \tag{A-2}$$

Using $\bar{p}_r \equiv (1/N_r^c) \sum_v \int_{\Omega_{vr}} p_{vr}(j) \mathrm{d}j$, expression (A-2) can be rewritten as follows:

$$\begin{aligned} q_{sr}(i) &= \frac{E_r}{N_r^c \bar{p}_r} - \frac{1}{\alpha} \ln p_{sr}(i) + \frac{1}{\alpha N_r^c \bar{p}_r} \sum_v \int_{\Omega_{vr}} \ln\left[p_{vr}(j)\right] p_{vr}(j) \mathrm{d}j \\ &= \frac{E_r}{N_r^c \bar{p}_r} - \frac{1}{\alpha} \ln\left[\frac{p_{sr}(i)}{N_r^c \bar{p}_r}\right] + \frac{1}{\alpha} \sum_v \int_{\Omega_{vr}} \ln\left[\frac{p_{vr}(j)}{N_r^c \bar{p}_r}\right] \frac{p_{vr}(j)}{N_r^c \bar{p}_r} \mathrm{d}j, \end{aligned}$$

which, given the definition of η_r, yields (2).

A.2. Derivation of the firm-level variables (9) and properties of W. Using $p_s^d = m_{rs}^x \tau_{rs} w_r$, the first-order conditions (6) can be rewritten as

$$\ln\left[\frac{m_{rs}^x \tau_{rs} w_r}{p_{rs}(m)}\right] = 1 - \frac{\tau_{rs} m w_r}{p_{rs}(m)}.$$

Taking the exponential of both sides and rearranging terms, we have

$$\mathrm{e} \frac{m}{m_{rs}^x} = \frac{\tau_{rs} m w_r}{p_{rs}(m)} \mathrm{e}^{\frac{\tau_{rs} m w_r}{p_{rs}(m)}}.$$

Noting that the Lambert W function is defined as $\varphi = W(\varphi)\mathrm{e}^{W(\varphi)}$ and setting $\varphi = \mathrm{e} m / m_{rs}^x$, we obtain

$$W\left(\mathrm{e} \frac{m}{m_{rs}^x}\right) = \frac{\tau_{rs} m w_r}{p_{rs}(m)},$$

which implies $p_{rs}(m)$ as given in expression (9). The expression for the quantities $q_{rs}(m) = (1/\alpha)\left[1 - \tau_{rs} m w_r / p_{rs}(m)\right]$ and the expression for the operating profits $\pi_{rs}(m) = L_s q_{rs}(m)\left[p_{rs}(m) - \tau_{rs} m w_r\right]$ are then straightforward to compute.

Turning to the properties of the Lambert W function, $\varphi = W(\varphi)\mathrm{e}^{W(\varphi)}$ implies that $W(\varphi) \geq 0$ for all $\varphi \geq 0$. Taking logarithms on both sides and differentiating yields

$$W'(\varphi) = \frac{W(\varphi)}{\varphi[W(\varphi) + 1]} > 0$$

for all $\varphi > 0$. Finally, we have $0 = W(0)\mathrm{e}^{W(0)}$, which implies $W(0) = 0$; and $\mathrm{e} = W(\mathrm{e})\mathrm{e}^{W(\mathrm{e})}$, which implies $W(\mathrm{e}) = 1$.

Appendix B: Integrals involving the Lambert W function.

To derive closed-form solutions for various expressions throughout the paper we need to compute integrals involving the Lambert W function. This can be done by using the change in variables suggested by Corless *et al.* (1996, p.341). Let

$$z \equiv W\left(\mathrm{e} \frac{m}{I}\right), \quad \text{so that} \quad \mathrm{e} \frac{m}{I} = z \mathrm{e}^z, \quad \text{where} \quad I = m_r^d, m_{rs}^x.$$

The subscript r can be dropped in the single city case. The change in variables then yields $\mathrm{d}m = (1+z)\mathrm{e}^{z-1}I\mathrm{d}z$, with the new integration bounds given by 0 and 1. Under our assumption of a Pareto distribution for productivity draws, the change in variables allows to rewrite integrals in simplified form.

B.1. First, consider the following expression, which appears when integrating firms' outputs:

$$\int_0^I m\left[1 - W\left(\mathrm{e}\frac{m}{I}\right)\right]\mathrm{d}G_r(m) = \kappa_1\left(m_r^{\max}\right)^{-k} I^{k+1},$$

where $\kappa_1 \equiv k\mathrm{e}^{-(k+1)}\int_0^1 (1-z^2)\left(z\mathrm{e}^z\right)^k \mathrm{e}^z\mathrm{d}z > 0$ is a constant term which solely depends on the shape parameter k.

B.2. Second, the following expression appears when integrating firms' operating profits:

$$\int_0^I m\left[W\left(\mathrm{e}\frac{m}{I}\right)^{-1} + W\left(\mathrm{e}\frac{m}{I}\right) - 2\right]\mathrm{d}G_r(m) = \kappa_2\left(m_r^{\max}\right)^{-k} I^{k+1},$$

where $\kappa_2 \equiv k\mathrm{e}^{-(k+1)}\int_0^1 (1+z)\left(z^{-1}+z-2\right)\left(z\mathrm{e}^z\right)^k \mathrm{e}^z\mathrm{d}z > 0$ is a constant term which solely depends on the shape parameter k.

B.3. Third, the following expression appears when deriving the (expenditure share) weighted average of markups:

$$\int_0^I m\left[W\left(\mathrm{e}\frac{m}{I}\right)^{-2} - W\left(\mathrm{e}\frac{m}{I}\right)^{-1}\right]\mathrm{d}G_r(m) = \kappa_3\left(m_r^{\max}\right)^{-k} I^{k+1},$$

where $\kappa_3 \equiv k\mathrm{e}^{-(k+1)}\int_0^1 (z^{-2}-z^{-1})(1+z)(z\mathrm{e}^z)^k\mathrm{e}^z\mathrm{d}z > 0$ is a constant term which solely depends on the shape parameter k.

B.4. Finally, the following expression appears when integrating firms' revenues:

$$\int_0^I m\left[W\left(\mathrm{e}\frac{m}{I}\right)^{-1} - 1\right]\mathrm{d}G_r(m) = \kappa_4\left(m_r^{\max}\right)^{-k} I^{k+1},$$

where $\kappa_4 \equiv k\mathrm{e}^{-(k+1)}\int_0^1 (z^{-1}-z)\left(z\mathrm{e}^z\right)^k \mathrm{e}^z\mathrm{d}z > 0$ is a constant term which solely depends on the shape parameter k. Using the expressions for κ_1 and κ_2, one can verify that $\kappa_4 = \kappa_1 + \kappa_2$.

Appendix C: Equilibrium in the single city case.

C.1. Existence and uniqueness of the equilibrium cutoff m^d. To see that there exists a unique equilibrium cutoff m^d, we apply the Leibniz integral rule to the left-hand side of (14) and use $W(\mathrm{e}) = 1$ to obtain

$$\frac{\mathrm{e}L}{\alpha(m^d)^2} \int_0^{m^d} m^2 \left(W^{-2} - 1\right) W' \mathrm{d}G(m) > 0,$$

where the sign comes from $W' > 0$ and $W^{-2} \geq 1$ for $0 \leq m \leq m^d$. Hence, the left-hand side of (14) is strictly increasing. This uniquely determines the equilibrium cutoff m^d, because

$$\lim_{m^d \to 0} \int_0^{m^d} m \left(W^{-1} + W - 2\right) \mathrm{d}G(m) = 0 \quad \text{and} \quad \lim_{m^d \to \infty} \int_0^{m^d} m \left(W^{-1} + W - 2\right) \mathrm{d}G(m) = \infty.$$

C.2. Indirect utility in the single city. To derive the indirect utility, we first compute the (unweighted) average price across all varieties. Multiplying both sides of (6) by $p(i)$, integrating over Ω, and using (3), we obtain:

$$\overline{p} = \overline{m}w + \frac{\alpha E}{N}$$

where $\overline{m} \equiv (1/N) \int_\Omega m(j)\mathrm{d}j$ denotes the average marginal labor requirement of the surviving firms. Using $\overline{p}$, expression (4) can be rewritten as

$$U = \frac{N}{k+1} - \frac{S}{L}\frac{\alpha}{m^d}, \tag{A-3}$$

where we use $E = (S/L)w$, $p^d = m^d w$ and $\overline{m} = [k/(k+1)]m^d$. When combined with (17) and (18), we obtain the expression for U as given in (19).

C.3. Single-peakedness of indirect utility in the single city case. We now show that U is single-peaked with respect to L. To this end, we rewrite the indirect utility (20) as $U = b(S/L)L^{1/(k+1)}$, where b is a positive constant capturing k, α, and $\mu^{\max}$, and then consider a log-transformation, $\ln U = \ln b + \ln S - [k/(k+1)] \ln L$. It then follows that

$$\frac{\partial \ln U}{\partial \ln L} = \frac{LS'}{S} - \frac{k}{k+1}.$$

To establish single-peakedness, we need to show that

$$\frac{LS'}{S} = \frac{\theta^2(L/\pi)}{2\left(\mathrm{e}^{\theta\sqrt{L/\pi}} - 1 - \theta\sqrt{L/\pi}\right)}$$

cuts the horizontal line $k/(k+1) \in (0,1)$ only once from above. Notice that $LS'/S \to 1$ as $L \to 0$, whereas $LS'/S \to 0$ as $L \to \infty$. Single-peakedness therefore follows if

$$\frac{\mathrm{d}}{\mathrm{d}L}\left(\frac{LS'}{S}\right) = -\frac{2+\theta\sqrt{L/\pi}+\mathrm{e}^{\theta\sqrt{L/\pi}}\left(\theta\sqrt{L/\pi}-2\right)}{(4/\theta^2)\left[\sqrt{\pi}\left(\mathrm{e}^{\theta\sqrt{L/\pi}}-1\right)-\theta\sqrt{L}\right]^2} < 0, \quad \forall L.$$

For this to be the case, the numerator must be positive. Let $y \equiv \theta\sqrt{L/\pi} > 0$. Then we can show that $H(y) \equiv 2+y+\mathrm{e}^y(y-2) > 0$ for all $y > 0$. Obviously, $H(0) = 0$. So, if $H' > 0$ for all $y > 0$, the proof is complete. It is readily verified that $H' = 1 + y\mathrm{e}^y - \mathrm{e}^y > 0$ is equivalent to $\mathrm{e}^{-y} > 1-y$, which is true for all y by convexity of e^{-y} (observe that $1-y$ is the tangent to e^{-y} at $y=0$ and that a convex function is everywhere above its tangent).

Appendix D: Equilibrium in the urban system.

D.1. Equilibrium conditions using the Lambert W function. By definition, the zero expected profit condition for each firm in city r is given by

$$\sum_s L_s \int_0^{m_{rs}^x} \left[p_{rs}(m) - \tau_{rs} m w_r\right] q_{rs}(m)\mathrm{d}G_r(m) = F w_r. \tag{D-1}$$

Furthermore, each labor market clears in equilibrium, which requires that

$$N_r^E \left[\sum_s L_s \tau_{rs} \int_0^{m_{rs}^x} m q_{rs}(m)\mathrm{d}G_r(m) + F\right] = S_r. \tag{D-2}$$

Last, in equilibrium trade must be balanced for each city

$$N_r^E \sum_{s\neq r} L_s \int_0^{m_{rs}^x} p_{rs}(m) q_{rs}(m)\mathrm{d}G_r(m) = L_r \sum_{s\neq r} N_s^E \int_0^{m_{sr}^x} p_{sr}(m) q_{sr}(m)\mathrm{d}G_s(m). \tag{D-3}$$

We now restate the foregoing conditions (D-1)–(D-3) in terms of the Lambert W function.

First, using (9), the labor market clearing condition can be rewritten as follows:

$$N_r^E \left\{\frac{1}{\alpha}\sum_s L_s \tau_{rs} \int_0^{m_{rs}^x} m\left[1 - W\left(\mathrm{e}\frac{m}{m_{rs}^x}\right)\right]\mathrm{d}G_r(m) + F\right\} = S_r. \tag{D-4}$$

Second, plugging (9) into (D-1), zero expected profits require that

$$\frac{1}{\alpha}\sum_s L_s \tau_{rs} \int_0^{m_{rs}^x} m\left[W\left(\mathrm{e}\frac{m}{m_{rs}^x}\right)^{-1} + W\left(\mathrm{e}\frac{m}{m_{rs}^x}\right) - 2\right]\mathrm{d}G_r(m) = F. \tag{D-5}$$

Last, the trade balance condition is given by

$$N_r^E w_r \sum_{s\neq r} L_s \tau_{rs} \int_0^{m_{rs}^x} m \left[W\left(\mathrm{e}\,\frac{m}{m_{rs}^x}\right)^{-1} - 1 \right] \mathrm{d}G_r(m)$$

$$= L_r \sum_{s\neq r} N_s^E \tau_{sr} w_s \int_0^{m_{sr}^x} m \left[W\left(\mathrm{e}\,\frac{m}{m_{sr}^x}\right)^{-1} - 1 \right] \mathrm{d}G_s(m). \qquad \text{(D-6)}$$

Applying the city-specific Pareto distribution $G_r(m) = (m/m_r^{\max})^k$ to (D-4)–(D-6) yields, using the results of Appendix B, expressions (21)–(23) given in the main text.

D.2. The mass of varieties consumed in the urban system. Using N_r^c as defined in (8), and the external cutoff and the mass of entrants as given by (7) and (24), and making use of the Pareto distribution, we obtain:

$$N_r^c = \frac{\kappa_2}{\kappa_1+\kappa_2}\left(m_r^d\right)^k \sum_s \frac{S_s}{F(m_s^{\max})^k}\left(\frac{\tau_{rr}}{\tau_{sr}}\frac{w_r}{w_s}\right)^k = \frac{\alpha}{\kappa_1+\kappa_2}\frac{\left(m_r^d\right)^k}{\tau_{rr}} \sum_s S_s \tau_{rr}\left(\frac{\tau_{rr}}{\tau_{sr}}\frac{w_r}{w_s}\right)^k \frac{\kappa_2}{\alpha F(m_s^{\max})^k}.$$

Using the definition of $\mu_s^{\max}$, and noting that the summation in the foregoing expression appears in the equilibrium relationship (25), we can then express the mass of varieties consumed in city r as given in (26).

D.3. The weighted average of markups in the urban system. Plugging (9) into the definition (27), the weighted average of markups in the urban system can be rewritten as

$$\overline{\Lambda}_r = \frac{1}{\alpha E_r \sum_s N_s^E G_s(m_{sr}^x)} \sum_s N_s^E \tau_{sr} w_s \int_0^{m_{sr}^x} m\left(W^{-2} - W^{-1}\right) \mathrm{d}G_s(m),$$

where the argument $\mathrm{e}m/m_{sr}^x$ of the Lambert W function is suppressed to alleviate notation. As shown in Appendix B, the integral term is given by $\kappa_3 (m_s^{\max})^{-k}(m_{sr}^x)^{k+1} = \kappa_3 G_s(m_{sr}^x) m_{sr}^x$. Using this, together with (7) and $E_r = (S_r/L_r)w_r$, yields the expression in (27).

D.4. Indirect utility in the urban system. To derive the indirect utility, we first compute the (unweighted) average price across all varieties sold in each market. Multiplying both sides of (6) by $p_{rs}(i)$, integrating over Ω_{rs}, and summing the resulting expressions across r, we obtain:

$$\overline{p}_s \equiv \frac{1}{N_s^c}\sum_r \int_{\Omega_{rs}} p_{rs}(j)\mathrm{d}j = \frac{1}{N_s^c}\sum_r \tau_{rs} w_r \int_{\Omega_{rs}} m_r(j)\mathrm{d}j + \frac{\alpha E_s}{N_s^c},$$

where the first term is the average of marginal delivered costs. Under the Pareto distribution, $\int_{\Omega_{sr}} m_s(j)\mathrm{d}j = N_s^E \int_0^{m_{sr}^x} m\mathrm{d}G_s(m) = [k/(k+1)]m_{sr}^x N_s^E G_s(m_{sr}^x)$. Hence, the (unweighted) average price can be rewritten for city r as follows

$$\overline{p}_r = \frac{1}{N_r^c}\sum_s \tau_{sr} w_s \left(\frac{k}{k+1}\right) m_{sr}^x N_s^E G_s(m_{sr}^x) + \frac{\alpha E_r}{N_r^c} = \left(\frac{k}{k+1}\right) p_r^d + \frac{\alpha E_r}{N_r^c}, \tag{D-7}$$

where we have used (8) and $p_r^d = \tau_{sr} w_s m_{sr}^x$. Plugging (D-7) into (4) and using (7), the indirect utility is then given by

$$U_r = \frac{N_r^c}{k+1} - \frac{\alpha}{\tau_{rr}} \frac{S_r}{L_r m_r^d},$$

which together with (26) and (27) yields (28).

Appendix E: The case with two cities.

E.1. Market equilibrium in the two city case. Recall that $\tau_{12} = \tau_{21} = \tau$, $\tau_{11} = \tau_{22} = t$, and $\tau \geq t$ by assumption. For given city sizes L_1 and L_2, the market equilibrium is given by a system of three equations (22)–(24) with three unknowns (the two internal cutoffs m_1^d and m_2^d, and the relative wage $\omega \equiv w_1/w_2$) as follows:

$$\mu_1^{\max} = L_1 t \left(m_1^d\right)^{k+1} + L_2 \tau \left(\frac{t}{\tau}\frac{1}{\omega} m_2^d\right)^{k+1} \tag{E-1}$$

$$\mu_2^{\max} = L_2 t \left(m_2^d\right)^{k+1} + L_1 \tau \left(\frac{t}{\tau}\omega m_1^d\right)^{k+1} \tag{E-2}$$

$$\omega^{2k+1} = \frac{\rho}{\sigma}\left(\frac{m_2^d}{m_1^d}\right)^{k+1}, \tag{E-3}$$

where $\rho \equiv \mu_2^{\max}/\mu_1^{\max}$ and $\sigma \equiv h_2/h_1 = (S_2/L_2)/(S_1/L_1)$.

When $\tau > t$, equations (E-1) and (E-2) can be uniquely solved for the cutoffs as a function of ω:

$$(m_1^d)^{k+1} = \frac{\mu_1^{\max}}{L_1 t}\frac{1-\rho(t/\tau)^k \omega^{-(k+1)}}{1-(t/\tau)^{2k}} \quad \text{and} \quad (m_2^d)^{k+1} = \frac{\mu_2^{\max}}{L_2 t}\frac{1-\rho^{-1}(t/\tau)^k \omega^{k+1}}{1-(t/\tau)^{2k}}. \tag{E-4}$$

Substituting the cutoffs (E-4) into (E-3) yields, after some simplification, the following expression:

$$\text{LHS} \equiv \omega^k = \rho \frac{S_1}{S_2}\frac{\rho - (t/\tau)^k \omega^{k+1}}{\omega^{k+1} - \rho\,(t/\tau)^k} \equiv \text{RHS}. \tag{E-5}$$

The RHS of (E-5) is non-negative if and only if $\underline{\omega} < \omega < \overline{\omega}$, where $\underline{\omega} \equiv \rho^{1/(k+1)} (t/\tau)^{k/(k+1)}$ and $\overline{\omega} \equiv \rho^{1/(k+1)} (\tau/t)^{k/(k+1)}$. Furthermore, the RHS is strictly decreasing in $\omega \in (\underline{\omega}, \overline{\omega})$ with $\lim_{\omega \to \underline{\omega}+}$ RHS $= \infty$ and $\lim_{\omega \to \overline{\omega}-}$ RHS $= 0$. Since the LHS of (E-5) is strictly increasing in $\omega \in (0, \infty)$, there exists a unique equilibrium relative wage $\omega^* \in (\underline{\omega}, \overline{\omega})$. The internal cutoffs are then uniquely determined by (E-4).

When $\tau = t$, we can also establish the uniqueness of ω, m_1^d and m_2^d. The proof is relegated to E.4. (i).

E.2. Market equilibrium: $L_1 > L_2$ implies $\omega > 1$ and $m_1^d < m_2^d$. Assume that $\overline{h}_1 = \overline{h}_2 = \overline{h}$, $\theta_1 = \theta_2 = \theta$, and $\rho = 1$. Observe that $L_1/L_2 = 1$ implies $S_1/S_2 = 1$, so that the unique equilibrium wage is $\omega^* = 1$ by (E-5) if the two cities are equally large. Now suppose that city 1 is larger than city 2, $L_1/L_2 > 1$, which implies $S_1/S_2 > 1$. Then, the equilibrium relative wage satisfies $\omega^* > 1$ because an increase in S_1/S_2 raises the RHS of (E-5) without affecting the LHS. Finally, expression (E-3), together with the foregoing assumption, yields $\omega^{2k+1} = (1/\sigma) \left(m_2^d/m_1^d\right)^{k+1}$. As $L_1 > L_2$ implies $\omega > 1$ and $\sigma > 1$ (recall that $h \equiv S/L$ is decreasing in L), it follows that $m_1^d < m_2^d$. Hence, the unique market equilibrium is such that the larger city has the higher wage and the lower cutoff. Note that the proof relies on (E-5), which is obtained under $\tau > t$. However, we can establish the same properties for $\tau = t$ by using the expressions in E.4. (i) below.

E.3. Spatial equilibrium: No urban frictions. We have claimed that the third and the fourth term in (32) are negative because $m_1^d < \widetilde{m}_1^d < \widetilde{m}_2^d < m_2^d$. To verify these inequalities, notice at first that the reduction in θ from any given positive value to zero raises S_1/S_2. This is straightforward to prove: In a world with urban frictions (where $\theta > 0$), and given that $\overline{h}_1 = \overline{h}_2 = \overline{h}$ and $\theta_1 = \theta_2 = \theta$, the term S_1/S_2 is given by

$$\frac{S_1}{S_2} = \frac{1 - \left(1 + \theta\sqrt{L_1/\pi}\right) \mathrm{e}^{-\theta\sqrt{L_1/\pi}}}{1 - \left(1 + \theta\sqrt{L_2/\pi}\right) \mathrm{e}^{-\theta\sqrt{L_2/\pi}}}. \tag{E-6}$$

In a world without urban frictions ($\theta = 0$), we have $\widetilde{S}_1 = L_1\overline{h}$ and $\widetilde{S}_2 = L_2\overline{h}$, so that $\widetilde{S}_1/\widetilde{S}_2 = L_1/L_2$. Letting $y_r \equiv \theta\sqrt{L_r/\pi} > 0$, proving that L_1/L_2 is larger than the term S_1/S_2 given in (E-6) is equivalent to proving that $y_1^2/\left(1 - \mathrm{e}^{-y_1} - y_1\mathrm{e}^{-y_1}\right) > y_2^2/\left(1 - \mathrm{e}^{-y_2} - y_2\mathrm{e}^{-y_2}\right)$. We thus need to show that $y^2/(1 - \mathrm{e}^{-y} - y\mathrm{e}^{-y})$ is increasing because $y_1 > y_2$. By differentiating, we have the derivative

$$\frac{y\mathrm{e}^{-y}}{(1 - \mathrm{e}^{-y} - y\mathrm{e}^{-y})^2} Y, \quad \text{where} \quad Y \equiv 2\mathrm{e}^{y} - [(y+1)^2 + 1].$$

Noting that $Y = 0$ at $y = 0$ and $Y' = 2[\mathrm{e}^y - (y+1)] > 0$ for all $y > 0$, we know that the derivative is positive for all $y > 0$. Hence, $\widetilde{S}_1/\widetilde{S}_2 = L_1/L_2 > S_1/S_2$. The elimination of urban frictions thus raises S_1/S_2, and thereby the relative wage ω by shifting up the RHS of (E-5). We hence observe *wage divergence*. The expressions in (E-4) then indeed imply $m_1^d < \widetilde{m}_1^d < \widetilde{m}_2^d < m_2^d$ as ω increases.

E.4. Spatial equilibrium: No trade frictions. Our aim is to show the condition for $\widetilde{\Upsilon} < \Upsilon$ to hold in (33), and we proceed in two steps. First, we show that the elimination of trade frictions implies a lower cutoff in both regions. Second, we show under which conditions the elimination of trade frictions lead to a decrease in $\mathbb{P}_1$.

(i) Setting $\tau = t$, the market equilibrium conditions (E-1)–(E-3) can be rewritten as

$$\frac{\mu_1^{\max}}{t} = L_1 X_1 + L_2 \frac{X_2}{\Omega} \tag{E-7}$$

$$\frac{\mu_2^{\max}}{t} = L_2 X_2 + L_1 \Omega X_1 \tag{E-8}$$

$$\Omega = \left(\frac{\rho}{\sigma}\frac{X_2}{X_1}\right)^{\frac{k+1}{2k+1}}, \tag{E-9}$$

where $X_1 \equiv (m_1^d)^{k+1}$, $X_2 \equiv (m_2^d)^{k+1}$, and $\Omega \equiv \omega^{k+1}$. From (E-7) and (E-8), we thus have $\Omega \mu_1^{\max}/t = \mu_2^{\max}/t = L_1 \Omega X_1 + L_2 X_2$. Hence, $\Omega = \rho$ must hold when $\tau = t$, and ω is uniquely determined. We know by (E-9) that $X_2 = (\sigma/\rho)\Omega^{\frac{2k+1}{k+1}} X_1 = \sigma \rho^{\frac{k}{k+1}} X_1$. Plugging this expression into (E-7) yields the unique counterfactual cutoffs

$$\widetilde{X}_1 = (\widetilde{m}_1^d)^{k+1} = \frac{\mu_1^{\max}/(L_1 t)}{1 + \sigma \rho^{-\frac{1}{k+1}}(L_2/L_1)} \quad \text{and} \quad \widetilde{X}_2 = (\widetilde{m}_2^d)^{k+1} = \frac{\mu_2^{\max}/(L_2 t)}{1 + \sigma^{-1} \rho^{\frac{1}{k+1}}(L_1/L_2)}. \tag{E-10}$$

Establishing that $\widetilde{X}_1 < X_1$, i.e., that $\widetilde{m}_1^d < m_1^d$ requires

$$\frac{1 - \rho(t/\tau)^k \omega^{-(k+1)}}{1 - (t/\tau)^{2k}} > \frac{1}{1 + \sigma \rho^{-\frac{1}{k+1}}(L_2/L_1)}$$

$$\Rightarrow \quad \sigma \rho^{-\frac{1}{k+1}} \left(\frac{L_2}{L_1}\right) \left[1 - \rho \left(\frac{t}{\tau}\right)^k \omega^{-(k+1)}\right] > \left(\frac{t}{\tau}\right)^k \left[\rho \omega^{-(k+1)} - \left(\frac{t}{\tau}\right)^k\right]$$

$$\Rightarrow \quad \rho^{-\frac{1}{k+1}} \left(\frac{S_2}{S_1}\right) \omega^{-(k+1)} \left[\omega^{k+1} - \rho \left(\frac{t}{\tau}\right)^k\right] > \left(\frac{t}{\tau}\right)^k \omega^{-(k+1)} \left[\rho - \left(\frac{t}{\tau}\right)^k \omega^{k+1}\right]$$

$$\Rightarrow \quad \rho \rho^{-\frac{1}{k+1}} \left(\frac{\tau}{t}\right)^k > \rho \left(\frac{S_1}{S_2}\right) \frac{\rho - (t/\tau)^k \omega^{k+1}}{\omega^{k+1} - \rho(t/\tau)^k} = \omega^k,$$

where the last equality holds by (E-5). We thus need to prove $\rho^{k/(k+1)}(\tau/t)^k > \omega^k$ or $\rho^{1/(k+1)}(\tau/t) > \omega$, which is straightforward since $\rho^{1/(k+1)}(\tau/t) > \rho^{1/(k+1)}(\tau/t)^{k/(k+1)} \equiv \overline{\omega} > \omega$. Hence, $\widetilde{m}_1^d < m_1^d$ must hold. Using a similar approach, it can be shown that $\widetilde{m}_2^d < m_2^d$. The elimination of trade frictions thus leads to lower cutoffs in *both* regions.

(ii) Now we want to show under which conditions we have $\widetilde{\Upsilon} < \Upsilon$ in (33). Let $\Delta m_r^d \equiv m_r^d - \widetilde{m}_r^d > 0$. Then, proving $h_1(1/\widetilde{m}_1^d - 1/m_1^d) < h_2(1/\widetilde{m}_2^d - 1/m_2^d)$ is equivalent to proving that

$$\frac{h_1 \Delta m_1^d}{m_1^d \widetilde{m}_1^d} < \frac{h_2 \Delta m_2^d}{m_2^d \widetilde{m}_2^d} \quad \Leftrightarrow \quad \frac{m_1^d \widetilde{m}_1^d \Delta m_2^d}{m_2^d \widetilde{m}_2^d \Delta m_1^d} \frac{h_2}{h_1} > 1. \tag{E-11}$$

This can be done by the following steps. First, we prove cutoff convergence, i.e., $\widetilde{m}_2^d/\widetilde{m}_1^d < m_2^d/m_1^d$. Using (E-10), the counterfactual cutoff ratio is given by $(\widetilde{m}_2^d/\widetilde{m}_1^d)^{k+1} = \sigma\rho^{k/(k+1)}$, whereas using (E-4), the cutoff ratio with trade frictions is

$$\left(\frac{m_2^d}{m_1^d}\right)^{k+1} = \frac{L_1}{L_2}\frac{1}{\omega^{-(k+1)}}\frac{\rho - (t/\tau)^k \omega^{k+1}}{\omega^{k+1} - \rho(t/\tau)^k} = \frac{L_1}{L_2}\frac{1}{\omega^{-(k+1)}}\frac{\omega^k}{\rho}\frac{S_2}{S_1} = \frac{\sigma}{\rho}\omega^{2k+1},$$

where we use (E-5) to obtain the second equality. Taking their difference, showing that $\widetilde{m}_2^d/\widetilde{m}_1^d < m_2^d/m_1^d$ boils down to showing that $\rho^{1/(k+1)} < \omega$ at the market equilibrium. This can be done by evaluating (E-5) at $\omega = \rho^{1/(k+1)}$. The LHS is equal to $\rho^{k/(k+1)}$, which falls short of the RHS given by $\rho S_1/S_2$ (because $\rho \geq 1$, $k \geq 1$, and $S_1/S_2 > 1$). Since the LHS is increasing and the RHS is decreasing, it must be that $\rho^{1/(k+1)} < \omega^*$. Thus, we have proved $\widetilde{m}_2^d/\widetilde{m}_1^d < m_2^d/m_1^d$. Turning to the second step, this cutoff convergence then implies

$$\frac{m_2^d}{m_1^d} > \frac{\widetilde{m}_2^d}{\widetilde{m}_1^d} \quad \Rightarrow \quad \frac{m_1^d}{m_2^d}\frac{\Delta m_2^d}{\Delta m_1^d} > 1 \quad \Rightarrow \quad \left(\frac{m_1^d}{m_2^d}\frac{\widetilde{m}_1^d}{\widetilde{m}_2^d}\frac{\Delta m_2^d}{\Delta m_1^d}\frac{h_2}{h_1}\right)\frac{\widetilde{m}_2^d}{\widetilde{m}_1^d}\frac{h_1}{h_2} > 1. \tag{E-12}$$

Recall from (E-11) that we ultimately want to prove that $\left(\frac{m_1^d}{m_2^d}\frac{\widetilde{m}_1^d}{\widetilde{m}_2^d}\frac{\Delta m_2^d}{\Delta m_1^d}\frac{h_2}{h_1}\right) > 1$. A sufficient condition for this to be satisfied, given condition (E-12), is that $(\widetilde{m}_2^d/\widetilde{m}_1^d)(h_1/h_2) \leq 1$, i.e., that $[\sigma\rho^{k/(k+1)}]^{1/(k+1)}(1/\sigma) = [\rho^{1/(k+1)}/\sigma]^{k/(k+1)} \leq 1$. This is the case if $\rho^{1/(k+1)} \leq \sigma$. In words, the elimination of trade frictions leads to a decrease in the size of the large city if the two cities are not too different in terms of their technological possibilities. In the simple case where $\rho = 1$, the large city always becomes smaller as $\sigma > 1$.

Appendix F: Quantification – Data, procedure, and model fit.

F.1. Data. We summarize the data used for the quantification of our model.

i) MSA *data*

We construct a dataset for 356 MSAs (see Table 4 below for a full list). The bulk of our MSA-level data comes from the 2007 American Community Survey (ACS) of the US Census, from the Bureau of Economic Analysis (BEA), and from the Bureau of Labor Statistics (BLS). The geographical coordinates of each MSA are computed as the centroid of its constituent counties' geographical coordinates. The latter are obtained from the 2000 US Census Gazetteer county geography file, and the MSA-level aggregation is carried out using the county-to-MSA concordance tables for 2007. We then construct our measure of distance between two MSAs as $d_{rs} = \cos^{-1}\big(\sin(\mathrm{lat}_r)\sin(\mathrm{lat}_s) + \cos(|\mathrm{lon}_r - \mathrm{lon}_s|)\cos(\mathrm{lat}_r)\times \cos(\mathrm{lat}_s)\big) \times 6{,}378.137$ using the great circle formula, where lat_r and lon_r are the geographical coordinates of the MSA. The internal distance of an MSA is defined as $d_{rr} \equiv (2/3)\sqrt{\mathrm{surface}_r/\pi}$ as in Redding and Venables (2004). All MSA surface measures are given in square kilometers and include only land surface of the MSA's forming counties. That data is obtained from the 2000 US Census Gazetteer, and is aggregated from the county to the MSA level.

We further obtain total gross domestic product by MSA from the BEA metropolitan GDP files. Total employment at the MSA level is obtained from the 2007 BLS employment flat files (we use aggregate values for 'All occupations'). Using gross domestic product, total employment, and the average number of hours worked allows us to recover our measure of average MSA productivity (GDP per employee), which is proportional to $1/m_r^d$ because of the Pareto distribution. Wages at the MSA level for 2007 are computed as total labor expenses (compensation of employees plus employer contributions for employee pension and insurance funds plus employer contributions for government social insurance) divided by total MSA employment. Data to compute total labor expenses is provided by the BEA.

ii) Amenity data

Next, county-level data on natural amenities refer to the year 1999 and are provided by the US Department of Agriculture (USDA). The USDA data includes six measures of climate, topography, and water area that reflect environmental attributes usually valued by people. We use the standardized amenity score from that data as a proxy for our observed amenities A_r^o. We aggregate the county-level amenities up to the MSA level by using the county-to-MSA concordance table and by weighting each county by its share in the total MSA land surface.

iii) Urban frictions data

Data is taken from the 2007 ACS which provides total MSA population, average weekly hours worked and average (one-way) commuting time in minutes. Those pieces of information are used to compute the aggregate labor supply $\overline{h}_r L_r$, and the effective labor supply S_r.

iv) Trade frictions data

Finally, we use aggregate bilateral trade flows X_{rs} from the 2007 Commodity Flow Survey (CFS) of the Bureau of Transportation Statistics (BTS) for the lower 48 contiguous US states, as these are the states containing the MSAs that will be used in our analysis. We work at the state level since MSA trade flows from the CFS public files can only be meaningfully exploited for a relatively small sample of large 'CFS regions'. The distance between r and s in kilometers is computed using the great circle formula given above. In that case, lat_r and lon_r denote the coordinates of the capital of state r, measured in radians, which are taken from Anderson and van Wincoop's (2003) dataset.

F.2. Quantification procedure. As explained in the main text, the quantification procedure for the market equilibrium consists of five steps that we now explain in detail.

i) Urban frictions θ_r

To obtain the city-specific commuting technology parameters $\widehat{\theta}_r$ that constitute urban frictions, we rewrite equation (12) as

$$L_r \frac{h_r}{\overline{h}_r} = \frac{2\pi}{\theta_r^2} \left[1 - \left(1 + \theta_r \sqrt{L_r/\pi}\right) \mathrm{e}^{-\theta_r \sqrt{L_r/\pi}}\right], \tag{F-1}$$

where we use $S_r = L_r h_r$. We compute h_r as the average number of hours worked per week in MSA r. The gross labor supply per capita, $\overline{h}_r$, which is the endowment of hours available for work and commuting, is constructed as the sum of h_r and hours per week spent by workers in each MSA for travel-to-work commuting in 2007. Given h_r, $\overline{h}_r$, as well as city size L_r, the above equation can be uniquely solved for the city-specific commuting parameter $\widehat{\theta}_r$. Table 4 below provides the values for the 356 MSAs.

ii) Trade frictions τ_{rs}

To estimate the distance elasticity $\widehat{\gamma}$ that constitutes trade frictions, we consider the value of sales from r to s:

$$X_{rs} = N_r^E L_s \int_0^{m_{rs}^x} p_{rs}(m) q_{rs}(m) \mathrm{d}G_r(m). \tag{F-2}$$

Using (7), (9), (24), and the result from Appendix B.4, we then obtain the following gravity equation: $X_{rs} = S_r L_s \tau_{rs}^{-k} \tau_{ss}^{k+1} (w_s/w_r)^{k+1} w_r \left(m_s^d\right)^{k+1} (\mu_r^{\max})^{-1}$. Turning to the specification of trade costs τ_{rs}, we stick to standard practice and assume that $\tau_{rs} \equiv d_{rs}^{\gamma}$, where d_{rs} stands for the distance from r to s. The gravity equation can then be rewritten in log-linear stochastic form:

$$\ln X_{rs} = \text{const.} - k\gamma \ln d_{rs} + I_{rs}^0 + \zeta_r^1 + \zeta_s^2 + \varepsilon_{rs}, \tag{F-3}$$

where all terms specific to the origin and the destination are collapsed into fixed effects ζ_r^1 and ζ_s^2, where I_{rs}^0 is a zero-flow dummy, and ε_{rs} is an error term with the usual properties for OLS consistency.[26] Using aggregate bilateral trade flows X_{rs} in 2007 for the 48 contiguous US states that cover all MSAs used in the subsequent analysis, we estimate the gravity equation on state-to-state trade flows. Given a value of k, we then obtain an estimate of the distance elasticity $\widehat{\gamma}$ that constitutes trade frictions.

iii) Market equilibrium conditions $(w_r, \mu_r^{\max})$

Observe that expressions (22) and (25) can be rewritten as:

$$\mu_r^{\max} = \sum_s L_s \tau_{rs} \left(m_s^d \frac{\tau_{ss}}{\tau_{rs}} \frac{w_s}{w_r} \right)^{k+1} \tag{F-4}$$

$$\frac{S_r}{L_r} \frac{1}{\left(m_r^d\right)^{k+1}} = \sum_s S_s \tau_{rr} \left(\frac{\tau_{sr}}{\tau_{rr}} \frac{w_s}{w_r} \right)^{-k} \frac{1}{\mu_s^{\max}}. \tag{F-5}$$

Ideally, we would use data on technological possibilities $\mu_r^{\max}$ to solve for the wages and cutoffs. Yet, $\mu_r^{\max}$ is unobservable. We thus solve for wages and technological possibilities $(\widehat{w}_r, \widehat{\mu}_r^{\max})$ by using the values of m_r^d that are obtained as follows. Under the Pareto distribution, we have $(1/\overline{m}_r) = [k/(k+1)](1/m_r^d)$, where $1/\overline{m}_r$ is the average productivity in MSA r. The latter can be computed as GDP per employee, using data on GDP of MSA r and the total number of hours worked in that MSA (hours worked per week times total employment). Given an estimate of $1/\overline{m}_r$ and the value of k, we can compute the cutoffs m_r^d. Using the value of k, the cutoffs m_r^d, the city-specific commuting technologies $\widehat{\theta}_r$, the observed MSA populations L_r, as well as trade frictions $\widehat{\tau}_{rs} = d_{rs}^{\widehat{\gamma}}$, we can solve

[26] There are 179 'zero flows' out of 2,304 in the data, i.e., 7.7% of the sample. We control for them by using a standard dummy-variable approach, where I_{rs}^0 takes value 1 if $X_{rs} = 0$ and 0 otherwise.

(F-4) and (F-5) for the wages and unobserved technological possibilities $(\widehat{w}_r, \widehat{\mu}_r^{\max})$ that are consistent with the market equilibrium.

iv) Firm size distribution and Pareto shape parameter k

The quantification procedure described thus far has assumed a given value of the shape parameter k. To estimate k structurally, we proceed as follows. First, given a value of k, we can compute trade frictions $\widehat{\tau}_{rs}$ and the wages and cutoffs $(\widehat{w}_r, \widehat{\mu}_r^{\max})$ as described before. This, together with the internal cutoff m_r^d computed from data, yields the external cutoffs $\widehat{m}_{rs}^x$ by (7). With that information in hand, we can compute the share $\widehat{\nu}_r$ of surviving firms in each MSA as follows:

$$\widehat{\nu}_r \equiv \frac{\widehat{N}_r^p}{\sum_s \widehat{N}_s^p}, \quad \text{where} \quad \widehat{N}_r^p = \widehat{N}_r^E G_r\left(\max_s \widehat{m}_{rs}^x\right) = \frac{\alpha}{\kappa_1 + \kappa_2} S_r \left(\widehat{\mu}_r^{\max}\right)^{-1} \left(\max_s \widehat{m}_{rs}^x\right)^k$$

denotes the number of firms operating in MSA r. The total effective labor supply S_r is computed as described above in *i)*. Note that $\widehat{\nu}_r$ is independent of the unobservable constant scaling $\alpha/(\kappa_1 + \kappa_2)$ that multiplies the number of firms.

Second, we draw a large sample of firms from our calibrated MSA-level productivity distributions $\widehat{G}_r(m) = \left(m/m_r^d\right)^k$. For that sample to be representative, we draw firms in MSA r in proportion to its share $\widehat{\nu}_r$. For each sampled firm with marginal labor requirement m in MSA r, we can compute its employment as follows:[27]

$$\text{employment}_r(m) = m \sum_s \widehat{\chi}_{rs} L_s q_{rs}(m) = \frac{m}{\alpha} \sum_s \widehat{\chi}_{rs} L_s \left[1 - W\left(\mathrm{e}\frac{m}{\widehat{m}_{rs}^x}\right)\right],$$

where $\widehat{\chi}_{rs} = 1$ if $m < \widehat{m}_{rs}^x$ (the establishment can sell to MSA s) and zero otherwise (the establishment cannot sell to MSA s). Since we can identify employment only up to some positive constant (which depends on the unobservable α) we choose, without loss of generality, that coefficient such that the average employment per firm in our sample of establishments matches the observed average employment in the 2007 CBP. Doing so allows us to readily compare the generated and observed data as we can sort the sampled firms into the same size bins as those used for the observed firms. We use four standard employment size bins from the CBP: $\iota = \{1\text{–}19, 20\text{–}99, 100\text{–}499, 500+\}$ employees. Let $N_{(\iota)}^{\text{SIM}}$ and $N_{(\iota)}^{\text{CBP}}$ denote the number of firms in each size bin ι in our sample and in the CBP, respectively. Let also N^{SIM} and N^{CBP} denote our sample size and the observed number of establishments in the CBP. Given a value of k, the following statistic is a natural measure

[27] We exclude the labor used for shipping goods and the sunk initial labor requirement.

Table 3: Shipment shares and shipping distances – summary for observed and simulated data.

Employment	Number of establishments		Shipment shares by distance shipped to destination						Mean distance shipped		
			< 100 miles		100–500 miles		> 500 miles				
	Observed	Model	Observed	Model	Observed	Model	Observed	Model	Observed	Model	Model (wgt)
All	6,431,884	6,431,886	0.261	0.506	0.288	0.277	0.348	0.217	529.6	71.98	739.8
1–19	5,504,463	5,498,328	0.561	0.984	0.204	0.016	0.194	0.000	327.2	38.5	61.2
20–99	769,705	755,275	0.382	0.835	0.288	0.162	0.276	0.004	423.8	157.9	194.4
100–499	141,510	153,021	0.254	0.420	0.318	0.440	0.342	0.139	520.4	556.0	740.3
500+	16,206	25,255	0.203	0.079	0.272	0.332	0.388	0.590	588.6	1450.6	1519.1

Notes: Shipping distance and shipping share columns are adapted from calculations by Holmes and Stevens (2012, Table 1) who use confidential Census microdata from the 1997 Commodity Flow Survey. The small difference (of 2 units) between the observed and model total number of establishments is due to rounding in our sampling procedure. The last column reports distances shipped weighted by establishments' sales shares in total sales.

of the goodness-of-fit of the simulated establishment-size distribution:

$$\text{SS}(k) = \sum_{\iota=1}^{4} \left[\frac{N_{(\iota)}^{\text{SIM}}}{N^{\text{SIM}}} - \frac{N_{(\iota)}^{\text{CBP}}}{N^{\text{CBP}}} \right]^2, \tag{F-6}$$

the value of which depends on the chosen k. It is clear from (F-6) that we can choose any large sample size N^{SIM} since it would not affect the ratio $N_{(\iota)}^{\text{SIM}}/N^{\text{SIM}}$. Without loss of generality, we choose the sample size such that the total number of simulated firms operating matches the observed total number of establishments ($N^{\text{SIM}} = N^{\text{CBP}}$). There are 6,431,884 establishments across our 356 MSAs in the 2007 CBP, and we sample the same number of firms from our quantified model.[28] We finally choose k by minimizing $\text{SS}(k)$.

F.3. Model fit. We now provide details about our model fit with respect to trade frictions. Figure 10 below is analogous to Figures 1-3 in Hillberry and Hummels (2008) who provide micro evidence on the spatial structure of firms' shipping patterns. The figure reports kernel regressions of various predicted shipment characteristics on distance. Specifically, we consider that the value of sales from an establishment in city r to city s represents one shipment characterized by an origin MSA, a destination MSA, a shipping value, a unit price, and a shipping distance. We then draw a representative sample of 40,000 establishments from all MSAs, which yields a total of $40{,}000 \times 356^2$ potential shipments.[29] Most of these shipments do of course not occur, and there are only 243,784 positive shipments in our sample. As in Hillberry and Hummels (2008), we then use a Gaussian kernel with optimal bandwidth and calculated on 100 points.

We illustrate the results for distances greater than about 10 miles (the minimum in our sample) and up to slightly below 3,000 miles (the maximum in our sample). Note that

[28] Doing so allows for a direct comparison of $N_{(\iota)}^{\text{SIM}}$ and $N_{(\iota)}^{\text{CBP}}$ for each ι. The very small differences in the aggregate numbers in Tables 1 and 3 are due to rounding as the number of firms has to be an integer.

[29] The sample size is immaterial for our results provided that it is large enough. Given that the number of shipments is substantially larger than the number of firms, drawing a large sample of 6.5 million firms as before proves computationally infeasible.

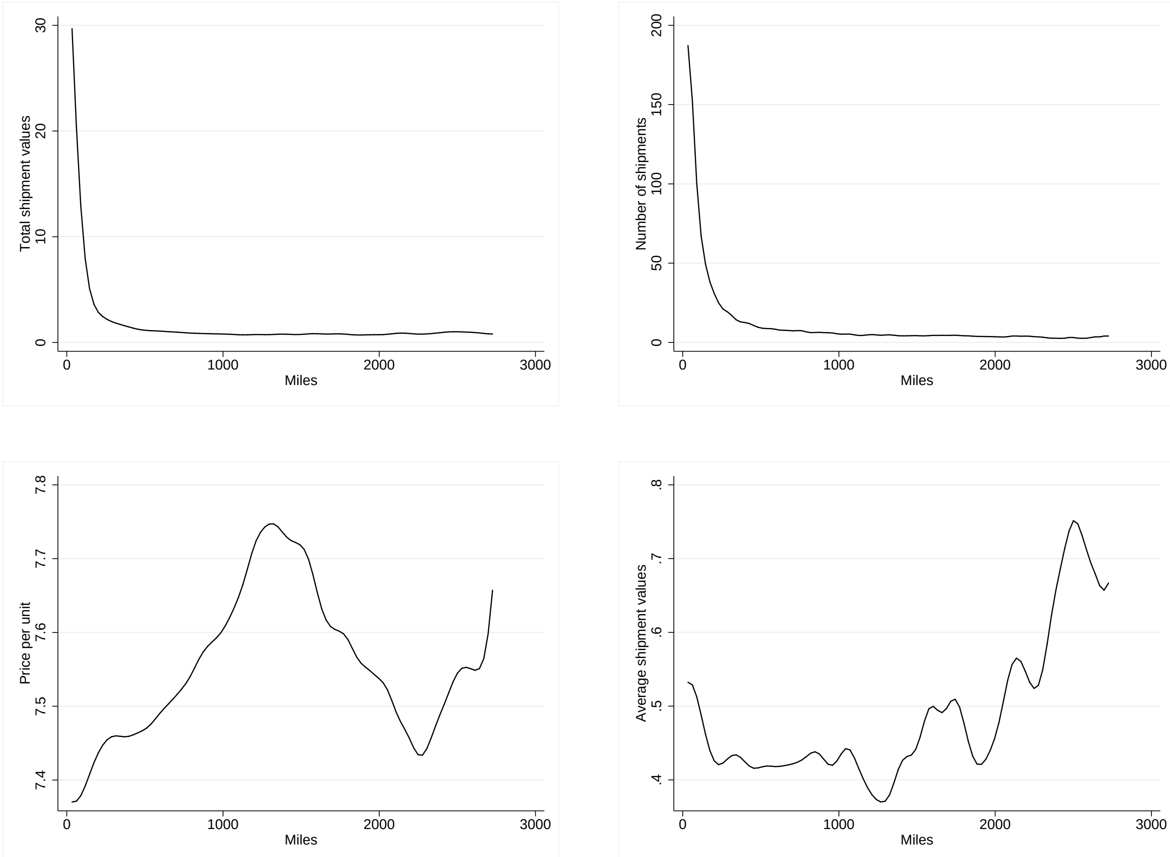

Figure 10: Micro-fit for establishment-level shipments across MSAs (kernel regressions on distance)

we have less variation in distances than Hillberry and Hummels (2008) who use either 3-digit or 5-digit zip code level data instead of MSA data. In line with the micro evidence presented in Hillberry and Hummels (2008), we find that both aggregate shipment values and the number of shipments predicted by our model fall off very quickly with distance – becoming very small beyond a threshold of about 200 miles – whereas price per unit first rises with distance and average shipment values do not display a clear pattern.

Next, we compare shipping shares and shipping distances by establishment size class predicted by our model, and their empirically observed counterparts. The former are obtained as follows. First, for each establishment with labor requirement m in MSA r, we compute the value of its sales:

$$\text{sales}_r(m) = \sum_s \chi_{rs} L_s p_{rs}(m) q_{rs}(m) = \frac{\widehat{w}_r m}{\alpha} \sum_s \chi_{rs} L_s d_{rs}^{\widehat{\gamma}} [W(\mathrm{e}m/\widehat{m}_{rs}^x)^{-1} - 1].$$

We then classify all 6,431,886 establishments in our sample by employment size class, and disaggregate the value of sales for each establishment by distance shipped to compute the shares reported in Table 3.[30] The observed patterns in Table 3 come from Holmes and Stevens (2012) who use confidential CFS microdata from 1997 to compute the shares of shipping values by distance as well as average shipping distances. As can be seen, our model can qualitatively reproduce the observed shipment shares, and it can also explain the tendency that the mean distance shipped increases with establishment size.

Appendix G: Unique solution for D_r and the spatial equilibrium.

Letting $D_r = (U_r + A_r)/\beta$, the spatial equilibrium condition can be written as

$$\frac{\exp(D_r)}{\sum_{s=1}^{K} \exp(D_s)} = \frac{L_r}{\sum_{s=1}^{K} L_s}, \quad \text{with } D_1 = 0. \tag{G-1}$$

Taking the ratio for regions r and 1, we have

$$\frac{\exp(D_r)}{\exp(D_1)} = \exp(D_r) = \frac{L_r}{L_1}, \quad \forall r. \tag{G-2}$$

Hence, D_r is uniquely determined as $D_r = \ln(L_r/L_1)$ for all r.

Appendix H: Numerical procedure for counterfactual analyses.

For simplicity, we only explain the procedure for the 'no urban frictions' case, as it works analogously for the 'no trade frictions' scenario. First, we let $\widehat{\theta}_r = 0$ for all r and keep the initial population distribution fixed. This parameter change induces changes in the indirect utility levels. Let $\widetilde{U}_r^0$ denote the new counterfactual utility in MSA r, evaluated at the initial population and $\widehat{\theta}_r = 0$. Second, we replace $\widehat{U}_r$ with its new counterfactual value $\widetilde{U}_r^0$ to obtain $\widetilde{D}_r^0 = \widehat{\alpha}_0 + \widehat{\alpha}_1 \widetilde{U}_r^0 + \widehat{\alpha}_2 A_r^o + \widehat{A}_r^u$. The spatial equilibrium conditions (34) will then, in general, no longer be satisfied, and hence city sizes must change.

We thus consider the following iterative adjustment procedure to find the new counterfactual spatial equilibrium:

1. Consider the new choice probabilities

$$\widetilde{\mathbb{P}}_r^0 = \frac{\exp(\widetilde{D}_r^0)}{\sum_s \exp(\widetilde{D}_s^0)} \tag{H-1}$$

   induced by the change in spatial frictions, which yield a new population distribution $\widetilde{L}_r^0 = L\widetilde{\mathbb{P}}_r^0$ for all $r = 1,...,K$.

[30] Since we work with shares, the unobservable scaling parameter α does not affect our results.

2. Given the intial $\widehat{\mu}_r^{\max}$, the new population distribution $\widetilde{L}_r^0$ for all $r = 1,...,K$, as well as the counterfactual value for the commuting technology parameter $\widehat{\theta}_r = 0$, the market equilibrium conditions generate new wages and cutoffs $\{\widetilde{w}_r^1, (\widetilde{m}_r^d)^1\}$. Expression (28) then yields new utility levels $\widetilde{U}_r^1$.

3. Using $\widetilde{D}_r^1 = \widehat{\alpha}_0 + \widehat{\alpha}_1 \widetilde{U}_r^1 + \widehat{\alpha}_2 A_r^o + \widehat{A}_r^u$, the choice probabilities can be updated as in (H-1), which yields a new population distribution $\widetilde{L}_r^1 = L\widetilde{\mathbb{P}}_r^1$ for all $r = 1,...,K$.

4. We iterate over steps 2–3 until convergence of the population distribution to obtain $\{\widetilde{L}_r, \widetilde{w}_r, \widetilde{m}_r^d\}$ for all $r = 1,...,K$.

Appendix I: Agglomeration economies.

We compute $\widehat{\mu}_r^{\max}$ in the initial equilibrium. Call it $\widehat{\mu}_r^{\max,0}$. Assume now that the population of MSA r changes from L_r^0 to L_r^1. The new $\widehat{\mu}_r^{\max}$ is then given by $\widehat{\mu}_r^{\max,1} = c \cdot (L_r^1/\text{surface}_r)^{-k\xi} \cdot \widehat{\psi}_r^{\max}$. Hence, it is easy to see that, given the initial estimates $\widehat{\mu}_r^{\max,0}$ we have $\widehat{\mu}_r^{\max,1} = \widehat{\mu}_r^{\max,0}\left(L_r^1/L_r^0\right)^{-k\xi}$. Thus, we can integrate agglomeration economies in a straightforward way into our framework by replacing $\widehat{\mu}_r^{\max}$ by $\widehat{\mu}_r^{\max}\left(L_r^1/L_r^0\right)^{-k\xi}$ in the market equilibrium conditions (F-4) and (F-5) when running the counterfactuals:

$$\widehat{\mu}_r^{\max}\left(\frac{L_r^1}{L_r^0}\right)^{-k\xi} = \sum_s L_s^1 \tau_{rs}\left(m_s^d \frac{\tau_{ss}}{\tau_{rs}}\frac{w_s}{w_r}\right)^{k+1} \tag{I-1}$$

$$\frac{S_r^1}{L_r^1}\frac{1}{\left(m_r^d\right)^{k+1}} = \sum_s S_s^1 \tau_{rr}\left(\frac{\tau_{sr}}{\tau_{rr}}\frac{w_s}{w_r}\right)^{-k}\frac{1}{\widehat{\mu}_s^{\max}\left(\frac{L_s^1}{L_s^0}\right)^{-k\xi}}. \tag{I-2}$$

Appendix J: Additional results tables.

Table 4: MSA variables and descriptives for the initial equilibrium.

FIPS	MSA name	State	$L_r/\overline{L}$	$\widehat{\mu}_r^{\max}$	$1/\overline{m}_r$	$\widehat{\theta}_r$	A_r^o	$\widehat{A}_r^u$
10180	Abilene	TX	0.2268	6.8852	0.8328	0.3925	1.3141	-0.6556
10420	Akron	OH	0.9956	17.4352	0.8212	0.2473	-2.2749	1.0062
10500	Albany	GA	0.2336	28.3000	0.7182	0.4608	-0.0435	-0.4451
10580	Albany-Schenectady-Troy	NY	1.2149	15.6558	0.8722	0.2015	-0.2432	1.1317
10740	Albuquerque	NM	1.1889	11.6475	0.8694	0.2232	3.7322	0.9275
10780	Alexandria	LA	0.2133	14.7747	0.7632	0.5445	-0.2067	-0.5842
10900	Allentown-Bethlehem-Easton	PA-NJ	1.1444	22.9469	0.8678	0.3088	0.3026	0.9760
11020	Altoona	PA	0.1787	28.9660	0.6877	0.5223	-0.8600	-0.7009
11100	Amarillo	TX	0.3449	7.1209	0.8305	0.3277	1.6304	-0.2289
11180	Ames	IA	0.1207	0.7978	0.9817	0.6556	-3.5400	-1.1175
11300	Anderson	IN	0.1869	6.1621	0.8247	0.8718	-3.4700	-0.6463
11340	Anderson	SC	0.2562	16.3593	0.7543	0.5571	0.7100	-0.4872
11460	Ann Arbor	MI	0.4983	2.9986	0.9738	0.2977	-2.1900	0.1721
11500	Anniston-Oxford	AL	0.1610	13.1516	0.7430	0.5613	0.2200	-0.9536
11540	Appleton	WI	0.3104	9.1579	0.7999	0.3684	-2.7304	-0.0904
11700	Asheville	NC	0.5756	31.3698	0.7609	0.3163	2.1012	0.2978
12020	Athens-Clarke County	GA	0.2668	15.4460	0.7858	0.4865	-1.0511	-0.3069
12060	Atlanta-Sandy Springs-Marietta	GA	7.5152	7.9312	1.0828	0.1174	0.2253	2.7880
12100	Atlantic City-Hammonton	NJ	0.3853	4.3460	0.9247	0.3301	-0.0400	-0.2364
12220	Auburn-Opelika	AL	0.1858	14.1079	0.7298	0.6358	-0.2400	-0.7240
12260	Augusta-Richmond County	GA-SC	0.7524	23.6409	0.8053	0.2920	-0.0192	0.6829
12420	Austin-Round Rock	TX	2.2752	5.6156	0.9979	0.1860	1.6141	1.5231
12540	Bakersfield	CA	1.1257	8.3291	0.9841	0.2453	4.8400	0.6741
12580	Baltimore-Towson	MD	3.7983	12.0935	0.9856	0.1519	-0.3557	2.1378
12620	Bangor	ME	0.2118	5.6207	0.8107	0.5506	-0.5200	-0.5302
12700	Barnstable Town	MA	0.3163	2.9345	0.8556	0.4759	1.5200	-0.4993
12940	Baton Rouge	LA	1.0962	3.7242	1.0012	0.2569	-0.6186	0.9311
12980	Battle Creek	MI	0.1945	7.2642	0.8301	0.4982	-2.7300	-0.6453
13020	Bay City	MI	0.1531	6.5755	0.7780	0.7995	-1.5300	-0.9167
13140	Beaumont-Port Arthur	TX	0.5356	8.3601	0.8672	0.2801	0.9407	0.1728
13380	Bellingham	WA	0.2748	1.1589	0.9747	0.4955	5.2600	-0.7955
13460	Bend	OR	0.2193	2.3869	0.8996	0.4620	6.1000	-1.0336
13740	Billings	MT	0.2131	7.1640	0.7761	0.3735	2.4532	-0.6830
13780	Binghamton	NY	0.3508	56.9535	0.6866	0.3785	-0.9289	0.0588
13820	Birmingham-Hoover	AL	1.5777	5.8973	1.0014	0.2055	0.5780	1.2351
13900	Bismarck	ND	0.1470	12.2467	0.7085	0.4403	-1.6258	-0.7564
13980	Blacksburg-Christiansburg-Radford	VA	0.2244	10.1677	0.8144	0.5208	0.5141	-0.5979
14020	Bloomington	IN	0.2616	14.7889	0.8140	0.5467	-0.4507	-0.3408
14060	Bloomington-Normal	IL	0.2338	2.4247	0.9891	0.3871	-3.5700	-0.4375
14260	Boise City-Nampa	ID	0.8367	10.6193	0.8491	0.2399	2.2919	0.6976
14460	Boston-Cambridge-Quincy	MA-NH	6.3819	2.7007	1.1870	0.1098	0.1444	2.4955
14500	Boulder	CO	0.4132	0.6188	1.1168	0.3373	5.8200	-0.6755
14540	Bowling Green	KY	0.1651	12.3177	0.7702	0.5611	-0.2160	-0.8510
14740	Bremerton-Silverdale	WA	0.3370	1.2068	1.0491	0.7249	2.6100	-0.6981
14860	Bridgeport-Stamford-Norwalk	CT	1.2742	0.0329	1.8325	0.2506	2.2500	-0.2081
15180	Brownsville-Harlingen	TX	0.5512	55.3719	0.5912	0.3178	2.4600	0.3482
15260	Brunswick	GA	0.1449	13.3594	0.7523	0.6313	1.3530	-1.0593
15380	Buffalo-Niagara Falls	NY	1.6061	15.4178	0.8225	0.1730	-0.6399	1.4505
15500	Burlington	NC	0.2069	16.5166	0.7377	0.6324	-0.9600	-0.6176
15540	Burlington-South Burlington	VT	0.2952	2.2778	0.9027	0.4271	-0.1238	-0.3845
15940	Canton-Massillon	OH	0.5797	27.4059	0.7541	0.3382	-1.4796	0.4955
15980	Cape Coral-Fort Myers	FL	0.8407	2.0378	0.9635	0.3210	5.2300	0.1676
16220	Casper	WY	0.1021	0.0797	1.3629	0.4917	2.4900	-1.9697
16300	Cedar Rapids	IA	0.3599	6.3374	0.8708	0.3126	-3.3035	0.0590
16580	Champaign-Urbana	IL	0.3145	14.7922	0.8363	0.3848	-4.3383	0.0884
16620	Charleston	WV	0.4327	6.2623	0.9251	0.3322	-0.7294	0.0286
16700	Charleston-North Charleston-Summerville	SC	0.8970	8.8536	0.8690	0.2777	0.5686	0.7409
16740	Charlotte-Gastonia-Concord	NC-SC	2.3512	0.6377	1.3186	0.1561	0.1000	1.3196
16820	Charlottesville	VA	0.2744	7.2636	0.9001	0.4341	-0.0364	-0.4526
16860	Chattanooga	TN-GA	0.7326	8.8814	0.8897	0.2830	0.2832	0.5342
16940	Cheyenne	WY	0.1229	2.1311	0.9176	0.5112	3.0500	-1.4960
16980	Chicago-Naperville-Joliet	IL-IN-WI	13.5596	7.6522	1.1400	0.0867	-2.1021	3.4958
17020	Chico	CA	0.3115	5.1269	0.8541	0.5341	5.1100	-0.5608
17140	Cincinnati-Middletown	OH-KY-IN	3.0376	14.2620	0.9455	0.1438	-0.7916	2.0448
17300	Clarksville	TN-KY	0.3727	1.4179	1.0663	0.5319	0.0733	-0.3729
17420	Cleveland	TN	0.1582	3.0055	0.9115	0.7279	0.8781	-1.1302
17460	Cleveland-Elyria-Mentor	OH	2.9846	7.3233	0.9836	0.1352	-1.4310	1.9676
17660	Coeur d'Alene	ID	0.1914	8.3418	0.7161	0.6066	3.5000	-0.9011
17780	College Station-Bryan	TX	0.2895	47.5407	0.7123	0.4095	0.8622	-0.2296
17820	Colorado Springs	CO	0.8671	7.0613	0.8860	0.2838	5.3867	0.3780
17860	Columbia	MO	0.2311	16.7125	0.7364	0.4196	0.1054	-0.4706
17900	Columbia	SC	1.0194	22.2288	0.8323	0.2385	0.5017	0.9371
17980	Columbus	GA-AL	0.4025	8.7851	0.8541	0.3100	-0.2353	-0.0490

Table 4 (continued).

FIPS	MSA name	State	$L_r/\overline{L}$	$\widehat{\mu}_r^{\max}$	$1/\overline{m}_r$	$\widehat{\theta}_r$	A_r^o	$\widehat{A}_r^u$
18020	Columbus	IN	0.1064	2.9595	0.8788	0.4856	-2.3800	-1.3775
18140	Columbus	OH	2.4975	11.5892	0.9535	0.1398	-1.9162	1.8984
18580	Corpus Christi	TX	0.5899	5.0627	0.8543	0.2746	2.8551	0.1577
18700	Corvallis	OR	0.1159	0.1014	1.2152	0.7211	3.1000	-1.8133
19060	Cumberland	MD-WV	0.1414	56.7425	0.6576	0.7389	1.0076	-0.9889
19100	Dallas-Fort Worth-Arlington	TX	8.7483	3.2987	1.2029	0.0923	0.6857	2.8079
19140	Dalton	GA	0.1908	15.8567	0.7386	0.3339	0.4652	-0.8035
19180	Danville	IL	0.1156	13.3585	0.7769	0.7748	-3.2100	-1.0515
19260	Danville	VA	0.1506	34.1566	0.7025	0.6804	-0.3000	-0.8908
19340	Davenport-Moline-Rock Island	IA-IL	0.5355	8.2798	0.8791	0.2759	-2.6893	0.4377
19380	Dayton	OH	1.1895	14.1872	0.8640	0.1988	-2.1260	1.1962
19460	Decatur	AL	0.2125	3.5335	0.9214	0.6612	0.7910	-0.8247
19500	Decatur	IL	0.1548	2.7975	0.8839	0.4092	-2.7900	-0.9344
19660	Deltona-Daytona Beach-Ormond Beach	FL	0.7124	22.2777	0.7462	0.3743	3.4500	0.3884
19740	Denver-Aurora	CO	3.4326	2.2957	1.1516	0.1477	4.1942	1.7018
19780	Des Moines-West Des Moines	IA	0.7782	2.2274	1.0158	0.2050	-2.0346	0.6429
19820	Detroit-Warren-Livonia	MI	6.3602	8.3299	1.0380	0.1089	-1.6704	2.7501
20020	Dothan	AL	0.1986	49.5100	0.6561	0.4212	-0.4149	-0.5370
20100	Dover	DE	0.2168	1.9540	1.0020	0.5895	-0.0700	-0.8842
20220	Dubuque	IA	0.1315	5.7814	0.7869	0.3977	-0.7900	-1.1171
20260	Duluth	MN-WI	0.3905	18.6402	0.7996	0.3678	-0.8127	0.1938
20500	Durham	NC	0.6828	0.8200	1.1939	0.2552	0.0966	0.1845
20740	Eau Claire	WI	0.2247	12.7566	0.7611	0.4796	-2.6695	-0.3365
20940	El Centro	CA	0.2304	19.7182	0.7872	0.4081	6.4500	-0.8598
21060	Elizabethtown	KY	0.1589	3.7636	0.8891	0.5914	-0.8465	-1.0560
21140	Elkhart-Goshen	IN	0.2818	9.4337	0.7923	0.2901	-2.7200	-0.2450
21300	Elmira	NY	0.1253	16.7836	0.7000	0.6243	-1.1300	-1.0690
21340	El Paso	TX	1.0459	2.2083	0.9271	0.2441	4.4600	0.5021
21500	Erie	PA	0.3973	18.7253	0.7395	0.3204	-0.5700	0.0764
21660	Eugene-Springfield	OR	0.4891	13.2218	0.7821	0.3197	4.2900	0.0543
21780	Evansville	IN-KY	0.4979	8.0962	0.8860	0.2898	-1.6375	0.2844
22020	Fargo	ND-MN	0.2739	4.1400	0.8364	0.3067	-4.5908	-0.0388
22140	Farmington	NM	0.1743	0.2874	1.2203	0.5778	2.8300	-1.3307
22180	Fayetteville	NC	0.4968	0.7242	1.1132	0.3601	-0.9161	-0.1293
22220	Fayetteville-Springdale-Rogers	AR-MO	0.6203	13.9314	0.8230	0.2715	0.8552	0.4160
22380	Flagstaff	AZ	0.1814	41.4362	0.7797	0.4704	4.9300	-0.8937
22420	Flint	MI	0.6189	11.2936	0.8235	0.4086	-1.9000	0.4963
22500	Florence	SC	0.2829	14.4850	0.7801	0.4358	-0.2137	-0.3219
22520	Florence-Muscle Shoals	AL	0.2038	22.0682	0.7281	0.6420	0.8059	-0.6681
22540	Fond du Lac	WI	0.1411	5.1570	0.8386	0.6231	-1.9200	-1.0104
22660	Fort Collins-Loveland	CO	0.4094	9.8391	0.8295	0.3890	5.6200	-0.3039
22900	Fort Smith	AR-OK	0.4124	21.2879	0.7892	0.3342	1.6228	-0.0124
23020	Fort Walton Beach-Crestview-Destin	FL	0.2584	0.3985	1.1155	0.4967	2.0100	-0.9455
23060	Fort Wayne	IN	0.5838	20.3049	0.7882	0.2692	-3.0754	0.5929
23420	Fresno	CA	1.2803	22.9506	0.8468	0.2171	6.0300	0.8406
23460	Gadsden	AL	0.1469	27.7629	0.6669	0.7121	0.9600	-1.0397
23540	Gainesville	FL	0.3660	7.8664	0.8210	0.3731	2.0892	-0.2095
23580	Gainesville	GA	0.2565	4.7162	0.8383	0.6287	0.9600	-0.6703
24020	Glens Falls	NY	0.1835	53.2073	0.6769	0.6495	-0.3136	-0.6305
24140	Goldsboro	NC	0.1617	4.7743	0.8234	0.6350	-1.4100	-0.9470
24220	Grand Forks	ND-MN	0.1391	7.5933	0.7678	0.4540	-4.2873	-0.6426
24300	Grand Junction	CO	0.1980	14.4225	0.7324	0.5205	2.2600	-0.7599
24340	Grand Rapids-Wyoming	MI	1.1058	14.8202	0.8746	0.2091	-2.1226	1.1623
24500	Great Falls	MT	0.1164	3.0799	0.7954	0.5633	2.2000	-1.3183
24540	Greeley	CO	0.3470	11.1165	0.8543	0.6195	1.7000	-0.2422
24580	Green Bay	WI	0.4287	7.7067	0.8387	0.2912	-1.3945	0.1489
24660	Greensboro-High Point	NC	0.9944	12.2863	0.8764	0.2038	-0.2512	0.8794
24780	Greenville	NC	0.2455	8.4053	0.8048	0.4570	-1.9108	-0.3848
24860	Greenville-Mauldin-Easley	SC	0.8739	29.0690	0.7805	0.2293	1.3467	0.7392
25060	Gulfport-Biloxi	MS	0.3296	3.7705	0.8944	0.4062	0.1310	-0.3076
25180	Hagerstown-Martinsburg	MD-WV	0.3718	29.3045	0.7547	0.6204	0.3042	-0.0839
25260	Hanford-Corcoran	CA	0.2119	4.4956	0.8817	0.5882	3.4800	-0.9992
25420	Harrisburg-Carlisle	PA	0.7529	15.7008	0.8614	0.2220	-0.0004	0.5819
25500	Harrisonburg	VA	0.1674	3.5773	0.9210	0.4938	1.2500	-1.0739
25540	Hartford-West Hartford-East Hartford	CT	1.6929	0.6312	1.3157	0.1934	1.4760	0.8809
25620	Hattiesburg	MS	0.1967	14.5668	0.7576	0.6026	-0.2014	-0.6437
25860	Hickory-Lenoir-Morganton	NC	0.5132	43.2249	0.7227	0.3150	1.5055	0.2302
25980	Hinesville-Fort Stewart	GA	0.1022	0.0097	1.7152	1.4824	0.8063	-2.4818
26100	Holland-Grand Haven	MI	0.3690	4.6934	0.8693	0.4246	-0.0400	-0.1742
26300	Hot Springs	AR	0.1372	11.9767	0.7219	0.7581	1.6400	-1.1335
26380	Houma-Bayou Cane-Thibodaux	LA	0.2863	2.3685	0.9718	0.4086	0.3192	-0.5579
26420	Houston-Sugar Land-Baytown	TX	8.0123	0.7875	1.4273	0.1036	0.8426	2.4951
26580	Huntington-Ashland	WV-KY-OH	0.4043	18.9859	0.7879	0.3638	-0.1699	0.0365
26620	Huntsville	AL	0.5504	4.8277	0.9105	0.2864	-0.9066	0.2760
26820	Idaho Falls	ID	0.1700	14.9270	0.6994	0.6242	1.7783	-0.8152
26900	Indianapolis-Carmel	IN	2.4131	6.4117	1.0203	0.1453	-2.5367	1.8239
26980	Iowa City	IA	0.2093	3.0028	0.9098	0.4185	-2.9476	-0.5311
27060	Ithaca	NY	0.1439	7.6229	0.7882	0.5491	-0.2800	-0.9925
27100	Jackson	MI	0.2321	5.6531	0.8683	0.6124	-2.4500	-0.4931
27140	Jackson	MS	0.7603	9.3264	0.8735	0.2701	-0.6024	0.6792
27180	Jackson	TN	0.1604	8.0248	0.7820	0.4913	-1.6345	-0.8225
27260	Jacksonville	FL	1.8519	6.0828	0.9489	0.1930	2.0244	1.3020
27340	Jacksonville	NC	0.2317	0.1526	1.2201	0.6158	0.7400	-1.3510
27500	Janesville	WI	0.2272	17.1165	0.7514	0.5567	-2.6200	-0.3910
27620	Jefferson City	MO	0.2074	21.2752	0.7585	0.4518	0.3296	-0.5943

Table 4 (continued).

FIPS	MSA name	State	$L_r/\overline{L}$	$\widehat{\mu}_r^{\max}$	$1/\overline{m}_r$	$\widehat{\theta}_r$	A_r^o	$\widehat{A}_r^u$
27740	Johnson City	TN	0.2755	15.4626	0.7613	0.4448	1.5055	-0.4559
27780	Johnstown	PA	0.2064	47.5556	0.6679	0.5599	-0.2300	-0.5483
27860	Jonesboro	AR	0.1657	19.0537	0.7332	0.4910	-2.2503	-0.6718
27900	Joplin	MO	0.2438	33.7469	0.6737	0.4025	-1.3200	-0.2872
28020	Kalamazoo-Portage	MI	0.4602	10.9030	0.8445	0.3422	-1.3239	0.2034
28100	Kankakee-Bradley	IL	0.1576	66.9572	0.6773	0.7130	-3.3000	-0.6326
28140	Kansas City	MO-KS	2.8265	9.2978	0.9719	0.1388	-1.3222	2.0201
28420	Kennewick-Pasco-Richland	WA	0.3260	1.7999	0.9386	0.4454	0.7491	-0.3261
28660	Killeen-Temple-Fort Hood	TX	0.5268	2.1655	1.0220	0.3488	1.5578	-0.0822
28700	Kingsport-Bristol-Bristol	TN-VA	0.4323	20.7011	0.7895	0.3835	0.3622	0.0800
28740	Kingston	NY	0.2589	38.4944	0.7621	0.7757	0.7000	-0.4394
28940	Knoxville	TN	0.9702	10.7076	0.8633	0.2284	1.0960	0.7774
29020	Kokomo	IN	0.1421	4.4454	0.8611	0.4794	-4.4522	-0.9032
29100	La Crosse	WI-MN	0.1864	15.4794	0.7197	0.4276	-1.1484	-0.6119
29140	Lafayette	IN	0.2736	6.6786	0.8963	0.4269	-3.4119	-0.2047
29180	Lafayette	LA	0.3652	0.3936	1.1340	0.3333	-0.9092	-0.4845
29340	Lake Charles	LA	0.2732	0.2160	1.2988	0.4158	0.1230	-0.8452
29460	Lakeland-Winter Haven	FL	0.8182	41.3451	0.7338	0.3320	3.9800	0.5254
29540	Lancaster	PA	0.7096	23.6630	0.8138	0.2773	0.4500	0.4974
29620	Lansing-East Lansing	MI	0.6498	8.5097	0.9034	0.3102	-3.3358	0.6664
29700	Laredo	TX	0.3319	40.7539	0.6586	0.3942	1.1200	-0.0710
29740	Las Cruces	NM	0.2830	14.1950	0.7658	0.4945	4.7700	-0.5204
29820	Las Vegas-Paradise	NV	2.6143	5.7538	0.9982	0.1449	4.8600	1.4990
29940	Lawrence	KS	0.1616	9.0883	0.7461	0.6893	0.3600	-0.9008
30020	Lawton	OK	0.1620	1.7247	0.9186	0.4717	2.2900	-1.2620
30140	Lebanon	PA	0.1821	21.6701	0.7301	0.6784	-0.6600	-0.7918
30340	Lewiston-Auburn	ME	0.1521	6.7201	0.7348	0.6650	-0.3200	-0.9631
30460	Lexington-Fayette	KY	0.6366	7.4339	0.8874	0.2408	-2.0342	0.5128
30620	Lima	OH	0.1498	6.3170	0.7978	0.4620	-2.3700	-0.9154
30700	Lincoln	NE	0.4160	6.3780	0.8194	0.2917	-2.8183	0.2242
30780	Little Rock-North Little Rock-Conway	AR	0.9487	8.6504	0.8992	0.2235	-0.0673	0.8521
30860	Logan	UT-ID	0.1724	17.5016	0.6920	0.6184	2.2845	-0.8079
30980	Longview	TX	0.2899	3.1890	0.9405	0.4235	1.0970	-0.5565
31020	Longview	WA	0.1430	5.9983	0.8127	0.8130	4.5400	-1.3338
31100	Los Angeles-Long Beach-Santa Ana	CA	18.3301	4.3306	1.2309	0.0708	10.0712	2.8862
31140	Louisville/Jefferson County	KY-IN	1.7564	14.2754	0.9145	0.1752	-0.7687	1.5113
31180	Lubbock	TX	0.3804	12.8002	0.7377	0.3094	1.7950	-0.0905
31340	Lynchburg	VA	0.3468	21.0406	0.7998	0.4312	0.4764	-0.1345
31420	Macon	GA	0.3272	31.5646	0.7452	0.3784	0.9051	-0.1751
31460	Madera	CA	0.2086	6.7275	0.8891	0.8123	6.0000	-1.0943
31540	Madison	WI	0.7910	4.1702	0.9806	0.2343	-0.4945	0.6170
31700	Manchester-Nashua	NH	0.5727	0.1167	1.4554	0.5151	0.0700	-0.3611
31900	Mansfield	OH	0.1789	33.4517	0.6730	0.4979	-2.8800	-0.5658
32580	McAllen-Edinburg-Mission	TX	1.0115	78.4494	0.6015	0.2479	0.4600	1.0886
32780	Medford	OR	0.2837	7.3664	0.7742	0.3762	4.5000	-0.5412
32820	Memphis	TN-MS-AR	1.8230	5.5326	0.9880	0.1653	-0.7140	1.4824
32900	Merced	CA	0.3495	3.4046	0.9806	0.6661	4.5100	-0.5673
33100	Miami-Fort Lauderdale-Pompano Beach	FL	7.7064	5.1829	1.0756	0.1063	5.2315	2.4562
33140	Michigan City-La Porte	IN	0.1563	21.9162	0.7391	0.6279	-1.8700	-0.8200
33260	Midland	TX	0.1800	0.0677	1.2915	0.3498	1.4200	-1.5392
33340	Milwaukee-Waukesha-West Allis	WI	2.1987	5.9256	0.9583	0.1410	-1.7072	1.6745
33460	Minneapolis-St. Paul-Bloomington	MN-WI	4.5673	4.2763	1.0673	0.1133	-2.1830	2.4717
33540	Missoula	MT	0.1504	2.8725	0.8180	0.4512	1.7400	-1.0344
33660	Mobile	AL	0.5757	9.1311	0.8016	0.3067	1.5200	0.2423
33700	Modesto	CA	0.7278	6.4113	0.9156	0.4128	7.2100	0.0268
33740	Monroe	LA	0.2453	9.2380	0.7899	0.4184	0.3390	-0.5074
33780	Monroe	MI	0.2187	2.0031	0.9750	0.9408	-1.4300	-0.7490
33860	Montgomery	AL	0.5210	12.6484	0.8354	0.3087	0.4625	0.2498
34060	Morgantown	WV	0.1677	4.0622	0.9172	0.6007	-0.5645	-0.9222
34100	Morristown	TN	0.1916	17.5432	0.7285	0.6252	1.4428	-0.8147
34580	Mount Vernon-Anacortes	WA	0.1657	0.7668	1.0340	0.7719	4.9400	-1.4000
34620	Muncie	IN	0.1643	21.3999	0.7009	0.5363	-2.6000	-0.6699
34740	Muskegon-Norton Shores	MI	0.2483	10.5424	0.7619	0.4962	-0.4000	-0.4569
34820	Myrtle Beach-North Myrtle Beach-Conway	SC	0.3558	14.1273	0.7514	0.3492	0.8800	-0.1685
34900	Napa	CA	0.1887	0.7977	1.1158	0.6025	7.5300	-1.5827
34940	Naples-Marco Island	FL	0.4496	0.8553	1.0987	0.3608	5.0000	-0.4961
34980	Nashville-Davidson–Murfreesboro–Franklin	TN	2.1660	8.8103	0.9775	0.1761	-0.8913	1.6814
35300	New Haven-Milford	CT	1.2037	0.3565	1.3393	0.3373	2.5200	0.3149
35380	New Orleans-Metairie-Kenner	LA	1.4669	0.3827	1.3139	0.1997	0.3337	0.8483
35620	New York-Northern New Jersey-Long Island	NY-NJ-PA	26.7870	2.3289	1.4318	0.0708	0.7740	3.7219
35660	Niles-Benton Harbor	MI	0.2272	4.2225	0.8899	0.4910	-0.3000	-0.7112
35980	Norwich-New London	CT	0.3806	2.5282	0.9939	0.3834	2.4300	-0.4626
36100	Ocala	FL	0.4625	26.5691	0.7385	0.4508	2.5900	0.0392
36140	Ocean City	NJ	0.1373	1.0674	0.9729	0.6085	0.0700	-1.4334
36220	Odessa	TX	0.1845	1.7012	0.8694	0.4434	2.5000	-1.1410
36260	Ogden-Clearfield	UT	0.7379	7.3733	0.8296	0.3433	4.0883	0.3479
36420	Oklahoma City	OK	1.6984	8.9525	0.9256	0.1702	0.1199	1.4212
36500	Olympia	WA	0.3396	2.6762	0.8761	0.5266	3.3200	-0.5078

Table 4 (continued).

FIPS	MSA name	State	$L_r/\overline{L}$	$\widehat{\mu}_r^{\max}$	$1/\overline{m}_r$	$\widehat{\theta}_r$	A_r^o	$\widehat{A}_r^u$
36540	Omaha-Council Bluffs	NE-IA	1.1815	4.6939	0.9594	0.1726	-1.6836	1.1351
36740	Orlando-Kissimmee	FL	2.8935	9.3348	0.9478	0.1484	3.6792	1.6530
36780	Oshkosh-Neenah	WI	0.2308	3.4099	0.8448	0.3631	-1.3700	-0.5731
36980	Owensboro	KY	0.1596	5.0431	0.8563	0.4904	-0.9396	-0.9497
37100	Oxnard-Thousand Oaks-Ventura	CA	1.1366	1.0892	1.1665	0.3101	11.1700	-0.0195
37340	Palm Bay-Melbourne-Titusville	FL	0.7633	7.0268	0.8433	0.3242	3.9300	0.3194
37460	Panama City-Lynn Haven	FL	0.2335	3.9684	0.8128	0.4859	2.1500	-0.7925
37620	Parkersburg-Marietta-Vienna	WV-OH	0.2287	20.4051	0.7635	0.4824	-0.0229	-0.5302
37700	Pascagoula	MS	0.2164	3.3176	0.8870	0.6623	0.1912	-0.7469
37860	Pensacola-Ferry Pass-Brent	FL	0.6455	10.5757	0.8059	0.3574	2.0978	0.3456
37900	Peoria	IL	0.5285	6.0365	0.9428	0.2890	-2.5036	0.3764
37980	Philadelphia-Camden-Wilmington	PA-NJ-DE-MD	8.2969	5.0519	1.1876	0.1023	-0.6748	2.8345
38060	Phoenix-Mesa-Scottsdale	AZ	5.9500	13.0025	0.9713	0.1114	4.3136	2.4388
38220	Pine Bluff	AR	0.1445	18.4953	0.7485	0.5508	-1.2731	-0.8725
38300	Pittsburgh	PA	3.3537	10.5364	0.9970	0.1425	0.4012	2.0415
38340	Pittsfield	MA	0.1848	0.0590	1.5480	0.7997	0.8100	-1.5454
38540	Pocatello	ID	0.1247	18.4792	0.6806	0.5365	1.9030	-1.1149
38860	Portland-South Portland-Biddeford	ME	0.7305	0.3729	1.2367	0.3868	0.9595	0.1744
38900	Portland-Vancouver-Beaverton	OR-WA	3.0966	2.5795	1.0900	0.1534	2.8130	1.7475
38940	Port St. Lucie	FL	0.5696	4.4925	0.8792	0.4656	5.1827	-0.0890
39100	Poughkeepsie-Newburgh-Middletown	NY	0.9537	57.5790	0.7869	0.3958	0.0107	0.8914
39140	Prescott	AZ	0.3027	55.8791	0.7200	0.5665	5.2100	-0.4084
39300	Providence-New Bedford-Fall River	RI-MA	2.2790	1.8282	1.1372	0.2242	1.2849	1.3694
39340	Provo-Orem	UT	0.7023	15.6423	0.8210	0.3378	3.0296	0.5132
39380	Pueblo	CO	0.2200	33.0571	0.6806	0.5804	2.1100	-0.5738
39460	Punta Gorda	FL	0.2176	4.7904	0.8279	0.6776	5.1000	-1.0319
39540	Racine	WI	0.2777	2.6053	0.9046	0.5556	-0.5100	-0.5717
39580	Raleigh-Cary	NC	1.4914	4.1913	0.9997	0.2143	-0.6762	1.1883
39660	Rapid City	SD	0.1712	10.5487	0.7744	0.4558	-0.3579	-0.7024
39740	Reading	PA	0.5722	12.9659	0.8697	0.3670	-0.7300	0.2974
39820	Redding	CA	0.2554	5.9179	0.8368	0.4672	5.6900	-0.7588
39900	Reno-Sparks	NV	0.5841	6.1702	0.9153	0.2685	6.7038	-0.0559
40060	Richmond	VA	1.7268	11.1761	0.9742	0.1846	-0.9568	1.4730
40140	Riverside-San Bernardino-Ontario	CA	5.8104	104.4265	0.8632	0.1695	4.3817	2.5456
40220	Roanoke	VA	0.4222	22.5390	0.7805	0.3012	0.9380	0.0199
40340	Rochester	MN	0.2578	7.1786	0.8243	0.3375	-3.3458	-0.2406
40380	Rochester	NY	1.4670	9.7948	0.9057	0.1746	-0.6948	1.3292
40420	Rockford	IL	0.5015	16.7848	0.7779	0.3553	-2.7901	0.3797
40580	Rocky Mount	NC	0.2073	6.0239	0.8554	0.4688	-1.7475	-0.6464
40660	Rome	GA	0.1361	17.3345	0.7232	0.6475	0.3300	-1.0785
40900	Sacramento–Arden-Arcade–Roseville	CA	2.9770	4.8303	1.0444	0.1708	5.4091	1.5526
40980	Saginaw-Saginaw Township North	MI	0.2880	16.5948	0.7583	0.3910	-3.3300	-0.0839
41060	St. Cloud	MN	0.2642	12.5971	0.7626	0.4347	-3.0004	-0.1386
41100	St. George	UT	0.1905	23.2639	0.6948	0.4957	2.5700	-0.7385
41140	St. Joseph	MO-KS	0.1756	10.6024	0.7922	0.5409	-1.4641	-0.7059
41180	St. Louis	MO-IL	3.9914	19.9079	0.9226	0.1312	-0.4277	2.3707
41420	Salem	OR	0.5505	9.5532	0.8053	0.3850	3.4215	0.1330
41500	Salinas	CA	0.5803	1.2221	1.1497	0.3426	9.2400	-0.5045
41540	Salisbury	MD	0.1703	13.6356	0.7665	0.6063	-0.3934	-0.8133
41620	Salt Lake City	UT	1.5660	5.5353	0.9849	0.1645	3.3545	1.1401
41660	San Angelo	TX	0.1539	11.3999	0.7550	0.5001	1.5945	-0.9984
41700	San Antonio	TX	2.8340	12.2914	0.9238	0.1656	2.1287	1.8188
41740	San Diego-Carlsbad-San Marcos	CA	4.2351	1.5943	1.2222	0.1332	9.7800	1.4266
41780	Sandusky	OH	0.1101	4.8876	0.7919	0.5651	-0.9100	-1.3725
41860	San Francisco-Oakland-Fremont	CA	5.9848	0.3531	1.4952	0.1203	7.3604	1.6192
41940	San Jose-Sunnyvale-Santa Clara	CA	2.5677	0.1447	1.5878	0.1526	5.5612	0.8121
42020	San Luis Obispo-Paso Robles	CA	0.3736	2.4081	1.0086	0.3809	7.8700	-0.6538
42060	Santa Barbara-Santa Maria-Goleta	CA	0.5754	0.8643	1.1438	0.2810	10.9700	-0.5659
42100	Santa Cruz-Watsonville	CA	0.3584	0.6286	1.1396	0.6419	8.4900	-1.0716
42140	Santa Fe	NM	0.2035	0.1706	1.2396	0.6477	3.0200	-1.2264
42220	Santa Rosa-Petaluma	CA	0.6612	1.8173	1.0370	0.3670	7.9300	-0.2054
42260	Bradenton-Sarasota-Venice	FL	0.9783	8.0869	0.8481	0.2326	4.7123	0.5228
42340	Savannah	GA	0.4688	9.2001	0.8077	0.3385	0.7595	0.0822
42540	Scranton–Wilkes-Barre	PA	0.7822	62.6807	0.7348	0.2540	0.3497	0.7451
42660	Seattle-Tacoma-Bellevue	WA	4.7113	1.1719	1.2432	0.1332	4.6088	1.8885
42680	Sebastian-Vero Beach	FL	0.1877	1.2555	0.9359	0.6381	4.7200	-1.2862
43100	Sheboygan	WI	0.1630	3.2650	0.8625	0.4794	-0.3700	-1.0073
43300	Sherman-Denison	TX	0.1689	20.5729	0.7343	0.7441	0.7800	-0.9061
43340	Shreveport-Bossier City	LA	0.5518	0.5061	1.2082	0.2672	0.4263	-0.0654
43580	Sioux City	IA-NE-SD	0.2033	6.7056	0.8078	0.3518	-1.6477	-0.5531
43620	Sioux Falls	SD	0.3234	0.9176	1.0383	0.3194	-3.1981	-0.1810
43780	South Bend-Mishawaka	IN-MI	0.4508	5.9962	0.9017	0.3487	-2.3182	0.1576
43900	Spartanburg	SC	0.3923	11.2840	0.7992	0.3525	0.5200	-0.1066
44060	Spokane	WA	0.6494	3.8173	0.8466	0.2893	1.3300	0.3953
44100	Springfield	IL	0.2941	14.5944	0.7757	0.3680	-2.6215	-0.1150
44140	Springfield	MA	0.9719	48.7269	0.7653	0.2673	-0.0296	0.9868
44180	Springfield	MO	0.5980	42.4428	0.7162	0.3118	-0.1019	0.5377
44220	Springfield	OH	0.2000	20.6803	0.7124	0.6353	-2.0300	-0.5560
44300	State College	PA	0.2059	5.6983	0.8980	0.4912	-0.4000	-0.6733
44700	Stockton	CA	0.9552	9.1216	0.8869	0.3999	4.7700	0.4709
44940	Sumter	SC	0.1480	5.4151	0.8191	0.6486	0.4500	-1.1196
45060	Syracuse	NY	0.9187	11.6878	0.8621	0.2285	-1.0878	0.9094

Table 4 (continued).

FIPS	MSA name	State	$L_r/\overline{L}$	$\widehat{\mu}_r^{\max}$	$1/\overline{m}_r$	$\widehat{\theta}_r$	A_r^o	$\widehat{A}_r^u$
45220	Tallahassee	FL	0.5016	15.0466	0.7887	0.3650	1.8418	0.1910
45300	Tampa-St. Petersburg-Clearwater	FL	3.8779	17.9295	0.8662	0.1303	4.0087	1.9781
45460	Terre Haute	IN	0.2411	20.4346	0.7766	0.5363	-2.2437	-0.3093
45500	Texarkana	TX	0.1911	11.9339	0.7701	0.4806	0.3401	-0.7535
45780	Toledo	OH	0.9267	18.0928	0.8282	0.2156	-2.2985	0.9937
45820	Topeka	KS	0.3256	22.9574	0.7672	0.3978	-1.2054	-0.0417
45940	Trenton-Ewing	NJ	0.5203	1.6191	1.0467	0.3137	-0.8000	-0.1181
46060	Tucson	AZ	1.3768	24.1671	0.8204	0.2328	4.0400	1.0965
46140	Tulsa	OK	1.2895	5.5205	0.9845	0.1913	0.4138	1.0760
46220	Tuscaloosa	AL	0.2922	7.7286	0.8737	0.3964	0.5956	-0.3554
46340	Tyler	TX	0.2829	3.5960	0.8892	0.4075	0.7200	-0.5192
46540	Utica-Rome	NY	0.4198	76.1905	0.6887	0.3637	-1.6177	0.3300
46660	Valdosta	GA	0.1853	33.3007	0.6831	0.4890	0.4906	-0.6906
46700	Vallejo-Fairfield	CA	0.5817	2.3184	1.0196	0.5800	5.8800	-0.2641
47020	Victoria	TX	0.1620	1.9775	0.9658	0.5431	0.7132	-1.1395
47220	Vineland-Millville-Bridgeton	NJ	0.2214	18.9165	0.7773	0.5472	0.3800	-0.6868
47260	Virginia Beach-Norfolk-Newport News	VA-NC	2.3615	6.6554	0.9682	0.1646	0.7721	1.5923
47300	Visalia-Porterville	CA	0.6001	20.2186	0.8264	0.3309	5.6500	0.1024
47380	Waco	TX	0.3248	14.4336	0.7623	0.3399	0.7600	-0.2405
47580	Warner Robins	GA	0.1865	2.0361	0.8817	0.5774	-0.0400	-0.9647
47900	Washington-Arlington-Alexandria	DC-VA-MD-WV	7.5546	2.1874	1.2875	0.1175	-0.5658	2.6267
47940	Waterloo-Cedar Falls	IA	0.2325	4.0817	0.8784	0.3123	-3.6928	-0.3363
48140	Wausau	WI	0.1850	8.5505	0.7840	0.4457	-3.3000	-0.5433
48260	Weirton-Steubenville	WV-OH	0.1745	12.5561	0.7784	0.6507	-0.4289	-0.8395
48300	Wenatchee	WA	0.1526	2.5064	0.9367	0.6415	1.1223	-1.0532
48540	Wheeling	WV-OH	0.2071	27.1680	0.7306	0.5045	-0.0508	-0.6087
48620	Wichita	KS	0.8491	7.0330	0.8959	0.2070	-0.5189	0.7748
48660	Wichita Falls	TX	0.2109	3.6100	0.9231	0.4866	-0.0733	-0.7295
48700	Williamsport	PA	0.1663	37.1189	0.7212	0.5359	0.3300	-0.8261
48900	Wilmington	NC	0.4833	4.2397	0.9124	0.3689	0.8620	0.0454
49020	Winchester	VA-WV	0.1725	8.0065	0.8765	0.8358	0.2643	-0.9449
49180	Winston-Salem	NC	0.6594	3.7013	0.9707	0.2738	-0.3283	0.3418
49340	Worcester	MA	1.1124	1.7596	1.1348	0.4121	0.2400	0.7079
49420	Yakima	WA	0.3318	3.8343	0.9066	0.4012	1.4800	-0.2958
49620	York-Hanover	PA	0.5994	20.5103	0.8111	0.4145	-0.5800	0.3817
49660	Youngstown-Warren-Boardman	OH-PA	0.8125	37.2035	0.7640	0.2679	-2.2828	0.9348
49700	Yuba City	CA	0.2337	1.2193	1.0373	0.9995	3.3821	-1.0057
49740	Yuma	AZ	0.2713	45.4247	0.6962	0.3985	4.2400	-0.5236

Notes: See Appendix F.2 for additional details on computations.