A stylized applied energy-economy model for France

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Abstract

We build, calibrate and simulate a stylized energy-economy model designed to evaluate the magnitude of the carbon tax that would allow the French economy to divide by four its CO_2 emissions at a forty year horizon. We estimate the substitution possibilities between fossil energy and other factors on the households' and firms' sides. Two versions of the model are built, the first one with exogenous technical progress, and the second one with an endogeneization of the direction of technical progress. We show that if the energy-saving technical progress rate remains at its recent historical value, the magnitude of the carbon tax is quite unrealistic. When the direction of technical progress responds endogenously to economic incentives, it is possible to reduce CO_2 emissions beyond what substitution possibilities would allow, but not to divide them by four. For that, an additional instrument is needed, namely a subsidy to fossil energy-saving research. The redirection of technical progress, engine of the energy transition, comes at a small expense in terms of the overall growth rate of the economy.

1 Introduction

The term Factor 4 coined in France corresponds to the commitment entered into in 2003 by the French President to divide by at least 75% French GHG emissions by 2050 compared to the 1990 level. The European Council and the European Parliament have endorsed this objective, and asserted on numerous occasions and in various documents the necessity to develop long term strategies to encourage the transition to a low carbon economy. Those unilateral commitments are not sufficient in such for efficiently tackling climate change, but are however possibly useful. If they prove to be successful, they would indeed likely incite other countries to act in turn and could unlock international negotiations. But at this stage, important uncertainties exist on the costs and even the feasibility of those reductions. The debate on the adequate mix of instruments to be implemented to attain such reductions at a reasonable cost is still ongoing: market instruments, standards, public investments, R&D efforts? Whatever mix is chosen, mechanisms based on the increase of the consumer price of fossil fuels will certainly be settled; a situation with low fossil fuels prices seems indeed incompatible with a sensible decrease of their utilization. One can think that the necessary price increase will naturally occur due to the supply-demand equation and that no additional policy is necessary. Even if this happens to be true, which is very unlikely, the question remains: what should be the consumer price of fossil fuels, for their consumption to be divided by four in the long run?

This question has of course been already addressed. In France, an official commission chaired by Alain Quinet has been settled in 2008, the purpose of which was to determine the carbon social value that should be used in public cost-benefit analysis by the French administration (see Quinet (2009)). The underlying goal was in fact to determine the carbon value that should be diffused in the whole economy, so as to achieve a 75% reduction of emissions. The commission relied on the results of simulations performed by three French integrated assessment models, GEMINI-E3 (Vielle & Bernard (1998)), POLES (Criqui et al. (2006)) and IMACLIM-R (Sassi et al. (2010)), which computed the initial level and the time path of the carbon value allowing European economies to divide their carbon emissions by four at a forty year horizon. The three models include (different) assumptions on the substitution possibilities in the economy and on the magnitude of energy-saving technical progress, which have a major influence on the results obtained. However, these assumptions are somehow "hidden", because of the complexity of the models, their large size and particularly their sectoral disaggregation. It has proved impossible to obtain from the three teams of modelers involved in the Quinet exercise an implicit average rate of energy-saving technical progress. For given substitution possibilities, energy-saving technical progress obviously alleviates the magnitude of the carbon value necessary to reach the objective of reduction of emissions. It is thus very important first to have an accurate estimation of the substitution possibilities, and second to disentangle clearly the role of the instrument and the one of technical progress on the way to the objective.

We build here a stylized macroeconomic model in which the assumptions on technical progress are explicit and their influence can be easily analyzed. We model an open economy producing a generic good, which can be consumed or invested, and importing fossil energy as its sole source of energy[1]. Whereas usually energy is only considered as an input in the production process, we also introduce here households' consumption of fossil fuels, and the fact that fossil fuels are used together with durable goods (residential energy, fuel for transport). The two sectors that are the larger emitters and have been until now unable to reduce their GHG emissions in France are indeed transport and to a lesser extent housing (see Table 1). Both rely heavily on fossil fuels, and this dependence is expected to last. So it seems important to take them properly into account. Final or intermediate fossil energy consumption can be reduced either by substitutions triggered by an increase of the consumer energy price via a tax on CO_2 emissions (a carbon tax), or by technical progress. Substitution possibilities exist, between energy, durable goods and non-durable goods on the households' side, and between energy, capital and labor on the production side. The structure of households' consumption and firms' production is not given once and for all. However, these substitution possibilities are limited. The other option is to rely on fossil energy-saving technological progress. Therefore, we introduce two forms of technical progress, respectively labor- and energy-saving. The energy-saving technical progress we have in mind consists both in improvements of energy efficiency and the replacement of fossil fuels by renewables. Thus we do not explicitly introduce renewables in the model.

We present two versions of the model.

[1] In 2009, fossil energy represents 67.5% of total final energy consumption in France, electricity 23.7% and renewables 8.8% (SOes, Bilan de l'énergie 2009).

Table 1: CO_2 emissions due to energy, France, 2009 (CSV)

	Mt CO_2	%	evolution between 1990 and 2009, %
transport	141	40.2	+15.2
housing-tertiary	92	26.2	-3.7
industry (non-energy)	61	17.4	-28.5
agriculture	10	2.8	-0.7
energy	47	33.3	-22.1
	351	100.0	-6.1

Source: SOes, Bilan de l'énergie 2009

In the first version (Section 2), technical progress is exogenous. The rates of labor-saving and energy-saving technical progress are estimated using French annual historical data. The question we want to answer is the following: considering that the rates of technical progress remain those observed in the (recent) past, what is the carbon price path necessary for dividing CO_2 emissions by four in 40 years[2]? The implicit assumption is that the policies put in place in our simulations the increase of fossil fuels prices , have no effect on the rate of fossil energy-saving technical progress, and that no specific policy aimed at increasing this rate is implemented. With this first exercise, we determine if the carbon price increase necessary to reach the objective is reasonable within the time available. We show that it is not. We then increase exogenously the rate of fossil energy-saving technical progress and determine to what extent the level of the carbon tax can be reduced. However, this exercise remains unsatisfactory since this increase of the technical progress rate is costless, and does not occur at the expense of the other rate of technical progress of the model, the labor-saving technical progress.

The rate of technical progress associated with energy use is in fact likely to be notably correlated with the level and variations of the energy price. Thus, in the second version of the model (Section 3), we incorporate an endogenous mechanism in which the rate of technical progress on fossil energy can be stimulated by a price effect and by a size effect of the research effort directed to saving fossil energy. We can analyze to what extent the endogeneization of the direction of technical change affects the results obtained in the first exercise. Technical progress is not fully endogeneized: the total amount of resources devoted to research is exogenous.

The results are the following. When technical progress is exogenous, the carbon tax of the Quinet Report is far from being sufficient to reduce by four CO_2 emissions at a forty year horizon. It yields a reduction of emissions of only 25%. Hence in large applied models there are more substitution possibilities and/or a lot of "hidden" energy-saving technical progress. When technical progress is exogenous, we must increase the rate of fossil energy-saving technical progress a lot (unreasonably) to reach Factor 4. It's costless. Too easy! When the direction of technical progress is endogenous, the introduction of the carbon tax induces a re-direction of the research effort towards energy-saving technical progress. Its rate increases a lot at once, and stabilizes in the medium run above its baseline value. It comes at a small expense in terms of overall growth. Nevertheless, the re-direction of technical progress is not sufficient to reach the Factor 4 objective. A supplementary measure is needed, namely a subsidy

[2] CO_2 emissions remained stable between 1990 and 2007. The financial crisis which began in 2008 brought a 6 % reduction (in 2009) in emissions compared to 1990 levels. As the model developed here is a long term model, we do not account for short term fluctuations and consider that the level of emission in 2010 is the same as in 1990, so that the Factor 4 objective consists in dividing emissions by four from 2010 to 2050.

to fossil energy-saving technical progress.

2 The model with exogenous technical progress

This first version of the model consists in a standard exogenous growth model integrating two types of technical progress, respectively labor and energy-saving, fossil fuel use both on the households and firms' side, and rigidities in the adjustment of the housing and the productive sectors. We describe successively households' and firms' behaviour and the closure of the model, the calibration method and results, and the simulations performed.

2.1 Households

Several macroeconomists have emphasized that distinguishing non-durable and durable goods is important to obtain an accurate representation, both on a theoretical and an empirical point of view, of households' consumption and savings decisions along their life cycle. Ogaki & Reinhart (1998) for instance show that introducing separately non-durable and durable goods modifies very significantly the estimation of the intertemporal elasticity of substitution of consumption. More recently, Fernández-Villaverde & Krueger (2011) survey the empirical literature and also conclude that the distinction is meaningful. These papers do not distinguish households' energy consumption from the consumption of other non-durable goods. We think that separating energy consumption considerably reinforces the importance of distinguishing between non-durable and durable goods, because durables (almost) only need energy to deliver their services whereas non-durables do not. These two types of goods are very different to that respect.

We thus consider that households have access to three types of goods: non-durable goods N, energy (fossil fuels) E and durable goods D. Non-durable goods are consumed during the period, whereas durable goods can be stored or have a long lifespan. Contrary to non-durables and energy, durable goods follow an accumulation process of the standard form[3]:

$$D_t = (1 - \delta_d) D_{t-1} + X_t \tag{1}$$

where X_t represents the investment in durable goods at period t and δ_d is the rate of depreciation.

Utility at period t is a function of the consumptions of non-durable goods N_t and energy $E_{h;t}$ in that period, and of the services provided by the stock of durable goods D_{t-1} at the beginning of the period, these services being supposed to be proportional to the stock itself:

$$U(N_t; D_{t-1}; E_{h;t}) = U(C_t)$$

where C_t is defined by:

$$C_t = \left[\alpha N_t^{\frac{\omega-1}{\omega}} + (1-\alpha) Z_{h;t}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \tag{2}$$

[3] The model is simulated with the Dynare software (Adjemian et al. (2011)), which adopts the convention that for stock variables the default is to use a stock at the end of the period. Our definition of D_t follows: D_t represents the stock of durable goods at the end of period t, i.e. that will be used by households in the following period $t+1$. The same convention will apply for productive capital and other stock variables.

$Z_{h,t}$ being a CES aggregate of services provided by durables goods D_{t-1} and efficient energy consumption $A^e_t E_{h,t}$:

$$Z_{h,t} = \left[\kappa D_{t-1}^{\frac{\rho-1}{\rho}} + (1-\kappa)(A^e_t E_{h,t})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{3}$$

A^e represents energy efficiency.

The type of aggregation chosen (CES aggregators) is the result of an estimation procedure of the elasticities of substitution (see below).

Changing the stock of durable goods induces adjustment costs. We make the assumption that these costs are nil along a balanced growth path, so that households bear them only if they have to deviate from the "normal" trajectory of the economy. These costs are classically specified as:

$$AC_{h,t} = \frac{\psi_d}{2}\left[\frac{D_t}{(1+g^{al}_t)D_{t-1}} - 1 \right]^2 (1+g^{al}_t)D_{t-1} \tag{4}$$

where g^{al} is the growth rate of labor productivity, which will turn out to be the long term growth rate of the economy[4].

At each period, the representative household can buy or sell bonds which pay or cost a nominal rate r_t. We denote A_{t-1} the nominal value of bonds possessed at the beginning of period t. Households revenues also consist in labour revenues and lump-sum transfers from the government T_t.

The budget constraint at period t reads, with obvious notations for the various prices:

$$(1+\tau^c_t)(P^n_t N_t + P^x_t X_t) + (P^e_t + \tau_{h,t})E_{h,t} + P^x_t AC_{h,t} + A_t = P^l_t L_t + T_t + (1+r_t)A_{t-1} \tag{5}$$

where τ^c is the tax rate on the consumption of goods and τ_h the additive tax on households' energy consumption.

The representative household seeks to maximize the discounted sum of its utilities under the intertemporal budget constraint:

$$\max \sum_{t=1}^{\infty} \frac{1}{(1+\beta)^t} U(C_t)$$

$$\text{s.c.} \sum_{t=1}^{\infty} \frac{(1+\tau^c_t)(P^n_t N_t + P^x_t X_t) + (P^e_t + \tau_{h,t})E_{h,t} + P^x_t AC_{h,t}}{\Pi_{s=1}^t (1+r_s)} = A_0 + \sum_{t=1}^{\infty} \frac{P^l_t L_t + T_t}{\Pi_{s=1}^t (1+r_s)}$$

where $\beta > 0$ is the discount rate. The no-Ponzi condition[5] reads:

$$\lim_{t \to \infty} \frac{A_t}{\Pi_{s=1}^t (1+r_s)} = 0$$

We choose a logarithmic utility function:

$$U(C_t) = \ln C_t$$

Let P^d_t be the user cost of the stock of durable goods:

$$P^d_t = P^x_t \left[(1+\tau^c)\left(\frac{1+r_t}{1+\pi^x_t} - (1-\delta_d) \right) + \psi_d \left(-\frac{1}{2}(1+g^{al}_t)\left[\left(\frac{D_t}{(1+g^{al}_t)D_{t-1}} \right)^2 - 1 \right] + \frac{1+r_t}{1+\pi^x_t}\left(\frac{D_{t-1}}{(1+g^{al}_{t-1})D_{t-2}} - 1 \right) \right) \right] \tag{6}$$

[4] In this first version of the model, g^{al} is exogenous and constant. We will endogeneize it in the second version of the model, where it will not be constant anymore. This is why we keep here the time index.

[5] We do not impose explicitly in Dynare the no-Ponzi condition, but simply verify that it is satisfied in the simulations.

with $\pi_t^x = \frac{P_t^x}{P_{t-1}^x} - 1$:

We deduce from the FOC one forward-looking inter-temporal arbitrage and two static arbitrages between the three consumption goods:

$$\frac{1}{1+\rho}\left(\frac{N_t}{N_{t+1}}\right)^{\frac{1}{\omega}}\left(\frac{C_t}{C_{t+1}}\right)^{\frac{\omega-1}{\omega}} = \frac{1+\pi^n_{t+1}}{1+r_{t+1}} \tag{7}$$

$$\frac{1-\beta-\gamma}{\gamma}\frac{D_t}{E_{h;t+1}}\left(\frac{A^e_{t+1}E_{h;t+1}}{D_t}\right)^{\frac{\varepsilon-1}{\sigma}} = \frac{P^e_{t+1}+\tau_{h;t+1}}{P^d_{t+1}} \tag{8}$$

$$\frac{(1-\alpha)(1-\beta)}{\gamma}\frac{N_t}{E_{h;t}}\left(\frac{Z_{h;t}}{N_t}\right)^{\frac{\omega-1}{\omega}}\left(\frac{A^e_tE_{h;t}}{Z_{h;t}}\right)^{\frac{\sigma-1}{\sigma}} = \frac{P^e_t+\tau_{h;t}}{(1+\pi^c_t)P^n_t} \tag{9}$$

2.2 Firms

Firms are perfectly competitive. They produce the generic good using capital, labor and fossil fuels, according to the following specification:

$$Y_t = \left[\alpha(A^l_tL_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)Z_{f;t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{10}$$

$$Z_{f;t} = \left[\beta K_{t-1}^{\frac{\sigma-1}{\sigma}} + (1-\beta)(A^e_tE_{f;t})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{11}$$

$$K_t = (1-\delta_k)K_{t-1} + I_t \tag{12}$$

As for the stock of durable goods, changing the stock of capital leads to adjustment costs, specified as:

$$AC_{f;t} = \frac{\phi_k}{2}\left(\frac{K_t}{(1+g^{al}_t)K_{t-1}} - 1\right)^2(1+g^{al}_t)K_{t-1} \tag{13}$$

The problem of the representative firm reads:

$$\max V_0 = \sum_{t=1} \frac{P^y_tY_t - P^l_tL_t - P^i_t(I_t+AC_t) - (P^e_t+\tau_{f;t})E_{f;t}}{\Pi_{s=1}^t(1+r_s)}$$

s.c. (12), (13)

with τ_f the tax on firms' energy consumption, possibly different from the tax τ_h paid by households.

Let P^k_t be the user cost of capital:

$$P^k_t = P^i_t\left[\frac{1+r_t}{1+\pi^i_t} - (1-\delta_k) + \phi_k\left[\frac{1+r_t}{1+\pi^i_t}\left(\frac{K_{t-1}}{(1+g^{al}_{t-1})K_{t-2}} - 1\right) - \frac{1}{2}\left(1+g^{al}_t\right)\left(\left(\frac{K_t}{(1+g^{al}_t)K_{t-1}}\right)^2 - 1\right)\right]\right] \tag{14}$$

FOC simply state that the marginal productivity of the inputs are equal to their real cost:

$$\alpha(A^l_t)^{\frac{\sigma-1}{\sigma}}\left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}} = \frac{P^l_t}{P^y_t} \tag{15}$$

$$(1-\alpha)\left(\frac{Y_t}{Z_{f;t}}\right)^{\frac{1}{\sigma}}(1-\beta)(A^e_t)^{\frac{\sigma-1}{\sigma}}\left(\frac{Z_{f;t}}{E_{f;t}}\right)^{\frac{1}{\sigma}} = \frac{P^e_t+\tau_{f;t}}{P^y_t} \tag{16}$$

$$(1-\alpha)\left(\frac{Y_t}{Z_{f;t}}\right)^{\frac{1}{\sigma}}\beta\left(\frac{Z_{f;t}}{K_{t-1}}\right)^{\frac{1}{\sigma}} = \frac{P^k_t}{P^y_t} \tag{17}$$

2.3 Government

The government receives tax receipts and reimburses them lump sum to households, so that its budget is balanced at each date:

$$\tau^c_t(P^n_t N_t + P^x_t X_t) + \tau_{h,t} E_{h,t} + \tau_{f,t} E_{f,t} = T_t \tag{18}$$

2.4 Closure

The equilibria on the generic good market and the labor market respectively read:

$$Y_t = N_t + X_t + I_t + AC_{h,t} + AC_{f,t} + EX_t \tag{19}$$

$$L_t = \overline{L} \tag{20}$$

Note that adjustment costs are costs in terms of the generic good. Exportations of the generic good are denoted EX. They are proportional to an exogenous foreign demand for the good $\overline{D}$, and respond to a relative price effect:

$$EX_t = \overline{D}_t \left(\frac{er_t \overline{P}_t}{P^y_t}\right)^{-\sigma} \tag{21}$$

where $\overline{P}$ is the exogenous price of the generic good in the rest of the world in foreign currency, er the exchange rate and σ the price elasticity of exports.

Fossil fuels are totally imported. We do not model the extraction behaviour of producers and consider that the producer price in foreign currency $\overline{P}^e$ is exogenous and grows at a constant rate, in order to reflect the increasing scarcity of non-renewable resources and to mimic a Hotelling-type behaviour. The price of fossil fuels in domestic currency is $P^e = er\overline{P}^e$. er adjusts at each date as to ensure the equilibrium of the trade balance:

$$P^y_t EX_t = P^e_t (E_{h,t} + E_{f,t}) \tag{22}$$

Let's go back to the household's budget constraint (5). Using successively the government's budget constraint (18), the zero profit condition of firms, the equilibrium condition on the good market (19), the expressions of I and AC_f by their expressions in terms of the capital stock K (equations (12) and (13)) and using the expression of the user cost of capital (14) allows us to obtain:

$$A_t - (1+r_t)A_{t-1} = P^i_t\left[1+\phi_k\left(\frac{K_t}{(1+g^{al}_t)K_{t-1}}-1\right)\right]K_t - P^i_t\frac{1+r_t}{1+\pi^i_t}\left[1+\phi_k\left(\frac{K_{t-1}}{(1+g^{al}_{t-1})K_{t-2}}-1\right)\right]K_{t-1}$$

This equation, together with the initial condition[6]

$$A_0 = P^i_0 K_0$$

allows us to obtain the relationship between household's financial wealth and the stock of capital at each date $t > 0$:

$$A_t = P^i_t\left[1+\phi\left(\frac{K_t}{(1+g^{al}_t)K_{t-1}}-1\right)\right]K_t \tag{23}$$

The model in standard variables is composed of equations (1) to (22). We choose the production price as numeraire: $P^y_t = 1\ \forall t$. Hence $P^n_t = P^i_t = P^x_t = 1$.

[6] This condition is satisfied at the steady state, see below.

2.5 Long term

In this first version of the model, the growth rates of labor productivity g^{al} and of energy efficiency g^{ae} are exogenous and constant. The price of energy in foreign currency, and hence its growth rate, is exogenous.

We want to describe an economy evolving in the long run along a balanced growth path[7]. The common growth rate of the real economic variables, including efficient energy demands, is necessarily g^{al}, the exogenous rate of labor-augmenting technical progress. Energy efficiency growing at rate g^{ae}, gross energy demands have to grow at rate $(1+g^{al})/(1+g^{ae})-1$. Prices (and the exchange rate) are stationary except for P^{l} and P^{e}, which respectively grow at rates g^{al} and g^{ae}. Moreover, the fact that A, T, $\tau_h E_h$ and $\tau_f E_f$ must grow in the long run yields that taxes on energy τ_h and τ_f have to grow at rate g^{ae}. Finally, foreign demand $\overline{D}$ must grow at rate g^{al}.

The requirement that the economy evolves along a balanced growth path in the long run is thus very restrictive. The energy price, in foreign currency and in domestic currency, must grow at the same rate than energy efficiency, which suggests introducing an endogenous technical progress induced by the energy price. Taxes on energy must also grow at this same rate[8]. Fossil energy consumption may decrease or increase, depending on the respective magnitudes of g^{al} and g^{ae}.

2.6 Model in intensive variables

We note $x_t = X_t/(A^l_t L_t)$ and $p^l_t = P^l_t/A^l_t$. We normalize $\overline{L} = 1$. We introduce new variables which are stationary in the long run: $(A^e e_h)_t = A^e_t e_{h,t}$, $(A^e e_f)_t = A^e_t e_{f,t}$, $(P^e/A^e)_t = P^e_t/A^e_t$, $(\tau_h/A^e)_t = \tau_{h,t}/A^e_t$, $(\tau_f/A^e)_t = \tau_{f,t}/A^e_t$. The equations of the model in intensive variables read:

$$c_t = \left[\alpha n_t^{\frac{\omega-1}{\omega}} + (1-\alpha) z_{h,t}^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \tag{E1}$$

$$z_{h,t} = \left[\beta \left(\frac{d_{t-1}}{1+g^{al}_t}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\beta)(A^e e_h)_t^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{E2}$$

$$\left(\frac{n_t}{n_{t+1}}\right)^{\frac{1}{\omega}} \left(\frac{c_t}{c_{t+1}}\right)^{\frac{\omega-1}{\omega}} = \frac{(1+g^{al}_{t+1})(1+\rho)}{1+r_{t+1}} \tag{E3}$$

$$P^d_t = (1+\tau^c)(r_t+\delta_d) + \phi_d\left[(1+r_t)\left(\frac{d_{t-1}}{d_{t-2}}-1\right) - \frac{1}{2}(1+g^{al}_t)\left(\frac{d_t}{d_{t-1}}-1\right)^2\right] \tag{E4}$$

$$\frac{d_t}{(1+g^{al}_{t+1})(A^e e_h)_{t+1}} = \left[\frac{\beta}{1-\beta}\,\frac{(P^e/A^e)_{t+1} + (\tau_h/A^e)_{t+1}}{P^d_{t+1}}\right]^{\varepsilon} \tag{E5}$$

$$\frac{n_t^{\frac{1}{\omega}} z_{h,t}^{\frac{1}{\varepsilon}-\frac{1}{\omega}}}{(A^e e_h)_t^{\frac{1}{\varepsilon}}} = \frac{\alpha}{(1-\alpha)(1-\beta)}\,\frac{(P^e/A^e)_t + (\tau_h/A^e)_t}{1+\tau^c_t} \tag{E6}$$

$$d_t = \frac{(1-\delta_d)\,d_{t-1}}{1+g^{al}_t} + x_t \tag{E7}$$

$$k_t = \frac{(1-\delta_k)\,k_{t-1}}{1+g^{al}_t} + i_t \tag{E8}$$

[7] Indeed, it is possible to perform numerical simulations of the model only if both the initial and the final states of the economy are steady states. Fot that, the model must be written in stationary variables, i.e. in variables deflated by labor in efficiency units $A^l_t L_t$.

[8] Remember that energy taxes are here unit taxes; ad valorem taxes would have to remain constant.

$$y_t = \left[\square + (1 \square \square) z_{f;t}^{\frac{\square \square 1}{\square}} \right]^{\frac{\square}{\square \square 1}} \tag{E9}$$

$$z_{f;t} = \left(\square \left(\frac{k_{t \square 1}}{1+g_t^{al}} \right)^{\frac{\square \square 1}{\square}} + (1 \square \square) (A^e e_f)_t^{\frac{\square \square 1}{\square}} \right)^{\frac{\square}{\square \square 1}} \tag{E10}$$

$$y_t = \square^{\square \square} \left(p_t^l \right)^{\square} \tag{E11}$$

$$\frac{y_t}{z_{f;t}} = (1 \square \square)^{\square \square} \left(P_t^{zf} \right)^{\square} \tag{E12}$$

$$\left(\frac{k_{t \square 1}}{1+g_t^{al} \; z_{f;t}} \right) = \square^{\square} \left(\frac{P_t^{zf}}{P_t^k} \right)^{\square} \tag{E13}$$

$$\frac{k_{t \square 1}}{(1+g^{al})(A^e e_f)_t} = \left(\frac{\square}{1 \square \square} \right)^{\square} \left(\frac{(P^e = A^e)_t + (\square_f = A^e)_t}{P_t^k} \right)^{\square} \tag{E14}$$

$$P_t^k = \square_k + r_t + \square_f \quad (1+r_t) \left(\frac{k_{t \square 1}}{k_{t \square 2}} \square 1 \right) \square \frac{1}{2} \left(1+g^{al} \right) \left(\left(\frac{k_t}{k_{t \square 1}} \right)^2 \square 1 \right) \tag{E15}$$

$$y_t = i_t + n_t + x_t + \frac{\square_d}{2} \frac{(d_t \square d_{t \square 1})^2}{d_{t \square 1}} + \frac{\square_f}{2} \frac{(k_t \square k_{t \square 1})^2}{k_{t \square 1}} + ex_t \tag{E16}$$

$$(1+\square_t^c)(n_t + x_t) + (A^e e_h)_t \left((P^e = A^e)_t + (\square_h = A^e)_t \right) + \frac{\square_d}{2} \frac{(d_t \square d_{t \square 1})^2}{d_{t \square 1}} + a_t = p_t^l + (1+r_t) \frac{a_{t \square 1}}{1+g_t^{al}} + t_t \tag{E17}$$

$$t_t = \square_t^c (n_t + x_t) + (A^e e_h)_t (\square_h = A^e)_t + (A^e e_f)_t (\square_f = A^e)_t \tag{E18}$$

$$(A^e e)_t = (A^e e_h)_t + (A^e e_f)_t \tag{E19}$$

$$ex_t = \overline{d}_t \left(er_t \overline{P}_t \right)^{\square \square} \tag{E20}$$

$$ex_t = (P^e = A^e)_t (A^e e)_t \tag{E21}$$

$$(P^e = A^e)_t = er_t (\overline{P}^e = A^e)_t \tag{E22}$$

$$wal_t = a_t \square \left(1 + \square_k \left(\frac{k_t}{k_{t \square 1}} \square 1 \right) \right) k_t \tag{E23}$$

Equation (E23) is here just to ensure that the model is well specified: wal_t must be at each date equal to 0.

2.7 Estimations and calibration

The elasticities of substitution are central parameters which largely influence the simulation results. In the same way, assumptions on the rates of technical progress are determining. Thus, both for households' utility function and firms' production function, we perform an estimation of these elasticities and rates on French data. On the households' side, we estimate the elasticity of substitution between durable goods and energy, together with the average rate of technical progress related to households' energy use. Concerning the elasticity of substitution between non-durables goods and the aggregation of durable goods and energy, we follow Dhawan & Jeske (2008) who choose a unitary elasticity. On the firms' side, we estimate the elasticities of substitution in each CES function, together with the rates of technical progress on labor and energy.

2.7.1 Elasticities of substitution of households' utility function

Method and results

We choose a unitary elasticity of substitution (ω in equation (2)) between non-durable goods and the aggregation of durable goods and energy, as Dhawan & Jeske (2008). Their choice is motivated by Fernández-Villaverde & Krueger (2011), who use a Cobb-Douglas aggregation between durable and non-durable goods in the households' utility function. They justify this choice after reviewing literature results, and noting that in most cases, estimated elasticities are not significantly different from one. For instance, Ogaki & Reinhart (1998) find an elasticity of 1.167, not significantly different from one at the 5% level. Contrary to Fernández-Villaverde & Krueger (2011), our specification is not a two goods - durables and non-durables - utility function, since we also include energy. Though, as Dhawan & Jeske (2008), we extend the result of Fernández-Villaverde & Krueger (2011) to a utility function between non-durable goods and an aggregate between the stock of durables and energy and choose $\omega = 1$.

Concerning the elasticity of substitution between durable goods and energy (σ), Dhawan & Jeske (2008) find an elasticity of 0.26 for the United States by matching the theoretical volatility of households' energy use to the one observed in the data. We do not rely on their result because contrary to them, we include building in durable stocks, which is likely to impact the value of the elasticity. We perform an estimation of this elasticity, using a cointegration relation as Ogaki & Reinhart (1998). They indeed stress that the long run information identified by the cointegration relation is appropriate when dealing with durable goods because adjustment costs, although significant, do not affect the long run behaviour of consumption of durable goods. Moreover, their method avoids the computation of the user cost of durable goods P^d on the estimation sample, involving an expectation operator which is difficult to deal with.

For the purpose of this estimation, we show in Appendix A that $\left(\frac{(1+\tau^c)P^x_t}{(P^e_t+\tau_{h,t})/A^e_t}\right)\left(\frac{X_t}{A^e_t E_t}\right)^{\frac{1}{\sigma}}$ is stationary and that the vector $\left[\ln\left(\frac{(1+\tau^c)P^x_t}{(P^e_t+\tau_{h,t})/A^e_t}\right); \ln\left(\frac{X_t}{A^e_t E_t}\right)\right]$ is cointegrated with a cointegrating vector $\left(1; \frac{1}{\sigma}\right)$. The estimation of the relation gives the elasticity of substitution σ between durable goods and fossil energy, together with the average rate of technical progress related to households' energy use. We obtain $\sigma = 0.50$ and $g^{ae} = 1.6\%$ per year. Thus, we find that fossil energy and the services from durables are poorly substitutable. There are two ways to reduce fossil energy consumption without harming welfare. First, it is possible to substitute non-durable goods to the aggregate durables and fossil energy. Second, a decrease in fossil fuel consumption can be offset by an increase of energy efficiency, thanks to fossil energy-saving technical progress. Notice that with a rate of fossil energy-saving technical progress of 1.6% per year, with no economic growth, fossil fuel consumption would be divided only by 1.9 in 40 years! And 87 years would be necessary to reach the Factor 4 objective. If the economy keeps on growing, the decrease in fossil fuel consumption allowed by fossil energy-saving technical progress, if this rate remains the same as in historical data, will be even further from the 75% reduction in 40 years.

Data

We use data from the French national statistics administration (INSEE) for the period 1959-2010. In national accounts, durable goods are composed of furniture and consumers' equipment such as cars, television sets, refrigerators, etc. To include housing in the stock of durables, we build X_t as the sum of consumption of durable goods and of households' investment, which corresponds to housing investment. The price index P^x_t is built using the chained price methodology with elementary price indexes of housing investment and durable goods consumption. For $E_{h;t}$, we cut off fossil energy consumption from consumption by product provided in national accounts. This cannot be performed exactly with the energy split provided, since electricity is considered as a whole. Taking into account the fact that the share of electricity from fossil origin is very small in France, we totally exclude electricity in our computation. The price is built with the chained price methodology. Note that index prices include taxes on consumption. In particular the price index of fossil energy includes accises on energy. In the simulation section, we provide details on the decomposition of the price between gross price and taxes.

2.7.2 Elasticities of substitution in the production function

Method and results

We follow van der Werf (2008), see Appendix B.

The rate of labor saving technical progress is found to lie between $g^{al} = 1{:}5\%$ and $g^{al} = 1{:}6\%$; depending on the data we take. We estimate g^{ae} between 2.4% and 2.7%. As we assume that the rate of fossil energy-saving technical progress is the same for households' consumption and for production, and given that we found a rate of technical progress of 1:6% for households, we retain a uniform rate of energy-saving technical progress $g^{ae} = 2{:}0\%$. We find an elasticity of substitution of $\square = 0{:}5$ between capital (K) and fossil fuel use for production (E_f). We find an elasticity of substitution of $\square = 0{:}5$ between L and Z_f. In order to reinforce the result, we also follow the methodology of Ogaki & Reinhart (1998) to estimate the long term elasticity between K and E_f and find similar results.

We find that fossil energy, capital and labor are rather complement for production. The only way to reduce fossil energy consumption without decreasing production is to increase energy efficiency, thanks to fossil energy-saving technical progress. Note that we find that the rate of fossil energy-saving technical progress is larger than the rate of labor-saving technical progress. As a result, without any intervention, fossil energy use is progressively reduced, but at a small rate: 0.4% per year. With this rate of decline in fossil fuel use, the 75% reduction target would be reached in 347 years.

Data

We use data on labour, labour cost, value-added and price of value-added from INSEE. We use data on the stock of capital from OECD. The user cost of capital is foregone interest plus depreciation minus capital gain. Here the interest rate is the nominal bond rate (IMF), depreciation is 3.5%, capital gain is the price of investment in capital from INSEE. In order to have the total stock of energy from fossil fuels, we use data from INSEE on intermediate good consumption. However, the disaggregation of these tables does not allow us to have the total use (and price) of energy from fossil fuels, because gas is aggregated with water and electricity. We use data on gas consumption from CEREN and gaz price from pegase (French ministry of sustainable development) in order to reconstitute total consumption of fossil fuel energy. We run the regression from year 1986 (data on gas are only available from this date) to year 2008.

2.7.3 Calibration of the other parameters

The calibration procedure is standard. We choose a rate of time preference $\square = 3\%$ which, together with $g^{al} = 1{:}6\%;$ corresponds to a steady state annual interest rate of 4.6%. We follow Fernández-Villaverde & Krueger (2011) for the annual depreciation rate of durable goods: $\square_d = 9\%$. We use a standard value for the depreciation of productive capital: $\square_k = 10\%$. We assume that the price elasticity of exports is equal to 0:6, as in Klein & Simon (2010). The average tax on consumption is $\square^c = 0{:}12{:}$ Considering the level of taxes on fossil energy in the economy, we have $\frac{\square_h}{A^e} = 0{:}77$ and $\frac{\square}{A^e} = 0{:}26$ (see Appendix C). We use steady state ratios to set the other parameters. These ratios are computed using annual data from national accounts between 1986 and 2008. We take arbitrarily $A_0^e = 1$ and $P_0^e = 1$ at the initial steady state. With g^{ae} equal to 2.7% and 1.6% respectively for households and firms, we obtain the following average ratios: $\frac{d}{A^e e_m} = 37$, $\frac{n}{A^e e_m} = 23$, $\frac{k}{y} = 2{:}2$, $\frac{y}{A^e e_f} = 33$.

Table 2 summarizes the value of the exogenous parameters.

Table 2: Value of the main parameters

$\square$	$\square_d$	$\square_k$	g^{al}	g^{ae}	$\square$	!	$\square$	$\square$	$\square$	$\square_d$	$\square_k$	$\square^c$
0.03	0.09	0.10	0.016	0.02	0.5	1	0.5	0.5	0.6	0	0	0.12

The calibrated parameters are: $\square = 0{:}9913$; $\square = 0{:}7780$; $\square = 0{:}0012$ and $\square = 0{:}6876$.

2.8 Simulations: carbon taxes for Factor 4

In France, the objective of the international community to keep the global temperature increase below 2□C has been associated since 2003 to an objective of division by four of GHG emissions at a forty year horizon, the so-called Factor 4. In 2003, President Chirac and his Prime Minister Raffarin have actually committed themselves and France to a division of emissions by four by 2050, from their 1990 level, and this commitment has been reassessed many times since then ("Stratégie nationale de développement durable" in June 2003, "Plan climat" in July 2004, "Loi de programme fixant les orientations de la politique énergétique" in July 2005, "Grenelle de l'environnement" in 2007). Several modeling exercises have been performed since 2003 to assess the feasibility of Factor 4, and to compute the carbon tax path that would allow the French economy to meet this objective. The first one is the Rapport De Boissieu (2006) "Division par quatre des émissions de gaz à effet de serre de la France à l'horizon 2050" in 2005. The last one is the Rapport De Perthuis (2012), "Trajectoires 2020-2050 Vers une économie décarbonée", released very recently.

In the Quinet Report, the proposed values across time for the carbon tax are the following:

Table 3: Tax scenario, in e /tCO_2 proposed in the Quinet Report

	2010	2020	2030	2050
Recommended value	32	56	100	200 (150-350)

Source: Rapport Quinet, 2009

This corresponds roughly to an increase of 3.9% and 6.2% to reach respectively 150 € and 350 € in 40 years, starting at 32 €. In order to implement scenarios, we need to link the price of carbon to taxes expressed as a percentage of fossil fuel energy price before tax, as defined in the model with τ_h and τ_f. For this purpose, we get from each fuel the emission factor, from Ademe, expressed in kg of CO_2 per hl. We can then deduce the price of a carbon tax per hl, which we can compare to the price before tax. Table 9 in Appendix C shows this for a tax of 32 € per ton of CO_2, which is the initial level proposed by Quinet. Weighting each value by the relative consumption, we obtain that a 32 € tax corresponds to an increase of 15% of the price before tax.

The results are proportional for any given level of the tax. Consequently, while 32 € per ton corresponds to 15% of price before tax, 100 € leads to about a 50% increase, and 200 € to a 100 % increase. Note that these numbers are the same, whether we consider firms or households, since the price before tax is almost the same.

We study three scenarios: the carbon tax of the Quinet Report, an oil shock of equivalent magnitude, and a combination of the Quinet carbon tax and energy-saving technical progress allowing the economy to divide by four its carbon emissions in forty years (Factor 4).

The simulations are performed without adjustment costs[9], on the durable side as well as on the capital side. We expect that it makes it easier to reach the desired objective in terms of emissions reduction, since the economy is flexible and can adapt its durable and capital stocks readily to the new energy price.

2.8.1 Method

At $t = 0$, the economy is on a balanced growth path. We note $(A^e e_h)_0$ and $(A^e e_f)_0$ the variables corresponding to initial energy consumption by households and firms. On the households' side, we recall that

$$(A^e e_h)_t = \frac{A^e_t E_{h;t}}{A^l_t}$$

It is important to note that long term limitations, which are inherent to this type of models, have important implications for the simulations. On a balanced growth path, all real variables necessarily grow at the same pace. It means that $A^e E_h$ and $A^e E_f$ grow at the same rate as other real variables, i.e g^{al}. The sign of the difference between g^{al} and g^{ae} has significant implications on long term energy consumption. If energy efficiency grows faster than the economy, then in the long run energy use will tend towards zero. On the contrary, if it grows slower, then energy use will tend towards infinity. If we refer to our estimation, the initial steady state is in the first case ($g^{ae} > g^{al}$). It means that without any intervention, we will gradually and regularly reduce energy use at a rate $\frac{1+g^{ae}}{1+g^{al}}$. Consequently, we do not analyze the transition between an initial and a final steady state because in the long run, we would have $E_h = E_f = 0$. We rather analyze the transition between the initial steady state and the economy after 40 years, because it is the horizon at which we want to reduce emissions by 75%.

From $t = 1$ on, taxes on energy consumption are implemented and the economy deviates from the initial steady state. We want to have $E_{h;40} = E_{h;0}$=4, knowing that A^l and A^e are exogenous and grow at rates g^{al} and g^{ae}

[9] We then increased adjustment costs, up to a speed of convergence of the stock of capital of 2% per year (see Fève et al. (2009)), to evaluate how they make economic policy harder. We found that adjustment costs have mainly impacts on the dynamics of the stock of durables and on the stock of capital, but adding these costs does not change a lot the results on other variables so that, for the sake of brevity, we do not present the results with adjustment costs here.

respectively. Thus a 75% reduction in E_h implies that

$$(A^e e_h)_{40} = \left(\frac{1+g^{ae}}{1+g^{al}}\right)^{40} \frac{A_0^e E_{h,0}/4}{A_0^l} = \frac{1}{4}\left(\frac{1+g^{ae}}{1+g^{al}}\right)^{40} (A^e e_h)_0$$

We have the same expression between $(A^e e_f)_{40}$ and $(A^e e_f)_0$. In order to reach these reductions, we simulate the effect of a permanent tax, starting today and proportional to the oil price (so that the tax grows at the same rate as P^e). Moreover, we add, in some simulations, an increase of the rate of technical progress directed toward energy A^e during 40 years, from 2010 until 2050.

2.8.2 Simulation 1: Carbon tax of the Quinet Report

In this first simulation, we simulate the effect of the carbon tax proposed in the Quinet Report. The initial level of this tax is 32 €/tCO_2 in 2010, growing then at a rate of 4% per year. It amounts to add 0.15 to the initial tax (of 0.77 for households and 0.26 for firms) in the model. We find that this tax, alone, would only entail a 25% reduction of emissions in 2050. This comes from the poor substitution possibilities and the low rate of fossil energy saving technical progress.

Results are presented in Tables 4 and 5. In Table 4, φ is a measure of the welfare gains/losses associated to the policy shock. It is calculated as the percentage loss/gain of total welfare. For instance, if $\varphi = -10\%$, the effect of the reform is the same as a permanent 10% welfare loss. Alternatively, given initial consumption and the choice of a log utility function, a permanent 10% welfare loss corresponds to a 2.5% permanent decrease in consumption[10]. The welfare loss associated with the first simulation (see the first column of Table 4), is equivalent to a permanent welfare loss of 0.92% or a permanent consumption loss of 0.23 %. The welfare loss is thus relatively small, as is the decrease in fossil fuel consumption. Unsurprisingly, the carbon tax of the Quinet report is not large enough to entail a 75% reduction in oil consumption.

Details on the effect of this Quinet tax on economic variables over time are presented in the two first columns of Table 5. Initially, the increase of energy price leads to a decrease of energy consumption by households (E_h) and energy use by firms (E_f). This price shock also entails a decrease in durables (D), as D and E_h are complements. On the other hand, as non durables N and the aggregate composed with durables and energy are more substitutable (the elasticity of substitution equals 1), the initial increase of energy price leads to an increase of non-durables consumption (substitution effect). On the production side, production factors are not very substitutable, so that production falls rapidly. At the end of the shock (40 years), all variables are decreased compared to the baseline, except for the exchange rate. The decrease in energy consumption leads to a decrease in the value of imports. The relative price of interior goods, compared to foreign goods, increases so that the decrease in exports offsets the decrease in imports.

Table 4: Simulations results (1), exogenous technical progress

	Simulation 1	Simulation 2	Simulation 3
E_{2050}/E_{2010}	0.74	0.74	0.25
φ (%)	-0.92	-2.39	9.99

[10] Initial consumption equals 1.28 given our calibration, so that, if $\frac{\ln(C)-\ln(C_0)}{\ln(C_0)} = \varphi$, then $\frac{C-C_0}{C_0} = C_0^{\varphi} - 1$.

Table 5: Simulations results (2), exogenous technical progress

% diff. with baseline	Simulation 1		Simulation 2		Simulation 3	
	1 year	40 years	1 year	40 years	1 year	40 years
C	0.04	-1.28	-0.70	-3.05	1.80	9.93
N	0.46	-0.13	-0.49	-2.04	1.40	3.84
D	-0.39	-2.66	-1.06	-4.23	3.00	17.86
E_h	-5.55	-12.52	-5.76	-13.16	14.84	-67.86
Y	-0.30	-0.90	-0.28	-0.79	0.58	2.68
E_f	-7.24	-14.18	-6.87	-13.05	17.70	-74.36
er	4.26	9.42	-15.86	-26.17	-44.42	125.96

2.8.3 Simulation 2: Oil shock

We want to know whether an exogenous increase in the producer price of oil would have the same effects as a carbon tax. We simulate the consequences of such a shock, calibrated so as to ensure a 26% reduction in emissions in 2050, for comparability with Simulation 1 (in Simulation 1, the emissions were reduces by 26% with a "Quinet tax"). Recall that, in the baseline, exogenous foreign oil price increases at a 2% rate. We assume that the shock on oil price is the following: from date 1 to date 40, foreign oil price is the sum of the baseline price (increasing at a 2% rate) and some additional price component increasing at a 4% rate (as the Quinet tax of Simulation 1). The initial value of this additional price component is equal to 40% of the baseline price at date 1, ensuring a 26% reduction in emissions in 2050.

The main results of Simulation 2 are presented in column 2 of Table 4. There are two main differences with Simulation 1: (1) the carbon tax of Simulation 1 provides the government with tax receipts, whereas the oil shock benefits to the foreign economy, (2) the exchange rate increases in the case of the carbon tax, whereas it decreases in the case of the oil shock. Because of the adjustment of the exchange rate, the additional price component needed to reach a 26% reduction in emissions is larger than the Quinet tax. Because of the possibility to recycle the tax proceeds in Simulation 1, the welfare loss in Simulation 2 is larger than the welfare loss, for the same emission reduction, in Simulation 1. This loss is equivalent to a permanent decrease of 2.39% in welfare, or a permanent decrease in consumption of 0.59%.

Details on the effect of this oil shock on economic variables over time are presented in columns 3 and 4 of Table 5. As in the former simulation, the increase of energy price leads to a decrease of energy consumption by households (E_h) and energy use by firms (E_f). This price shock also entails a decrease in durables (D), as D and E_h are complements. Contrary to Simulation 1, even if non durables N and the aggregate composed with durables and energy are more substitutable, the initial increase of energy price leads to a decrease of non-durables consumption. This is the case because the revenue effect is larger than the substitution effect. On the production side, production falls but less than in Simulation 1. This comes from the increase of exports (in Simulation1, there were a decrease of exports). Indeed, the exchange rate decreases in this simulation, contrary to Simulation 1. The increase of foreign energy price leads to an increase in the value of imports. The relative price of interior goods, compared to foreign goods, decreases so that the increase of exports offsets the increase of imports.

2.8.4 Simulation 3: Factor 4

We run another simulation and find that the initial level of the carbon tax which would allow the economy to reach a 75% reduction of emissions would be obviously too high to be acceptable (actually, +3.9 instead of +0.15, or 832€ per ton of CO_2 instead of 32). Applied models used to assess the effects of climate policy all introduce assumptions on the energy-saving technical progress. These assumptions are often not explicit, different in different sectors of the economy and at different periods of time. It is really difficult to sum up these assumptions to obtain the implicit average growth rate of the energy-saving technical progress at stake in the simulations. Nevertheless, this hidden assumption plays a major role. To see that, we simulate the effects of the carbon tax of the Quinet Report, as in Simulation 1, associated to an increase in g^{ae}; this increase being calibrated to ensure that emissions are divided by 4 in 2050 (the so-called Factor 4). We find that this new g^{ae} is equal to 7.4% per year, instead of 2% in the baseline. The result of this simulation (Simulation 3) are presented in the third column of Table 4 and in columns 5 and 6 of Table 5. It is interesting to note that the initial increase of g^{ae} leads to an initial increase of E_f and E_h, this is a rebound effect.

The main lesson from these simulations is that the target of 75% cannot be reached without additional energy-saving technical progress. In Simulation 3, the energy directed technical progress is exogenous and free, so that increasing g^{ae} only yields positive benefits. The welfare gain associated with this simulation is equivalent to a 9.99% permanent welfare gain, or a 2.5% permanent consumption gain. We think that this is misleading. That is why we proceed with a second version of the model, in which the direction of technical progress is endogeneized, so that increasing fossil energy saving technical progress is costly.

3 The model with directed technical change

We now want to model directed technical change, in the sense that an increase of the energy price (due to an exogenous supply shock or an increase in environmental taxation) induces R&D aimed at saving energy, at the expense of R&D aimed at increasing labor productivity. Popp (2004) for instance provides empirical evidence of this partial crowding out effect.

We make the assumption that the research effort of the economy is a given proportion of the output: we do not endogeneize the intensity of this effort[11]. Nevertheless, given the total amount of resources devoted at each date to R&D, we endogeneize the *direction* of technical progress, that is the allocation of this amount between an energy research sector enhancing the efficiency of energy and a labor research sector enhancing the efficiency of labor. This direction responds endogenously to economic incentives. It shapes very deeply the future characteristics of the economy, as there is now a trade-off between economic growth and energy transition. Indeed, a high labor-saving technical progress ensures a high growth rate of the economy at the expense of high CO_2 emissions, whereas a high energy-saving technical progress enables the economy to reduce CO_2 emissions beyond what substitution possibilities would allow, possibly at the expense of a high overall growth rate.

[11] See Hassler et al. (2011) for a similar simplifying assumption.

3.1 The direction of technical progress

The intensity of the research effort of the economy (in terms of the final good) is exogenous and denoted $\square$; with $0 < \square < 1$: $S_t = \square Y_t$ is the level of the research effort. By construction, it grows at the same rate than Y_t; i.e. at rate g_t^{al}:

We endogeneize the allocation of this amount to an "energy research" sector (S_t^e) and to a "labor research" sector (S_t^l), or more precisely the share $sh_t = S_t^l = S_t$.

We build on Smulders & de Nooij (2003), and Acemoglu et al. (2012).

At any date, final goods producers use an homogeneous stock of capital (K), labour services (Y_L) and energy services (Y_R) to produce final goods (Y). The three inputs are imperfect substitutes, and we adopt the same two-level CES disaggregation than in the first version of the model, with $\square = \square$:

$$Y = \left[\square Y_L^{\frac{\square\square 1}{\square}} + (1 \square \square)\left[\square K^{\frac{\square\square 1}{\square}} + (1 \square \square) Y_E^{\frac{\square\square 1}{\square}}\right]\right]^{\frac{\square}{\square\square 1}} \tag{24}$$

The price of the final good is normalized to one and the (accounting) prices of labour and energy services are denoted P^{y_L} and P^{y_E}, respectively.

The services of labor (energy) are obtained by combining[12] raw labor L (raw fossil energy E) and a continuum of sector-specific intermediates x_j^l of quality A_j^l (x_j^e; A_j^e):

$$Y_L = L^{1\square\square} \int_0^1 \left(A_j^l\right)^{1\square\square} \left(x_j^l\right)^{\square} dj \tag{25}$$

$$Y_E = E_f^{1\square\square} \int_0^1 \left(A_j^e\right)^{1\square\square} \left(x_j^e\right)^{\square} dj \tag{26}$$

Thus we suppose that there exist 3 types of machines: all-purpose homogeneous machines, which total stock is K; and specialized sector-specific machines x_k^i[13]. All those machines are produced using the final good only. The interpretation of these sector-specific intermediates is the following. For the production of labor services, these specialized machines are mainly machines embodying ITC. For the production of energy services, they can be either devices aimed at improving the energy efficiency of existing capital, or specialized capital allowing to produce renewable energy, like solar cells or wind mills. Both allow to produce the same amount of energy services with less fossil energy.

Final goods producers maximize profits, taking prices as given. They demand labour services, energy services and all-purpose capital up to the point where the marginal productivity of these inputs equals their cost. This yields:

$$\frac{Y_E}{Y_L} = \left(\frac{(1 \square \square)(1 \square \square)}{\square}\frac{P^{y_L}}{P^{y_E}}\right)^{\square} \tag{27}$$

$$K^{\square 1} = Y_E\left(\frac{\square}{1 \square \square}\frac{P^{y_E}}{P^k}\right)^{\square} \tag{28}$$

The demand for energy services relative to labor services is a function of their relative prices.

[12] We suppose that the parameter characterizing this combination, $\square$, is the same for labor and energy services. This assumptions does not have any theoretical or empirical basis and is only made for simplicity, as it is the case in the rest of the literature.

[13] As it is the case in the rest of the literature, again, we treat these sector-specific machines as a flow and not a stock, which would be obviously a better assumption.

In a second stage, final goods producers choose how to produce labor and energy services. For labor services for instance, they solve the following problem:

$$\max P^{y_L} L^{1-\alpha} \int_0^1 \left(A_j^l\right)^{1-\alpha} \left(x_j^l\right)^{\alpha} dj - P^l L - \int_0^1 P_j^l x_j^l \, dj$$

The FOC read:

$$x_j^l = \left(\frac{\alpha P^{y_L}}{P_j^l}\right)^{\frac{1}{1-\alpha}} A_j^l L \tag{29}$$

$$P^{y_L} Y_L = \frac{1}{1-\alpha} P^l L \tag{30}$$

and for energy services we obtain equivalently:

$$x_j^e = \left(\frac{\alpha P^{y_E}}{P_j^e}\right)^{\frac{1}{1-\alpha}} A_j^e E_f \tag{31}$$

$$P^{y_E} Y_E = \frac{1}{1-\alpha} P^e E_f \tag{32}$$

Specialized intermediates are supplied by firms in monopolistic competition. Producing one unit of these machines costs c units of the final good. Firms producing intermediates aimed at enhancing labor productivity maximize their profit, taking into account the inverse demand function:

$$\max \pi_j^l = (P_j^l - c) x_j^l$$
$$\text{s.c. } (29)$$

FOC yield:

$$P_j^l = \frac{c}{\alpha}; \quad x_j^l = \left(\frac{\alpha^2 P^{y_L}}{c}\right)^{\frac{1}{1-\alpha}} A_j^l L; \quad \pi_j^l = \frac{1-\alpha}{\alpha} c \left(\frac{\alpha^2 P^{y_L}}{c}\right)^{\frac{1}{1-\alpha}} A_j^l L$$

Hence

$$Y_L = \left(\frac{\alpha^2 P^{y_L}}{c}\right)^{\frac{\alpha}{1-\alpha}} A^l L \tag{33}$$

with

$$A^l = \int_0^1 A_j^l \, dj$$

the average productivity of specialized inputs, and, using the FOC on raw labor (30):

$$P^{y_L} = \frac{1}{(1-\alpha)^{1-\alpha} \alpha^{2\alpha}} \left(\frac{P^l}{A^l}\right)^{1-\alpha} c^{\alpha} \tag{34}$$

The accounting price of labor services is a Cobb-Douglas combination of the effective price of labor in efficiency units and of the unit production cost of intermediates.

We obtain equivalent equations for the other research sector:

$$P_j^e = \frac{c}{\alpha}; \quad x_j^e = \left(\frac{\alpha^2 P^{y_E}}{c}\right)^{\frac{1}{1-\alpha}} A_j^e E_f; \quad \pi_j^e = (1+\tau^r) \frac{1-\alpha}{\alpha} c \left(\frac{\alpha^2 P^{y_E}}{c}\right)^{\frac{1}{1-\alpha}} A_j^e E_f$$

with τ^r the subsidy to the energy research sector, which we introduce as a new potential economic policy instrument, and

$$Y_E = \left(\frac{\alpha^2 P^{y_E}}{c}\right)^{\frac{\alpha}{1-\alpha}} A^e E_f \tag{35}$$

$$P^{y_E} = \frac{1}{(1-\square)^{1-\square}\square^{2\square}}\left(\frac{P^e+\square_f}{A^e}\right)^{1-\square} c^{\square} \tag{36}$$

Dividing (36) by (34) yields:

$$\frac{P^{y_E}}{P^{y_L}} = \left(\frac{P^e+\square_f}{P^l}\frac{A^l}{A^e}\right)^{1-\square} \tag{37}$$

The relative price of energy and labor services depends positively on the relative price of raw inputs, but also depends on the relative productivities of the 2 types of specialized machines. The higher the relative productivity of fossil energy-saving specialized inputs, the lower the relative price of energy services.

Dividing (35) by (33) yields:

$$\frac{Y_E}{Y_L} = \left(\frac{P^{y_E}}{P^{y_L}}\right)^{\frac{\square}{1-\square}}\frac{A^eE_f}{A^lL} \tag{38}$$

Eliminating $Y_E=Y_L$ between (27) and (38) yields:

$$\frac{A^eE_f}{A^lL} = \left(\frac{(1-\square)(1-\square)}{\square}\right)^{\square}\left(\frac{P^{y_L}}{P^{y_E}}\right)^{\square+\frac{\square}{1-\square}} \tag{39}$$

Replacing in this equation P^{y_L} and P^{y_E} by their expressions in (34) and (36) yields:

$$\frac{A^eE_f}{A^lL} = \left(\frac{(1-\square)(1-\square)}{\square}\right)^{\square}\left(\frac{P^l=A^l}{P^e=A^e}\right)^{(1-\square)\square+\square} \tag{40}$$

As in Acemoglu et al. (2012), productivities evolve according to[14]:

$$A^l_t = (1+\square_L\square_L sh_{t-1})A^l_{t-1} \tag{41}$$

$$A^e_t = (1+\square_E\square_E(1-sh_{t-1}))A^e_{t-1} \tag{42}$$

where $\square_L$ and $\square_E$ are the sizes of an innovation in each research sector, sh the normalized research effort in the research sector aimed at enhancing labor productivity, $\square_L$ the probability of success in this research sector, and $\square_E$ the probability of success in the other research sector[15]. These equations read equivalently:

$$g^{al}_t = \square_L\square_L sh_{t-1} \tag{43}$$

$$g^{ae}_t = \square_E\square_E(1-sh_{t-1}) \tag{44}$$

The expected profit of research aimed at enhancing labor productivity is:

$$\Pi^l_t = \square_L(1+\square_L)\frac{1-\square}{\square}c\left(\frac{\square^2P^{y_L}_t}{c}\right)^{\frac{1}{1-\square}}L_tA^l_{t-1}$$

and we have an equivalent expression for Π^e_t: Dividing both equations yields:

$$\frac{\Pi^l_t}{\Pi^e_t} = \frac{1}{1+\square^r_t}\frac{(1+\square_L)\square_L}{(1+\square_E)\square_E}\left(\frac{P^{y_L}_t}{P^{y_E}_t}\right)^{\frac{1}{1-\square}}\frac{L_t}{E_{f,t}}\frac{A^l_{t-1}}{A^e_{t-1}} \tag{45}$$

[14] We do not adopt exactly the same timing as in Acemoglu et al. (2012). We make the assumption, which we find plausible, that the research effort of period $t-1$, and not of period t, determines the productivity level of period t.

[15] Note that the model can be also seen as an adoption model instead of a research model. In this case, France does not develop new technologies but adopts existing technologies from other countries. Adoption is costly. In order to incorporate a new intermediate good into the production process, it is necessary to invest resources: $\square$ is the probability of succeeding in adapting existing technologies to French production process and sh the relative spending to buy new patents from foreign countries. See Grossman & Helpman (1993).

Acemoglu et al. (2012) identify 3 effects in this relationship, shaping the incentive to innovate in labor-saving technologies versus fossil energy-saving technologies: the direct productivity effect (captured by the term A^l_{t-1}/A^e_{t-1}), which pushes towards innovating in the sector with higher productivity; the price effect (captured by the term $(P^{y_L}/P^{y_E})^{1/(1-\square)}$), encouraging innovation toward the sector with higher prices; the market size effect (captured by the term L/E_f), encouraging innovation in the sector with the larger market for specialized inputs.

Replacing in (45) P^{y_L}/P^{y_E} obtained in (37) and using (40) to eliminate E_f/L we get:

$$\frac{\Pi^e_t}{\Pi^l_t} = (1+\square^r_t)\frac{(1+\square_E)\square_E}{(1+\square_L)\square_L}\left(\frac{(1-\square)(1-\square)}{\square}\right)^{\square\square}\left(\frac{(P^e_t+\square_{f,t})/A^e_t}{P^l_t/A^l_t}\right)^{(1-\square)(1-\square)}\frac{1+g^{al}_t}{1+g^{ae}_t} \tag{46}$$

An interior solution is characterized by the same opportunities of profit in the two research sectors, i.e., using also (43) and (44):

$$(1+\square^r_t)\frac{(1+\square_E)\square_E}{(1+\square_L)\square_L}\left(\frac{(1-\square)(1-\square)}{\square}\right)^{\square\square}\left(\frac{(P^e_t+\square_{f,t})/A^e_t}{P^l_t/A^l_t}\right)^{(1-\square)(1-\square)}\frac{1+\square_L\square_L sh_{t-1}}{\square_E\square_E(1-sh_{t-1})} = 1$$

The existence of an interior solution requires $0 < sh < 1$; which we will check ex post.

The two research sectors are owned by the representative consumer so that the profits earned by these research sectors are redistributed lump sum to her. Total transfers are now:

$$\begin{aligned} T_t &= \square^c_t(N_t+X_t)+\square_{h,t}E_{h,t}+\square_{f,t}E_{f,t}-\square^r_t\square(P^e_t+\square_{f,t})E_{f,t}+\square P^l_tL_t+(1+\square^r)\square(P^e_t+\square_{f,t})E_{f,t}-S_t \\ &= \square^c_t(N_t+X_t)+\square_{h,t}E_{h,t}+\square_{f,t}E_{f,t}+\square\left[P^l_tL_t+(P^e_t+\square_{f,t})E_{f,t}\right]-S_t \end{aligned} \tag{47}$$

Finally, we make the assumption that there exist perfect knowledge spillovers such that the fossil energy-saving innovations made in the industrial sector perfectly diffuse to the durable goods sector.

3.2 Model in intensive variables

Equations (E1) to (E8), (E15), (E19) to (E23) are unchanged. The new equations are:

$$y_t = \left[\square y_{L,t}^{\frac{\square-1}{\square}} + (1-\square)\left[\square\left(\frac{k_{t-1}}{1+g^{al}_t}\right)^{\frac{\square-1}{\square}} + (1-\square)y_{E,t}^{\frac{\square-1}{\square}}\right]\right]^{\frac{\square}{\square-1}} \tag{EE1}$$

$$y_t = \square^{-\square}\left(P^{y_L}_t\right)^{\square} y_{L,t} \tag{EE2}$$

$$\frac{\square}{(1-\square)(1-\square)}\left(\frac{y_{E,t}}{y_{L,t}}\right)^{\frac{1}{\square}} = \frac{P^{y_L}_t}{P^{y_E}_t} \tag{EE3}$$

$$\frac{k_{t-1}}{(1+g^{al}_t)\,y_{E,t}} = \left(\frac{\square}{1-\square}\frac{P^{y_E}_t}{P^k_t}\right)^{\square} \tag{EE4}$$

$$y_{L,t} = \left(\square^2\frac{P^{y_L}_t}{c}\right)^{\frac{\square}{1-\square}} \tag{EE5}$$

$$P^{y_L}_t = \frac{1}{(1-\square)^{1-\square}\square^{2\square}}\left(p^l_t\right)^{1-\square}c^{\square} \tag{EE6}$$

$$y_{E,t} = \left(\square^2\frac{P^{y_E}_t}{c}\right)^{\frac{\square}{1-\square}}(A^e e_f)_t \tag{EE7}$$

$$P^{y_E}_t = \frac{1}{(1-\square)^{1-\square}\square^{2\square}}\left((P^e/A^e)_t+(\square_f/A^e)_t\right)^{1-\square}c^{\square} \tag{EE8}$$

$$sh_t^{\square} = \frac{1 + \square_E \square_E \square \left(1 + \square^r_{t+1} \frac{(1+\square_E)\square_E}{(1+\square_L)\square_L}\right)^{\frac{(1\square\square)(1\square\square)}{\square}} \square \left(\frac{(P^e/A^e)_{t+1} + (\square/A^e)_{t+1}}{p^l_{t+1}}\right)^{(1\square\square)(1\square\square)}}{\square_E \square_E + \square_L \square_L \left(1 + \square^r_{t+1} \frac{(1+\square_E)\square_E}{(1+\square_L)\square_L}\right)^{\frac{(1\square\square)(1\square\square)}{\square}} \square \left(\frac{(P^e/A^e)_{t+1} + (\square/A^e)_{t+1}}{p^l_{t+1}}\right)^{(1\square\square)(1\square\square)}} \tag{EE9}$$

$$g_t^{al} = \square\square_L sh_{t\square 1} \tag{EE10}$$

$$g_t^{ae} = \square\square_E (1 \square sh_{t\square 1}) \tag{EE11}$$

$$t_t = \square^c(n_t + x_t) + (\square_h/A^e)_t(A^e e_h)_t + (\square_f/A^e)_t(A^e e_f)_t + \square\left[p_t^l + (P^e/A^e)_t (A^e e_f)_t + (\square_f/A^e)_t(A^e e_f)_t\right] \square s_t \tag{EE12}$$

$$s_t = \square y_t \tag{EE13}$$

$$y_t = i_t + n_t + x_t + \frac{\square_d}{2}\frac{(d_t \square d_{t\square 1})^2}{d_{t\square 1}} + \frac{\square_f}{2}\frac{(k_t \square k_{t\square 1})^2}{k_{t\square 1}} + ex_t + \frac{\square^2}{1\square\square}\left[p_t^l + (P^e/A^e)_t (A^e e_f)_t + (\square_f/A^e)_t(A^e e_f)_t\right] + s_t \tag{EE14}$$

$$(P^e/A^e)_t = \frac{1+\square_t^e}{1+g_t^{ae}}(P^e/A^e)_{t\square 1} \tag{EE15}$$

3.3 Calibration

The elasticities and the parameters that are in common with the first version of the model have the same value in this second version.

As for the new parameters, we retain $\square = 0.3$, $c = 0.1$, and we assume that the probability of success is the same in both research sectors, i.e. $\square_L = \square_E$. The results of the simulations are quite robust with respect to these assumptions.

The truly important assumption is the value given to sh. It is crucial because according to the value of sh the split of research between the two sectors will be on one side or the other of the optimal split, which will have major consequences on the welfare effects of the simulations. Dechezleprêtre et al. (2011) suggests $sh = 0.99$ by counting the energy-saving related patents. We perform two sets of simulation. In the first one, we assume that $sh = 0.99$. We find that the reforms we simulate induce a welfare gain, absent any external effect! This means that, given the calibration, the research effort toward fossil energy-saving technologies is too low so that an increase of oil taxes increase welfare, even without any climate change consideration. We believe that this is a little too optimistic. This result is very sensitive to the initial value of sh. As we have no clue (except for Dechezleprêtre et al. (2011)) on the true value of this parameter and we believe that other patents may have positive effect on energy-saving technologies, we prefer assuming that, in the baseline situation, one cannot increase welfare by adding a uniform tax (or subsidy) on energy. This is an agnostic point of view: we do not know whether, absent any externality, one should increase or decrease current taxation in order to stimulate or deter research toward energy saving technology. This leads us to take $sh = 0.90$ in the second set of simulation, that we present here. Note that the choice of the initial sh has a large effect on welfare gains associated with the simulations, but other economic variables, such as production, investment, consumption of durables and energy do not vary a lot when changing the calibration of sh. In particular, the reform always goes with a decrease in GDP.

3.4 Simulations

The growth rate of energy efficiency g^{ae} and the deflator of intensive variables g^{al} are now endogenous (and non-constant). The exogenous variables we have are intensive variables, e.g. $(\square_f/A^e)_t$. We make a shock on $(\square_f/A^e)_t$. It entails a path for g_t^{ae}, from which we deduce the path $\square_{f,t}$. We iterate until we obtain the initial value and the time profile we want for $\square_{f,t}$.

3.4.1 Simulation 1: Carbon tax of the Quinet Report

Table 6: Simulations results (1), endogenous technical progress

	Simulation 1 (calibration 1)	Simulation 2 (calibration 2)
E_{2050}/E_{2010}	0.61	0.60
' (%)	0.92	-1.35

As explained in the previous paragraph, when we take the initial value of sh equal to 0.99, we find a welfare gain associated with the reform. When we take $sh = 0.9$, we find a welfare loss. The decrease in fossil fuel use is almost the same in both cases. In the following, we only present the simulations with $sh = 0.9$, for the reasons explained above. Fig.1 represents some economic variables, over time, when the carbon tax of the Quinet report is implemented, with exogenous technical progress (dashed line) and when the direction of technical progress is endogenous (solid line). The variable var_C stands for the percentage change of C, compared to its baseline value[16]. The Quinet tax entails an increase of the rate of energy saving technical progress, from 2% to more than 6% initially. This comes at a small expense in term of overall growth: the labor saving technical progress decreases from 1.6% to 1.2%. The reduction in energy consumption entailed by the reform is a 40% reduction, whereas it was a 26% reduction in the exogenous model, with the same shock. Two effects make energy consumption decrease: the decrease in demand, for a given energy efficiency, because of the increase of the price and the increase of the energy efficiency.

3.4.2 Simulation 2: oil shock

We simulate the same oil shock as in the exogenous model. The shock on oil price is the following: from date 1 to date 40, foreign oil price is the sum of the baseline price (increasing at a 2% rate) and some additional price component increasing at a 4% rate (as the Quinet tax of Simulation 1). The initial value of this additional price component is equal to 40% of the baseline price at date 1.

Fig.2 represents some economic variables over time, when the oil shock is simulated, with exogenous technical progress (dashed line) and when the direction of technical progress is endogenous (solid line). The oil shock entails an increase of the rate of energy saving technical progress, from 2% to 6% initially. This comes at a small expense in term of overall growth: the labor saving technical progress decreases from 1.6% to 1.2%.

[16] Note that it is not a percentage change compared to a date 0 value, but a percentage change compared to a baseline value at the same date. So that if $var_{E_f} = -0.3$ in 2050 for instance, this means that E_f is equal to 70% of its baseline value in 2050, which is less than 70% of its value in 2010, as the energy use decreases from 2010 to 2050 in the baseline.

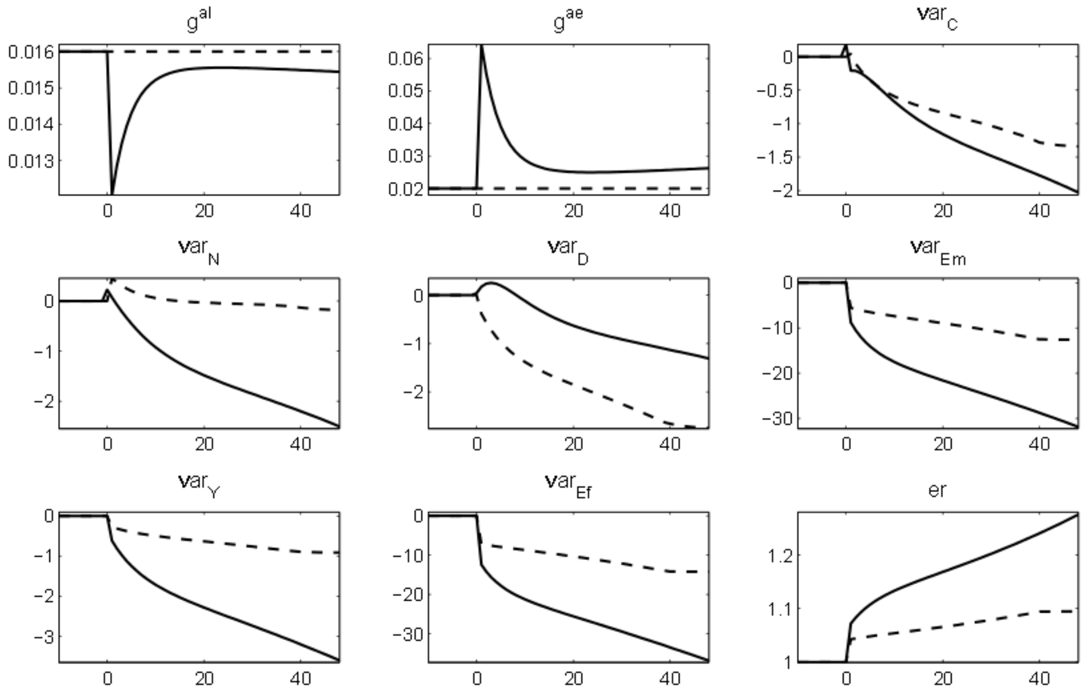

Figure 1: Carbon tax of the Quinet Report, exogenous (dashed line) and endogenous (solid line) TP

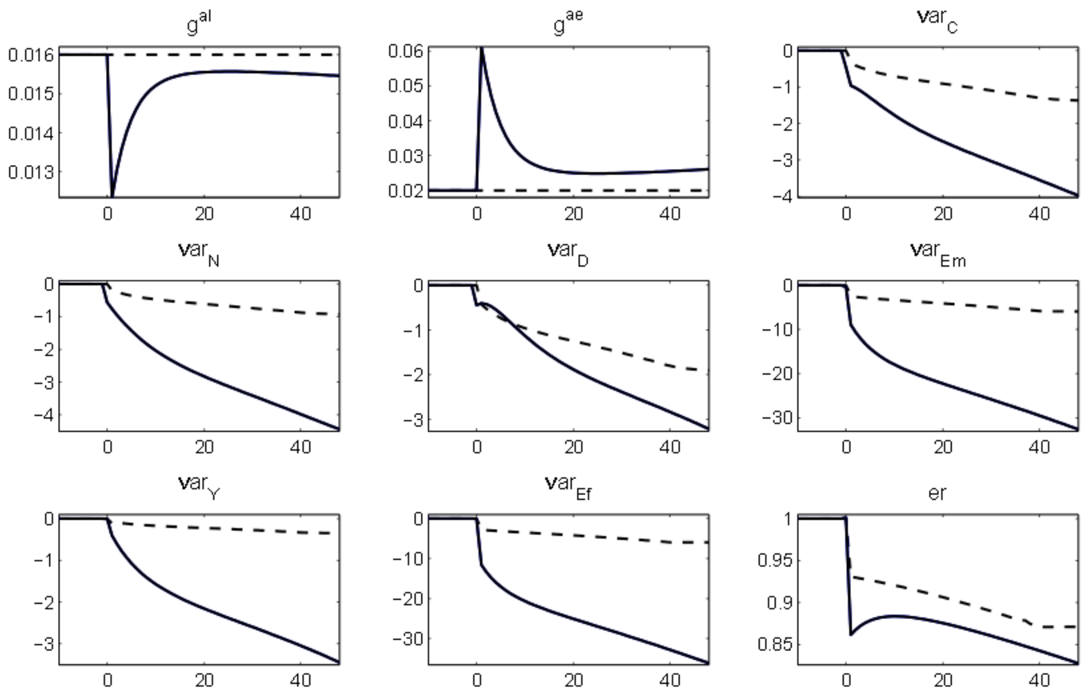

Figure 2: Oil shock, exogenous (dashed line) and endogenous (solid line) TP

3.4.3 Simulation 3: subsidy to energy saving R&D

Even when the direction of technical progress is endogenous, the Quinet tax alone is not sufficient to entail a 75% reduction in fossil fuel consumption. We simulate the effect of a Quinet tax associated with a subsidy σ_r of 20% on energy saving R&D, so that the research profit in the energy saving sector is increased by 20%. The results are presented in Fig.3 (dotted line). The increase in the rate of energy saving technical progress is huge, as it goes from 2% to almost 20%. The labor saving technical progress on the other hand decreases almost to zero. The decrease in production is almost twice as large as it was with the Quinet tax alone (solid line of Fig.3). Even with this huge effort toward energy saving technical progress, the energy consumption decreases only by 60% in forty years. Increasing the subsidy even more would redirect all research effort toward energy saving technical progress and make the rate of labor saving technical progress equal to zero for some time, which does not seem realistic neither acceptable. In order to reach the 75% reduction, we run another simulation, in which we increase the initial tax, as well as its rate of growth.

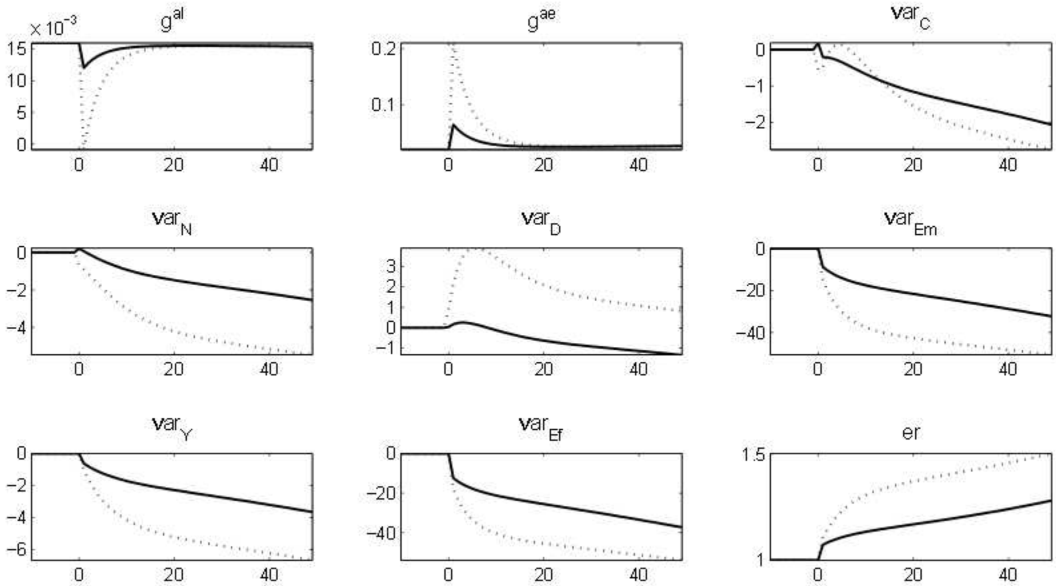

Figure 3: Carbon tax of the Quinet Report, endogenous model, with subsidy toward energy saving research (dotted line) and without (solid line)

3.4.4 Simulation 4: Factor 4

In this last simulation, we simulate the effect of the following carbon tax: its initial value is equal to 64€ per ton of CO_2, and it grows at a 8% rate. In Fig.4, the solid line represents the result of this simulation in the endogenous model, whereas the dashed line represents the results of the same simulation in the exogenous model. In the endogenous model, this carbon tax is enough to reach the 75% target. Contrary to previous simulations, the rate of labor saving technical progress is decreased for a long time period. As a result, the production Y continues to decrease over time compared to its baseline value.

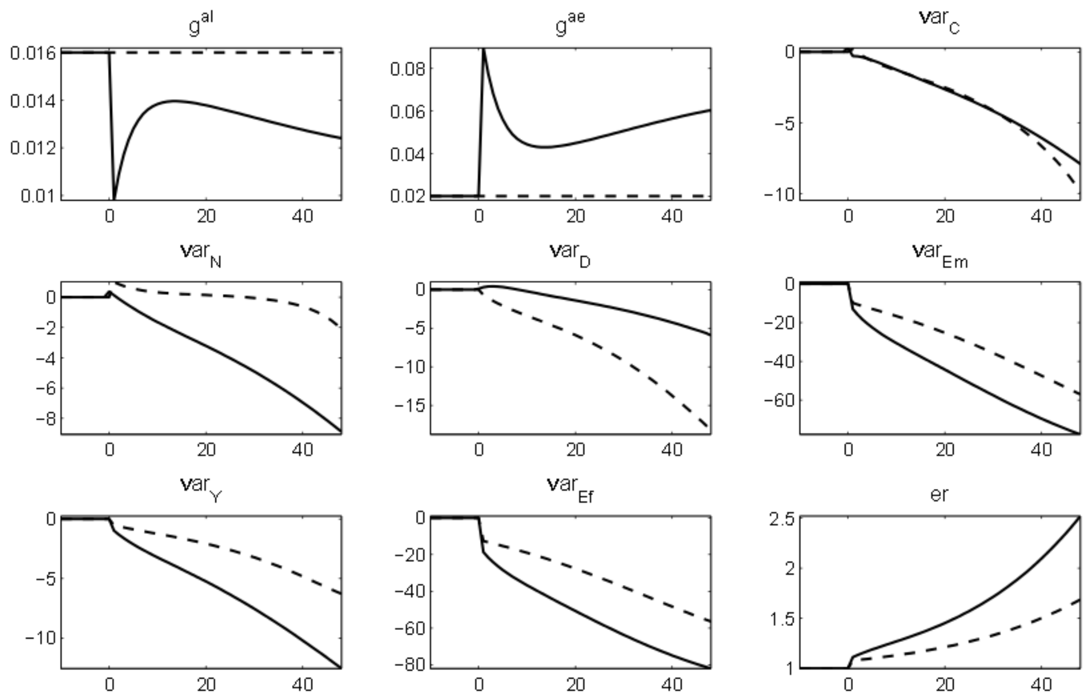

Figure 4: Carbon tax (initial value 0.3, increasing at 8% rate), exogenous (dashed line) and endogenous (solid line) TP

4 Conclusion

The lesson we can draw from our simulations, both in the exogenous version of our model and in its endogenous version, is that Factor 4 is a goal very difficult to reach for France. Even when we make the rate of energy-saving technical progress endogenous, a 75% reduction in emissions seems almost impossible to achieve. According to our model, it would require a high carbon tax and/or high subsidies to fossil energy-saving technical progress and it would necessarily entail reduced growth for some time.

The results depend of course on the calibration, as do the results from existing big applied models. We have tried to make the model and the calibration as simple and transparent as possible, in order to disentangle the different effects. What seems very robust is the need for fossil energy-saving technical progress to achieve a substantial reduction in emissions. And this technical progress is likely to come at some cost. We wonder if the existing big applied models commonly used to study environmental policy aren't misleading. They seem to under-estimate the magnitude of the effort required to reach the objective.

We do not take into account the external effect of pollution here. This is not very realistic. We chose to do that to focus on a worst-case scenario, and answer this question: what is the loss associated to the reform if it does not yield any positive benefit? We think this question is relevant as the estimates of the benefits from mitigation vary a lot across studies.

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A Estimation of the elasticity of substitution between durable goods and energy and of the rate of technical progress

Method

We follow Ogaki & Reinhart (1998) to identify a cointegration relation and estimate the intratemporal elasticity of substitution between E_h and D. Let's denote W_t the intertemporal welfare at date t. With obvious notations, first order conditions with respect to X_t and $E_{h,t}$ lead to:

$$\frac{(1+\tau_t^c)P_t^x}{P_t^e+\mu_{h,t}} = \frac{W_{X_t}}{W_{E_{h,t}}}$$

We have

$$W_{X_t} = \sum_{i=1}^{\infty} \frac{1}{(1+\rho)^{t+i}} \frac{\partial \ln C_{t+i}}{\partial X_t} = \sum_{i=1}^{\infty} \frac{1}{(1+\rho)^{t+i}} \frac{\partial \ln C_{t+i}}{\partial D_{t+i-1}} \frac{\partial D_{t+i-1}}{\partial X_t}$$

From $D_t = \sum_{j=0}^{\infty}(1-\delta_d)^j X_{t-j}$, we deduce $\frac{\partial D_{t+i-1}}{\partial X_t} = (1-\delta)^{i-1}$. Hence

$$W_{X_t} = \sum_{i=1}^{\infty} \frac{1}{(1+\rho)^{t+i}} (1-\delta)^{i-1} \frac{\partial \ln C_{t+i}}{\partial D_{t+i-1}}$$

The expression of $W_{E_{h,t}}$ is straightforward, and we obtain:

$$\frac{(1+\tau_t^c)P_t^x}{P_t^e+\mu_{h,t}} = \frac{\sum_{i=1}^{\infty} \frac{1}{(1+\rho)^{t+i}} (1-\delta)^{i-1} \frac{\partial \ln C_{t+i}}{\partial D_{t+i-1}}}{\frac{\partial \ln C_t}{\partial E_{h,t}}}$$

From F.O.C, we have the following relationship:

$$\frac{\partial \ln C_t}{\partial D_{t-1}} = \frac{\alpha}{1-\alpha} \frac{1}{A_t^e} \left(\frac{D_{t-1}}{A_t^e E_{h,t}} \right)^{-\frac{1}{\sigma}} \frac{\partial \ln C_t}{\partial E_{h,t}}$$

so that

$$\frac{(1+\tau_t^c)P_t^x}{P_t^e+\mu_{h,t}} = \sum_{i=1}^{\infty} \frac{1}{(1+\rho)^{t+i}} (1-\delta)^{i-1} \frac{\alpha}{1-\alpha} \frac{1}{A_{t+i}^e} \left(\frac{D_{t+i-1}}{A_{t+i}^e E_{h,t+i}} \right)^{-\frac{1}{\sigma}} \frac{\frac{\partial \ln C_{t+i}}{\partial E_{h,t+i}}}{\frac{\partial \ln C_t}{\partial E_{h,t}}}$$

Multiplying both sides by $A_t^e \left(\frac{X_t}{A_t^e E_t} \right)^{\frac{1}{\sigma}}$, we get:

$$\frac{(1+\tau_t^c)P_t^x}{(P_t^e+\mu_{h,t})/A_t^e} \left(\frac{X_t}{A_t^e E_t} \right)^{\frac{1}{\sigma}} = \sum_{i=1}^{\infty} \frac{1}{(1+\rho)^{t+i}} (1-\delta)^{i-1} \frac{\alpha}{1-\alpha} \frac{A_t^e}{A_{t+i}^e} \left(\frac{D_{t+i-1}}{X_t} \right)^{-\frac{1}{\sigma}} \left(\frac{A_t^e E_{h,t}}{A_{t+i}^e E_{h,t+i}} \right)^{-\frac{1}{\sigma}} \frac{\frac{\partial \ln C_{t+i}}{\partial E_{h,t+i}}}{\frac{\partial \ln C_t}{\partial E_{h,t}}} \quad (48)$$

If we show that the discounted sum in the right-hand side member of (48) is stationary, we can conclude that the left-hand side member is also stationary. Assuming that A_t^e grows at a constant growth rate, $A_t^e = A_0^e(1+g^{ae})^t$, we will hence be able to derive the following long run relationship:

$$\ln\left(\frac{E_t}{X_t}\right) = \sigma \ln\left(\frac{(1+\tau_t^c)P_t^x}{P_t^e + \tau_{h,t}}\right) + (\sigma - 1)\ln A_0^e + (\sigma - 1) g^{ae} t + c + u_t \qquad (49)$$

where c is a constant and u_t is a white noise.

Taking the logarithm of each term of the sum, we see that a necessary condition for stationarity is that $\ln A_t^e)$, $\ln(X_t)$, $\ln(A_t^e E_{h,t})$ and $\ln(\frac{\partial \ln C_{t+i}}{\partial E_{h,t+i}})$ are difference stationary. Indeed, if a variable V_t is difference stationary, $v_t = \ln(V_t)$ is also difference stationary and $\ln(\frac{V_t}{V_{t+i}}) = \ln(V_t) - \ln(V_t + i)$ is by definition stationary. This is straightforward for each term, except for the ratio between durable stock and expense. In that last case, we must note that $\frac{D_{t+i-1}}{X_t}$ can be written $\sum_{j=1}^{\infty}(1-\delta_d)^j \frac{X_{t+i-1-j}}{X_t}$, i.e a sum of stationary terms if X_t is difference stationary. Our assumption on the linear form of A_t^e implies that A_t^e is difference stationary with drift. Concerning $A^e E_{h,t}$, taking the logarithm shows that it is difference stationary if $E_{h,t}$ is difference stationary. Consequently, we perform below stationarity tests on X_t and $E_{h,t}$. We show that both series are different stationary. The last point concerns the marginal utility of energy consumption $\frac{\partial \ln C_t}{\partial E_{h,t}}$. We cannot prove empirically that its growth rate is stationary, but like Ogaki & Reinhart (1998), we consider that the non-stationarity of this growth rate is unlikely to be empirically important.

We assess the stationarity of $x_t = \ln(X_t)$ and $e_t = \ln(E_{h,t})$ by performing augmented Dickey Fuller tests. We use annual data between 1959 and 2010 for x_t; and between 1973 and 2010 for e_t. We conclude that x_t is difference stationary with a drift and a trend, and that $e_{h,t}$ is difference stationary with a drift, as presented in Table 7.

Table 7: Unit root tests

Variable	ADF test		Conclusion
	stat	$c_{5\%}$	
E_h	-3.13	-3.45	I(1)+trend
X	-2.87	-2.93	I(1)+drift

Estimation

Going back to equation (48), we can now draw the following conclusions: 1) $\frac{D_{t+i-1}}{X_t}$ is stationary as the ratio between two difference stationary variables with the same drift; 2) $\frac{A_t^e E_{h,t}}{A_{t+i}^e E_{h,t+i}}$ is stationary as the ratio of difference stationary variables. Consequently, we can now test the cointegration restriction. First, we estimate with OLS the following equation:

$$\ln\left(\frac{E_{h,t}}{X_t}\right) = \alpha + \beta t + \gamma \ln\left(\frac{(1+\tau_t^c)P_t^x}{P_t^e + \tau_{h,t}}\right) + u_t$$

Then we test the stationarity of residuals. We perform a Dickey Fuller test on residuals. We find a statistic $t = -2.37$ which is below the 5% critical value of -1.95. We conclude that we can reject the unit root hypothesis and that the residuals are stationary. Comparing the estimated equation with equation (49), we see that γ is the elasticity of substitution σ and that β is equal to $(\sigma - 1)g^{ae}$. So we have the following results: the elasticity of substitution between E_h and D is $\sigma = 0.50$ and the rate of technical progress is $g^{ae} = 1.6\%$.

Table 8: Estimation of the cointegration relation between $\frac{X}{E_h}$ and $\frac{(1+\square^e)P^x}{P^e+\square_h}$

sample period	□	□	□
1960-2010	15.105	-0.008138	0.495
	(4.7)	(-5.05)	(5.23)

B Estimation of the elasticities of substitution and the rates of technical progress in production

Method

The two-level production function is given by equations (10) and (11). We deduce from the first order conditions (15) (17), denoting $\hat{x} = \frac{\dot{X}}{X}$ and omitting the time index:

$$\hat{l} \square \hat{y} = (\square\square 1)\hat{a}^l + \square(\hat{p}^l \square \hat{p}^y) \tag{50}$$

$$\hat{z} \square \hat{y} = \square(\hat{p}^y \square \hat{p}^z) \tag{51}$$

$$\hat{e} \square \hat{z} = (\square\square 1)\hat{a}^e + \square(\hat{p}^z \square \hat{p}^e) \tag{52}$$

$$\hat{k} \square \hat{z} = \square(\hat{p}^z \square \hat{p}^k) \tag{53}$$

But $\hat{z}$ and $\hat{p}^z$ cannot be observed. To get rid of them, we first add $\hat{p}^k \square \hat{p}^z$ on both sides of (53):

$$\hat{p}^k + \hat{k} \square (\hat{p}^z + \hat{z}) = (\square\square 1)((\hat{p}^z \square \hat{p}^y) \square (\hat{p}^k \square \hat{p}^y))$$

then use (51) to obtain

$$(\square\square 1)(\hat{p}^z \square \hat{p}^y) = \hat{p}^y + \hat{y} \square (\hat{p}^z + \hat{z})$$

so that:

$$\hat{p}^k + \hat{k} \square (\hat{p}^z + \hat{z}) = (\square\square 1)\ \square \frac{\hat{p}^y + \hat{y} \square (\hat{p}^z + \hat{z})}{\square\square 1} \square (\hat{p}^k \square \hat{p}^y) \square$$

Then, denoting $\square_{KZ}$ the variation of the share in value of K in Z; we get:

$$\square_{KZ} = \frac{\square\square 1}{1\square\square}\square_{ZY} + (1\square\square)(\hat{p}^k \square \hat{p}^y)$$

Now adding to both sides of (52) the term $\hat{p}_e \square \hat{p}_z$ and using (51), we get:

$$\square_{EZ} = (\square\square 1)\hat{a}^e + \frac{\square\square 1}{1\square\square}\square_{ZY} + (1\square\square)(\hat{p}^e \square \hat{p}^y)$$

Finally, we have the following three equations:

$$\hat{l} \square \hat{y} = (\square\square 1)\hat{a}^l + \square(\hat{p}^l \square \hat{p}^y) \tag{54}$$

$$\square_{KZ} = \frac{\square\square 1}{1\square\square}\square_{ZY} + (1\square\square)(\hat{p}^k \square \hat{p}^y) \tag{55}$$

$$\square_{EZ} = (\square\square 1)\hat{a}^e + \frac{\square\square 1}{1\square\square}\square_{ZY} + (1\square\square)(\hat{p}^e \square \hat{p}^y) \tag{56}$$

so that we have to estimate the system:

$$y_1 = \square_1 + \square_1 x_1 + \square_1 \tag{57}$$

$$y_2 = \square_{21} x_{21} + \square_{22} x_{22} + \square_2 \tag{58}$$

$$y_3 = \square_3 + \square_{31} x_{31} + \square_{32} x_{23} + \square_3 \tag{59}$$

We proceed as in van der Werf (2008). We estimate first the first equation separately. Then we estimate the system of the two last equations, with the following restrictions on coefficients: $\square_{31} = \square_{21} = \square \frac{\square_{22}}{1 \square \square_1}$ and $\square_{22} = \square_{32}$:

Results

If we do not include gas in the data (data on gas come from another source, Ceren) and run the regression from 1986, we find similar results: the elasticity of substitution between L and Z_f is $\square = 0{:}52$; the elasticity of substitution between K and E_f is $\square = 0{:}48$; and the energy and labor efficiency growth rates are respectively $g^{ae} = 2{:}4\%$ and $g^{al} = 1{:}5\%$ (significant also).

Including gas, we find that the elasticity of substitution between L and Z_f is $\square = 0{:}52$. The elasticity of substitution between K and E_f is $\square = 0{:}52$. The energy efficiency growth rate is $g^{ae} = 2{:}7\%$ and the labour efficiency growth rate is $g^{al} = 1{:}5\%$. All the results are significant at 5% at least.

These results are consistent with those of Lalanne et al. (2009).

C Energy taxes

This appendix presents what taxes on fossil energy represent in comparison to the initial price without taxes, for households and firms. Fossil fuels in France are taxed at TICPE (former TIPP) for petroleum products, TICGN for gases, and VAT. TICP and TICGN are excise duties paid on the quantity consumed, and VAT applies on the price including those taxes. So the consumer price of oil products all taxes included is $P_{ATI} = (P + TICPE)(1 + VAT)$:

Many exemptions exist for the payment of TICPE for firms, but no one exists for households. On the contrary, the TICGN is not paid by households. Table 9 shows how the price is decomposed for each energy from fossil origin consumed by households. We do not present the details for gas consumption because it is only subject to VAT at normal rate 19.6%.

Table 9: Decomposition of the price of energy from fossil origin for 2010 in e/hl. and impact of a 32e/tCO$_2$ tax

	Price decomposition in e/hl				Emission	Extra cost of
	Price before taxes	Taxes on energy	VAT	Total price	factor in kg CO$_2$/ hl	a 32e/tCO$_2$ tax in e/ hl
Diesel	53.1	42.8	18.8	114.7	268	8.6
Gasoline	52.0	60.6	22.1	134.6	242	7.7
Domestic fuel	54.2	5.7	11.7	71.6	268	8.6
Liquefied gas	55.7	6.0	12.1	73.8	158	5.1

Source: Direction Generale de l'Energie et du Climat (DGEC), Ademe

Table 9 shows that the tax burden largely depends on the type of energy. We compute an average tax rate, including specific taxes on energy and VAT for households and firms. Households' consumption of fossil energies is available with details, as shown in Table 10. With the tax rates presented in Table 9, we find that the average tax rate for households' fossil energy consumption represents 77% of the price before tax (see Table 10).

Table 10: Households' energy consumption in billions of €

	Consumption incl. taxes	Consumption before taxes	Taxes on energy	VAT	Total taxes	As a % of cons. before taxes
Diesel	22.2	10.3	8.3	3.7	11.9	116
Gasoline	13.4	5.2	6.0	2.2	8.2	159
Domestic fuel	7.1	5.4	0.6	1.2	1.7	32
Liquefied gas	1.7	1.3	0.1	0.3	0.4	32
Natural gas	11.0	9.2	0	1.8	1.8	20
Total	55.4	31.3	15.0	9.1	24.1	77

Source: INSEE, DGEC and authors computation

Concerning firms, intermediate consumption only exists at an aggregate level, preventing us from applying the same procedure. We start from total energy taxes on petroleum products, and infer the part paid by firms by subtracting the part paid by households from the total. We then add TIGCN, which is only paid by firms, and compute the average rate of taxes, in regard to firms intermediate consumption of fossil energies. Total TICPE collected in 2010 is € 23.9 bn (source: DGEC), whereas TICGN is € 0.3 bn. From our € 15 bn estimation of TICPE paid by households, we obtain that TICPE paid by firms is € 8.9 bn. So total taxes is € 9.2 bn. Some firms are also covered by the European Trading Scheme, which puts constraint on their emissions. More precisely, the system puts an excess cost to fossil energy (emitting GHG), that can be interpreted as taxation. In 2010, the average price of permits was 13 €, and annual permits which were distributed amounted to 132 millions. We thus consider that it implies an excess cost of € 1.8 bn for firms.

We then retrieve from intermediate consumption of firms the amount paid for energy from fossil origin. This corresponds to € 51.3 bn (tax on energy included) in 2010. So the tax rate applied to firms on energy from fossil origin is $\frac{9.2+1.8}{51.3-9.2} = 26\%$.