Optimal Taxation and Human Capital Policies over the Life Cycle

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Human Capital and Taxes: A Two Way Interaction

- Interplay between HC policies and taxes.

- HC policies affect the income distribution - a key input for taxes.

- Taxes affect return and risk from HC investments.

- Calls for joint analysis of optimal taxation and HC policies.

- Optimal Taxation (Mirrlees) literature typically assumes exogenous ability

    - Mirlees 1971, Saez 2001...

Questions addressed in this paper

- How should the tax system take into account HC acquisition?

  - Should HC expenses be tax deductible?

  - What is the right tax treatment of cost of time?

  - What if HC unobservable to the govt?

- What parameters are important for HC policies, e.g., subsidies?

  - What is the lifetime evolution of optimal HC policies?

- What policy instruments implement the optimum?

  - How close can simpler policies come?

A Model to capture main features of HC acquisition

- Dynamic lifecycle model of labor supply and HC acquisition.

- Investment in HC through time (training) or monetary expenses.

- Heterogeneous and uncertain returns
  $\rightarrow$ Wage depends on endogenous HC and exogenous stochastic ability.

- Government faces asymmetric info about ability, evolution of ability, and labor effort.

    ▶ 2 cases: HC observable or unobservable to govt. (College vs. OJT?)

    ▶ Dynamic mechanism design with incentive compatibility constraints.

Preview of Findings

- Characterize constrained efficient allocations over life using "wedges."
  - Implementations proposed: Income Contingent Loans and "Deferred Deductibility" Scheme.

- Highlight important parameters for optimal policies:
  - Crucial how complementary HC is to ability and risk.
  - For training time: additional interaction with labor supply.

- Numerical analysis:
  - Full *dynamic risk-adjusted* deductibility close to optimal.
  - Simple age-dependent linear policies achieve bulk of welfare gain.

Related literature

Human Capital: Becker (1964), Ben-Porath (1967), Heckman (1976), Card (1995), Heckman and Cunha (2006), Goldin and Katz (2008), Lochner and Monge-Naranjo (2011).

Optimal Taxation: Mirrlees (1971), Saez (2001), Kocherlakota (2005), Albanesi and Sleet (2006), Golosov, Tsyvinski, Werning (2006), Scheuer (2013), Farhi and Werning (2013).

Taxation and HC: Bovenberg and Jacobs (2005), Kapicka and Neira (2013), Findeisen and Sachs (2013), Krueger and Ludwig (2013).

Dynamic Mechanism Design and First Order Appraoch: Spear and Srivastava (1987), Fernandes and Phelan (2000), Doepke and Townsend (2006), Pavan *et al.* (2013), **Farhi and Werning (2013)**.

Contribution: Lifecycle investment, money & time, heterogeneous & uncertain HC returns, unobservable HC stock, wage function.

Outline

Outline

Model: Risky investments in Human Capital

- **Wage**: $w_t = w_t\left(\theta_t, s_t, z_t\right)$

- **Ability** θ:
  stochastic, Markov $f^t\left(\theta_t | \theta_{t-1}\right)$, private info, privately uninsurable.

- Two ways of acquiring HC:

    1. **Expenses** e_t at cost $M_t\left(e_t\right)$. Stock of HC expenses s_t :

    $$s_t = s_{t-1} + e_t$$

    2. **Training time** i_t at disutility cost $\phi_t\left(l_t, i_t\right)$. Accomplished training z_t :

    $$z_t = z_{t-1} + i_t$$

- Cost composition of College versus OJT?

- Income: $y_t = w_t l_t$

Hicksian complementarity

- **Hicksian coefficients of complementarity**:

$$\rho_{\theta s} = \frac{w_{\theta s} w}{w_s w_\theta} \qquad \rho_{\theta z} = \frac{w_{\theta z} w}{w_z w_\theta}$$

- $\rho_{\theta s} \geq 0$: Marginal wage gain from HC $\uparrow$ in ability.

- $\rho_{\theta s} \geq 1$: Elasticity of wage to HC $\uparrow$ in ability.

- If separable $w = \theta + h(s, z) \Rightarrow \rho_{\theta s} = \rho_{\theta z} = 0$

- If multiplicative $w = \theta h(s, z) \Rightarrow \rho_{\theta s} = \rho_{\theta z} = 1$

- If CES $w = \left[\alpha_1 \theta^{1-\rho_t} + \alpha_2 s^{1-\rho_t} + \alpha_3 z^{1-\rho_t} \right]^{\frac{1}{1-\rho_t}} \Rightarrow \rho_{\theta s} = \rho_{\theta z} = \rho_t$

▸ Net Subsidy

Model: Preferences over Lifetime Allocations

- T periods of work, T_r periods of retirement.

- Per period utility: $u_t(c_t) - \phi_t(l_t, i_t)$.

- History $\theta^t = \{\theta_1, ..., \theta_t\} \in \Theta^t$, probability $P(\theta^t) = f^t(\theta_{t+1}|\theta_t) ... f(\theta_1)$.

- Allocation: $\{c(\theta^t), y(\theta^t), s(\theta^t), z(\theta^t)\}_{\theta^t}$.

- Expected lifetime utility from allocation:

$$
U\left(\{c(\theta^t), y(\theta^t), s(\theta^t), z(\theta^t)\}\right)
$$
$$
= \sum_{t=1}^{T+T_r} \int \beta^{t-1} \left[u_t(c(\theta^t)) - \phi_t\left(\frac{y(\theta^t)}{w_t(\theta^t)}, z(\theta^t) - z(\theta^{t-1})\right) \right] P(\theta^t) \, d\theta^t
$$

$$
w_t(\theta^t) \equiv w_t(\theta_t, s_t(\theta^t), z_t(\theta^t))
$$

Government's/Planner's Goals: Insurance and Redistribution

- Govt's/Planner's goal: max expected social welfare given Pareto weights.

  - Insurance against earnings risk.

  - Redistribution across intrinsic ability heterogeneity (persistent).

  - Incentives for efficient work and HC investment.

- Asymmetric information about:

$$\text{ability and its evolution} \qquad \text{labor supply}$$
$$\downarrow \qquad\qquad\qquad \downarrow$$
$$w_t(\theta_t, s_t, z_t) \times l_t = y_t$$
$$\uparrow \quad \uparrow$$

  2 cases: observable and unobservable HC.

  $\rightarrow$ **"direct revelation mechanism"** with incentive compatibility.

Government's/Planner's Program: Dual Formulation

- Min expected resource cost s.t. utility targets and incentive compatibility
  $\rightarrow$ **constrained efficiency**.

$$\min_{\{c,y,s,z\}} \sum_{t=1}^{T} \frac{1}{R^{t-1}} \int \left(c\left(\theta^t\right) - y\left(\theta^t\right) + M_t \left(s\left(\theta^t\right) - s\left(\theta^{t-1}\right)\right)\right) P\left(\theta^t\right) d\theta^t$$

$$\text{s.t.: } U\left(\{c,y,s,z\}\right) \geq \underline{U}$$

$$\{c,y,s,z\} \text{ is incentive compatible.}$$

- If initial heterogeneity and non-utilitarian welfare function set any Pareto weights through $\underline{U} = \left(\underline{U}\left(\theta_1\right)\right)_{\Theta}$.

Incentive Compatibility Defined

- Reporting strategy: $r = \left\{ r_t \left(\theta^t \right) \right\}_{t=1}^{T}$, with history $r^t \equiv \{ r_1, .., r_t \}$.

- **Continuation utility** under reporting strategy r:

$$
\omega^r \left(\theta^t \right) = u_t \left(c \left(r^t \left(\theta^t \right) \right) \right) - \phi_t \left(\frac{y \left(r^t \left(\theta^t \right) \right)}{w_t \left(\theta_t, s \left(r^t \left(\theta^t \right) \right), z \left(r^t \left(\theta^t \right) \right) \right)}, i \left(r^t \left(\theta^t \right) \right) \right)
$$
$$
+ \beta \int \omega^r \left(\theta^{t+1} \right) f^{t+1} \left(\theta_{t+1} | \theta_t \right) d\theta_{t+1}
$$

- **Under truth-telling**: $\omega \left(\theta^t \right)$ with $r_t \left(\theta^t \right) = \theta_t$ for all θ^t.

- **Incentive Compatibility**

$$
\omega \left(\theta^t \right) \geq \omega^r \left(\theta^t \right) \qquad \forall r, \forall \theta^t
$$

Solving the Government's Program: Method

1. Solving the direct revelation mechanism:
   - Step 1: Relax program using first order approach (FOA).
   - Step 2: Formulate relaxed program recursively.

2. Characterize optimal allocations using "wedges" or implicit taxes.

3. Decentralize or "implement" optimum using policy instruments.

Step 1. Relaxing the Program: First-Order Approach

- Consider deviating strategy σ^r with report r:

$$\omega\left(\theta^t\right) = \max_r(u_t\left(c\left(\theta^{t-1}, r\right)\right) - \phi_t\left(\frac{y\left(\theta^{t-1}, r\right)}{w_t\left(\theta_t, s\left(\theta^{t-1}, r\right), z\left(\theta^{t-1}, r\right)\right)}\right)$$
$$+ \beta \int \omega^{\sigma^r}\left(\theta^{t-1}, r, \theta_{t+1}\right) f^{t+1}\left(\theta_{t+1}|\theta_t\right) d\theta_{t+1})$$

- Replace by necessary **Envelope Condition**:

$$\frac{\partial \omega\left(\theta^t\right)}{\partial \theta_t} = \frac{\partial \phi_t}{\partial l_t}\frac{\partial w_t}{\partial \theta_t}\frac{l_t}{w_t} + \beta \int \omega\left(\theta^{t+1}\right)\frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_t\right)}{\partial \theta_t}d\theta_{t+1}$$

- **Sufficiency?**
  a) Conditions on allocations (Pavan *et al.* 2013).
  b) Ex-post verification (Werning, 2007, Farhi and Werning, 2013).

  ▸ Verification

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

$$\omega\left(\theta^t\right) = u_t\left(c\left(\theta^t\right)\right) - \phi_t\left(\frac{y\left(\theta^t\right)}{w_t\left(\theta^t\right)}, i\left(\theta^t\right)\right) + \beta \int \omega\left(\theta^{t+1}\right) f^{t+1}\left(\theta_{t+1}|\theta_t\right) d\theta_{t+1}$$

- Envelope Condition:

$$\frac{\partial\omega\left(\theta^t\right)}{\partial\theta_t} = \frac{\partial\phi_t}{\partial l_t}\frac{\partial w_t}{\partial\theta_t}\frac{l\left(\theta^t\right)}{w\left(\theta^t\right)} + \beta \int \omega\left(\theta^{t+1}\right) \frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_t\right)}{\partial\theta_t} d\theta_{t+1}$$

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$$v_t\left(\theta^t\right) = \int\omega\left(\theta^{t+1}\right)f^{t+1}\left(\theta_{t+1}|\theta_t\right)d\theta_{t+1}$$

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$$v_t\left(\theta^t\right) \;=\; \int\omega\left(\theta^{t+1}\right)f^{t+1}\left(\theta_{t+1}|\theta_t\right)d\theta_{t+1}$$

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- Envelope Condition:

$$\frac{\partial\omega\left(\theta^t\right)}{\partial\theta_t} = \frac{\partial\phi_t}{\partial l_t}\frac{\partial w_t}{\partial\theta_t}\frac{l\left(\theta^t\right)}{w\left(\theta^t\right)} + \beta\Delta_t\left(\theta^t\right)$$

$$v_t\left(\theta^t\right) = \int\omega\left(\theta^{t+1}\right)f^{t+1}\left(\theta_{t+1}|\theta_t\right)d\theta_{t+1}$$

Step 2. Recursive Formulation: Define States

- Definition of continuation utility:

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$$v_t\left(\theta^t\right) = \int \omega\left(\theta^{t+1}\right) f^{t+1}\left(\theta_{t+1}|\theta_t\right) d\theta_{t+1}$$

$$\Delta_t\left(\theta^t\right) = \int \omega\left(\theta^{t+1}\right) \frac{\partial f^{t+1}\left(\theta_{t+1}|\theta_t\right)}{\partial\theta_t} d\theta_{t+1}$$

Step 2. Recursive Formulation: Rewrite Recursively

- Definition of continuation utility, using $\theta_-, \theta, \theta'$.

$$\omega(\theta) = u_t(c(\theta)) - \phi_t\left(\frac{y(\theta)}{w_t(\theta)}, z(\theta) - z_-\right) + \beta v(\theta)$$

- Envelope Condition:

$$\frac{\partial \omega(\theta)}{\partial \theta} = \frac{\partial \phi_t}{\partial l}\frac{\partial w_t(\theta)}{\partial \theta}\frac{l(\theta)}{w_t(\theta)} + \beta \Delta(\theta)$$

$$v(\theta) = \int \omega(\theta') f^{t+1}(\theta'|\theta) d\theta'$$

$$\Delta(\theta) = \int \omega(\theta') \frac{\partial f^{t+1}(\theta'|\theta)}{\partial \theta} d\theta'$$

Recursive Formulation of Relaxed Program

$$K(v, \Delta, \theta_-, s_-, z_-, t) = \min \int (c(\theta) + M_t(s(\theta) - s_-) - w_t(\theta, s(\theta), z(\theta)) l(\theta)$$

$$+ \frac{1}{R} K(v(\theta), \Delta(\theta), \theta, s(\theta), z(\theta), t+1)) f^t(\theta|\theta_-) d\theta$$

$$\omega(\theta) = u_t(c(\theta)) - \phi_t(l(\theta), z(\theta) - z_-) + \beta v(\theta)$$
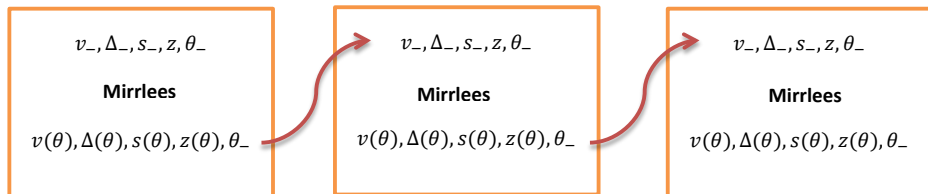
$$\dot{\omega}(\theta) = \frac{w_{\theta,t}}{w_t} l(\theta) \phi_{l,t}((\theta), z(\theta) - z_-) + \beta \Delta(\theta)$$

$$v = \int \omega(\theta) f^t(\theta|\theta_-) d\theta$$

$$\Delta = \int \omega(\theta) \frac{\partial f^t(\theta|\theta_-)}{\partial \theta_-} d\theta$$

over $(c(\theta), l(\theta), s(\theta), z(\theta), \omega(\theta), v(\theta), \Delta(\theta))$

Recursive Formulation of Relaxed Program

$$K\left(v, \Delta, \theta_-, s_-, z_-, t\right) = \min \int \left(c\left(\theta\right) + M_t\left(s\left(\theta\right) - s_-\right) - w_t\left(\theta, s\left(\theta\right), z\left(\theta\right)\right) l\left(\theta\right)\right.$$

$$\left. + \frac{1}{R} K\left(v\left(\theta\right), \Delta\left(\theta\right), \theta, s\left(\theta\right), z\left(\theta\right), t+1\right)\right) f^t\left(\theta | \theta_-\right) d\theta$$

$$\omega\left(\theta\right) = u_t\left(c\left(\theta\right)\right) - \phi_t\left(l\left(\theta\right), z\left(\theta\right) - z_-\right) + \beta v\left(\theta\right)$$

$$\dot{\omega}\left(\theta\right) = \frac{w_{\theta, t}}{w_t} l\left(\theta\right) \phi_{l, t}\left(\left(\theta\right), z\left(\theta\right) - z_-\right) + \beta \Delta\left(\theta\right)$$

$$v = \int \omega\left(\theta\right) f^t\left(\theta | \theta_-\right) d\theta$$

$$\Delta = \int \omega\left(\theta\right) \frac{\partial f^t\left(\theta | \theta_-\right)}{\partial \theta_-} d\theta$$

over $\left(c\left(\theta\right), l\left(\theta\right), s\left(\theta\right), z\left(\theta\right), \omega\left(\theta\right), v\left(\theta\right), \Delta\left(\theta\right)\right)$

Recursive Formulation of Relaxed Program

$$K\left(v, \Delta, \theta_-, s_-, z_-, t\right) = \min \int (c\left(\theta\right) + M_t\left(s\left(\theta\right) - s_-\right) - w_t\left(\theta, s\left(\theta\right), z\left(\theta\right)\right) l\left(\theta\right)$$

$$+ \frac{1}{R} K\left(v\left(\theta\right), \Delta\left(\theta\right), \theta, s\left(\theta\right), z\left(\theta\right), t+1\right)) f^t\left(\theta | \theta_-\right) d\theta$$

$$\omega\left(\theta\right) = u_t\left(c\left(\theta\right)\right) - \phi_t\left(l\left(\theta\right), z\left(\theta\right) - z_-\right) + \beta v\left(\theta\right)$$

$$\dot\omega\left(\theta\right) = \frac{w_{\theta,t}}{w_t} l\left(\theta\right) \phi_{l,t}\left(\left(\theta\right), z\left(\theta\right) - z_-\right) + \beta \Delta\left(\theta\right)$$

$$v = \int \omega\left(\theta\right) f^t\left(\theta | \theta_-\right) d\theta$$

$$\Delta = \int \omega\left(\theta\right) \frac{\partial f^t\left(\theta | \theta_-\right)}{\partial \theta_-} d\theta$$

over $\left(c\left(\theta\right), l\left(\theta\right), s\left(\theta\right), z\left(\theta\right), \omega\left(\theta\right), v\left(\theta\right), \Delta\left(\theta\right)\right)$

Recursive Formulation of Relaxed Program

$$K\left(v, \Delta, \theta_-, s_-, z_-, t\right) = \min \int \left(c\left(\theta\right) + M_t\left(s\left(\theta\right) - s_-\right) - w_t\left(\theta, s\left(\theta\right), z\left(\theta\right)\right) l\left(\theta\right)\right.$$
$$\left. + \frac{1}{R}K\left(v\left(\theta\right), \Delta\left(\theta\right), \theta, s\left(\theta\right), z\left(\theta\right), t+1\right)\right)f^t\left(\theta | \theta_-\right) d\theta$$

$$\omega\left(\theta\right) = u_t\left(c\left(\theta\right)\right) - \phi_t\left(l\left(\theta\right), z\left(\theta\right) - z_-\right) + \beta v\left(\theta\right)$$

$$\dot\omega\left(\theta\right) = \frac{w_{\theta,t}}{w_t}l\left(\theta\right)\phi_{l,t}\left(\left(\theta\right), z\left(\theta\right) - z_-\right) + \beta\Delta\left(\theta\right)$$

$$v = \int \omega\left(\theta\right) f^t\left(\theta | \theta_-\right) d\theta$$

$$\Delta = \int \omega\left(\theta\right) \frac{\partial f^t\left(\theta | \theta_-\right)}{\partial \theta_-} d\theta$$

over $\left(c\left(\theta\right), l\left(\theta\right), s\left(\theta\right), z\left(\theta\right), \omega\left(\theta\right), v\left(\theta\right), \Delta\left(\theta\right)\right)$

Method summary:
Repeated Mirrlees Nested in Dynamic Programming
+ Endogenous Human Capital Formation

$v_-, \Delta_-, s_-, z, \theta_-$		$v_-, \Delta_-, s_-, z, \theta_-$		$v_-, \Delta_-, s_-, z, \theta_-$
Mirrlees	→	**Mirrlees**	→	**Mirrlees**
$v(\theta), \Delta(\theta), s(\theta), z(\theta), \theta_-$		$v(\theta), \Delta(\theta), s(\theta), z(\theta), \theta_-$		$v(\theta), \Delta(\theta), s(\theta), z(\theta), \theta_-$

Outline

1 Model and Solution Approach

2 Human Capital Expenses

3 Training Time

4 Unobservable Human Capital

5 Implementation

Implicit taxes and subsidies: Definition

Implicit marginal labor income tax:

$$\tau_{Lt} \equiv 1 - \underbrace{\frac{\phi_{l,t}(l_t, i_t)}{w_t u'_t(c_t)}}_{\text{MRS/MRT } l_t \text{ and } c_t}$$

Implicit marginal savings tax:

$$\tau_{kt} \equiv 1 - \underbrace{\frac{1}{\beta R} \frac{u'_t(c_t)}{E_t(u'_{t+1}(c_{t+1}))}}_{(\text{MRS } c_t \text{ and } c_{t+1})/\text{Return on savings}}$$

Implicit marginal HC subsidy:

$$\tau_{St} \equiv \underbrace{M'_t(e_t) - \beta E_t \left(\frac{u'_{t+1}(c_{t+1})}{u'_t(c_t)} M'_{t+1}(e_{t+1}) \right)}_{\text{Dynamic (risk-adjusted) Cost}} - \underbrace{(1 - \tau_{Lt}) w_{s,t} l_t}_{\text{Marginal Benefit}}$$

Implicit marginal bonus for training time:

$$\tau_{Zt} \equiv \underbrace{\frac{\phi_{i,t}(l_t, i_t)}{u'_t(c_t)} - \beta E_t \left(\frac{u'_{t+1}}{u'_t} \frac{\phi_{i,t+1}(l_{t+1}, i_{t+1})}{u'_{t+1}(c_{t+1})} \right)}_{\text{Dynamic (risk-adjusted, monetary) cost}} - \underbrace{(1 - \tau_{Lt}) w_{z,t} l_t}_{\text{Marginal Benefit}}$$

A Net Human Capital Subsidy to Capture Real Incentives

Definition (Net Wedge)

$$t_{st} \equiv \frac{\tau_{St} - \tau_{Lt} M_t'^d + P_t}{\left(M_t'^d - \tau_{St}\right)\left(1 - \tau_{Lt}\right)}$$

$M_t'^d \equiv M_t' - \beta E_t \left(\frac{u_{t+1}'}{u_t'} M_{t+1}'\right)$: risk adjusted dynamic cost

$P_t \equiv \frac{1}{R} \frac{\tau_k}{1 - \tau_k} \left(1 - \tau_{Lt}\right) E \left(\beta \frac{u_{t+1}'}{u_t'} M_{t+1}'\right)$: risk adjusted savings distortion

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$t_{st} = 0 \rightarrow$ **Full dynamic risk-adjusted deductibility of expenses Neutrality of tax system wrt HC.**

Static model: $\tau_{St}^* = M_t' \tau_{Lt}^*$, standard deductibility (BJ, 2005).

Dynamic model + uncertainty: $\tau_{St}^* = M_t'^d \tau_{Lt}^* - P_t^*$
i) dynamic cost, ii) risk adjustment, iii) savings wedge.

$t_{st} > 0 \rightarrow$ positive net subsidy beyond deductibility.

Optimal Net Subsidy: the Formula

$$t_{st}^* \left(\theta^t \right) = \frac{\left(\kappa \left(\theta^t \right) + \eta \left(\theta^t \right) \right) u_t' \left(c \left(\theta^t \right) \right)}{f^t \left(\theta_t | \theta_{t-1} \right)} \frac{\varepsilon_{w\theta,t}}{\theta_t} \left(1 - \rho_{\theta s,t} \right)$$

$\epsilon_{w\theta,t}$: wage elasticity wrt ability.

1. Insurance Motive

$$t_{st}^* \left(\theta^t \right) = \frac{\left(\kappa \left(\theta^t \right) + \eta \left(\theta^t \right) \right) u_t' \left(c \left(\theta^t \right) \right)}{f^t \left(\theta_t | \theta_{t-1} \right)} \frac{\varepsilon_{w\theta,t}}{\theta_t} \left(1 - \rho_{\theta s,t} \right)$$

$$\downarrow$$

$$\kappa \left(\theta^t \right) \equiv \int_{\theta_t}^{\bar{\theta}} \left(\frac{1}{u_t' \left(c \left(\theta^{t-1}, \theta_s \right) \right)} - E_{t-1} \left(\frac{1}{u_t' \left(c \left(\theta^t \right) \right)} \right) \right) f \left(\theta_s | \theta_{t-1} \right) d\theta_s$$

Insurance Motive

$\kappa \left(\theta^t \right)$ captures dispersion in marginal utilities

$\kappa \left(\theta^t \right) = 0$ if quasilinear utility or no uncertainty (fully persistent types).

2. Persistence and the Redistributive Motive

$$t_{st}^* \left(\theta^t \right) = \frac{\left(\kappa \left(\theta^t \right) + \eta \left(\theta^t \right) \right) u_t' \left(c \left(\theta^t \right) \right)}{f^t \left(\theta_t | \theta_{t-1} \right)} \frac{\varepsilon_{w\theta,t}}{\theta_t} \left(1 - \rho_{\theta s,t} \right)$$

$$\downarrow$$

$$\eta \left(\theta^t \right) \equiv t_{st-1}^* \left(\theta^{t-1} \right) \left[\frac{R\beta}{u_{t-1}'} \frac{1}{\left(1 - \rho_{\theta s,t-1} \right)} \frac{\theta_{t-1}}{\varepsilon_{w\theta,t-1}} \int_{\theta_t}^{\bar{\theta}} \frac{\partial f \left(\theta_s | \theta_{t-1} \right)}{\partial \theta_{t-1}} d\theta_s \right]$$

Persistence and the Redistributive Motive

Persistence of ability $\rightarrow$ persistence of policy.

$$\eta \left(\theta^t \right) = 0 \text{ with iid shocks.}$$

Redistributive motive against initial heterogeneity remains active if persistence.

▸ Initial heterogeneity

3. Complementarity Between HC and Ability

$$t^*_{st}\left(\theta^t\right) = \frac{\left(\kappa\left(\theta^t\right)+\eta\left(\theta^t\right)\right)u'_t\left(c\left(\theta^t\right)\right)}{f^t\left(\theta_t|\theta_{t-1}\right)}\frac{\varepsilon_{w\theta,t}}{\theta_t}\left(1-\rho_{\theta s,t}\right)$$

$$t^*_{st}\left(\theta^t\right) \geq 0 \Leftrightarrow \rho_{\theta s,t} \leq 1$$

Labor Supply Effect:
subsidy increases wage
$\rightarrow \uparrow$ labor
$\rightarrow \uparrow$ resources.

$+$

Inequality Effect:
if $\rho_{\theta s} \geq 0$
HC benefits more able agents more
$\rightarrow \uparrow$ *pre-tax* inequality and risk.

_____

$\rho_{\theta s} \leq 1 \Rightarrow$ subsidy $\downarrow$ *post-tax* inequality
$\Rightarrow$ has **positive redistributive and insurance effects**.

$\rho_{\theta s} = 1 \Rightarrow t^*_{st}\left(\theta^t\right) = 0$
Benchmark case in literature $w_t = \theta_t s_t$
Full dynamic risk-adjusted deductibility $\approx$ Atkinson-Stigliz result. ▸ $\rho_{\theta s}$

Evolution of the net subsidy over time

- If $\log(\theta_t) = p \log(\theta_{t-1}) + \psi_t$, with $f^\psi(\psi|\theta^{t-1})$ and $E(\psi|\theta^{t-1}) = 0$.

$$E_{t-1}\left(t_{st} \frac{\varepsilon_{w\theta,t-1}}{\varepsilon_{w\theta,t}} \frac{(1-\rho_{\theta s,t-1})}{(1-\rho_{\theta s,t})}\left(\frac{1}{R\beta}\frac{u'_{t-1}}{u'_t}\right)\right)$$
$$= (1-\rho_{\theta s,t-1})\varepsilon_{w\theta,t-1} Cov\left(\frac{1}{R\beta}\frac{u'_{t-1}}{u'_t}, \log(\theta_t)\right) + pt_{st-1}$$

- If HC has **positive insurance value** ($\rho_{\theta s} \leq 1$): positive drift.
    - Fading drift term $\rightarrow$ "**Subsidy smoothing.**"

- Persistence of shocks $\rightarrow$ persistence of policy t_{st}.

Empirical Estimates of the Hicksian Complementarity

- **Labor/Human Capital Literature**:
  Heckman, Cunha et al., 2006, Ashenfelter and Rouse, 1998, ...

  - Early Childhood investments level playing field $\Rightarrow \rho_{\theta s} \leq 1$.

  - Evidence suggests $\rho_{\theta s}$ changes over life.

  - College benefits already able people $\Rightarrow \rho_{\theta s} \geq 0$ and $\rho_{\theta s} \geq 1$ possible.

- **Structural Macro Literature**:
  Huggett, Ventura, Yaron, 2010, Heathcote, Perri, Violante, 2010:

  - Ability as the residual, assume log separability $\Leftrightarrow \rho_{\theta s} = 1$.

- **OJT**? Investments later in life? Scarce empirical evidence.

▸ Sufficient Stats

Numerical Analysis: Setup

	Functional Form	**Values**	**Source**
Wage	$w_t = \left(\theta_t^{1-\rho} + c_s s_t^{1-\rho}\right)^{\frac{1}{1-\rho}}$	$\rho = \{0.2, 1.2\}$ $c_s = \{0.09, 0.1\}$	CHLM (2006) Match wage premium (AKK, 1998)
Utility	$\log\left(c_t\right) - \frac{1}{\gamma}\left(\frac{y_t}{w_t}\right)^{\gamma}$	$\gamma = 3$	Chetty (2012)
Stochastic process	$\log\theta_t = \log\theta_{t-1} + \psi_t$ $\psi_t \sim N\left(-\frac{1}{2}\sigma_\psi^2, \sigma_\psi^2\right)$	$\sigma_\psi^2 = 0.0095$	HSV (2005)
Cost	$M_t\left(e_t\right) = c_l e_t + 2\left(\frac{e_t}{s_{t-1}}\right)^2$	$c_l = 0.5$	Match expenses OECD (2013) US DoE (2010)

table 1

- $T = 20$, $T_r = 10$, $\beta = 0.95$, $R = 1/\beta$.
- Select zero net cost allocation, utilitarian planner. ▸Match ▸Baseline ▸Vol

Optimal Gross and Net Human Capital Wedges
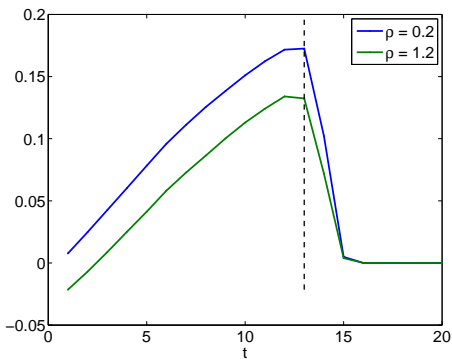
(a) Gross Wedge τ_{St}
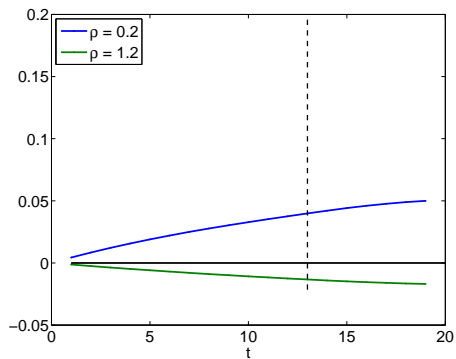
(b) Net Wedge t_{st}

If $\rho_{\theta s} < 1$, τ_S higher and grows faster; $t_s > 0$ and growing.
But: Full dynamic risk adjusted deductibility close to optimal.

▸ Effects of $\rho_{\theta s}$ ▸ Effects of volatility

Labor and Capital Wedges with Human Capital
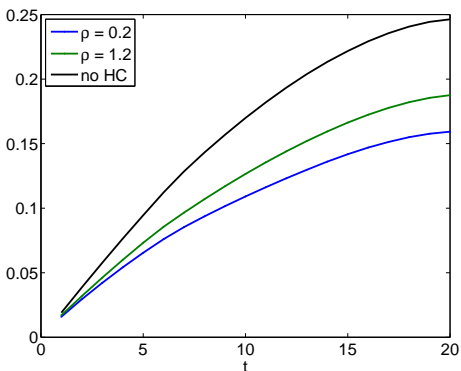
(a) Labor Wedge over time
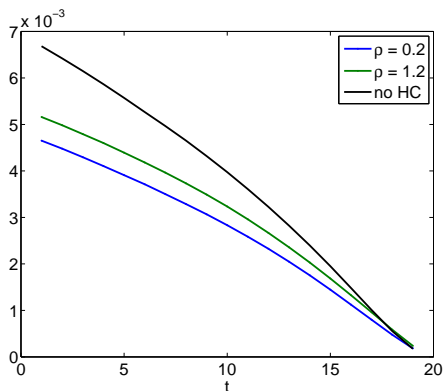
(b) Capital Wedge over time

Labor and capital wedges are smaller in the presence of HC.

Standard Inverse Euler Equation holds.

▸ Tax Regressivity   ▸ Variance Cons Growth

Subsidy Smoothing over Life

t_{st} against t_{st-1}
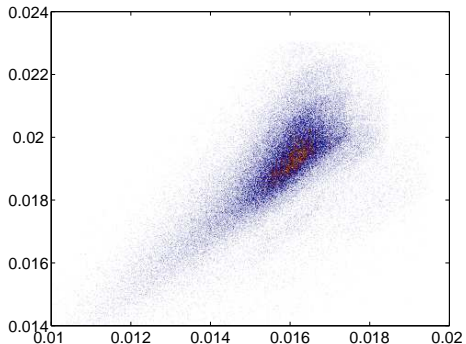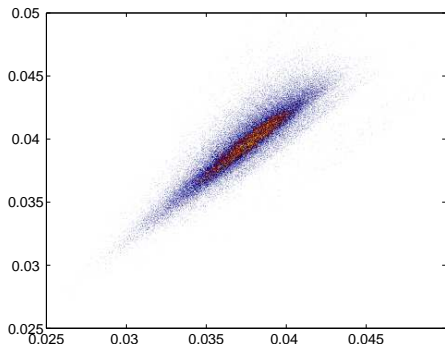
(a) at $t = 5$ (b) at $t = 13$

t_s becomes more correlated over time as age increases
because the variance of consumption growth vanishes.

▸ Subsidy Regressivity   ▸ Tax Smoothing

Insurance and HC Over the Life Cycle
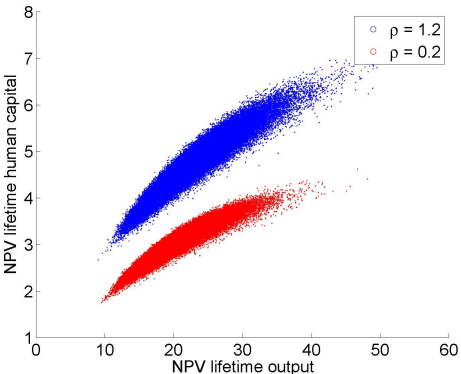
(a) HC against lifetime income
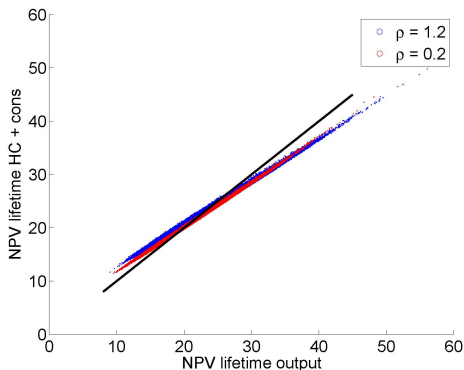
(b) Consumption + HC against lifetime income

Outline

The net bonus on training time

Measures real incentive on training, beyond purely compensating for income and savings tax distortions.

Definition

$$t_{zt} \equiv \frac{\tau_{Zt} - \tau_{Lt} \left(\phi_{zt}/u_t'\right)^d + P_{Zt}}{(1 - \tau_{Lt}) \left(\left(\phi_{zt}/u_t'\right)^d - \tau_{Zt}\right)}$$

$\left(\phi_{zt}/u_t'\right)^d \equiv \frac{\phi_{z,t}}{u_t'} - \beta E_t \left(\frac{u_{t+1}'}{u_t'} \frac{\phi_{z,t+1}}{u_{t+1}'}\right)$ dynamic risk adjusted cost

$P_{Zt} \equiv \frac{1}{R} \frac{\tau_k}{1-\tau_k} E \left(\beta \frac{u_{t+1}'}{u_t'} \frac{\phi_{z,t+1}'}{u_{t+1}'}\right)$, risk adjusted savings distortion.

Optimal Net Bonus: Special Case of Separable Disutility

- Subsidize training on net iff positive redistributive and insurance values:

$$t_{zt}^* \left(\theta^t \right) \geq 0 \Leftrightarrow \rho_{\theta z, t} \leq 1$$

- **Inverse elasticity rules** for implicit taxes:

$$t_{zt}^* = \frac{\tau_{Lt}^*}{1 - \tau_{Lt}^*} \frac{\varepsilon_t^c}{1 + \varepsilon_t^u} \left(1 - \rho_{\theta z, t} \right), \qquad t_{st}^* = \frac{\tau_{Lt}^*}{1 - \tau_{Lt}^*} \frac{\varepsilon_t^c}{1 + \varepsilon_t^u} \left(1 - \rho_{\theta s, t} \right)$$

- Bonus and subsidy set **proportionally to their redistributive effects**:

$$\frac{t_{zt}^*}{t_{st}^*} = \frac{\left(1 - \rho_{\theta z, t} \right)}{\left(1 - \rho_{\theta s, t} \right)}$$

- If CES wage, identical tax treatment of expenses and time: $t_{st}^* = t_{zt}^*$.

Optimal Net Bonus on Training Time

At the optimum, the net bonus is given by:

$$t_{zt}^* \left(\theta^t\right) = \frac{\tau_{Lt}^* \left(\theta^t\right)}{1 - \tau_{Lt}^* \left(\theta^t\right)} \frac{\varepsilon_t^c}{1 + \varepsilon_t^u} \left((1 - \rho_{\theta z,t}) - \frac{\varepsilon_{\phi l z,t}}{\varepsilon_{wz,t}} \right)$$
$$+ \frac{1}{R} E_t \left(\frac{\tau_{Lt+1}^* \left(\theta^{t+1}\right)}{1 - \tau_{Lt}^* \left(\theta^t\right)} \frac{\varepsilon_{t+1}^c}{1 + \varepsilon_{t+1}^u} \frac{w_{z,t+1}}{w_{z,t}} \frac{l_{t+1}}{l_t} \frac{\varepsilon_{\phi l z,t+1}}{\varepsilon_{wz,t+1}} \right)$$

$$t_{zt}^* \left(\theta^t\right) \geq 0 \Leftarrow \underbrace{(1 - \rho_{\theta z,t})}_{\text{Insurance effect}} \geq \underbrace{\varepsilon_{\phi l z,t} / \varepsilon_{wz,t}}_{\text{Interaction w/ Labor}} \geq 0$$

- Same "Labor Supply" and "Inequality Effect" as for HC expenses.

- In addition: Direct interaction with labor supply.
  "**Learning-and-Doing**" versus "**Learning-or-Doing**?"

Outline

Unobservable HC: Modified Program

- With unobservable expenses: Euler Equation for HC ($\tau_{St} \equiv 0$).

$$u_t'(c_t) M_t'(e_t) = \frac{w_{s,t}}{w_t} l_t \phi'(l_t) + \beta E_t \left(u_{t+1}'(c_{t+1}) M_{t+1}'(e_{t+1}) \right)$$

Unobservable HC: Modified Program

- With unobservable expenses: Euler Equation for HC ($\tau_{St} \equiv 0$).

$$u_t'\left(c_t\right) M_t'\left(e_t\right) = \frac{w_{s,t}}{w_t} l_t \phi'\left(l_t\right) + \beta E_t \left(u_{t+1}'\left(c_{t+1}\right) M_{t+1}'\left(e_{t+1}\right)\right)$$

Unobservable HC: Modified Program

- With unobservable expenses: Euler Equation for HC ($\tau_{St} \equiv 0$).

$$u_t'\left(c_t\right) M_t'\left(e_t\right) = \frac{w_{s,t}}{w_t} l_t \phi'\left(l_t\right) - \beta \Delta^s\left(\theta^t\right)$$

Unobservable HC: Modified Program

- With unobservable expenses: Euler Equation for HC ($\tau_{St} \equiv 0$).

$$u'_t(c_t) M'_t(e_t) = \frac{w_{s,t}}{w_t} l_t \phi'(l_t) - \beta \Delta^s(\theta^t)$$

$$\Delta^s(\theta^t) \equiv -E_t\left(u'_{t+1}(c_{t+1}) M'_{t+1}(e_{t+1})\right)$$

Unobservable HC: Modified Program

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$$u_t'(c_t) M_t'(e_t) = \frac{w_{s,t}}{w_t} l_t \phi'(l_t) - \beta \Delta^s (\theta^t)$$

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$$\Delta^s \equiv -E_{t-1}\left(u_t'\left(c_t\right) M_t'\left(e_t\right)\right)$$

- With unobservable training: Euler Equation for training ($\tau_{Zt} \equiv 0$).

$$\phi_{z,t} = w_{z,t} l_t \frac{1}{w_t} \phi_{l,t} + \beta E_t\left(\phi_{z,t+1}\right)$$

Unobservable HC: Modified Program

- With unobservable expenses: Euler Equation for HC ($\tau_{St} \equiv 0$).

$$u'_t(c_t) M'_t(e_t) = \frac{w_{s,t}}{w_t} l_t \phi'(l_t) - \beta \Delta^s(\theta^t)$$

$$\Delta^s \equiv -E_{t-1}\left(u'_t(c_t) M'_t(e_t)\right)$$

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Unobservable HC: Modified Program

- With unobservable expenses: Euler Equation for HC ($\tau_{St} \equiv 0$).

$$u_t'(c_t) M_t'(e_t) = \frac{w_{s,t}}{w_t} l_t \phi'(l_t) - \beta \Delta^s(\theta^t)$$

$$\Delta^s \equiv -E_{t-1}(u_t'(c_t) M_t'(e_t))$$

- With unobservable training: Euler Equation for training ($\tau_{Zt} \equiv 0$).

$$\phi_{z,t} = w_{z,t} l_t \frac{1}{w_t} \phi_{l,t} - \beta \Delta_t^z(\theta)$$

Unobservable HC: Modified Program

- With unobservable expenses: Euler Equation for HC ($\tau_{St} \equiv 0$).

$$u_t'(c_t) M_t'(e_t) = \frac{w_{s,t}}{w_t} l_t \phi'(l_t) - \beta \Delta^s(\theta^t)$$

$$\Delta^s \equiv -E_{t-1}\left(u_t'(c_t) M_t'(e_t)\right)$$

- With unobservable training: Euler Equation for training ($\tau_{Zt} \equiv 0$).

$$\phi_{z,t} = w_{z,t} l_t \frac{1}{w_t} \phi_{l,t} - \beta \Delta_t^z(\theta)$$

$$\Delta_t^z \equiv -E_t\left(\phi_{z,t+1}\right)$$

Unobservable HC: Modified Program

- With unobservable expenses: Euler Equation for HC ($\tau_{St} \equiv 0$).

$$u'_t(c_t) M'_t(e_t) = \frac{w_{s,t}}{w_t} l_t \phi'(l_t) - \beta \Delta^s(\theta^t)$$

$$\Delta^s \equiv -E_{t-1}\left(u'_t(c_t) M'_t(e_t)\right)$$

- With unobservable training: Euler Equation for training ($\tau_{Zt} \equiv 0$).

$$\phi_{z,t} = w_{z,t} l_t \frac{1}{w_t} \phi_{l,t} - \beta \Delta_t^z(\theta)$$

$$\Delta^z \equiv -E_{t-1}(\phi_{z,t})$$

Unobservable HC expenses: Results

- τ_L and τ_K indirectly provide incentives for HC accumulation.
  Lower τ_L and higher τ_K mimic $t_s > 0$.

- τ_L: Lower if $\rho_{\theta s}$ lower.

- τ_K: "Modified Inverse Euler" holds (hybrid model).

$$\frac{\beta R(1 - \tilde{\gamma}_t^E (\theta^t) M_t' u_t'')}{u_t'} = \int_{\underline{\theta}}^{\bar{\theta}} \frac{\left(1 - \tilde{\gamma}_{t+1}^E (\theta^{t+1}) M_{t+1}' u_{t+1}''\right)}{u_{t+1}'} f^{t+1} (\theta_{t+1}|\theta_t)$$

$\tilde{\gamma}_t^E (\theta^t)$: multiplier on agent's Euler for HC.

Unobservable Training, Observable HC Expenses: Results

$$\underbrace{\frac{\tau_{Lt}}{1 - \tau_{Lt}} - \frac{\tau_{Lt}^*}{1 - \tau_{Lt}^*}}_{\text{Adjustment of labor tax}} = \underbrace{\frac{1}{(1 - \rho_{zs,t})} \frac{1 + \varepsilon_t^u}{\varepsilon_t^c}}_{\text{Relative efficiency}} \underbrace{(t_{st} - t_{st}^*)}_{\text{Adjustment of subsidy}}$$

- Inverse elasticity rule for available instruments.

- Sharpest instrument adjusts more to compensate for "missing bonus."

- Subsidy for HC expenses changed iff $\rho_{zs,t} \neq 1$.

Outline

Implementation: Idea

- From direct revelation mechanism to policy instruments.

- "Taxation principle" $\rightarrow T_t \left(y^t, s^t \right)$ implements optimum.

- Indeterminacy of instruments in theory:
  Administrative constraints or political preferences in practice?

- Propose ICLs and Deferred Deductibility Scheme.

Income Contingent Loans (ICLs)

- Loan covers HC cost: $L_t(e_t) = M_t(e_t)$.
- Repayment based on history of loans and earnings: $R_t(L^{t-1}, y^{t-1}, e_t, y_t)$.
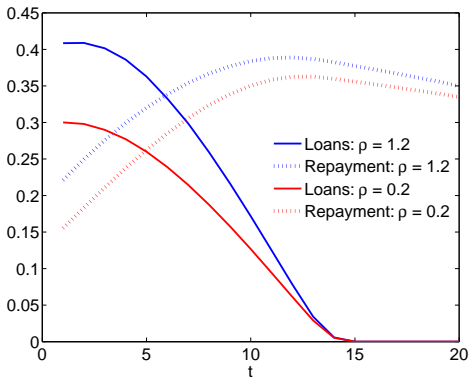
(a) Loans and repayment as % of income
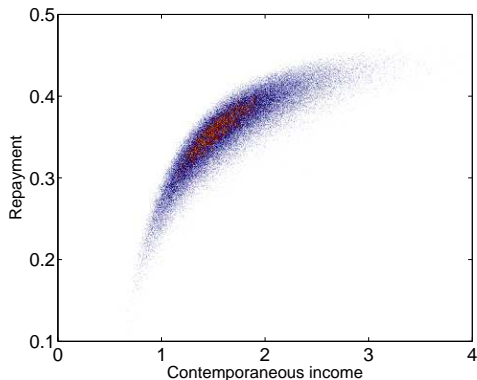
(b) Insurance role and history contingency of repayments ▸ Agent's program

Simplified versions of ICLs exist

- Proposed by J. Tobin and M. Friedman.

- Tried in Sweden, Australia, NZ, UK, US, Chile, SA, Thailand.

  - US: ICR for public sector (1994), IBR since 2007 (CRRAA).

  - Australia: HECS - automatic, collected by tax authority.

- Main differences of scheme proposed here:

  - Not only for College

  - Longer history-dependence

  - Focus on both downside and upside.

- "Yale Plan" debacle (1970s): need tax power of govt (adverse selection).

Deferred Deductibility Scheme

- Part of expense made at t deducted at $t + j$.

    - Nonlinear cost: deduct at MC effective at $t + j$, not historic MC. ▸ Nonlinear
    - Linear cost: $(1 - \beta)$% of cost. ▸ Linear
    - Plus "no arbitrage" for physical capital taxation.

- Not sufficient to make HC expenses **contemporaneously deductible**:

    - Changing nonlinear tax rates
    - Savings tax
    - Risk adjustment (varying u').

Out-of-pocket HC costs at different income levels
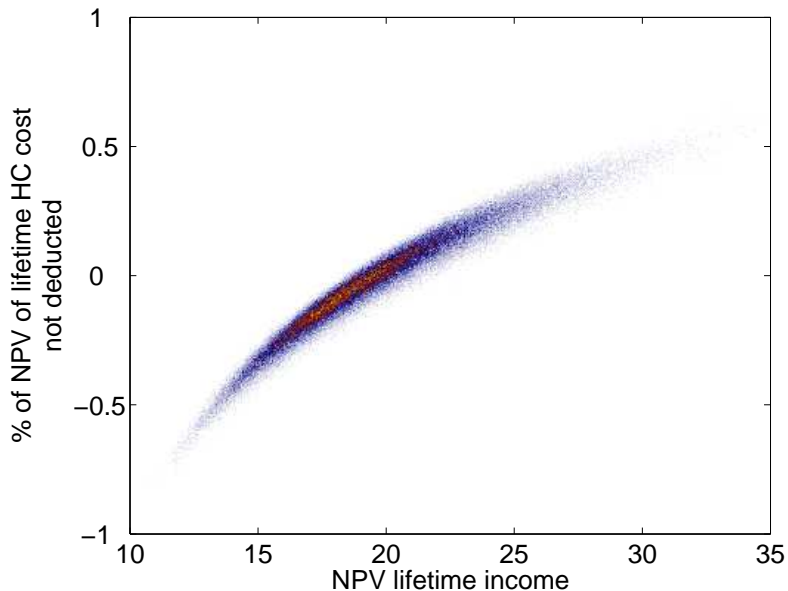
What welfare gain can simpler policies achieve?

Policy studied:
Set linear $\tau_{Lt}, \tau_{St}, \tau_{Kt}$ to cross-sectional average (across all histories θ^t at age t).

Table 2: Welfare Gains

Volatility	$\rho_{\theta s} = 0.2$		$\rho_{\theta s} = 1.2$	
	Medium	High	Medium	High
Welfare gain from second best	0.85%	1.60%	0.98%	1.76%
Welfare gain from linear				
age-dependent policies	0.79%	1.53%	0.94%	1.74%
as % of second best	93%	95.6%	95.5%	98.5%

Conclusion

- Applications: entrepreneurial taxation, bequest taxation, health care?

- Open empirical questions:
  i) Estimate $\rho_{\theta s}$ (also later in life investments).
  ii) How strongly does HC respond to taxes (weaker: to net returns)?

  Bottomline

- Crucial consideration: complementarity of HC to ability and risk + direct interaction with labor time.

- Numerically: i) Full dynamic risk-adjusted deductibility close to optimal.
  ii) Simpler age-dependent linear policies perform very well.

Appendix

Verification Procedure

Policy Functions:

$c^p\left(v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t\right)$, $y^p\left(v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t\right)$,
$s^p\left(v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t\right)$, $z^p\left(v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t\right)$,
$\omega^p\left(v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t\right)$, $\Delta^p\left(v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t\right)$,
$v^p\left(v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t\right)$

Check for all states $v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}$, report r_{t-1}, t, and θ_t that:

$$
\begin{aligned}
\theta_t \in argmax_r \, u_t &\left(c^p\left(v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, r_{t-1}, r, t\right)\right) \\
&- \phi_t\left(y^p\left(v_{t-1}, \Delta_{t-1}, s_{t-1}, z_{t-1}, \theta_{t-1}, \theta_t, t\right)\right) \\
&+ \beta \int \left(\omega^p\left(v^p, \Delta^p, s^p, z^p, r, \theta_{t+1}, t+1\right)\right)
\end{aligned}
$$

with ω^p, v^p, s^p, z^p as defined above evaluated at $\theta_t = r$.

First Period Heterogeneity

With a non-utilitarian objective, if θ_1 is interpreted as heterogeneity:

$$\kappa(\theta_1) = \int_{\theta_1}^{\bar{\theta}} \frac{1}{u_1'(c_1(\theta_s))} \left(1 - \lambda_0(\theta_s) u_1'(c_1(\theta_s))\right) f(\theta_s)$$

$$\eta(\theta_1) = 0$$

where $\lambda_0(\theta_s)$ is the multiplier (scaled by $f(\theta_s)$) on type θ_s target utility.

With linear utility: $1 = \int_{\underline{\theta}}^{\bar{\theta}} \lambda_0(\theta_s) f(\theta_s)$.

Sufficient Statistics for $\rho_{\theta s}$?

$$t_s \equiv \frac{\tau_S - \tau_L}{(1 - \tau_S)(1 - \tau_L)} = \left(\underbrace{\frac{\varepsilon'_{LS}}{\left(\varepsilon'_{LT} \frac{\varepsilon_{YE} e}{y} \right)}}_{\text{Labor supply effect}} - \underbrace{\frac{(1 - \bar{e})}{(1 - \bar{y})}}_{\rho_{\theta s}} \right) terms$$

$$\bar{y} \equiv \frac{E(u'_{ti}(c_{ti}) y_{ti})}{E\left(u'_{ti}(c_{ti}) \right) y_t}, \quad \bar{e} \equiv \frac{E(u'_{ti}(c_{ti}) e_{ti})}{E\left(u'_{ti}(c_{ti}) \right) e_t}$$

$\rho_{\theta s}$ captured by relative redistributive effect of HC versus income.
Is education less or more concentrated than income among high consumption (high ability?) people? Note that relative concentration matters.

Endogenously targeted moments

- Calibrate wage and cost function parameters (c_s and c_l) in **"baseline" economy**:
  - ► Free saving and borrowing
  - ► Linear $\tau_S = 35\%$ for first 2 periods; $\tau_L = 13\%$; $\tau_K = 25\%$.
- **Wage Premium**: The top 42.7% in the population of baseline economy, ranked by educational expenses, are "college-goers" (Autor *et al,* 1998).
  - ► Their average wage relative to bottom 38.6% must match college wage premium.
  - ► Estimates: 1.58 (Murphy and Welch, 1992), 1.66 to 1.73 (Autor *et al,* 1998), 1.80 (Heathcote *et al.*, 2010). Target: 1.7.
- **NPV education expenses/NPV lifetime income**:
  - ► For College is 13%: $\approx \$30,000$ resource cost per year (OECD, 2013) for 4 yr college (67%) or 2 yr college (33%) (NCES, 2010).
    Mean income $\$47,000$.
  - ► Add allowance for later-in-life investments $\rightarrow 19\%$.

► Numerical Analysis

Baseline allocations
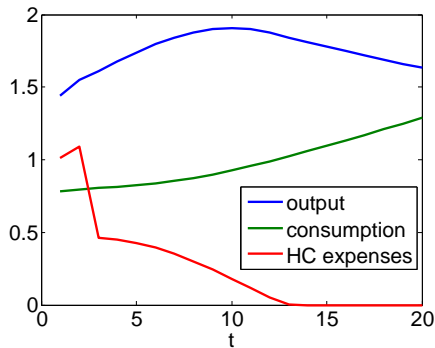
(a) $\rho_{\theta s} = 0.2$
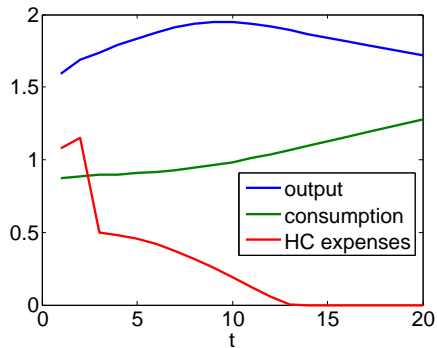
(b) $\rho_{\theta s} = 1.2$

Baseline volatilities
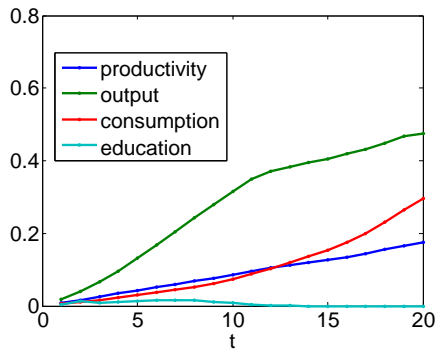
(a) $\rho_{\theta s} = 0.2$
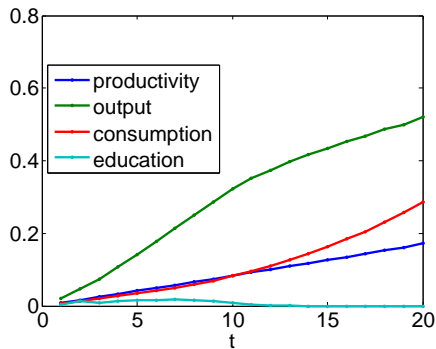
(b) $\rho_{\theta s} = 1.2$

Subsidy Regressivity

Net wedge t_{st} against θ_t
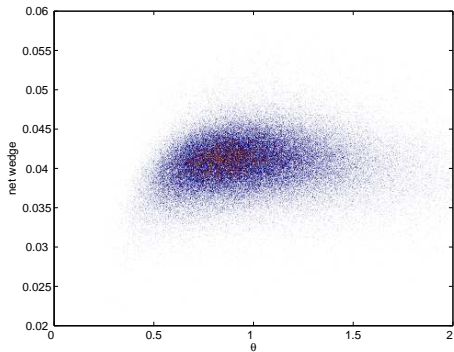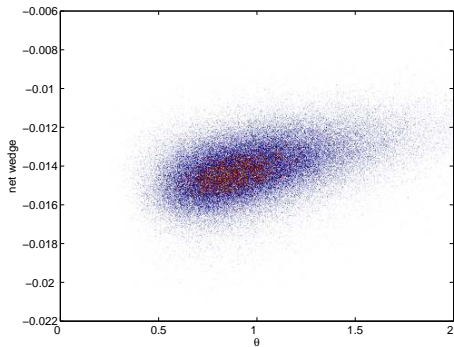
(a) $\rho_{\theta s} = 0.2$

(b) $\rho_{\theta s} = 1.2$

Tax Progressivity

Labor wedge τ_{Lt} against θ_t
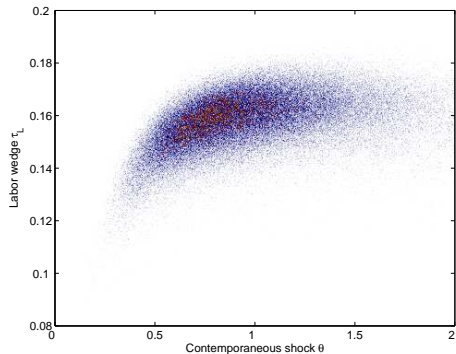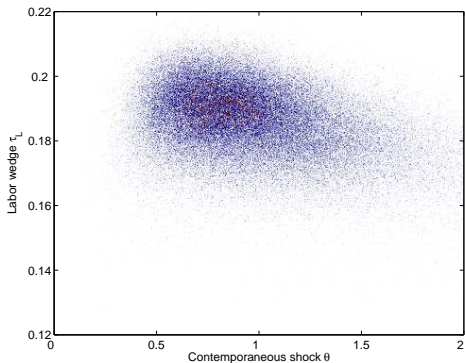
(a) $\rho_{\theta s} = 0.2$

(b) $\rho_{\theta s} = 1.2$

Tax Smoothing

τ_{Lt} against τ_{Lt-1}
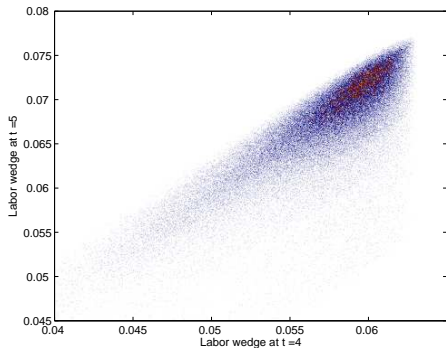
(a) at $t = 5$
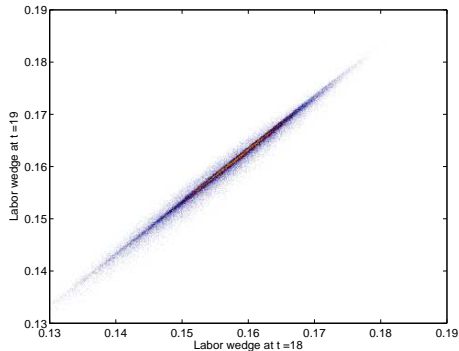
(b) at $t = 19$

Variance of Consumption Growth
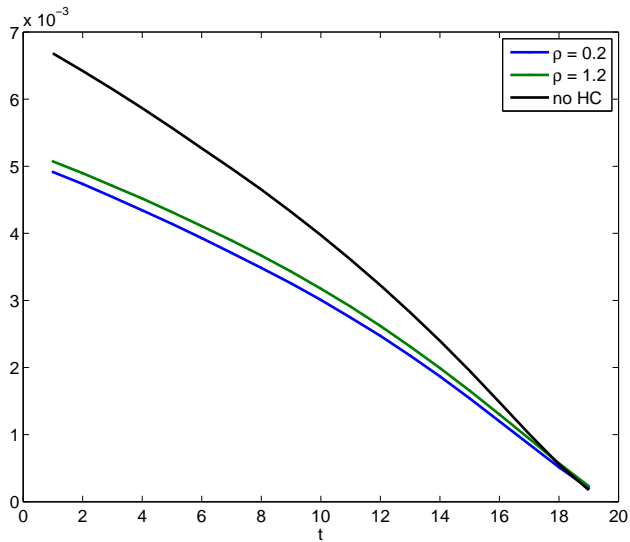
Allocations: Consumption, HC, and output
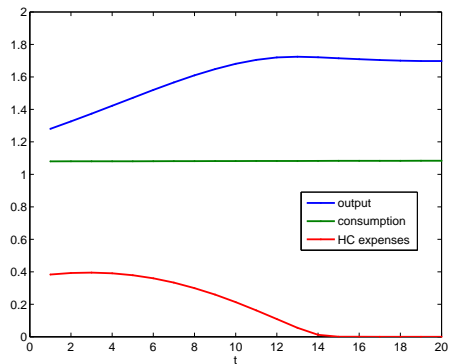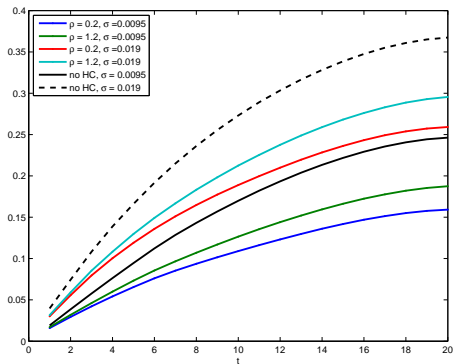

(a) $\rho_{\theta s} = 0.2$
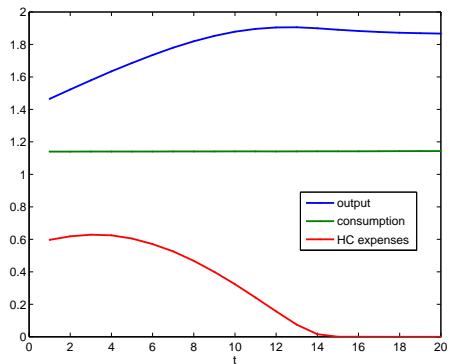
(b) $\rho_{\theta s} = 1.2$

Effects of volatility (I)
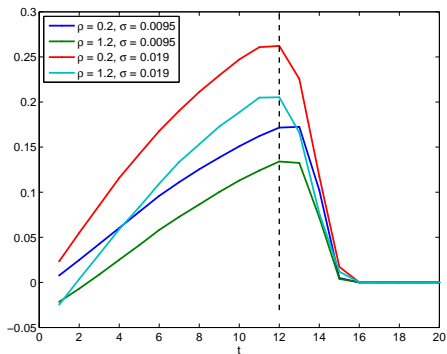
(a) Gross Wedge τ_S
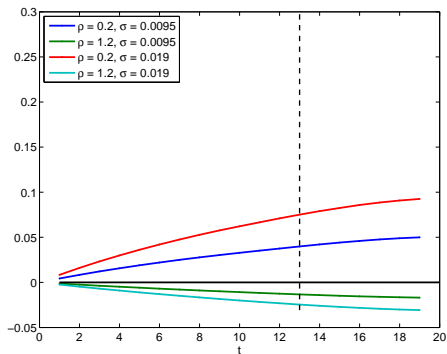
(b) Net Wedge t_s

Effects of volatility (II)

(c) Labor Wedge τ_L
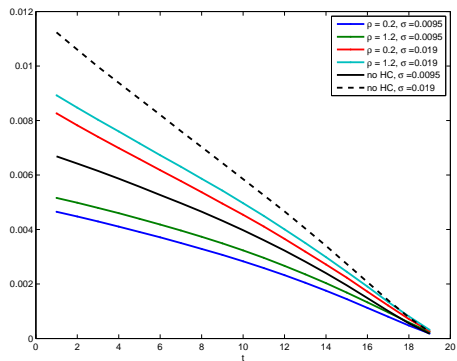
(d) Capital Wedge τ_K
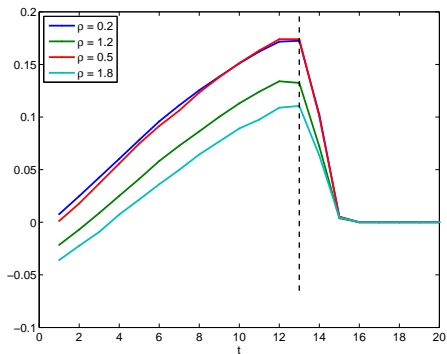
Effects of $\rho_{\theta s}$ (I)

(a) Gross Wedge τ_S
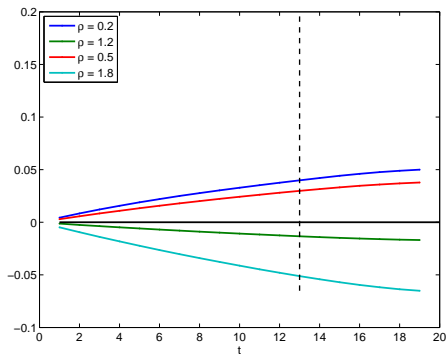
(b) Net Wedge t_s

Effects of $\rho_{\theta s}$ (II)
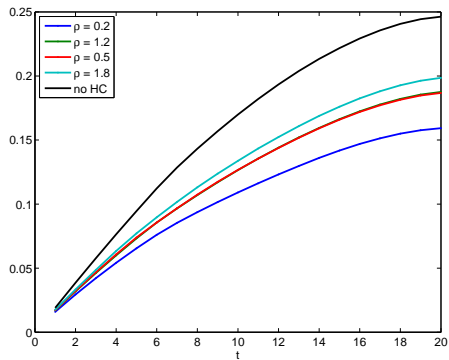
(c) Labor Wedge τ_L
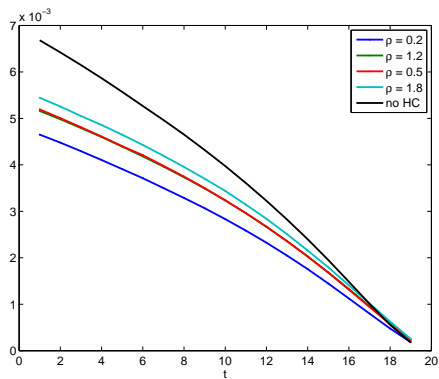
(d) Capital Wedge τ_K

Agent's program with ICLs

$$V_1\left(b_0,\theta_0\right) = \sup \sum_{t=1}^{T} \int \left[u_t\left(c_t\left(\theta^t\right)\right) - \phi_t\left(\frac{y_t\left(\theta^t\right)}{w_t\left(\theta_t, s_{t-1}\left(\theta^{t-1}\right) + e_t\left(\theta^t\right)\right)} \right) \right] P\left(\theta^t\right) d\theta^t$$

$$c_t\left(\theta^t\right) + \frac{1}{R} b_t\left(\theta^t\right) + M_t\left(e_t\left(\theta^t\right)\right) - b_{t-1}\left(\theta^{t-1}\right) - L_t\left(e_t\left(\theta^t\right)\right)$$
$$\leq y_t\left(\theta^t\right) - D_t\left(L^{t-1}\left(\theta^{t-1}\right), y^{t-1}\left(\theta^{t-1}\right), e_t\left(\theta^t\right), y_t\left(\theta^t\right)\right) - T_Y\left(y_t\left(\theta^t\right)\right) - T_K\left(b_t\right)$$
$$s_t\left(\theta^t\right) = s_{t-1}\left(\theta^{t-1}\right) + e_t\left(\theta^t\right)$$
$$s_0 \text{ given}, \, e_t\left(\theta^t\right) \geq 0, b_0 = 0, \, b_T \geq 0$$

$$L_t\left(e_t\right) = M_t\left(e_t\right) \quad \forall t, \, \forall e_t$$

$$D_t\left(L^{t-1}, y^{t-1}, e_t^*\left(\theta^{t-1}, \theta\right), y_t^*\left(\theta^{t-1}, \theta\right)\right) + T_Y\left(y_t^*\left(\theta^{t-1}, \theta\right)\right)$$
$$= y_t^*\left(\theta^{t-1}, \theta\right) - c_t^*\left(\theta^{t-1}, \theta\right)$$

for all $\left(L^{t-1}, y^{t-1}\right)$ such that $\theta^{t-1} \in \Theta^{t-1}\left(\left\{M_1^{-1}\left(L_1\right), ..., M_{t-1}^{-1}\left(L_{t-1}\right)\right\}, y^{t-1}\right) \neq \varnothing$, and all $\theta \in \Theta$, where the history of education e^{t-1} is inverted from L^{t-1}. ▶ Back to Main

Deductibility Scheme with Linear cost

$$
-\frac{\partial T_t}{\partial e_t} = \underbrace{(1-\beta)\sum_{j=1}^{T-t}\beta^{j-1}E_t\left(\frac{u'_{t+j-1}}{u'_t}\frac{\partial T_{t+j-1}}{\partial y'_{t+j-1}}\right) + \beta^{T-t}E\left(\frac{u'_T}{u'_t}\left(\frac{\partial T_T}{\partial y_T}\right)\right)}_{\text{Staggered deductions}}
$$

$$
\underbrace{-\sum_{j=1}^{T-t}\beta^j E\left(\frac{u'_{t+j}}{u'_j}\left(\frac{\partial T_{t+j}}{\partial b_{t+j-1}} - \frac{\partial T_{t+j}}{\partial s_{t+j-1}}\right)\right)}_{\text{No arbitrage with Physical capital}}
$$

▸ Back to Main

General Deductibility Scheme with nonlinear cost

$$-\frac{\partial T_t}{\partial e_t} = \sum_{j=1}^{T-t} \beta^{j-1} E_t \left(\frac{u'_{t+j-1}}{u'_t} \frac{\partial T_{t+j-1}}{\partial y_{t+j-1}} \left(M'_{t+j-1} - \frac{1}{R} M'_{t+j} \right) \right) + \beta^{T-t} E_t \left(\frac{u'_T}{u'_t} \left(\frac{\partial T_T}{\partial y_T} \right) \right)$$
$$- \sum_{j=1}^{T-t} \beta^j E_t \left(\frac{u'_{t+j}}{u'_t} \left(\left(1 - \zeta'_{M',t+j} \right) E_{t+j-1} \left(M'_{t+j} \right) \frac{\partial T_{t+j}}{\partial b_{t+j-1}} - \frac{\partial T_{t+j}}{\partial s_{t+j-1}} \right) \right)$$

- Marginal cost not constant: deduction in period $t+j$ occurs at dynamic marginal cost effective then $(M'_{t+j} - \frac{1}{R} M'_{t+j+1})$, not at "historic" marginal cost at time of the purchase M'_t.
  - purchase of Δe at time t is deducted as $(M'_{t+j} - \frac{1}{R} M'_{t+j+1})\Delta e$ from y_{t+j} at $t+j$.
- "No-arbitrage" term takes into account differential tax increases from physical capital versus human capital
  - risk adjusted:
    $$\zeta'_{M',t+1} \equiv -Cov \left(\frac{\beta u'_{t+1}}{u'_t} - 1, M'_{t+1} \right) \Big/ \left(E_t \left(\frac{\beta u'_{t+1}}{u'_t} - 1 \right) E_t \left(M'_{t+1} \right) \right).$$