Optimal income taxation of lone mothers: an empirical comparison for Germany and the UK

by

Richard Blundell[b,c], Mike Brewer[b], Peter Haan[a] and Andrew Shephard[b]

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Abstract

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a) German Institute for Economic Research, Berlin (DIW Berlin) and Free University of Berlin (FU Berlin), phaan@diw.de
b) Institute for Fiscal Studies
c) UCL

Correspondents: phaan@diw.de and mike_b@ifs.org.uk

1 Introduction

In most developed countries there exists large government financed and managed income transfer programs. At the same time, there are ongoing discussions and controversies about the size and the design of income transfer programs. This controversy can be best described by the trade off between *equity and efficiency*. Whereas income transfers increase the disposable income of the disadvantaged, and thus increases their well-being, these programs introduce distortions that might lead to substantial disincentives on the labour market, and thus to behavioural adjustment of labour supply.

Individuals can adjust their labour supply along two margins, i) the decision to participate on the labour market (extensive margin), and ii) the decision about working time (intensive margin). Although labour supply effects on the extensive margin tend to be more important (Heckman 1993) it is necessary to study the intensive margin as well when analysing the labour supply behaviour. This is in particular important for the evaluation of welfare programs such as in work credits as these reforms might provide opposite incentives for the labour market participation and the working hours.

The design of transfer programs, and the trade-off between equity and efficiency has been intensively analysed in the economic literature. The seminal theoretical contribution is Mirrlees (1971). In that framework, which focuses exclusively on the case where agents chose only how much to work (i.e., on the intensive margin), it can be shown that negative marginal tax rates can never be optimal, ruling out in-work credits.[1] Diamond (1981) extended the model of optimal income taxation by focussing exclusively on the extensive labour supply margin. In this framework, the optimality results derived within the Mirrlees framework no longer hold. Instead, Diamond shows that for some income ranges, optimal marginal taxes may be negative. Saez (2002) puts forward a model that combines the ideas in both Mirrlees and Diamond, and allows for workers to choose whether and (to a degree) how much to work; he shows that it is more likely that optimal tax rates may turn negative the larger is the extensive elasticity relative to the intensive elasticity.

[1] At least, ruling out in-work credits that look like the earned income tax credit from the US, which has a phase-in portion where the size of the transfer increases with earnings (i.e., there is a negative marginal rate). In this paper, we use the phrase “in-work credit” to mean a tax system that redistributes more to people with strictly positive earnings than it does to those who do not work.

The aim of this paper is to apply the theoretical model presented in Saez (2002) to analyse empirically the design of income taxation, and to discuss its optimality. We focus on the tax and transfer systems in Germany and the UK. More specifically, we want to assess and compare the design of the tax and transfer system for lone mothers in both countries.

We chose to focus on lone mothers for a number of reasons. First, in both countries, lone mothers are eligible for generous transfer programmes, and the interaction of transfer programmes and the income tax system can generate budget constraints with high and variable effective marginal tax rates. Second, there is a (partly emotional) debate in both countries about the extent to which lone mothers should be supported by the state, even when they do not work. Lastly, in practical terms, focusing on lone adult households allows us to avoid the substantial complexity to both models of labour supply as well as optimal tax theory that arise when dealing with household decisions of labour supply. So far the optimal tax literature has not suggested a theoretical framework accounting for the simultaneous decision of households that can be empirically analysed. By the same token, focussing of lone mothers who are in general a relatively low-skilled, low-wage group gives greater justification to focusing exclusively on labour supply responses to taxation, rather than responses involving other factors that might affect taxable earnings (Gruber and Saez, 2002). In both, Britain and Germany, lone mothers are of important size. According to the German population survey, in 2003 more than 16% of all families with the youngest child younger 18 years are households with a single parent (Statistisches Bundesamt, 2004). This implies that about 15% of all children younger 16 are raised by single parents. In Britain the share of lone parents is even higher: Roughly 25% of families with children are lone parents (Office for National Statistics 2005). In both countries the majority of lone parents are mothers, only about 10% of lone parents household have only a father.

We analyse the optimality of income taxation in a comparative setting for Germany and the UK. A comparison between these countries is interesting for several reasons. Most important, the transfer and benefit systems for families with children are quite different in both countries. In Britain, there is a clear dichotomy between out-of-work support (provided through traditional means-tested benefits) and in-work support, provided through refundable tax credits, and the importance of the latter has grown substantially over the past decade. Germany, however, relies on the more traditional means-tested social assistance, with very high positive marginal withdrawal rates (however, as we show later, the budget constraints facing lone mothers in the two countries are in practice rather similar, even if different ideas

underpin the two designs). Furthermore, there exist enormous differences in the institutional structure, the distribution of earnings and the structure of labour markets.

We address two questions: first, following Bourgignon and Spadaro (2005), we want to assess the welfare weights that a social planner would assign to different groups (defined by their income) given that the tax and transfer system that we observe in each country is optimal, and given the labour supply elasticities that we estimate. Second, we want to derive the optimal tax schedules in each country given various assumed social welfare functions.

Based on the theoretical literature of optimal taxation, there exist several empirical studies employing microsimulation models that analyse and compare welfare and tax systems of different countries.[2] A recent application about the design of the tax and benefit systems is most closely related to our research on the optimality of tax in transfer systems: Immervoll et al. (2006) apply a basic framework of optimal taxation to the analysis of two different transfer programmes for 14 Western European countries: the first reform is traditional means tested welfare that covers all; the second reform proposal is an in-work tax credit that focuses exclusively on the working poor. The authors use the microsimulation model EUROMOD that mimics the current welfare and tax system of 14 European countries, and calibrate labour supply elasticities on the intensive and extensive margin. Their results are strongly in favour of the in-work tax credit: they conclude that in particular in countries with large current welfare programmes, such as Germany, a purely means-tested benefit programme is not desirable. Eissa, Kleven, Kreiner, (2005): evaluate the welfare effects of four tax reform acts on single mothers in the United States over the last 20 years. They find that the tax reforms reduced the tax burden for this and thereby causing welfare gains. The findings of this study underline the importance to account for both the intensive and the extensive margin when analysing the welfare effects of tax reforms as the effect are mainly driven by the effects on the extensive margin. Yet, as in Immervoll et al (2006)., this study does not allow for heterogeneity in the behaviour of individuals but assumes labour supply effects to be constant.

Thus, the key advance in this paper, in contrast to the previous literature on optimality of the tax and benefit system, is that we combine the theory of optimal taxation with both country-specific tax and benefit micro-simulation models and (country-specific) structural models of labour supply. This enables us to recognise fully the complexity (and heterogeneity) in the tax

[2] There exists numerous empirical studies on welfare effects of tax reforms (e.g. Aarberge and Columbino, 2005). However these studies differ from the models closely linked to the optimal income tax theory as they are not derived from an optimal tax formula but rather from structural econometric models of labour supply behaviour.

and transfer system within each country, and it also allows us to estimate, rather than calibrate, the key behavioural inputs (labour supply elasticities) in the expression for optimal tax rates; we are also able to reflect heterogeneity in household behaviour.

2 The theoretical model

We base our analysis on the framework outlined in Saez (2002), slightly modified for our research questions.

Generally, the problem of optimal income taxation can be described as follows: a social planner (the government) maximises a social welfare function given its budget constraint. The social welfare function is a transformed function of individual utilities which themselves depend on net household income (consumption) and leisure. The functional form of the social welfare function is based on normative assumptions ranging from a Rawlsian to a Utilitarian welfare function. In a Rawlsian society, the social planner cares only about the worst off individual; in an Utilitarian world, the social planner weights the utility of all individuals equally.

In the framework of optimal taxation, the margin along which individuals can adjust their behaviour is their labour supply. This leads to the trade-off between equity and efficiency. Whereas transfer programs (or negative tax payments) can increase the disposable income of the disadvantaged, and thus increase their wellbeing, financing these programs with positive income tax rates introduces disincentives to work, and, in general, will lead to a reduction in labour supply. Therefore, when analysing the optimality of the tax and transfer system, it is vital to have a detailed understanding of how individuals' labour supply responds to tax changes.

Saez (2002) sets up an optimal tax problem where there are *I+1* groups in the labour market: *I* groups of individuals who do work, plus one group consisting of those who do not work. In Individuals choose whether or not to participate (the extensive margin), and which group to choose (the intensive margin). In this framework, optimal taxation has the following form:

$$\frac{T_i - T_{i-1}}{C_i - C_{i-1}} = \frac{1}{\mu_i h_i} \sum_{j \geq i}^{I} h_j \left[1 - g_j - \eta_j \frac{T_j - T_0}{C_j - C_0} \right].$$

In this expression, T_i is net tax paid by group i and C_i is the net household income of this group, so the term on the left-hand side is the extra tax paid when moving from group i-1 to i divided by the gain in net income. Non-workers receive benefits $-T_0$, by definition identical to C_0. The gross earnings of group i, equal to $C_i + T_i$, are exogenously fixed. h_i measures the share of group i in the population. The social welfare function is summarised by g_i, the weight the government assigns to group i.

The intensive elasticity, μ_i, is defined as:

$$\mu_i = \frac{C_i - C_{i-1}}{h_i} \frac{dh_i}{d(C_i - C_{i-1})}.$$

This mobility elasticity captures the percentage increase in supply of group i when C_i-C_{i-1} is increased by 1%, and is defined under the assumption that individuals are restricted to adjust their labour supply to the neighbouring choice.

Finally, η_i is a measure of the extensive elasticity, and is defined as the percentage of individuals in group i who stop working when the difference between the net household income out of work and at earnings point i is reduced by 1%:

$$\eta_i = \frac{C_i - C_0}{h_i} \frac{dh_i}{d(C_i - C_0)}.$$ [3]

The main implication of the optimal tax rule above is that the optimal tax system depends heavily on whether labour supply responses are concentrated at the intensive or extensive margin. When the extensive elasticity is assumed to be zero, Saez' model gives results similar to Mirrlees', where negative marginal tax rates are never optimal. However, the greater is the extensive elasticity compared to the intensive elasticity, the more likely it is that the optimal schedule will feature relative smaller guaranteed income for non-workers, and negative marginal taxes at low levels of earnings.

We apply the model outlined above to a comparative analysis of optimal income taxation. The focus of this analysis is on the tax and transfer system of lone mothers in Germany and the UK.[4] As Saez (2002) does in the empirical example, we define the groups by gross earnings.

[3] As we show empirically in the following section, this is different from the conventional extensive elasticity, or elasticity of labour force participation, which is (usually) defined as the proportional increase in workers when net incomes rise by 1%.

[4] At first glance it might seem problematic to derive an optimal tax schedule for a sub population. However, the government can distinguish lone mothers and explicitly targets transfers towards this group: income tax

A first-best solution of income tax would be based on measures of skill or productivity captured by the hourly wage, but in practice this cannot be observed, and so optimal tax models assume that the income tax has to be a function solely of gross earnings. We condition the optimal income schedule on this information as we aim to mimic the taxation decision the government faces.

3 Lone mothers in Germany and UK: The tax and transfer system and labour market behaviour

Over all, the female employment rate[5] is with 66.3% higher in the UK than in Germany, where 58.8% of the relevant population is employed (OECD, 2005). However, as Haan and Myck (2006) shows, the picture is different for lone mothers. This is partly due to compositional differences - lone mothers in Germany have older children – but is also due to other factors: conditional on the age of their children, lone mothers in Germany work more than in the UK. Table 1 gives more detail, based on the samples used in the subsequent analysis.[6]

Table 1: Employment rates of lone mothers. UK and Germany, 2002/03.

	Share		Employment rate in %	
	Germany	**UK**	**Germany**	**UK**
with children <17			71.37	52.53
with children: youngest 0-3	9.06	27.28	27.96	28.82
with children: youngest 4-6	19.18	19.35	53.81	48.63
with children: youngest 7-16	71.76	52.79	81.55	65.46

Note: In Germany, roughly 16% of families with children are lone parents households, in Britain about 25%.
Source: FRS 2002/3 and GSOEP 2003. See Appendix for precise details of sample.

In both samples, only a small minority of lone mothers with young children (defined as "any children under 4") work: 28% in the UK and 29% in Germany. But employment rates are markedly higher in Germany than in the UK once children start school: 65.46% of lone

legislation in Germany and the UK discriminates between households with and without children, and by marital status. In other words, in this analysis we derive a tax schedule for single adults with children, taking taxation of the rest of the population as exogenous and constant.

[5] Employment rates are defined as the share of employed and self-employed people over the whole population in this age group.

[6] The population for our analysis is lone mothers aged between 17 and 60 with at least one dependent child under 17 years, but excluding self-employed, and those in full-time education. Additionally, the UK sample excludes those receiving disability benefits, and the German sample excludes those declaring themselves to be retired. More information about the data employed and the sample is provided in the Appendix.

mothers with no children under 7 in the UK work, compared with 81.56% in Germany.[7] Average hours worked are higher in Germany except for lone mothers with young children, who work slightly more hours on average in the UK.

However, an important compositional difference is that lone mothers in the UK tend to have younger children than in Germany. In our samples, three times as many lone mothers in the UK have children under 4 than in Germany (28 % compared with 9 %), and almost a half of lone parents in the UK have a child under 7, compared to just under a third in Germany. This considerable compositional difference gives an additional reason why the mean employment rate, and mean hours worked, of lone mothers in the UK is considerably lower than in Germany (52% compared with 71% in our sample).

Regarding the working hours, we observe a similar picture when comparing the two countries (Table 2). Over all groups average working hours are higher in Germany. Yet, decomposed by age of the child we can demonstrate that higher working hours of lone mother are driven by those mothers with children older than 3 years. Lone mothers with children younger than three work, on average, slightly less in Germany than in the UK.

Table 2. Working hours of lone mothers. UK and Germany, 2002/03.

	Weekly working hours	
	Germany	**UK**
With children <17	21.10	15.24
Youngest child 0-3	5.88	7.06
Youngest child 4-6	14.54	12.98
Youngest child 7-16	24.77	20.39

Source: FRS 2002/3 and GSOEP 2003. See Appendix for precise details of sample.

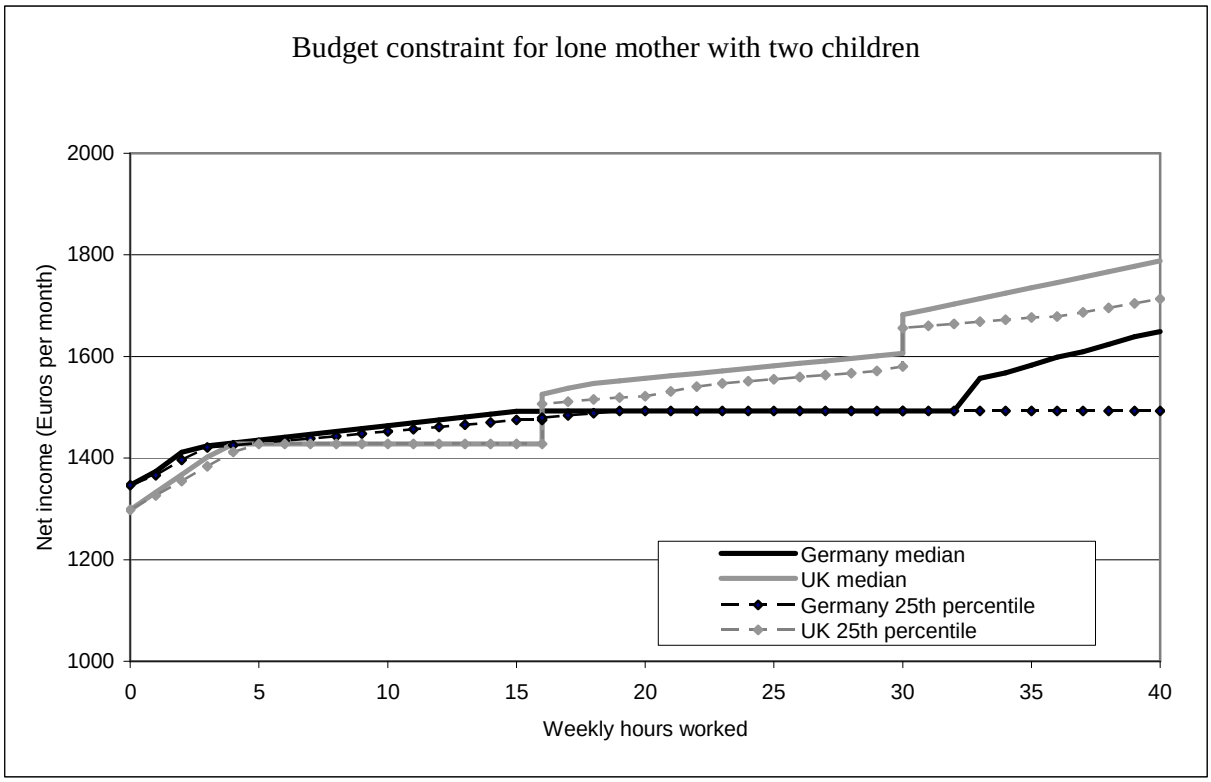
A comparative analysis of the tax and benefit design and its optimality in Germany and the UK is insightful as there exist substantial differences in the transfer and benefit systems. In the UK, as well as means tested out of work benefits, a large amount of transfers are made conditional on working through in-work credits (WFTC during the period covered by our data). In contrast, the German tax and transfer system almost exclusively relies on more traditional means-tested social assistance, with very high withdrawal rates. Thus, the German transfer system is mainly targeted towards the non working poor. This difference is in particular true for lone mothers, as in both countries several programs are in particularly

[7] Children in the UK start full-time education no later than the term after their 5th birthday; in Germany, school starts in general after the 6th birthday.

targeted at this group. In the UK, the amount of financial support through in work credits is dependent on the number of dependent children and as this transfer is withdrawn based on household income, it affects single and couple household differently. The latter is true as well for the means tested income support programs in the UK. In Germany, both income taxation and transfer programs target lone parents differently to couple households with children. There exists an additional tax exemption for taxable income that is conditioned on being lone parent. For single households with children means tested benefits are more generous due to an extra transfer.

The effects of the tax and transfer system on the net household income for lone mothers is best described by looking at stylised budget lines for these families.

Figure 1: Budget constraints for lone mother with two children in UK and Germany, 2002



Notes: For each country we consider a lone mother working at 25th percentile, and at the median hourly wage, renting at the cost of median rent. 25th percentile wage for lone mothers in Britain is €6.45 and in Germany €8.80 median wage is in Britain €7.96. and in Germany €10.90
Source: authors' calculations using TAXBEN and STSM.

Figure 1 presents comparisons of budget constraints for a lone mother with two children for the fiscal year 2002. The budget lines are drawn under the assumption that the woman is earning the 25th percentile or the median female gross hourly wage. For the UK, they do not include housing benefit or council tax benefit. At the lowest levels of earnings, i.e. in scenarios where the family qualify for the basic means tested support, disposable incomes of

families in Germany and the UK are very similar. Differences become apparent only at hours levels beyond about 16 per week. As mentioned above the transfer system in Germany is mainly based on means tested benefits that are withdrawn with almost 100%. Therefore, the budget line for a low wage lone mother with two children is hardly affected by her working hours. Still at 40 weekly working hours net household she receives full means tested benefits and her earnings are fully withdrawn. The budget constraint of a comparable lone mother earning median wage is similar. Her net household income starts to increase after about 32 working hours when the means tested benefits run out. In contrast, the budget line of a low wage mother in the UK is affected by her labour supply. This is the result of generous in-work support which these families are eligible for in the form of the WFTC. This is true regardless of earning the 25 percentile or the median wage. Interesting to note is that despite the wages are markedly higher in Germany, the net household income of lone mothers is higher after working more than 16 hours. This is the effect of the in work credits. As we will show in the next section, taking all programmes the UK government is more generous towards lone mothers as the German. Over most of the earnings distribution, lone mothers in the UK receive higher transfers than they pay in income and payroll taxes. On average the British government transfer about 200 Euros per week to a lone mother, in Germany the average transfers are with 85 Euros per week markedly lower.

For the empirical analysis we employ detailed country specific microsimulation models, TAXBEN for the UK and STSM for Germany that allow us to derive the amount of tax payments and transfers and the resulting disposable net household income for all lone mothers.[8] This allows us to derive the net income distribution for the lone mothers under the current tax legislation and hypothetical reform scenarios which is necessary to derive the optimal tax schedule for this group.

4 Estimating the labour supply elasticities

One key innovation in this paper is that, rather than calibrating the labour supply elasticities of various groups, we make use of labour supply elasticities derived from comparable, country-specific, structural models of labour supply. Full details of the two models used are given in Bargain et al. (2006) and Haan (2006) for Germany, and Brewer et al. (2005, 2006) for the UK, but the salient features are that both are discrete choice models of labour supply,

[8] These micro simulation have been extensively used in previous research and are best described in IFS for TAXBEN (Giles and McCrae, 1995), and for STSM in Steiner, et al. (2005).

where each individual *i* is assumed to choose between not working and a finite number (*J*) of positive hours choices, with each choice *j*=0,...*J* corresponding to a level of disposable income C_{ij} (choice *j*=0 corresponds to not working). The attraction of this approach is that it can easily allow for non-linear and non-convex budget sets (see Blundell and MaCurdy 1999). Both models specify the direct utility function as a quadratic in net income and hours worked. The utility is allowed to vary with observable and unobservable effects. A detailed specification for both countries can be found in the Appendix.

Labour Supply Elasticities on the Extensive and Intensive Margin

As is the case in the empirical example in Saez (2002), we see the optimal tax model in terms of groups defined with respect to gross earnings. However, the two discrete choice labour supply models are defined with respect to (weekly) hours worked. The way that we use the structural labour supply models to calculate the intensive and extensive elasticities required by the Saez formula is detailed described in the Appendix, and the resulting elasticities shown in Table 3 below. For the UK, elasticities are estimated from a sample of lone mothers in 2002/3, for Germany, from 2002-2004.[9] Note that the definition of the extensive elasticities given in section 2 differs from that of the conventional extensive elasticity (sometimes called the participation elasticity, or the elasticity of labour force participation), which measures the proportional increase in labour force participation in response to a 1% increase in net income in work: for comparison with other studies, therefore, we show values of this conventional elasticity of labour force participation.[10]

The Table shows that the estimated elasticities differ between the countries, being generally higher in the UK. The intensive elasticities decline as weekly hours increase, but the extensive elasticities increase. That the overall labour market behaviour of lone mothers in the UK differs from those in Germany is confirmed by our estimates of the conventional elasticity of labour force participation, which stands at 1.36 for lone mothers in the UK, compared with 0.4 in Germany.

[9] Given this information we estimate the elasticities for the fiscal years 2001 to 2003. The tax and benefit system in Germany did hardly change during that time, so the panel dimension provides more information and variation for the analysis.

[10] In practice, we estimate this by increasing net incomes at all positive hours choices.

Table 3: Employment rates of lone mothers. UK and Germany, 2002/03.

	Labour Supply Elasticities			
	UK		Germany	
	extensive	intensive	extensive	intensive
Hours choice 1	0.22	0.22	0.12	0.12
Hours choice 2	0.44	0.04	0.13	0.01
Hours choice 3	0.49	0.02	0.16	0.03
Hours choice 4	0.65	0.03	0.24	0.02
Hours choice 5	0.66	0.02	0.28	0.04
Elasticity of LFP	1.36		0.40	

Notes: : For Germany, the intervals for working hours were 0-5, 6-14, 15-21, 22-27, 28-3, 34+, with corresponding hours points 0,10,20,25,30,38. For the UK, the intervals are 0, 1-15, 16-22, 23-29, 30-36, 37+, with corresponding hours points 0,10,19,26,33,40 (the median of each band).

5 Numerical Simulation

For the numerical simulation of the optimal tax schedule we define I+1 discrete groups along the gross earnings distribution, I groups for positive earnings and in addition the group of non workers which have zero gross earnings. For comparative reasons we define the same income classes for Germany and the UK.[11]

Given the derived elasticities and the defined discrete earning points we can apply Saez framework of optimal taxation to analyse optimal transfer and tax schedule for lone mothers in Germany and the UK. Therefore, we need to solve the optimal tax schedule defined above. The optimal schedule is derived subject to two constraints.

$$\sum_{0}^{I} h_i T_i = H;$$

$$\sum_{0}^{I} h_i g_i = 1.$$

The first is the government's budget constraint, that is the weighted sum of net taxes has to sum up to the budget constraint. As stressed above, for lone parents the budget constraint is

[11] The income classes are defined to be deciles of a hypothetical earnings distribution. The hypothetical earnings distribution was constructed by assuming that each lone mother in our German sample has a 20% probability of working at the 5 positive values of hours a week, and then estimating the resulting distribution of weekly earnings.

negative in both countries, because lone parents receive a net transfer financed by the rest of the society. The second constraint is a normalisation necessary for identification.

We make use of the duality of optimal income taxation framework and analyse two questions. First, we follow Bourgignon and Spadaro (2005) and derive the welfare weights assigned to the different groups along the income distribution that make the actual tax and transfer system in both countries optimal. Second, assuming a specific welfare function we design the optimal tax and transfer system for lone mothers in Germany and the UK.

6.1 Optimal Weights

In an application for France, Bourgignon and Spadaro (2005) invert the Mirrlees model and find that, if intensive elasticities are low (compared to those we have estimated for Germany and the UK), then the French tax and transfer schedule is optimal under a Paretian government. However, when they assume higher elasticities, they show that the actual French tax and transfer system is only optimal if the authority imputes negative social welfare weights to individuals at the upper end of the income distribution: speaking very loosely, tax rates at the upper end of the income distribution are distorting "too much" compared to the revenue raised, and so can only be optimal if the social planner actively wishes to have penal rates of tax at the top of the income distribution. We develop this sort of analysis by using estimated, not calibrated, labour supply elasticities along the extensive and the intensive margin.

As mentioned above, we have defined groups over the gross earnings distribution.[12] Table 4 shows, for each group, mean net tax, mean net income, mean elasticities, and the actual share of the population located in each band.[13] First, the share of lone mothers at the discrete earnings points differs markedly between Germany and the UK. As shown in the previous section, almost half of the lone mothers in the UK are located at zero gross earnings. The distribution over positive earnings is fairly even, with about 5 % at each point. However, in Germany, about one third of lone mothers have zero earnings point, few women are at the low- to middle- earnings points, and the majority of working lone mothers are at the top three points. The higher labour market participation, higher hours of work given labour market

[12] With a common set of cut-off points. The exchange rate was 1.466.

[13] As the Appendix sets out, the mean net tax, disposable income and elasticities shown in Table 4 are over the whole sample, not just those lone mothers who are observed to have gross earnings in each band. This is because we are able to estimate elasticities for each individual at each discrete band, and we can calculate net taxes for every individual for any level of gross earnings.

participation, and higher hourly wages together all mean that average gross earnings are considerably higher in Germany than in the UK.

Second, net transfers are higher (net taxes are lower) for lone mothers in the UK than in Germany at every earnings point, showing that the UK has a more generous transfer system for lone mothers. This fact, combined with the fact that lone mothers in Germany have higher gross earnings, means that net transfers received by lone mothers in the UK are around 200 Euro per week, but only 85 euro a week in Germany.

Lastly, there are also important differences in the estimated labour supply elasticities. Most importantly, the extensive elasticities in the UK are high, relative both to the intensive elasticity in the UK, and the extensive elasticities in Germany.

Table 4: Optimal weights for the taxation of lone mothers. UK versus Germany

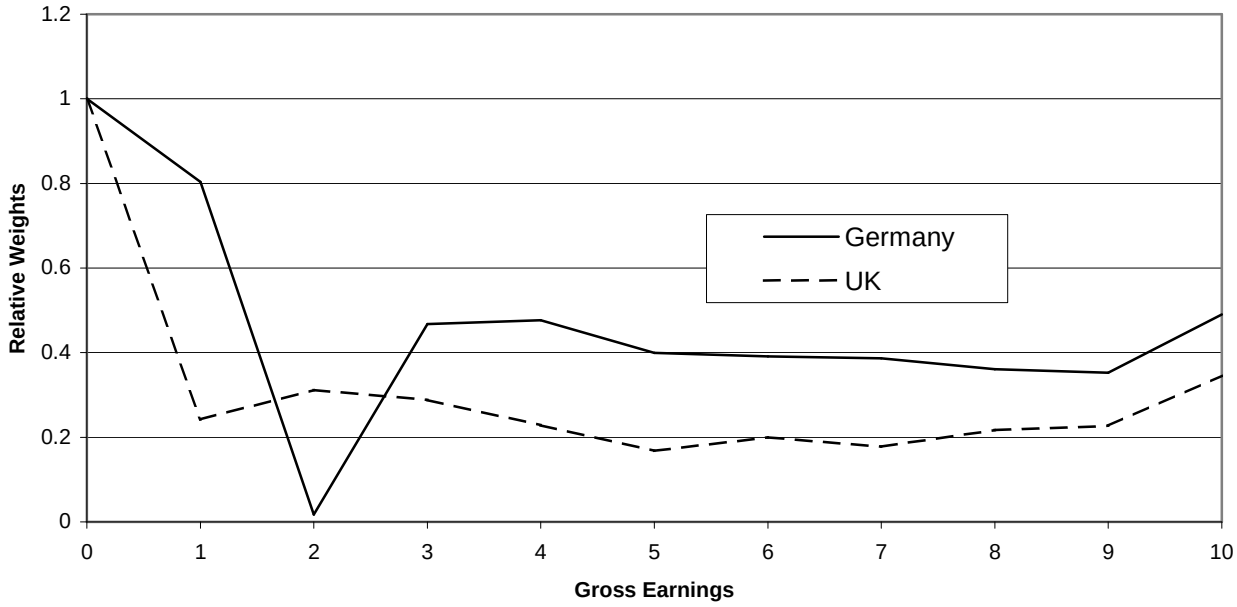
	Gross Earnings in €	Net Income in €	Net Tax in €	Marginal Tax Rate	Share	Intensive Elasticity	Extensive Elasticity	Opt. Weights	Relative Opt. Weights
					United Kingdom				
0	0.00	274.78	-274.78		0.48	0.00	0.00	1.64	1.00
1	76.25	305.75	-229.49	0.59	0.05	0.20	0.26	0.40	0.24
2	130.81	335.38	-204.58	0.46	0.07	0.06	0.40	0.51	0.31
3	173.26	359.64	-186.38	0.43	0.06	0.03	0.50	0.47	0.29
4	210.55	377.80	-167.25	0.51	0.05	0.02	0.61	0.37	0.23
5	245.79	392.23	-146.44	0.59	0.05	0.02	0.66	0.27	0.17
6	281.73	409.01	-127.28	0.53	0.04	0.03	0.62	0.33	0.20
7	320.46	425.26	-104.80	0.58	0.04	0.02	0.63	0.29	0.18
8	371.33	448.56	-77.23	0.54	0.05	0.03	0.58	0.36	0.22
9	446.10	477.30	-31.20	0.62	0.05	0.03	0.52	0.37	0.23
10	642.02	583.40	58.62	0.46	0.05	0.05	0.36	0.57	0.35
					Germany				
0	0.00	244.54	-244.54		0.29	0.00	0.00	1.69	1.00
1	86.00	294.98	-208.98	0.41	0.05	0.12	0.12	1.36	0.80
2	129.84	299.09	-169.25	0.91	0.03	0.08	0.14	0.03	0.02
3	173.68	320.02	-146.34	0.52	0.02	0.05	0.13	0.79	0.47
4	211.04	336.52	-125.48	0.56	0.04	0.01	0.20	0.81	0.48
5	246.44	343.98	-97.53	0.79	0.05	0.01	0.20	0.67	0.40
6	282.22	358.27	-76.05	0.60	0.07	0.01	0.24	0.66	0.39
7	321.93	380.23	-58.31	0.45	0.06	0.04	0.31	0.65	0.39
8	373.03	391.70	-18.67	0.78	0.08	0.03	0.22	0.61	0.36
9	447.39	430.04	17.35	0.48	0.11	0.04	0.29	0.60	0.35
10	659.19	546.76	112.44	0.45	0.20	0.03	0.13	0.83	0.49

Notes: Cut off points for the positive earnings points (in €): 107, 153, 193, 228, 264, 300, 344, 405,and 502. All income ant tax information are the mean average values per week. Marginal tax rate is calculated as change in net tax over change in gross earnings between adjacent groups.

Source: SOEP and FRS.

The weights under which the UK and German tax and transfer system for lone mothers are optimal, given our estimated labour supply elasticities, are presented in the last two columns of table 4 and in figure 2.

Figure 2: Optimal weights by gross earning groups: UK and Germany



We find that both countries' tax and transfer systems are optimal only if the government has strong concern for redistributing to non-workers: the weights for non-working women are relatively high, and those for working women are low, and decline by little as earnings rise.

There is one striking feature of the weights calculated for Germany: a sharp drop in the weight for the third group. This drop coincides with a part of the budget constraint where there are very high marginal tax rates (due to a high withdrawal rate of means-tested benefits). With such high marginal tax rates, it makes little sense for lone mothers to choose to work if their earnings would fall in this band, and the only way that the optimal tax model can rationalise this is if the government actively wants to prevent lone mothers from choosing this point, and therefore the social weight is very low. When we characterise the earnings distribution with fewer points, this result disappears, because the portion of earnings over which the very high marginal rates apply is relatively small (see Appendix).

To anchor the social welfare weights, Saez (2002) requires that the sum of weights, weighted by the share of the population that choose each band of earnings, is equal to one. This scaling,

though, makes it difficult to compare the weights estimates for two countries with such different patterns of work. In other words, the unweighted mean welfare weight in the UK will be lower than in Germany because more lone mothers in the UK do not work. To provide a better cross-country comparison, we show the derived optimal weights expressed relative to the weight given to the non-workers This reveals that in Germany, the government seems to assign higher relative welfare weights to working lone mothers than the government in the UK: Compared to the weight for non-workers, the welfare weight for working lone mothers is about 0.4 in Germany, but only 0.2 in the UK, and from this we conclude that the government in the UK has stronger preferences for redistribution to the non-workers than in Germany.

It is worth considering how this result arrives: it is driven by the relatively high extensive elasticities in the UK. A shift in the tax burden from the working poor to the non-workers (i.e. a reduction in net taxes for the working poor, and an increase in net taxes for non-workers) in the UK would induce relatively large numbers of lone parents to work because extensive elasticities are high, but not have a large negative impact on the labour supply of those already in work because intensive elasticities are low.[14] The only way that the expression for optimal taxes can rationalise the UK government choosing not to do this is by assigning a much higher weight to the incomes of the non-workers than the incomes of the working poor. In Germany, extensive elasticities are relatively lower and therefore a tax schedule with an earned income tax credit is less favourable than in the UK.

6.2 The Optimal Tax Schedule

As discussed in the previous section, neither in the UK nor in Germany the tax and transfer system has negative marginal tax rates. However, negative marginal tax rates can become optimal when extensive elasticities are relatively important compared with intensive elasticities. It is therefore of interest to find out under what social welfare functions would increased transfers to the working poor become optimal. Recall that rationalising the current

[14] Although the current British tax system conditions some transfers on working (16 or more hours a week), the transfer system on average (ie across all lone mothers) does not generally give larger transfers to the working poor than to non-workers (ie, marginal tax rates are generally non-negative). In the 2002/3 transfer system, low-wage part-time workers could receive higher net transfers in work than if they did not work, but only if they had two or more children, and – crucially – only if they would not receive housing benefit or council tax benefit if they did not work. In practice, the vast majority of non-working lone parents receive at least one of these.

transfer system in both countries requires the government to have relatively strong desires to redistribute to non-working lone mothers.

We therefore derive the optimal tax schedule across the gross earnings points under a class of social welfare weights, g_i, that decrease with gross earnings as follows:

$$g_i = \frac{1}{p \exp(\tilde{y}_i)^v - 0.25},$$

where y~ is the gross earnings at point *i* relative to the gross earnings at the highest earnings point. We follow Saez (2002) and interpret *p* as the marginal value of public expenditure. The redistributive taste of the government is expressed with *v*: the higher *v,* the higher is the redistributive taste, and we provide three scenarios with varying taste for redistribution: a scenario with low redistributive taste, v=0.5, medium taste v=1, and high redistributive taste v=1.5. As in Table 6.1, we present the weights in absolute and in relative (i.e., scaled to the weight given to the non-workers) to provide a better country comparison.

Table 5: Optimal tax rates for lone mothers. UK versus Germany

Gross Earnings	Net Tax	Opt. Weights	Relative Weight	Absolute Weights	Relative Weight	Optimal Net Tax	Absolute Weights	Relative Weight	Optimal Net Tax	Absolute Weights	Relative Weight	Optimal Net Tax
						Britain						
				v=0.5			**v=1**			**v=1.5**		
0.00	-274.78	1.64	1.00	1,33	1.00	-148,13	1.33	1.00	-245.63	1.33	1.00	-262.97
76.25	-229.49	0.40	0.24	1,28	0.92	-287,73	1.14	0.86	-268.34	1.06	0.79	-262.25
130.81	-204.58	0.51	0.31	1,25	0.87	-301,50	1.02	0.77	-252.81	0.90	0.68	-239.61
173.26	-186.38	0.47	0.29	1,22	0.84	-282,49	0.94	0.71	-230.78	0.80	0.60	-216.33
210.55	-167.25	0.37	0.23	1,20	0.81	-261,36	0.88	0.66	-210.13	0.72	0.54	-195.92
245.79	-146.44	0.27	0.17	1,18	0.78	-246,94	0.82	0.62	-189.72	0.66	0.49	-174.76
281.73	-127.28	0.33	0.20	1,15	0.75	-240,80	0.77	0.58	-165.27	0.59	0.45	-148.86
320.46	-104.80	0.29	0.18	1,13	0.73	-232,18	0.72	0.54	-136.76	0.54	0.40	-119.23
371.33	-77.23	0.36	0.22	1,10	0.69	-222,41	0.65	0.49	-97.83	0.47	0.35	-78.76
446.10	-31.20	0.37	0.23	1,06	0.64	-198,49	0.57	0.43	-37.30	0.39	0.29	-16.47
642.02	58.62	0.57	0.35	0,97	0.54	-133,44	0.41	0.30	109.26	0.24	0.18	135.44
						Germany						
				v=0.5			**v=1**			**v=1.5**		
0.00	-244.54	1.69	1.00	1.33	1.00	-175.84	1.33	1.00	-266.60	1.33	1.00	-290.81
86.00	-208.98	1.36	0.80	1.22	0.92	-310.51	1.12	0.84	-278.11	1.04	0.78	-268.37
129.84	-169.25	0.03	0.02	1.17	0.88	-284.78	1.03	0.78	-250.11	0.91	0.69	-238.23
173.68	-146.34	0.79	0.47	1.12	0.84	-246.76	0.95	0.71	-212.73	0.81	0.61	-201.12
211.04	-125.48	0.81	0.48	1.08	0.81	-210.74	0.89	0.67	-177.16	0.73	0.55	-166.08
246.44	-97.53	0.67	0.40	1.05	0.78	-177.56	0.83	0.62	-144.83	0.67	0.50	-134.76
282.22	-76.05	0.66	0.39	1.01	0.76	-144.13	0.78	0.58	-112.35	0.61	0.45	-103.57
321.93	-58.31	0.65	0.39	0.97	0.73	-109.81	0.72	0.54	-78.95	0.55	0.41	-71.26
373.03	-18.67	0.61	0.36	0.93	0.70	-63.68	0.66	0.50	-32.03	0.48	0.36	-24.16
447.39	17.35	0.60	0.35	0.87	0.65	-0.76	0.58	0.44	32.90	0.40	0.30	41.11
659.19	112.44	0.83	0.49	0.71	0.54	174.89	0.41	0.30	221.41	0.24	0.18	232.79

Notes: Cut off points for the positive earnings points (in €): 107, 153, 193, 228, 264, 300, 344, 405, and 502. Source: SOEP 2001-2003 and FRS 2002/3.

Assuming a low redistributive taste, negative marginal tax rates become optimal in both countries. Achieving such a tax and transfer system in the UK would mean increasing taxes for the poorest (i.e. the non-workers) , and reducing them for all other points compared to the actual system in 2002/3. The tax credit would be of enormous size: even lone mothers in the second highest group would still receive an in-work credit, with higher net transfers than the non-working lone mothers. A similar result would hold for Germany, yet at a lower scale: net taxes would be higher for non-working lone parents than for working lone parents until gross earnings reached about 250 Euros a week.

In a scenario with medium redistributive taste. in the UK, in the optimum tax credits are lower but still of substantial size. Households earning up to about 150 Euros would receive higher net transfers than when out of work. In contrast in Germany, in this scenario tax credits only small tax credits for the poorest working group (of about 12 € per week) are optimal.

The difference between the UK and Germany becomes again evident in the scenario with relatively high redistribution taste. For Germany the optimal tax schedule does not contain a tax credit component. Net taxes are monotonously increasing with gross earnings. For the UK, in this welfare scenario it is optimal for the government to set net taxes for the non-working lone mothers to be the same as those of the poorest lone mothers in work (in other words, the effective marginal tax rate should be zero over this earnings range).

6 Conclusion

In this paper we apply the optimal tax rule suggested by Saez (2002) to empirically discuss the optimal tax and transfer design in Germany and the UK. The key advance on this paper is that we combine the theoretical model with a structural estimation of households` labour supply. Thus we are able to allow for heterogeneity between groups regarding their behaviour adjustment rather than calibrating an overall labour supply elasticity for the whole society.

When focusing on lone parents we have shown that in-work credits for this group are optimal from a social welfare perspective with relatively low and medium taste for redistribution in both Germany and the UK. Even with a high taste for distribution it is optimal in the UK to tax the non working and the poorest working women at the same rate. These results are driven by relatively high elasticities on the extensive margin which imply a high positive participation response of the non working.

By the same token we show that the given tax schedules in both countries, without an explicit in-work credit, are only optimal if the government has a high welfare value for the non working lone mothers and a relatively low taste for redistribution towards the working lone mothers. These findings have been derived with respect to a specific group, lone mothers, as in the current political debate this is the main target group for in-work credits. However, the main findings of this analysis might carry over to other groups or even to the whole population. As mentioned above, so far the optimal tax literature has not developed a theoretical framework incorporating the joint decision of households that can be empirically analysed. However, as we have shown, when elasticities on the extensive margin are relatively high relative to the potential negative reactions on the intensive margin, labour supply effects of in-work credits will be positive and depending on the distributive taste of the government are optimal.

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Appendix 1: Data and descriptive statistics

The database used for Germany is the German Socio Economic Panel (SOEP), a representative sample of over 12,000 households living in Germany interviewed annually. (Haisken De-New and Frick, 2001) For the empirical analysis, an unbalanced panel for the years 2001 - 2003 is used. The population consists of lone women with at least one dependent child that are aged between 20 and 60 years. Excluded are adults in full-time education, the self-employed or retired, and households with missing information, leaving 1,009 lone mothers.

According to the empirical distribution of working hours we have chosen 6 discrete working hours alternative, inactivity three part time and two full time working alternatives. The following table yields descriptive statistics about the variables that enter the estimation. Alternative specific variables are listed by working hours.

	Germany			UK	
	Working hours	Share	Net income	Working hours	Share Net income
inactivity	0	0.29	1094.99	0	
part time 1	10	0.06	1379.19	10	
part time 1	20	0.11	1498.84	20	
part time 3	25	0.07	1630.34	25	
full time 1	30	0.13	1764.94	30	
full time 2	40	0.34	1960.87	40	
Variables used in UK model					
Variables used in German model					
Age	39.18				
Share with children younger 3	0.11				
Share with children between 3 and 6	0.19				
Share living in East Germany	0.25				
Share with a medium educ. degree	0.82				
Share with a high educ. degree	0.16				

Notes: Germany: the following intervals for working hours have been chosen 0-5, 5-15, 15-22, 22-28, 28-35, >=35. The sample consists of 1009 observation, organised in an unbalanced panel for the years 2001-203.
UK:

Source: Soep 2001 - 2003

Appendix 2: Discrete Choice Labour Supply Estimation

Discrete choice models of labour supply are based on the assumption that a household can choose among a finite number J+1 of working hours (J positive hours points and non-employment); each hour j=0,…,J corresponds to a given level of disposable income Cij and each discrete bundle of leisure and income provides a different level of utility. The utility Vij derived by household i from making choice j is assumed to depend on a function U of the woman's leisure term Lfij, her disposable income Cij and household characteristics Zi, and on a random term εij. When the error term εij is assumed to be identically and independently distributed across alternatives and households according to the Extreme Value distribution, McFadden (1974) proves that the probability that alternative k is chosen by household i is given by:

$$\Pr_{ik} = \frac{\exp(V_{ik})}{\sum_{j=0}^{J} \exp(V_{ij})}, k \in J$$

The likelihood for a sample of observed choices can be derived from that expression and maximised to estimate the parameters of function *U*. We assume a quadratic specification of the utility function as in Blundell et al. (2000). In the estimation we include observed and unobserved heterogeneity by allowing income and leisure to vary with observed and unobserved characteristics. The observed characteristics include, age, region, number and age of children, educational information and nationality.

Appendix 3: how we calculated the elasticities

Labour supply elasticities can be derived numerically from our two discrete choice models. Recall that Saez (2002) formula for the optimal tax is written in terms of intensive and extensive elasticities respectively defined as:

$$\mu_i = \frac{C_i - C_{i-1}}{h_i} \frac{dh_i}{d(C_i - C_{i-1})};$$

and:

$$\eta_i = \frac{C_i - C_0}{h_i} \frac{dh_i}{d(C_i - C_0)};$$

where such elasticities are implicitly averages across the relevant population, and $i = 0…J$ indexes the choice ($i=0$ corresponds to not working).

To use this model to say something about the optimal tax function in practice requires us to view the different groups as different groups defined with respect to gross earnings (just as Saez (2002) does in his numerical example). For each individual k in our sample, we therefore estimate the elasticities $\tilde{\mu}_{i,k}$ and $\tilde{\eta}_{i,k}$, where $i = 0…J$ indexes the hours choice. [15] By definition, the intensive and extensive elasticity are identical for $i=1$ (the first choice of positive hours worked).

We then translate these elasticities in terms of weekly hours worked into elasticities in terms of gross weekly earnings by calculating:

$$\mu_i = \sum_{\forall k: H_i . w_k \in Y_i} \overline{\tilde{\mu}_{i,k}},$$

(and equivalently for the extensive elasticity), where the bar denotes the mean, w_k is the (actual or predicted) hourly wage for each individual, $H_i.w_k$ measures gross (weekly) earnings for individual k at choice i, and the set of Y_i defines intervals of gross earnings, and i=1…J (where J = 5 or 10) indexes the intervals of gross earnings.[16]

[15] We are able to estimate an elasticity for each individual by taking repeated draws from the extreme value errors, and calculating (for example) the fraction of times a given individual's preferred choice would change from choice i to choice $i-1$ in response to a 1% change in $C_i - C_{i-1}$ divided by the fraction of times the individual's preferred choice is choice i (and equivalently for the extensive elasticities). See 5.2.7 in Creedy et al (2002). When estimating the elasticities using the labour supply model for the UK and Germany, we assume full take-up of (complete program participation in) all benefits and tax credits.

[16] One drawback from having to perform this translation from elasticities defined wrt hours worked to elasticities defined wrt gross earnings is that it is not the case that the estimated intensive elasticity is identical to the estimated extensive elasticity in the first gross earnings interval.

Appendix 3: Optimal weights for the taxation of lone mothers: UK versus Germany

	Gross Earnings	Net Income	Net Tax	Marginal Tax Rate	Share	Intensive Elasticity	Extensive Elasticity	Opt. Weights	Relative Opt. Weights
					United Kingdom				
0	0.00	274.78	-274.78		0.48	0.00	0.00	1.65	1.00
1	100.08	318.69	-218.61	0.56	0.12	0.14	0.32	0.43	0.26
2	190.24	367.91	-177.67	0.45	0.11	0.03	0.55	0.43	0.26
3	261.58	399.60	-138.03	0.56	0.09	0.02	0.64	0.30	0.18
4	343.88	435.99	-92.11	0.56	0.09	0.03	0.61	0.33	0.20
5	530.19	522.84	7.35	0.53	0.10	0.04	0.45	0.44	0.27
					Germany				
0	0.00	244.54	-244.54		0.29	0.00	0.00	1.73	1.00
1	108.40	297.08	-188.68	0.52	0.08	0.10	0.13	0.79	0.46
2	192.63	328.39	-135.75	0.63	0.06	0.03	0.17	0.79	0.46
3	264.39	351.15	-86.75	0.68	0.12	0.01	0.22	0.70	0.41
4	347.94	386.07	-38.13	0.58	0.15	0.03	0.27	0.63	0.36
5	553.54	488.53	65.00	0.50	0.31	0.03	0.21	0.77	0.45

Notes: All income ant tax information are the mean average values per week. Marginal tax rate is calculated as change in net tax over change in gross earnings between adjacent groups.
Source: SOEP and FRS.