Endogenous Public Information and Welfare in Market Games

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Abstract

This paper performs a welfare analysis of market games with private information in which agents can condition on noisy prices in the rational expectations tradition. Price-contingent strategies introduce two externalities in the use of private information: a payoff (pecuniary) externality related to aggregate volatility and a learning externality. The impact of the first depends on whether competition is of the strategic substitutes or complements variety and the second induces agents to put too little weight on private information. We find that with strategic substitutes and when the learning externality is not very strong agents put too much weight on private information and prices are too informative. This will happen in the normal case where the allocational role of price prevails over its informational role. Under strategic complementarity there is always under-reliance on private information. The welfare loss at the market solution may be increasing in the precision of private information. These results extend to the internal efficiency benchmark (accounting only for the collective welfare of the active players). Received results—on the relative weights placed by agents on private and public information, when the latter is exogenous— may be overturned.

Keywords: learning externality, market games, complementarity and substitutability, asymmetric information, pecuniary externality, excess volatility, team solution, rational expectations

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1. <u>Introduction</u>

We show that when agents can condition on prices the presumption that they will put too little weight on private information, and consequently prices will contain too little information, need not hold. Agents may put too much weight on private information and prices may contain too much information for reasons other than the well-known Hirshleifer (1971) effect of destruction of insurance opportunities.[1] This happens, in fact, in common scenarios under strategic substitutes competition. Somewhat paradoxically, the fact that agents condition on the price may impair the (social) value of learning from it.

There has been a recent surge of interest in the welfare analysis of economies with private information and in particular on the role of public information in such economies (see, e.g., Morris and Shin 2002; Angeletos and Pavan 2007; Amador and Weill 2010). Morris and Shin (2005) point to the paradox that a central bank by publishing aggregate statistics makes those less reliable by inducing agents in the economy to rely less on their private signals. The same paradox of public information has been pointed out by Vives (1993, 1997). Agents may fail to place welfare-optimal weights on private and public information owing to payoff and information externalities.

In many markets agents compete in demand and/or supply schedules and therefore condition on prices. This is very common in financial markets, asset auctions, and some goods markets such as wholesale electricity. Prices are main providers of endogenous public information. In financial markets, prices are noisy statistics that arise from the decisions of traders. In goods markets, prices aggregate information on the preferences of consumers and the quality of the products. On the empirical front, initial evidence of herding of analysts forecasts (see Gallo et al. 2002 for GDP forecasts; Trueman 1994, Hong et al. 2000 for securities), and therefore of "insufficient" weight placed on private information, has been reversed by subsequent work. For example, both Bernardt et al. (2006) and Pierdzioch et al. (2013b) find strong evidence of anti-herding behavior by, respectively, professional financial

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[1]   See the general analysis of the value of public information in Schlee (2001).

analysts and oil-price forecasters.[2] According to those authors forecasters issue predictions biased towards their private information. Effinger and Polborn (2001) and Levy (2004) explain anti-herding behavior with reputational concerns. In the paper we will offer a novel explanation of why agents may put excessive weight on private information in the context of market games.

Any welfare analysis of rational expectations equilibria faces several difficulties. First of all, it must employ a model capable of dealing in a tractable way with the dual role of prices as conveyors of information and determinants of traders' budget constraints. Grossman and Stiglitz (1980) were pioneers in this respect with their CARA-normal model. Second, we require a welfare benchmark against which to test market equilibria in a world with asymmetric information. An appropriate benchmark for measuring inefficiency at the market equilibrium is the team solution in which agents internalize collective welfare but must still rely on private information when making their own decisions (Radner 1979; Vives 1988; Angeletos and Pavan 2007). This is in the spirit of Hayek (1945), where the private signals of agents cannot be communicated to a center. The team-efficient solution internalizes the payoff and information externalities associated with the actions of agents in the market. Collective welfare may refer to the surplus of all market participants, active or passive, or may be restricted to the internal welfare of the active agents. The third challenge for such welfare analysis is dealing with and disentangling the interaction of payoff and informational externalities. If we take as a benchmark a pure prediction model with no payoff externalities, then agents will typically rely too much on public information. The reason is that agents do not take into account that their reaction to private information affects the informativeness of public statistics and general welfare. In other words, agents do not internalize an information externality. Pure information externalities will make agents insufficiently responsive to their private information (Vives 1993, 1997; Amador and Weill 2012) and, in the limit to disregard it (Banerjee 1992, Bikhchandani et al. 1992).

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[2]   Other papers with anti-herding results by forecasters on asset prices and macro variables are Pierdzioch and Rülke (2012), Pierdzioch et al. (2013a), and Frenkel et al. (2012).

The under-reliance on private information result extends to some classes of economies with endogenous public information. Indeed, consider an economy in which equilibria are restricted efficient when public information is exogenous as, for example, a Cournot market with a continuum of firms and private information (Vives 1988). Then increasing public information has to be good marginally, and under regularity conditions the result is global. This implies that more weight to private information is needed (Angeletos and Pavan 2009). This logic breaks down in a market game where agents condition on the price, say firms competing in supply functions, because then there is a payoff (pecuniary) externality related to aggregate volatility which makes the market inefficient even if public information were to be exogenous. This payoff externality may counteract the learning from the price externality.

We consider a tractable linear-quadratic-Gaussian model that allows us to address the three challenges just described when public information is endogenously generated and influenced by the actions of agents. The context is a market game, where external effects go through the price. There is uncertainty about a common valuation parameter (say cost shock) about which agents have private information, and the price is noisy (say because of a demand shock). We use a model with a rational expectations flavor but in the context of a well-specified game, where a continuum of agents compete in schedules, and allow actions to be strategic substitutes or complements. We focus our attention on linear Bayesian equilibria. The model is flexible and admits several interpretations in terms of firms competing in a homogenous product market, monopolistic competition, trading in a financial market, and asset auctions. (We will follow the first interpretation until the extensions section.)

Let us discuss the results in some more detail. For concreteness, consider a homogenous product market with random demand and a continuum of ex ante identical firms competing in supply schedules with increasing marginal costs with uncertain intercept. Each firm receives a private signal about the marginal cost intercept. We show that there is a unique and symmetric linear equilibrium. Firms correct the slope of their strategy according to what they learn from the price and the character of competition. Under strategic substitutes competition (downward sloping demand) the price's informational and allocational roles conflict. In this case a high

price is bad news (high cost) and the equilibrium schedule is steeper than with full information. In fact, in equilibrium schedules may slope downwards when the informational role of prices dominates their allocational role. This will occur when there is little noise in the price. With strategic complements (upward sloping demand for a network good) there is no conflict: a high price is good news, and the equilibrium schedule is flatter than with full information.

In the economy considered the full information equilibrium is efficient since it is competitive. In this equilibrium all firms produce the same amount since they all have full information on costs, which are symmetric. With private information there is both aggregate and productive inefficiency. Aggregate inefficiency refers to a distorted total output and productive inefficiency refers to a distorted distribution of a given total output. The team-efficient solution in an economy with asymmetric information optimally trades off the tension between the two sources of welfare loss, aggregate and productive inefficiency. The somewhat surprising possibility that prices are too informative arises then since at the market solution firms may respond excessively to private information generating too much productive inefficiency. This happens under strategic substitutability, when the dual role of prices conflict, in the normal case where the allocational role of prices dominates the information role and the equilibrium supply is upward sloping. In the opposite case prices convey too little information. At the boundary of those situations there is a knife-edge case where parameters are such that firms use vertical schedules (as in a Cournot game), non contingent on the price. In this particular case constrained efficiency is restored. Under strategic complementarity, prices always convey too little information.

The explanation of the results is as follows. Price-contingent strategies create a double-edged externality in the use of private information. One is the traditional learning externality, which leads to under-reliance on private information and prices with too little information. Another is a payoff externality which obtains even if public information is exogenous (i.e. even if firms disregard the information content of the price). More specifically, consumers dislike aggregate output volatility under strategic substitutes but enjoy it under strategic complements but this is not internalized by the firms (precisely because they are protected from the aggregate volatility by conditioning on the price). The result is that firms respond too much

(little) to private information in the strategic substitutes (complements) case. When combined with the learning externality, which always leads firms to underweight private information, we obtain that with strategic complements we have always underweighting of private information. However, under strategic substitutes depending on the strength of the learning externality we may overcome or not the overweighting result due to the payoff externality. The point where both externalities cancel each other is when firms use vertical supply schedules. When supply is upward sloping, which happens when noise in demand is high, the allocational effect of the price prevails and the learning externality is weak. In this case the payoff externality effect wins over the learning externality and the weight to private information is too large. When the supply function is downward sloping, which happens when noise in demand is low, the informational component of the price prevails and the learning externality is strong. In this case the learning externality wins over the payoff externality and the weight to private information is too small.

More precise information, be it public or private, reduces the welfare loss at the team-efficient solution. The reason is that the direct impact of the increased precision is to decrease the welfare loss and this is the whole effect since at the team-efficient solution the responses to private and public information are already (socially) optimized (this is as in Angeletos and Pavan 2009). In contrast, at the market solution an increase in, say, the precision of private information will increase the response of a firm to its private signal and this will tend to increase the welfare loss when the market calls already for a too large response to private information. If this indirect effect is strong enough the welfare loss may be increasing with the precision of private information. In principle the same effect could happen with the precision of public information but we can show that the indirect effect of changes in both the exogenous public precision of information and the precision of the noise in the endogenous public signal are always dominated by the direct effect. The result is that the welfare loss at the market solution is always decreasing with the precisions of public information.

The results can be extended to the internal team-efficient benchmark (where only the collective welfare of the players is taken into account, for example, ignoring passive consumers). Then the full information market does not achieve an efficient outcome.

In this case also, endogenous public information may overturn conclusions reached using exogenous information models (e.g., Angeletos and Pavan 2007) when the informational role of the price is in conflict and dominates its allocational role.

The paper follows the tradition of the literature on the welfare analysis of private information economies (Palfrey 1985, Vives 1988, Angeletos and Pavan 2007, 2009), extending the analysis to endogenous public information. The results qualify the usual intuition of informational externality models (Vives 1997, Amador and Weill 2010, 2012) in a market game model. It is worth noting that pecuniary externalities are associated to inefficiency in competitive but incomplete markets and/or in the presence of private information since then the conditions of the first fundamental welfare theorem are not fulfilled. Competitive equilibria are not constrained efficient in those circumstances (Greenwald and Stiglitz 1986). For example, pecuniary externalities in markets with financial frictions (borrowing or collateral constraints) can explain market failure (see, e.g., Caballero and Krishnamurthy 2001 and Jeanne and Korinek 2010). In our paper (as in Laffont 1985) competitive noisy rational expectations equilibria (REE), in which traders take into account information from prices, are not constrained efficient. If REE where to be fully revealing then there would be (ex-post) Pareto optimal (Grossman 1981) and in our case, since we have quasilinear utility, also ex ante Pareto optimal. In our quasilinear utility model there is no room for the Hirshleifer (1971) effect according to which fully revealing REE may destroy insurance opportunities by revealing too much information (and then REE need not be ex ante efficient). We provide therefore an instance of REE which may reveal too much information on a fundamental on which agents have private information which is independent of the Hirshleifer effect.

Recent literature has examined the circumstances under which more public information actually reduces welfare (as in Burguet and Vives 2000; Morris and Shin 2002; Angeletos and Pavan 2007; Amador and Weill 2010, 2011). In Burguet and Vives (2000) a higher (exogenous) public precision may discourage private information acquisition and lead to a higher welfare loss in a purely informational externality model. In Morris and Shin (2002) the result is driven by a socially excessive incentive to coordinate by agents. Angeletos and Pavan (2007) qualify this result and relate it to the payoff externalities present in a more general model. In

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Amador and Weill (2010) a public release of information reduces the informational efficiency of prices and this effect may dominate the direct information provision effect. Their model is purely driven by information externalities in the presence of strategic complementarities in terms of responses to private information.[3] In our model more public information is not damaging welfare but more private precision may be. This happens when at the market solution there is already too much dispersion of actions and an increase in private precision exacerbates the problem.

The plan of the paper is as follows. Section 2 presents the model and the leading interpretation of firms competing in a homogenous product market. Section 3 characterizes the equilibrium. Section 4 performs the welfare analysis, and Section 5 studies extensions: the internal team-efficient benchmark and comparative static properties of the equilibrium. Section 6 presents alternative interpretations of the model and applications. Concluding remarks are provided in Section 7. Proofs are gathered in the Appendix.

2. The market game

Consider a continuum of firms indexed within the interval $[0,1]$ (endowed with the Lebesgue measure), x_i is the output of firm i, produced at cost $C(x_i) = \theta x_i + (\lambda/2) x_i^2$ where θ is potentially random and $\lambda > 0$. Firms face an inverse demand for an homogenous product $p = \alpha + u - \beta \tilde{x}$, where u is a demand shock, $\alpha > 0$, and $\tilde{x} = \int_0^1 x_i \, di$ is the aggregate output.

Firm i has therefore the payoff function

$$\pi(x_i, \tilde{x}) = (p - \theta) x_i - \frac{\lambda}{2} x_i^2,$$

where $p = \alpha + u - \beta \tilde{x}$ for given parameters θ and u.

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[3]   Ganguli and Yang (2009) develop the implications of strategic complementarities for information acquisition in noisy rational expectations models.

We have then that $\partial^2 \pi / (\partial x_i)^2 = -\lambda < 0$ and $\partial^2 \pi / \partial x_i \partial \tilde{x} = -\beta$, and the slope of the best reply of a firm is $m \equiv \left(\partial^2 \pi / \partial x_i \partial \tilde{x} \right) / \left(-\partial^2 \pi / (\partial x_i)^2 \right) = -\beta / \lambda$. Thus we have strategic substitutability (complementarity) for $\beta > 0$ (for $\beta < 0$), and m can be understood as the degree of complementarity in the payoffs. (In the rest of this paper, when discussing strategic substitutability or complementarity we refer to this meaning). We assume that $m < 1/2$ (i.e., $2\beta + \lambda > 0$), limiting the extent of strategic complementarity. The condition $2\beta + \lambda > 0$ guarantees that $\pi(x, x)$ is strictly concave in x ($\partial^2 \pi / (\partial x)^2 = -(2\beta + \lambda) < 0$).

If $\beta > 0$, then demand is downward sloping and we have strategic substitutability in the usual partial equilibrium market. If $\beta < 0$, we have strategic complementarity and demand is upward sloping. The latter situation may arise in the case of a network good with compatibility.

The parameter θ has prior Gaussian distribution with mean $\bar{\theta}$ and variance σ_θ^2 (we write $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$ and, to ease notation, set $\bar{\theta} = 0$). Player i receives a signal $s_i = \theta + \varepsilon_i$ with $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$. Error terms are uncorrelated across players, and the random variables $\{\theta, \varepsilon_i, u\}$ are mutually independent. Given a random variable y we denote by $\tau_y \equiv 1/\sigma_y^2$ its precision. We follow the convention that error terms cancel in the aggregate: $\int_0^1 \varepsilon_i \, di = 0$ almost surely (a.s.). Then the aggregation of all individual signals will reveal the underlying uncertainty: $\int_0^1 s_i \, di = \theta + \int_0^1 \varepsilon_i = \theta$.[4]

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[4] Suppose that $(q_i)_{i \in [0,1]}$ is a process of independent random variables with means $\mathbb{E}[q_i]$ and uniformly bounded variances $\text{var}[q_i]$. Then we let $\int_0^1 q_i \, di = \int_0^1 \mathbb{E}[q_i] \, di$ (a.s.). This convention will be used while taking as given the usual linearity property of integrals. (Equality of random variables is assumed to hold almost surely always.) In short, we assume that the strong law of large numbers (SLLN) holds for a continuum of independent random variables with uniformly bounded variances.

Firms use supply functions as strategies and at the time of submitting the supply function firm i has a noisy estimate s_i of the intercept of marginal cost. As an example, the cost parameter θ could be a unit ex post pollution damage that is assessed on firm i, say an electricity generator, and for which the firm has an estimate s_i before submitting its supply function.[5]

The timing of the game is as follows. At $t = 0$, the random variables θ and u are drawn but not observed. At $t = 1$, each firm observes his own private signal s_i and submits a schedule $X_i(s_i, \cdot)$ with $x_i = X_i(s_i, p)$, where p is the price. The strategy of a player is a map from the signal space to the space of schedules. Finally, the market clears, the price is formed by finding a p that solves $p = \alpha + u - \beta\left(\int_0^1 X_j(s_j, p)\, dj\right)$, and payoffs are collected at $t = 1$.

Let us assume that there is a unique price $\hat{p}\left((X_j(s_j, \cdot))_{j \in [0,1]}\right)$ for any realization of the signals.[6] Then, for a given profile $(X_j(s_j, \cdot))_{j \in [0,1]}$ of firms' schedules and realization of the signals, the profits for firm i are given by

$$\pi_i = (p - \theta)x_i - \frac{\lambda}{2}x_i^2,$$

where $x_i = X_i(s_i, p)$, $\tilde{x} = \int_0^1 X_j(s_j, p)\, dj$, and $p = \hat{p}\left((X_j(s_j, \cdot))_{j \in [0,1]}\right)$. This formulation has a rational expectations flavor but in the context of a well-specified schedule game. We will restrict our attention to linear Bayesian equilibria of the schedule game.

It is worth to remark that in the market game both payoff and informational (learning) externalities go through the market price p, which has both an allocational and an

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5   Normality of random variables means that prices and quantities can be negative with positive probability. The probability of this event can be controlled, if necessary, by an appropriate choice of means and variances. Furthermore, for this analysis the key property of Gaussian distributions is that conditional expectations are linear. Other prior-likelihood conjugate pairs (e.g., beta-binomial and gamma-Poisson) share this linearity property and can display bounded supports.

6   We assign zero payoffs to the players if there is no p that solves the fixed point problem. If there are multiple solutions, then the one that maximizes volume is chosen.

informational role. Indeed, when $\beta = 0$, there are neither payoff nor informational externalities among players. The dual role of β as both a parameter in the payoff function and in the public statistic should be noted. This situation arises naturally in market games.

The model admits other interpretations than the one presented in terms of monopolistic competition or demand function competition (see Section 6). [7]

3. Equilibrium

We are interested in a linear (Bayesian) equilibrium—equilibrium, for short—of the schedule game for which the public statistic functional is of type $P(\theta, u)$. Since the payoffs and the information structure are symmetric and since payoffs are strictly concave, there is no loss of generality in restricting our attention to symmetric equilibria. Indeed, the solution to the problem of firm i,

$$\max_{x_i} \mathbb{E}\left[\left(p - \theta - \frac{\lambda}{2}x_i\right)x_i \,\middle|\, s_i, p\right],$$

is both unique (given strict concavity of profits) and symmetric across firms (since the cost function and signal structure are symmetric across firms):

$$X(s_i, p) = \lambda^{-1}\left(p - \mathbb{E}[\theta | s_i, p]\right),$$

where $p = P(\theta, u)$. A strategy for firm i may be written as

$$x_i = \hat{b} + \hat{c}p - as_i,$$

in which case the aggregate action is given by

$$\tilde{x} = \int_0^1 x_i \, di = \hat{b} + \hat{c}p - a\theta.$$

It then follows from $p = \alpha + u - \beta\tilde{x}$ that, provided $\hat{c} \neq -\beta^{-1}$,

$$p = P(\theta, u) = (1 + \beta\hat{c})^{-1}\left(\alpha - \beta\hat{b} + z\right);$$

here the random variable $z = \beta a\theta + u$ is informationally equivalent to the price p. Because u is random, z (and the price) will typically generate a noisy signal of the

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unknown parameter θ. Let τ denote the precision of the price p or of z in the estimation of θ, $\tau \equiv \left(\text{var}\left[\theta | z \right] \right)^{-1}$. From the properties of Gaussian random variables it is immediate that $\tau = \tau_\theta + \tau_u \beta^2 a^2$.

The information available to player i is $\{ s_i, p \}$ or, equivalently, $\{ s_i, z \}$. Since $\mathbb{E}\left[\theta | s_i, p \right] = \mathbb{E}\left[\theta | s_i, z \right]$, we can posit strategies of the form

$$X\left(s_i, z \right) = b - a s_i + cz$$

and obtain that $p = \alpha - \beta b + \left(1 - \beta c \right) z$. If $1 + \beta \hat{c} > 0$ then $1 - \beta c > 0$ (since $\hat{c} = \left(c^{-1} - \beta \right)^{-1}$ and $1 + \beta \hat{c} = \left(1 - \beta c \right)^{-1}$) and so p and z will move together. The strategy of player i is then given by

$$X\left(s_i, z \right) = \lambda^{-1} \left(\alpha - \beta b + \left(1 - \beta c \right) z - \mathbb{E}\left[\theta | s_i, z \right] \right).$$

We can solve for the linear equilibrium in the usual way: identifying coefficients with the candidate linear strategy $x_i = b - a s_i + cz$ by calculating $\mathbb{E}\left[\theta | s_i, z \right]$ and using the supply function of a firm. In equilibrium, firms take public information z, with precision $\tau \equiv \left(\text{var}\left[\theta | z \right] \right)^{-1}$, as given and use it to form probabilistic beliefs about the underlying uncertain parameter θ. We have that $\mathbb{E}\left[\theta / s_i, z \right] = \gamma s_i + \left(1 - \gamma \right) \mathbb{E}\left[\theta / z \right]$. The market chooses the weight to private information $\gamma = \tau_\varepsilon \left(\tau_\varepsilon + \tau \right)^{-1}$ in a Bayesian way. This is the γ that minimizes the mean square error of predicting θ with the private and public signals:

$$\min_\gamma \frac{1}{2\lambda} \left(\frac{\left(1 - \gamma \right)^2}{\tau} + \frac{\gamma^2}{\tau_\varepsilon} \right).$$

Revised beliefs and optimization determine thus the coefficients a $\left(= \lambda^{-1} \gamma \right)$ and c for private and public information, respectively. In equilibrium, the informativeness of public information z depends on the sensitivity of strategies to private information $a : \tau = \tau_\theta + \tau_u \beta^2 a^2$. Firms behave as information takers and so, from the perspective of an individual firm, public information is exogenous. This fact is at the root of a learning externality: firms fail to account for the impact of their own actions on public

information (the price) and hence on other firms. A second, payoff (pecuniary) externality in the use of information arises even if firms with private signals do not to learn from prices but still use price-contingent strategies (for example, it arises even if the price is extremely noisy, $\tau_u = 0$). We will deal with them in the welfare analysis section.

The following proposition characterizes the equilibrium.

<u>Proposition 1</u>. *There is a unique (and symmetric) equilibrium*

$$X\left(s_i, p\right) = \lambda^{-1}\left(p - \mathbb{E}\left[\theta | s_i, p\right]\right) = \hat{b} - as_i + \hat{c}p,$$

where a is the unique (real) solution of the equation $a = \tau_\varepsilon \lambda^{-1}\left(\tau_\varepsilon + \tau\right)^{-1}$ where

$\tau = \tau_\theta + \tau_u \beta^2 a^2$, $\hat{c} = \left(\left(\beta + \lambda\right)\left(1 - \beta\lambda\tau_u a^2 \tau_\varepsilon^{-1}\right)^{-1} - \beta\right)^{-1}$, *and $\hat{b} = \alpha\left(1 - \lambda\hat{c}\right)/\left(\beta + \lambda\right)$. In*

equilibrium, $a \in \left(0, \tau_\varepsilon \lambda^{-1}\left(\tau_\theta + \tau_\varepsilon\right)^{-1}\right)$ and $1 + \beta\hat{c} > 0$.

<u>Remark 1.</u> We have examined linear equilibria of the schedule game for which the price functional is of type $P\left(\theta, u\right)$. In fact, these are the equilibria in strategies with bounded means and with uniformly (across players) bounded variances. (See Claim 1 in the Appendix.)

<u>Remark 2.</u> We can show that the equilibrium in the continuum economy is the limit of equilibria in replica economies that approach the limit economy. Take the homogenous market interpretation with a finite number of firms n and inverse demand $p_n = \alpha + u - \beta\tilde{x}_n$, where $\tilde{x}_n$ is the average output per firm, and with the same informational assumptions. In this case, given the results in Section 5.2 of Vives (2011), the supply function equilibrium of the finite n-replica market converges to the equilibrium in Proposition 1.

In equilibrium, the "price impact" (how much the price moves to accommodate a unit increase in u or inverse of the depth of the market [8]) is always positive,

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[8]   See, for example, Kyle (1985).

$\partial P/\partial u = (1+\beta\hat{c})^{-1} > 0$, and excess demand $\varXi(p)$ is downward or upward sloping depending on β: $\varXi' = -(\beta^{-1} + \hat{c})$ and $\mathrm{sgn}\{\varXi'\} = \mathrm{sgn}\{-\beta\}$.[9] That is, the slope's direction depends on whether the competition is in strategic substitutes or in strategic complements.

The price serves a dual role as index of scarcity and conveyor of information. Indeed, a high price has the direct effect of increasing a firm's competitive supply, but it also conveys news about costs—namely, that costs are high (low) if $\beta > 0$ ($\beta < 0$). Consider as a benchmark the full information case with perfectly informative signals ($\tau_\varepsilon = \infty$). This is a full information competitive equilibrium and we have $c = (\beta + \lambda)^{-1}$, $a = \hat{c} = \lambda^{-1}$, and $X(\theta, p) = \lambda^{-1}(p - \theta)$. In this case, agents have nothing to learn from the price. If signals become noisy ($\tau_\varepsilon < \infty$) then $a < \lambda^{-1}$ and $\hat{c} < \lambda^{-1}$ for $\beta > 0$, with supply functions becoming steeper (lower $\hat{c}$) as agents protect themselves from adverse selection. The opposite happens ($\hat{c} > \lambda^{-1}$ and flatter supply functions) when $\beta < 0$, since then a high price is good news (entailing lower costs).[10] There is then "favorable" selection.

There are several other cases in which $\hat{c} = \lambda^{-1}$ and there is no learning from the price: (i) When signals are uninformative about the common parameter θ ($\tau_\varepsilon = 0$) or when there is no uncertainty ($\tau_\theta = \infty$ and $\theta = \overline{\theta}$ (a.s.)), the price has no information to convey; $a = 0$ and $X(s_i, p) = \lambda^{-1}(p - \overline{\theta})$; (ii) When the public statistic is extremely noisy ($\tau_u = 0$) or when $\beta = 0$ (in which case there is no payoff externality, either), then public information is pure noise, $a = \lambda^{-1}\tau_\varepsilon(\tau_\theta + \tau_\varepsilon)^{-1}$, with $X(s_i, p) = \lambda^{-1}(p - \mathbb{E}[\theta|s_i])$.

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[9] $\varXi(p) = \beta^{-1}(\alpha + u - p) - \hat{b} + a\theta - \hat{c}p = \beta^{-1}(\alpha + u) - \hat{b} + a\theta - (\beta^{-1} + \hat{c})p$.

[10] This follows because, with upward-sloping demand, we assume that $2\beta + \lambda > 0$ and therefore $\lambda > -\beta$.

As τ_u tends to ∞, the precision of prices τ also tends to ∞, the weight given to private information a tends to 0, and the equilibrium collapses (with market depth $1 + \beta\hat{c} \to 0$). Indeed, the equilibrium becomes fully revealing and is not implementable. The informational component of the price increases with τ_u and decreases with τ_θ (since firms are endowed with better prior information with a larger τ_θ). With $\beta > 0$, as τ_u increases from 0, $\hat{c}$ decreases from λ^{-1} (and the slope of supply increases) because of the price's increased informational component (a high price indicates higher costs). As τ_u increases more, $\hat{c}$ becomes zero at some point and then turns negative; as τ_u tends to ∞, $\hat{c}$ tends to $-\beta^{-1}$.[11] At the point where the allocational and informational effects balance, agents place zero weight ($\hat{c} = 0$) on the price. In this case the model reduces to a quantity-setting model à la Cournot (however, not reacting to the price is optimal). If τ_θ increases then the informational component of the price diminishes and we have a more elastic supply (higher $\hat{c}$).

When $\beta < 0$ then a high price conveys the good news that average quantity tends to be high and that costs therefore tend to be low. In this case, increasing τ_u, which reinforces the informational component of the price, increases $\hat{c}$—the opposite of what happens when τ_θ increases. It follows that in either case ($\beta > 0$ or $\beta < 0$) market depth $\left(\partial P / \partial u\right)^{-1} = 1 + \beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ. (See Proposition 5 for a complete statement of the comparative statics properties of the equilibrium.)

4. Welfare analysis

Consider the homogeneous product market with quadratic production costs. The inverse demand $p = \alpha + u - \beta\tilde{x}$ arises from a benefit or surplus function $\left(\alpha + u - (\beta/2)\tilde{x}\right)\tilde{x}$, and the welfare criterion is total surplus:

$$TS = \left(\alpha + u - \beta\frac{\tilde{x}}{2}\right)\tilde{x} - \int_0^1 \left(\theta x_i + \frac{\lambda}{2}x_i^2\right)di .$$

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[11] Downward sloping supply bids have been allowed in some wholesale electricity markets (e.g. in the Nord Pool before 2007).

Under our assumptions, $\beta + \lambda > 0$ and the TS function is strictly concave for symmetric solutions. It is immediate that the first-best (full information) allocation is symmetric and given by $x^o = (\lambda + \beta)^{-1}(\alpha + u - \theta)$.

The equilibrium is partially revealing (with $0 < \tau_u < \infty$ and $0 < \tau_\varepsilon < \infty$), so expected total surplus should be strictly greater in the first-best (full information) allocation, which is just the market solution with full information, than at the equilibrium. (That is, in our market the full information equilibrium is (first best) efficient.) The reason is that suppliers produce under uncertainty and rely on imperfect idiosyncratic estimation of the common cost component; hence they end up producing different amounts even though costs are identical and strictly convex. However, since producers are competitive they produce in expected value the right amount at the equilibrium: $\mathbb{E}[\tilde{x}] = \mathbb{E}[x^o] = \alpha(\lambda + \beta)^{-1}$.

The welfare benchmark we use is the team solution maximizing expected total surplus subject to employing linear decentralized strategies (as in Vives 1988; Angeletos and Pavan 2007). This team-efficient solution internalizes the payoff and information externalities of the actions of agents, and it is restricted to using the same type of strategies (decentralized and linear) that the market employs.

In the economy considered if firms would not condition on prices, i.e. if each firm would set quantities conditioning only on its private information as in a Cournot market, then the market solution would be team-efficient (Vives 1988). That is, in the Cournot economy, the private information equilibrium is team-efficient for given public information. We will see that this is not the case in our market game with price-contingent strategies even with exogenous public information because of a pecuniary externality in the use of private information.

We deal first with the Cournot case in order to fix ideas and review received results, and continue our analysis of the supply function market with the characterization of the payoff externality in the use of information when public information is exogenous,

the information externality induced by endogenous public information and, finally, the combined effect of the two externalities.

4.1 Efficiency in the Cournot market

In this section we assume that firms compete in quantities contingent only on their private information as well as a normally distributed public signal z about θ, with precision τ, for the moment exogenous. A strategy for firm i is a mapping from signals into outputs: $X_i(s_i, z)$. This is the model considered in Vives (1988) (and Angeletos and Pavan, Section 6.1, 2007) from where it follows that, under the same distributional assumptions as in Section 2, there is a unique Bayesian Cournot equilibrium and it is symmetric and linear:

$$X(s_i, z) = \frac{\alpha}{\beta + \lambda} - \left(as_i + \left((\beta + \lambda)^{-1} - a\right)\mathbb{E}[\theta \,|\, z]\right) \text{ where } a = \frac{\tau_\varepsilon}{\lambda(\tau + \tau_\varepsilon) + \beta\tau_\varepsilon}.$$

Note that $a \leq (\beta + \lambda)^{-1}$. The equilibrium follows immediately from the optimization problem of firm i

$$\max_{x_i} \mathbb{E}\left[px_i - C(x_i)\big|s_i, z\right]$$

and the associated FOC (which are also sufficient):

$$\mathbb{E}[p - MC(x_i) \,|\, s_i, z] = 0 \text{ or } \mathbb{E}[p - \theta \,|\, s_i, z] = \alpha - \beta\mathbb{E}[\tilde{x} \,|\, s_i, z] - \mathbb{E}[\theta \,|\, s_i, z] = \lambda x_i,$$

where the difference from our market game is that firms do *not* condition on the price.

It follows that the market solution is team efficient since the same FOC hold also for the maximization of $\mathbb{E}[\text{TS}]$ subject to decentralized production strategies. Indeed, under our assumptions, it is easily seen that the solution is symmetric and with the same FOC as the market

$$\mathbb{E}\left[\frac{\partial \text{TS}}{\partial x_i}\big|s_i, z\right] = \mathbb{E}[p - MC(x_i) \,|\, s_i, z] = 0.$$

In the terminology of Angeletos and Pavan (2007), the economy in which agents use non-price contingent strategies displays exactly the right degree of coordination or complementarity. Note that firms in the Cournot market, in contrast to the supply

function market, are subject to aggregate volatility since they have to estimate the price.

Suppose now that public signal z comes from an endogenous noisy quantity signal $q = \tilde{x} + \eta$ where $\eta \sim N\left(0, \tau_\eta^{-1}\right)$ is independent of the other random variables in the model. Then positing that firms use a linear strategy $x_i = b - as_i - \hat{c}q$ it is easily seen that $q = \left(1 + \hat{c}\right)^{-1}\left(b - z\right)$ where $z = a\theta - \eta$. Letting $\hat{z} = \mathbb{E}\left[\theta \mid z\right]$, the strategy $X\left(s_i, z\right)$ has the same form as before but now $\tau = \tau_\theta + \tau_\eta a^2$ is endogenous.

We may conjecture that the endogenous quantity signal will lead firms to put too little weight on their private information due to the presence of an information externality (Vives (1997), Amador and Weill (2012)). We confirm that this is indeed the case. It can be checked that candidate team strategies are of the same form as the market but with potentially a different response a to private information. We have that:

$$\frac{\partial \mathbb{E}\left[\mathrm{TS}\right]}{\partial a} = \mathbb{E}\left[\left(p - \mathrm{MC}\left(x_i\right)\right)\frac{\partial x_i}{\partial a}\bigg|_{\hat{z}\ \mathrm{ct.}}\right] + \mathbb{E}\left[\left(p - \mathrm{MC}\left(x_i\right)\right)\left(\frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a}\right)\right].$$

At the (Cournot) market solution $\mathbb{E}\left[\left(p - \mathrm{MC}\left(x_i\right)\right)\frac{\partial x_i}{\partial a}\bigg|_{\hat{z}\ \mathrm{ct.}}\right] = 0$ since firms take z as given and the learning externality term is positive, $\mathbb{E}\left[\left(p - \mathrm{MC}\left(x_i\right)\right)\left(\frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a}\right)\right] > 0$, and therefore $\partial \mathbb{E}\left[\mathrm{TS}\right]/\partial a > 0$. This indicates that a has to be increased from the market level and, since $\mathbb{E}\left[\mathrm{TS}\right]$ is strictly concave in a, we conclude as expected that the information externality leads to a too small response to private information.[12] It is, however, natural to think that the noisy quantity signal comes from the price. In this case firms would condition on prices. We return to the supply function model in the rest of the paper.

4.2 <u>Efficiency in the supply function market</u>

Let us go back to our market game where firms condition on prices on top of their private information. In this context, at the team-efficient solution, expected total

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[12]   See Lemma A1 in the Appendix for proof of the statements.

surplus $\mathbb{E}[\text{TS}]$ is maximized under the constraint that firms use decentralized linear production strategies contingent on endogenous public information (price p or the equivalent variable z). That is,

$$\max_{a,b,c} \mathbb{E}[\text{TS}] \qquad (\dagger)$$

subject to $\quad x_i = b - as_i + cz$, $\tilde{x} = b - a\theta + cz$, and $z = u + \beta a\theta$.

Equivalently, the team-efficient solution minimizes, over the restricted strategies, the expected welfare loss WL with respect to the full information first best TS^o. (Indeed we have that $\mathbb{E}[\text{TS}] = \mathbb{E}[\text{TS}^o] - \text{WL}$.) It is possible to show that

$$\text{WL} = \left((\beta + \lambda) \mathbb{E}\left[(\tilde{x} - x^o)^2 \right] + \lambda \mathbb{E}\left[(x_i - \tilde{x})^2 \right] \right) \Big/ 2,$$

where the first term in the sum corresponds to aggregate inefficiency (how distorted is the average quantity $\tilde{x}$ while producing in a cost-minimizing way), which is proportional to $\mathbb{E}\left[(\tilde{x} - x^o)^2 \right]$, and the second term to productive inefficiency (how distorted is the distribution of production of a given average quantity $\tilde{x}$), which is proportional to the dispersion of outputs $\mathbb{E}\left[(x_i - \tilde{x})^2 \right]$. Let p^o be the full information first best price. Note that the non-fundamental price volatility is given by $\mathbb{E}\left[(p - p^o)^2 \right] = \beta^2 \mathbb{E}\left[(\tilde{x} - x^o)^2 \right]$ and therefore it is proportional to aggregate inefficiency.

It is easily seen that the form of the optimal team strategy is

$$x_i = \lambda^{-1} \left(p - \left(\gamma s_i + (1 - \gamma) \mathbb{E}[\theta \mid z] \right) \right)$$

where the weight to private information $\gamma = \lambda a$ may differ from the market weight. This means that at a team strategy public information is optimally used but that the weight to private information may differ from the Bayesian one. It follows then that the welfare loss at any candidate team solution will depend only on the response to private information a since we have

$$\mathbb{E}\left[(\tilde{x} - x^o)^2 \right] = (1 - \lambda a)^2 \Big/ \left(\tau (\beta + \lambda)^2 \right), \ \tau = \tau_\theta + \tau_u \beta^2 a^2$$

and

$$\mathbb{E}\left[\left(x_i - \tilde{x}\right)^2\right] = a^2 / \tau_\varepsilon \ .$$

This yields a strictly convex WL as a function of a:

$$\mathrm{WL}\left(a; \tau(a)\right) = \frac{1}{2}\left(\frac{\left(1 - \lambda a\right)^2}{\tau\left(\beta + \lambda\right)} + \frac{\lambda a^2}{\tau_\varepsilon}\right) \text{ with } \tau = \tau_\theta + \tau_u \beta^2 a^2 \ .$$

Changing a has opposite effects on both sources of the welfare loss since allocative inefficiency decreases with a, as price informativeness τ increases and the average quantity gets close to the full information allocation, but productive inefficiency increases with a as dispersion increases. A more informative price reduces allocative inefficiency and non-fundamental price volatility but increases productive inefficiency. The team solution, denoted a^{T}, minimizes WL and optimally trades off the sources of inefficiency among decentralized strategies.

In order to compare the market and team solutions we consider first the case of exogenous public information and the associated externality to which we add the learning externality to complete the analysis.

4.2.1 <u>Exogenous public information: the payoff externality in the use of information</u>
Suppose that firms receive a public signal with exogenous precision τ (including the prior precision). The market solution is then a "naive" competitive equilibrium where firms condition on the market price but do not learn from it.

The market chooses the weight to private information to minimize the mean square error in predicting θ, yielding $a = \tau_\varepsilon / (\lambda(\tau_\varepsilon + \tau))$. This is the same solution as when firms learn from the price but then τ is endogenous. The team solution for given with τ, $a^*_{exo}(\tau)$, minimizes $\mathrm{WL}(a; \tau)$ instead. This is a strictly convex function of a with a unique minimum

$$a^{\mathrm{T}}_{\mathrm{exo}}(\tau) = \frac{\tau_\varepsilon}{\lambda\left(\tau_\varepsilon + \tau\right) + \beta\tau} \ .$$

It follows that $a^{\mathrm{T}}_{\mathrm{exo}}(\tau) \le a^*_{\mathrm{exo}}(\tau)$ if and only if $\beta \ge 0$. The solution depends indeed on β, the term $\beta\tau$ reflects a payoff externality at the market solution. The following Lemma states the result.

<u>Lemma 1</u>. *Let $\infty > \tau_\varepsilon > 0$, suppose that firms receive a public signal of precision τ and ignore the information content of the price. Then firms respond more (less) to private information $a_{\text{exo}}^*(\tau)$ than at the team optimal solution $a_{\text{exo}}^{\text{T}}(\tau)$ in the strategic substitutes (complements) case, i.e. $\text{sgn}\left\{a_{\text{exo}}^*(\tau) - a_{\text{exo}}^{\text{T}}(\tau)\right\} = \text{sgn}\left\{\beta\right\}$.*

The team solution takes into account the payoff externalities induced by the aggregate action, how the volatility of the average quantity affects welfare, depending on whether we have strategic substitutes ($\beta > 0$) or complements ($\beta < 0$). Consumers dislike output volatility under strategic substitutes and enjoy it under strategic complements since their gross surplus is given by $(\alpha + u - (\beta/2)\tilde{x})\tilde{x}$. When $\beta > 0$, the team solution diminishes the response to private information from the Bayesian benchmark to moderate output volatility. When $\beta < 0$, the opposite occurs. When $\beta = 0$ there is no payoff externality and agents use information efficiently. Obviously, when there is no private information ($\tau_\varepsilon = 0$) or information is perfect ($\tau_\varepsilon = \infty$) the market is efficient since it is competitive and pecuniary externalities are internalized.

In summary, there is a pecuniary externality in the use of private information at the (naive) competitive equilibrium since firms use price-contingent strategies but they do not take into account how their response to private information affects the price and therefore aggregate volatility. Indeed, since the strategy for firm i is of the form $x_i = \lambda^{-1}\left(p - (\gamma s_i + (1-\gamma)\hat{z})\right)$ where $\hat{z} = \mathbb{E}[\theta \mid z]$ and z here is an exogenous signal, we have that

$$\frac{\partial x_i}{\partial a} = \left.\frac{\partial x_i}{\partial a}\right|_{p,\hat{z}\text{ ct.}} + \lambda^{-1}\frac{\partial p}{\partial a}$$

and

$$\frac{\partial \mathbb{E}[\text{TS}]}{\partial a} = \mathbb{E}\left[\left.(p - \text{MC}(x_i))\frac{\partial x_i}{\partial a}\right|_{p,\hat{z}\text{ ct.}}\right] + \mathbb{E}\left[(p - \text{MC}(x_i))\left(\lambda^{-1}\frac{\partial p}{\partial a}\right)\right],$$

where the first term is what the market equates to zero and the second corresponds to the payoff externality in the use of private information. Consistently with the result in

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Lemma 1 it can be checked that at the market solution for given τ the sign of the payoff externality depends on the sign of β,

$$\text{sgn}\left\{\mathbb{E}\left[\left(p - \text{MC}(x_i)\right)\left(\lambda^{-1}\frac{\partial p}{\partial a}\right)\right]\right\} = \text{sgn}\{-\beta\},$$

and $\text{sgn}\{\partial\mathbb{E}[\text{TS}]/\partial a\} = \text{sgn}\{-\beta\}$.[13]

4.2.2 Endogenous public information: The learning externality

Let us consider now the case where firms do take into account the information content of the price. Then there is a learning externality and an added reason for the market solution to be inefficient. We know from the received literature that the learning externality will tend to make agents put too little weight on private information (Vives 1997, Amador and Weill 2012). The reason is that an agent when responding to its private information does not take into account the improved informativeness of public statistics. This effect will also be present in our case. Indeed, when the informational value of the price is accounted for, public information is endogenous ands the response to private information a affects the precision of the price $\tau = \tau_\theta + \tau_u \beta^2 a^2$.

As stated before, $\text{WL}(a; \tau(a))$ is a strictly convex function of a and the following FOC characterizes the team solution a^T

$$\frac{d\text{WL}}{da} = \frac{\partial\text{WL}}{\partial a} + \underbrace{\frac{\partial\text{WL}}{\partial\tau}}_{-}\underbrace{\frac{\partial\tau}{\partial a}}_{+} = 0$$

The first term $\partial\text{WL}/\partial a$ corresponds to the direct effect of changing a for a fixed τ and the second corresponds to the indirect effect through the public precision τ. This second term is the effect of the learning externality and it is negative since $\partial\text{WL}/\partial\tau < 0$ and $\partial\tau/\partial a > 0$. This implies that for any given τ we want to increase a from the optimal level with exogenous information. Indeed, we have that $\partial\text{WL}\left(a_{\text{exo}}^T(\tau); \tau\right)/\partial a = 0$ and therefore, $d\text{WL}\left(a_{\text{exo}}^T(\tau); \tau\right)/da < 0$.

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[13]   And, in fact, the result of Lemma 1 follows since $\mathbb{E}[\text{TS}]$ is strictly concave in a.

This confirms the idea that the learning externality biases the market solution towards putting too little weight on private information. The following lemma states the result.

<u>Lemma 2</u>. *Let $\tau_u > 0$. At the team solution with exogenous public precision τ by increasing a the welfare loss is reduced; i.e.* $\mathrm{dWL}\left(a_{\mathrm{exo}}^{\mathrm{T}}(\tau);\tau\right)/da < 0$.

4.2.3 <u>The combined effect of the externalities</u>

We examine now the combined effect of the two (payoff and learning) externalities in the use of information characterized in Lemmas 1 and 2. We know that the learning externality always leads agents to underweight private information and that the payoff externality leads to overweight or to underweight depending on whether competition is of the strategic substitutes or complements variety. From this it follows that with strategic complements we would have always underweighting of private information. However, under strategic substitutes depending on the strength of the learning externality we may overcome or not the overweighting result due to the payoff externality.

From the FOC $d\mathrm{WL}\left(a;\tau(a)\right)/da = 0$ with

$$\frac{\partial \mathrm{WL}}{\partial a} = \frac{\lambda a}{\tau_\varepsilon} - \frac{\lambda(1-\lambda a)}{(\beta+\lambda)\tau} \quad \text{and} \quad \frac{\partial \mathrm{WL}}{\partial \tau}\frac{\partial \tau}{\partial a} = -\frac{(1-\lambda a)^2 \beta^2 a \tau_u}{(\beta+\lambda)\tau^2},$$

we obtain that a^T fulfills

$$a = \frac{\tau_\varepsilon}{\lambda(\tau(a)+\tau_\varepsilon)+\beta\tau(a)-\Delta(a)}$$

where $\beta\tau(a)$ corresponds to the payoff externality and $\Delta(a) = \frac{(1-\lambda a)^2 \beta^2 \tau_u \tau_\varepsilon}{\lambda \tau(a)} \geq 0$ to the learning externality.

At the market solution, denoted by *, with strategic substitutes ($\beta > 0$) the payoff and learning externalities cancel each other exactly when $\beta\tau = \Delta$, in which case $a^* = a^{\mathrm{T}}$. This happens when $c^* = 0$. We have that $\beta\tau - \Delta > 0$ when $c^* > 0$ and $\beta\tau - \Delta < 0$ when $c^* < 0$ (see Lemma A2 in the Appendix). This suggests that $a^* < a^{\mathrm{T}}$ when $c^* < 0$ and $a^* > a^{\mathrm{T}}$ when $c^* > 0$. The first case happens when τ_u is large, the supply

function is downward sloping because the informational component of the price prevails, and the learning externality wins over the payoff externality.[14] The second case happens when τ_u is low, the supply function is upward sloping because the allocational effect of the price prevails, and the learning externality is overpowered by the payoff externality. With strategic complements ($\beta < 0$) the payoff and learning externalities reinforce each other and $a^* < a^{\mathrm{T}}$.

When $\beta > 0$ and firms do not respond to the price ($c^* = 0$), the model is equivalent to a quantity-setting model with private information. Indeed, the strategy used by a firm reduces to a Cournot strategy because, in the given parameter constellation, the allocation weight to the price in the supply function $X(s_i, p) = \lambda^{-1}(p - \mathbb{E}[\theta | s_i, p])$, equal to 1, exactly matches its informational weight (the weight to the price in $\mathbb{E}[\theta | s_i, p]$).

The result is given in the following proposition (see the Appendix for a detailed proof).

<u>Proposition 2</u>. *Let $\infty > \tau_\varepsilon > 0$. Then the team problem has a unique solution with*

$$\lambda^{-1} > a^{\mathrm{T}} > 0 \text{ and } \mathrm{sgn}\{a^* - a^{\mathrm{T}}\} = \mathrm{sgn}\{\beta c^*\}.$$

From the expression for WL we obtain directly that $\left. \dfrac{d\mathrm{WL}}{da} \right|_{a=a^*} = \lambda a^* \sigma_\varepsilon^2 \beta c^*$ and WL is strictly convex with one minimum. The result follows since $a^* > 0$ when $\tau_\varepsilon > 0$. An alternative argument which isolates the effect of the externalities associated to the use of private information because agents use price-contingent strategies is as follows. The strategy for firm i is of the form $x_i = \lambda^{-1}(p - (\lambda a s_i + (1 - \lambda a)\hat{z}))$, where $\hat{z} = \mathbb{E}[\theta / z]$, $z = \beta a \theta + u$. We have that

$$\frac{\partial x_i}{\partial a} = \left. \frac{\partial x_i}{\partial a} \right|_{p,\hat{z} \text{ ct.}} + \frac{\partial x_i}{\partial p} \left. \frac{\partial p}{\partial a} \right|_{\hat{z} \text{ ct.}} + \frac{\partial x_i}{\partial \hat{z}} \frac{\partial \hat{z}}{\partial a}$$

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[14]   Recall that c^* is decreasing in τ_u.

where the first term corresponds to market behavior, the second to the payoff externality with exogenous public information, and the last term to the learning externality:

$$\frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a} = \left(\frac{\partial x_i}{\partial p}\frac{\partial p}{\partial \hat{z}} + \frac{\partial x_i}{\partial \hat{z}} \right)\frac{\partial \hat{z}}{\partial a}.$$

We obtain that at the market solution the learning externality has the expected sign

$$\mathbb{E}\left[(p - \mathrm{MC}(x_i))\left(\frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a} \right) \right] > 0$$

and adding up both externality effects delivers the desired result $\mathrm{sgn}\{\partial \mathbb{E}[\mathrm{TS}]/\partial a\} = \mathrm{sgn}\{-\beta c^*\}$.

If $\beta = 0$ then there is neither a learning nor a payoff externality, and the team and market solutions coincide. For $\beta \neq 0$, $\tau_\varepsilon > 0$, and $\tau_u > 0$, the solutions coincide only if $c^* = 0$. When signals are uninformative ($\tau_\varepsilon = 0$) or perfect ($\tau_\varepsilon = \infty$) there is no private information, there is no learning externality and the payoff externality is internalized at the competitive equilibrium. As a result the team and the market solution coincide (with $a = 0$ when $\tau_\varepsilon = 0$). When the price contains no information ($\tau_u = 0$) there is no learning externality, $c = 1/(\beta + \lambda)$ both for the team and the market solutions, and only the payoff externality remains with the result that $\mathrm{sgn}\{a^* - a^{\mathrm{T}}\} = \mathrm{sgn}\{\beta\}$ (as in Lemma 1).

In conclusion, in the usual case with strategic substitutability, $\beta > 0$, and upward sloping supply functions, $c^* > 0$, the price is too informative about θ, and there is too much dispersion and productive inefficiency. In the electricity example, the price of electricity would be too informative about the pollution damage. With downward sloping supply functions, $c^* < 0$, the price contains too little information about θ, and there is too much aggregate inefficiency. With strategic complementarity ($\beta < 0$) agents give insufficient weight to private information and the market displays too much aggregate inefficiency.

<u>Corollary (market quality)</u>. *At the market solution:*

- *In relation to the team optimum, when $\beta c^* > 0$ price informativeness τ and dispersion $\mathbb{E}\left[(x_i - \tilde{x})^2\right]$ are too high, and market depth $1 + \beta\hat{c}$ and non-fundamental volatility $\mathbb{E}\left[(\tilde{x} - x^o)^2\right]$ too low. The opposite is true when $\beta c^* < 0$.*

- *In relation to the first best (where $\mathbb{E}\left[(\tilde{x} - x^o)^2\right] = \mathbb{E}\left[(x_i - \tilde{x})^2\right] = 0$), price informativeness and market depth are too low, and non-fundamental volatility and dispersion are too high.*

<u>Remark 3.</u> If the signals of agents can be communicated to a center, then questions arise concerning the incentives to reveal information and how welfare allocations may be modified. This issue is analyzed in a related model by Messner and Vives (2006), who use a mechanism design approach along the lines of Laffont (1985).

4.3 <u>Can more information hurt</u>?

The question arises as of how the welfare loss WL at the market solution depends on information precisions τ_ε, τ_u and τ_θ. We know that WL is a strictly convex function of a attaining a minimum at the team-efficient solution a^T. It is immediate then that $\text{WL}(a^T)$ is decreasing in τ_ε, τ_u and τ_θ. This is so since WL is decreasing in τ_ε, τ_u and τ_θ for a given a and $d\text{WL}(a^T)/da = 0$. Things are potentially different at the market solution a^* since then $d\text{WL}(a^*)/da$ is positive or negative depending on whether $a^* > a^T$ or $a^* < a^T$. Since a^* is decreasing in τ_u and τ_θ, and increasing in τ_ε (see Proposition 5) we have thus that $\text{WL}(a^*)$ is decreasing in τ_u and τ_θ when $a^* > a^T$ and in τ_ε when $a^* < a^T$. It would be possible in principle that increasing precisions of public information τ_u and τ_θ increases the welfare loss when $a^* < a^T$ when the direct effect of the increase of τ_u or τ_θ is dominated by the indirect effect via the induced decrease in a^* (and similarly for an increase in τ_ε when $a^* > a^T$). We can check, however, that $\text{WL}(a^*)$ is always decreasing in τ_θ and τ_u. This need not be the case when changing τ_ε. In any case, as the information precisions τ_θ, τ_u,

and τ_ε tend to infinity $\mathrm{WL}\left(a^*\right)$ tends to 0.[15] The following proposition summarizes the results.

Proposition 3. *The welfare loss at the team-efficient solution is decreasing in* τ_ε, τ_u *and* τ_θ. *The welfare loss at the market solution is also decreasing in* τ_θ *and* τ_u, *and it may be decreasing or increasing in* τ_ε *(it will be increasing for* $\beta > \lambda$ *and* $\tau_\varepsilon / \tau_\theta$ *small enough). As any of the information precisions* τ_θ, τ_u, *and* τ_ε *tend to infinity welfare losses tend to zero.*

More precise public or private information reduces the welfare loss at the team-efficient solution. This is in accordance with the results in Angeletos and Pavan (2007, 2009) where more information can not hurt when it is used efficiently. The welfare loss at the market solution is also always decreasing with the precision of public information. However, the welfare loss at the market solution may be increasing with the precision of private information when the market calls already for a too large response to private information. The reason is that an increase in the precision of private information will increase the response of an agent to his private signal and this indirect effect may dominate.

The welfare result of the market solution is in contrast with received results in the literature where more *public* information may be damaging to welfare (Burguet and Vives 2000; Morris and Shin 2002; Angeletos and Pavan 2007; Amador and Weill 2010, 2011). In those papers more public information discourages the use and/or acquisition of private information. In the present paper this also happens but the direct effect of public information provision prevails.

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[15] This follows since as $\tau_\varepsilon \to \infty$, $a^* \to \lambda^{-1}$; and as τ_θ or $\tau_u \to \infty$, $a^* \to 0$ and $\tau \to \infty$.

5. <u>Extensions and complementary results</u>

5.1 <u>Internal welfare benchmark</u>

A different benchmark is provided by the collective welfare of the players, the producers in our case. At the internal team–efficient solution, expected average profit $\mathbb{E}[\tilde{\pi}]$ (where $\tilde{\pi} = \int_0^1 \pi_i \, di$ and $\pi_i = (\alpha + u - \beta\tilde{x} - \theta) x_i - (\lambda/2) x_i^2$) is maximized under the constraint that agents use decentralized linear strategies. Since the solution is symmetric we have that $\mathbb{E}[\tilde{\pi}] = \mathbb{E}[\pi_i]$. This is the cooperative solution from the firms' perspective. That is,

$$\max_{a,b,c} \mathbb{E}[\pi_i]$$

$$\text{subject to} \quad x_i = b - as_i + cz, \quad \tilde{x} = b - a\theta + cz, \quad \text{and} \quad z = u + \beta a\theta.$$

It should be clear that the market solution, not even with complete information, will attain the full information cooperative outcome (denoted M for monopoly, for which $x^M = (\lambda + 2\beta)^{-1}(\alpha + u - \theta)$) where joint profits are maximized under full information. This is so since the market solution does not internalize the competition (payoff) externality and therefore if $\beta \neq 0$ it will produce an expected output $\mathbb{E}[\tilde{x}^*] = \alpha(\beta + \lambda)^{-1}$ which is too high (low) with strategic substitutes (complements) in relation to the optimal $\mathbb{E}[x^M] = \alpha(2\beta + \lambda)^{-1}$. Furthermore, the market solution does not internalize the externalities in the use of information arising from price-contingent strategies. At the *internal team* (IT) benchmark, joint profits are maximized and information-related externalities internalized with decentralized strategies.[16] The question is whether the market solution allocates the correct weights (from the players' collective welfare viewpoint) to private and public information. We show that the answer to this question is qualitatively similar to the one derived when analyzing the *total surplus* team benchmark but in this case with a larger bias towards the market displaying too much weight on private information.

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[16] Indeed, when $\beta = 0$ there are no externalities (payoff or informational) and the internal team and market solutions coincide.

As before, it can be seen that the internal team-efficient solution minimizes, over the restricted strategies, the expected loss Ω with respect to the full information cooperative outcome x^M, and that

$$\Omega = \left(\left(2\beta + \lambda \right) \mathbb{E}\left[\left(\tilde{x} - x^M \right)^2 \right] + \lambda \mathbb{E}\left[\left(x_i - \tilde{x} \right)^2 \right] \right) \Big/ 2.$$

The first term in the sum corresponds to aggregate inefficiency in the average quantity, which is proportional to $\mathbb{E}\left[\left(\tilde{x} - x^M \right)^2 \right]$, and the second term to productive inefficiency, which is proportional to $\mathbb{E}\left[\left(x_i - \tilde{x} \right)^2 \right]$.

It can be checked that the form of the internal optimal team strategy is $x_i = \left(\lambda + \beta \right)^{-1} \left(p - \left(\gamma s_i + \left(1 - \gamma \right) \mathbb{E}\left[\theta \mid z \right] \right) \right)$ where $\gamma = \left(\lambda + \beta \right) a$ (while at the market solution we have that $\gamma = \lambda a$). The loss at any candidate internal team solution (which internalizes the competition payoff externality and for which $\mathbb{E}\left[\tilde{x} \right] = \alpha \left(2\beta + \lambda \right)^{-1}$) will depend only on the response to private information a since at this candidate solution we have $\mathbb{E}\left[\left(\tilde{x} - x^M \right)^2 \right] = \left(1 - \left(\lambda + \beta \right) a \right)^2 \Big/ \left(\tau \left(2\beta + \lambda \right)^2 \right)$ and $\mathbb{E}\left[\left(x_i - \tilde{x} \right)^2 \right] = a^2 / \tau_\varepsilon$. This yields a strictly convex Ω as a function of a. As before, changing a has opposite effects on both sources of the loss. Now the internal team solution optimally trades off the sources of the loss with respect to the responsiveness to private information among decentralized strategies which internalize the competition payoff externality.

In this case at the market solution there is as before a combined, payoff and learning, price-contingent strategy externality (PE+LE), and also a competition payoff (CE) externality through the impact of aggregate output on price in the use of information, since even with full information the market solution is not efficient (i.e. cooperative). The impact of the externalities on the response to private information can be assessed similarly as before. The market takes the public statistic z or p as given while the internal team solution takes into account all externalities:

$$\frac{\partial \mathbb{E}[\pi_i]}{\partial a} = \underbrace{\mathbb{E}\left[(p - MC(x_i))\left(\frac{\partial x_i}{\partial a}\right)_{z\ ct.}\right]}_{\text{Market}} +$$

$$\underbrace{\mathbb{E}\left[(p - MC(x_i))\left(\frac{\partial x_i}{\partial z}\frac{\partial z}{\partial a}\right)\right]}_{\text{PE+LE}} +$$

$$\underbrace{\mathbb{E}\left[x_i\left(\frac{\partial p}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial a}\right)\right]}_{\text{CE}}.$$

The market term is null at the market solution and the sum of the PE+LE and CE terms can be evaluated as follows:

$$\frac{\partial \mathbb{E}[\pi_i]}{\partial a}\bigg|_{a=a^*} = -\beta a^* \left(c^* \lambda \sigma_\varepsilon^2 + \left(c^* \beta - 1\right)^2 \sigma_\theta^2\right).$$

It is worth noting that while, as before, $\mathrm{sgn}\{\text{PE+LE}\} = \mathrm{sgn}\{-\beta c^*\}$ we have that $\mathrm{sgn}\{\text{CE}\} = \mathrm{sgn}\{-\beta\}$ since $\left(c^* \beta - 1\right)^2 \sigma_\theta^2 > 0$, and therefore the CE term will call for a lower (higher) response to private information with strategic substitutes (complements) than the market solution. If $\beta > 0$ a high price indicates high costs. If, say, costs are high ($\theta - \bar{\theta} > 0$) then an increase in a will increase p ($\frac{\partial p}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial a} = -\beta(c\beta - 1)(\theta - \bar{\theta}) > 0$ since at the market solution $c\beta - 1 < 1$) while x_i will tend to be low (since at the market solution $\mathbb{E}\left[(\theta - \bar{\theta})x_i\right] = a\sigma_\theta^2(c\beta - 1) < 0$). This means that if $\beta > 0$, $\text{CE} < 0$ and a must be reduced. Similarly, we have that $\text{CE} > 0$ if $\beta < 0$. The results on the payoff externality CE are in line with the results obtained by Angeletos and Pavan (Section 6.5, 2007) with non price-contingent strategies. We will see how the effect of the PE+LE term may overturn this result when $c < 0$.

The next proposition characterizes the response to private information.

Proposition 4. *Let* $\infty > \tau_\varepsilon > 0$. *Then the internal team problem has a unique solution with* $(\lambda + \beta)^{-1} > a^{\text{IT}} > 0$, *and* $\mathrm{sgn}\{a^* - a^{\text{IT}}\} = \mathrm{sgn}\left\{\beta\left(c^* \lambda \sigma_\varepsilon^2 + \left(c^* \beta - 1\right)^2 \sigma_\theta^2\right)\right\}$.

If $c^* \geq 0$ then $\mathrm{sgn}\{a^* - a^{\mathrm{IT}}\} = \mathrm{sgn}\{\beta\}$. Therefore, as before, under strategic complements ($\beta < 0$), there is too little response to private information, $a^* < a^{\mathrm{IT}}$. Indeed, the characterization yields the same qualitative result as in the previous section if $c^* > 0$: too much or too little response to private information in the presence of (respectively) strategic substitutability or strategic complementarity. In this case, however, if agents use Cournot strategies (i.e., if $c^* = 0$) then the market is not internal team–efficient. This should not be surprising when one considers that, when $c^* = 0$, the combined externality for the use of price-contingent strategies is nil, yet the competition payoff externality is not internalized, as agents set a quantity that is too large (small) under strategic substitutability (complementarity). If $\beta > 0$ and $c^* < 0$, then $c^* \lambda \sigma_\varepsilon^2 + (c^* \beta - 1)^2 \sigma_\theta^2 > 0$ for c^* close to zero or sufficiently negative (τ_u large). Only for intermediate values of c^* we have $c^* \lambda \sigma_\varepsilon^2 + (c^* \beta - 1)^2 \sigma_\theta^2 < 0$ and $a^{\mathrm{IT}} > a^*$. With strategic substitutes the market will bias the solution more towards putting too high a weight on private information since we may have $c^* \lambda \sigma_\varepsilon^2 + (c^* \beta - 1)^2 \sigma_\theta^2 > 0$ even if $c^* < 0$.

This is the same qualitative result concerning the response to private information as derived previously using the total surplus team benchmark—with the following proviso: when $c^* < 0$, it need not be the case that there is too little response to private information.

Remark 4. The weights to private information in the internal team and market solutions are, respectively, $\gamma^{\mathrm{IT}} = (\lambda + \beta) a^{\mathrm{IT}}$ and $\gamma^* = \lambda a^*$. It is easy to see that for τ_u small enough (and $\tau_\theta > 0$) we have that $\gamma^{\mathrm{IT}} > \gamma^*$. The same result applies when $\beta > 0$ and $c^* \lambda \sigma_\varepsilon^2 + (c^* \beta - 1)^2 \sigma_\theta^2 < 0$ in which case $a^{\mathrm{IT}} > a^*$ and therefore $\lambda \gamma^{\mathrm{IT}} > (\lambda + \beta) \gamma^* > \lambda \gamma^*$.

5.2. Comparative statics and the value of information

This section studies the comparative statics properties of the equilibrium and how the weights and the responses to public and private information vary with underlying

parameters. The following proposition presents a first set of results. The effects of changes in the degree of complementarity are dealt with afterwards.

<u>Proposition 5</u>. *Let* $\tau_\varepsilon > 0$ *and* $\tau_u > 0$. *In equilibrium, the following statements hold.*

(i) *Responsiveness to private information* a *decreases from* $\lambda^{-1}\tau_\varepsilon(\tau_\theta + \tau_\varepsilon)^{-1}$ *to* 0 *as* τ_u *ranges from* 0 *to* ∞, *decreases with* τ_θ, $|\beta|$ *and* λ, *and increases with* τ_ε.

(ii) *Responsiveness to the public statistic* $\hat{c}$ *goes from* λ^{-1} *to* $-\beta^{-1}$ *as* τ_u *ranges from* 0 *to* ∞. *Furthermore,* $\mathrm{sgn}\{\partial\hat{c}/\partial\tau_u\} = \mathrm{sgn}\{-\partial\hat{c}/\partial\tau_\theta\} = \mathrm{sgn}\{-\beta\}$ *and* $\mathrm{sgn}\{\partial\hat{c}/\partial\tau_\varepsilon\} = \mathrm{sgn}\{\beta(\beta^2\tau_u\tau_\varepsilon^2 + 4\lambda^2\tau_\theta^2(\tau_\varepsilon - \tau_\theta))\}$. *Market depth* $1 + \beta\hat{c}$ *is decreasing in* τ_u *and increasing in* τ_θ.

(iii) *Price informativeness* τ *is increasing in* $|\beta|$, τ_u, τ_θ *and* τ_ε, *and decreasing in* λ.

(iv) *Dispersion* $\mathbb{E}\left[(x_i - \tilde{x})^2\right]$ *decreases with* τ_u, τ_θ, $|\beta|$ *and* λ.

How the equilibrium weights to private and public information vary with the deep parameters of the model help to explain the results. We have that $\mathbb{E}[\theta/s_i, z] = \gamma s_i + hz$ where $h = \beta a\tau_u(\tau_\varepsilon + \tau)^{-1}$. Identify the informational component of the price with the weight $|h|$ on public information z, with $\mathrm{sgn}\{h\} = \mathrm{sgn}\{\beta\}$. When $\beta > 0$ there is adverse selection (a high price is bad news about costs) and $h > 0$ while when $\beta < 0$, $h < 0$ and there is favorable selection (a high price is good news). We have that $\mathrm{sgn}\{\partial|h|/\partial\beta\} = \mathrm{sgn}\{\beta\}$. As β is decreased from $\beta > 0$ adverse selection is lessened, and when $\beta < 0$ we have favorable selection with $h < 0$ and $\partial|h|/\partial\beta < 0$. The result is that an increase in $|\beta|$ increases the public precision[17] τ and decreases the response to private information. We have also that increasing the

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[17] An increase in $|\beta|$ has a direct positive effect on τ and an indirect negative effect via the induced change in a. The direct effect prevails. Note that changing β modifies not only the public statistic p but also the degree of complementarity in the payoff.

precision of the prior decreases the informational component of the price, $\partial |h|/\partial \tau_\theta < 0$, while that increasing the precision of the noise in the price increases it, $\partial |h|/\partial \tau_u > 0$. (See Claim 2 in the Appendix.) The effect of τ_ε is ambiguous.

Consider first the case $\beta > 0$. As τ_u increases from 0, $\hat{c}$ decreases from λ^{-1} (and the slope of supply increases) because of the price's increased informational component $h > 0$. Agents are more cautious when seeing a high price because it may mean higher costs. As τ_u increases more, $\hat{c}$ becomes zero at some point and then turns negative; as τ_u tends to ∞, $\hat{c}$ tends to $-\beta^{-1}$.[18] At the point where the scarcity and informational effects balance, agents place zero weight ($\hat{c} = 0$) on the price. If τ_θ increases then the informational component of the price diminishes since the agents are now endowed with better prior information, and induces a higher $\hat{c}$ (and a more elastic supply). An increase in the precision of private information τ_ε always increases responsiveness to the private signal but has an ambiguous effect on the slope of supply. The parameter $\hat{c}$ is U-shaped with respect to τ_ε. Observe that $\hat{c} = \lambda^{-1}$ not only when $\tau_\varepsilon = \infty$ but also when $\tau_\varepsilon = 0$ and that $\hat{c} < \lambda^{-1}$ for $\tau_\varepsilon \in (0, \infty)$. If τ_ε is high, then a further increase in τ_ε (less noise in the signals) lowers adverse selection (and h) and increases $\hat{c}$. If τ_ε is low then the price is relatively uninformative, and an increase in τ_ε increases adverse selection (and h) while lowering $\hat{c}$.

If $\beta < 0$ then a high price conveys goods news in terms of both scarcity effects and informational effects, so supply is always upward sloping in this case. Indeed, when $\beta < 0$ we have $\hat{c} > \lambda^{-1}$. A high price conveys the good news that average quantity tends to be high and that costs therefore tend to be low ($h < 0$). In this case, increasing τ_u, which reinforces the informational component of the price, increases $\hat{c}$—the opposite of what happens when τ_θ increases. An increase in the precision of private information τ_ε increases responsiveness to the private signal but, as before,

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[18]   See Wilson (1979) for a model in which adverse selection makes demand schedules upward sloping.

has an ambiguous effect on the slope of supply. Now the parameter $\hat{c}$ is hump-shaped with respect to τ_ε because $\hat{c} > \lambda^{-1}$ for $\tau_\varepsilon \in (0, \infty)$ and $\hat{c} = \lambda^{-1}$ in the extremes of the interval $(0, \infty)$.

In either case ($\beta > 0$ or $\beta < 0$) market depth $(\partial P / \partial u)^{-1} = 1 + \beta \hat{c}$ is decreasing in τ_u and increasing in τ_θ.

Table 1 summarizes the comparative statics results on the equilibrium strategy.

<u>Table 1:</u> Comparative Statics on the Equilibrium Strategy

sgn	$\partial \tau_u$	$\partial \tau_\theta$	$\partial \tau_\varepsilon$
∂a	$-$	$-$	$+$
$\partial \hat{c}$	$-\beta$	β	$\beta \left(\beta^2 \tau_u \tau_\varepsilon^2 + 4\lambda^2 \tau_\theta (\tau_\varepsilon - \tau_\theta) \right)$

The degree of complementarity $m \equiv -\beta / \lambda$ depends on λ for a fixed β (it makes sense to keep β fixed since β also affects the public statistic $p = \alpha + u - \beta \tilde{x}$). For fixed β we have that $\text{sgn}\{\partial m / \partial \lambda\} = \text{sgn}\{\beta\}$. From Proposition 5 we have then that $\text{sgn}\{\partial a / \partial m\} = \text{sgn}\{-\beta\}$, $\text{sgn}\{\partial \gamma / \partial m\} = \text{sgn}\{\beta\}$, and $\text{sgn}\{\partial \tau / \partial m\} = \text{sgn}\{\partial \mathbb{E}[(x_i - \tilde{x})^2] / \partial m\} = \text{sgn}\{-\beta\}$. The results are summarized in Table 2.

<u>Table 2:</u> Effects of a Change in the Degree of Complementarity (β fixed)

sgn	∂a	$\partial \gamma$	$\partial \tau$	$\mathbb{E}\left[(x_i - \tilde{x})^2\right]$
∂m	$-\beta$	β	$-\beta$	$-\beta$

Increased reliance on public information as complementarity increases is a general theme in the work of Morris and Shin (2002) and Angeletos and Pavan (2007) when public signals are exogenous. In stylized environments more complementarity

increases the value of public information in forecasting aggregate behavior and decreases the dispersion of actions (e.g., Cor. 1 in Angeletos and Pavan 2007). In our model this happens in the strategic substitutes case ($\beta > 0$). With strategic complements ($\beta < 0$) an increase in m (a lower λ) makes agents rely less on private information (γ decreases) but respond more to private information (a increases), and increases dispersion as well as increases the precision of public information. (See Table 2.)

6. Other interpretations of the model and applications.

In this section we extend the interpretation of the model to other applications.

6.1 *Monopolistic competition*. The model applies also to a monopolistically competitive market with quantity-setting firms; in this case, either $\beta > 0$ (goods are substitutes) or $\beta < 0$ (goods are complements). Firm i faces the inverse demand for its product, $p_i = \alpha + u - \beta \tilde{x} - (\lambda/2) x_i$, and has costs θx_i . Each firm uses a supply function that is contingent on its own price: $X(s_i, p_i)$ for firm i . It follows then that observing the price p_i is informationally equivalent (for firm i) to observing $p \equiv \alpha + u - \beta \tilde{x}$.

Under monopolistic competition, the total surplus function (consistent with the differentiated demand system) is slightly different:

$$ \text{TS} = (\alpha + u - \theta)\tilde{x} - \left(\beta \tilde{x}^2 + (\lambda/2)\int_0^1 x_i^2 \, di\right) \Big/ 2 . $$

Here the market is not efficient under complete information because price is not equal to marginal cost. Each firm has some residual market power. The results of Section 4 do not apply but those of Section 5.1 apply when firms collude. It is interesting to note then that, if agents cannot use price-contingent strategies (as in the cases of Cournot or Bertrand competition), Angeletos and Pavan (Section 6.5, 2007) argue that the strategic substitutability case would exhibit always excessive response to private information in contrast with the case with supply functions as strategies, where either excessive or insufficient response to private information is possible.

6.2 *Demand schedule competition*. Let a buyer of a homogenous good with unknown ex post value θ face an inverse supply $p = \alpha + u + \beta \tilde{y}$, where $\tilde{y} = \int_0^1 y_i \, di$ and y_i is the demand of buyer i. The suppliers face a cost of supply of $\left(\alpha + u + \beta \frac{\tilde{y}}{2} \right) \tilde{y}$. The buyer's net benefit is given by $\pi_i = (\theta - p) y_i - (\lambda/2) y_i^2$, where λy_i^2 is a transaction or opportunity cost (or an adjustment for risk aversion). The model fits this setup if we let $y_i \equiv -x_i$. Some examples follow.

Firms purchasing labor. A firm purchases labor whose productivity θ is unknown— say, because of technological uncertainty—and faces an inverse linear labor supply (with $\beta > 0$) and quadratic adjustment costs in the labor stock. The firm has a private assessment of the productivity of labor, and inverse supply is subject to a shock. In particular, the welfare analysis of Section 4 applies letting $y_i \equiv -x_i$.

Traders in a financial market. Traders compete in demand schedules for a risky asset with liquidation value θ and face a quadratic adjustment cost in their position (alternatively, the parameter λ proxies for risk aversion). Each trader receives a private signal about the liquidation value of the asset. There are also liquidity suppliers who trade according to the elastic aggregate demand $(\alpha + u - p)/\beta$, where u is random. We can interpret $1/|\beta|$ as the mass of liquidity suppliers. When $\beta > 0$, liquidity suppliers buy (sell) when the price is low (high); when $\beta < 0$, liquidity suppliers buy (sell) when the price is high (low). In this case liquidity suppliers may be program traders following a portfolio insurance strategy.[19] Our inverse supply follows from the market-clearing equation. It is worth noting that the normal case with $\beta > 0$ induces strategic substitutability in the actions of informed traders, while when $\beta < 0$ we have strategic complementarity in the actions of informed traders and the slope of excess demand $\Xi' = -\left(\beta^{-1} + \hat{c} \right)$ is positive.[20]

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[19] As in Gennotte and Leland (1990). Hendershott and Seasholes (2009) find that program trading accounts for almost 14% of the average daily market volume at the NYSE in 1999-2005 and that program traders lose money on average.

[20] See Shin (2010) for an explanation of upward sloping asset demand based on risk management considerations.

Increasing the mass of liquidity suppliers (i.e. decreasing $|\beta|$) increases the weight given to the private information by informed traders but decreases the informativeness of prices: $\tau = \tau_\theta + \beta^2 a^2 \tau_u$ (and $\beta^2 a^2$ is increasing in $|\beta|$, see Proposition 5). The direct effect on τ prevails over the indirect effect.

In the normal case with $\beta > 0$ and downward-sloping demand schedules for informed traders, prices will contain too much information (from the perspective of total surplus) about the value of the asset. This will happen when the volume of liquidity trading is high (i.e. when $\beta^2 \tau_u$ is low). In this case the response of informed traders induces excessive volatility of prices/quantities from perspective of liquidity traders (not protected by price contingent trades) and total surplus. In the region where demand schedules for the informed are upward sloping, prices will contain too little information about θ (with the total surplus benchmark). The same applies in the case $\beta < 0$.

Asset auctions. Consider the auction of a financial asset for which (inverse) supply is price elastic: $p = \alpha + \beta \hat{y}$ with $\beta > 0$, where $\hat{y}$ is the total quantity bid. The liquidation value θ of the asset may be its value in the secondary market (say, for a central bank liquidity or Treasury auction). The marginal valuation of a bidder is decreasing in the amount bid.[21] Each bidder receives a private signal about θ, and there are noncompetitive bidders who bid according to u/β. This setup yields $\hat{y} = \tilde{y} + u/\beta$, where $\tilde{y}$ is the aggregate of competitive (informed) bids, and an effective inverse supply for the competitive bidders: $p = \alpha + u + \beta \tilde{y}$.

From the viewpoint of general welfare or of competitive bidders prices contain too much information in the usual case of downward-sloping demand schedules, which obtain when the volume of *noncompetitive bidding* is large (low τ_u). When the volume generated by noncompetitive bids is small (high τ_u), demand schedules for the competitive bidders are upward sloping and prices will contain too little information from the perspective of total surplus.

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[21]   A justification for the case of liquidity auctions is given in Ewerhart, Cassola, and Valla (2009).

7. <u>Concluding remarks</u>.

We find that price-contingent strategies, on top of the usual learning externality, introduce a pecuniary externality in the use of private information which sign depends on whether competition is of the strategic substitutes or complements variety. Under strategic substitutability, prices will convey too much information in the normal case where the allocational role of prices prevails over their informational role (weak learning externality) and too little in the opposite situation (strong learning externality). Under strategic complementarity prices always convey too little information. The inefficiency of the market solution opens the door to the possibility that more precise public or private information will lead to an increased welfare loss. This is the case when the market already calls for a too large response to private information, then more precise private information exacerbates the problem (but not more precise public information).

The practical implication of the result is that in market games with strategic substitutes the presumption that prices will contain too little information will typically not hold. Therefore, subsidizing information acquisition is not warranted.

The results extend to an economy which is not efficient with full information. Then the potential bias towards putting too much weight on private information is increased. It follows that received results on the optimal relative weights to be placed on private and public information (when the latter is exogenous) may be overturned when the informational role of the price conflicts with its allocational role and the former is important enough.

Several extensions are worth considering. Examples include exploring tax-subsidy schemes to implement team-efficient solutions along the lines of Angeletos and Pavan (2009), Lorenzoni (2010), and Angeletos and La'O (2012); and studying incentives to acquire information (as in Vives 1988; Burguet and Vives 2000; Hellwig and Veldkamp 2009; Myatt and Wallace 2012; Llosa and Venkateswaran 2012; Colombo et al. 2012).

Appendix

<u>Proof of Proposition 1</u>: From the posited strategy $X\left(s_{i},z\right)=b-as_{i}+cz$, where $z=u+\beta a\theta$ and $1-\beta c\neq0$, we obtain that $p=\alpha-\beta b+\left(1-\beta c\right)z$. From the first-order condition for player i we have

$$X\left(s_{i},z\right)=\lambda^{-1}\left(\alpha-\beta b+\left(1-\beta c\right)z-\mathbb{E}\left[\theta|s_{i},z\right]\right).$$

Here $\mathbb{E}\left[\theta/s_{i},z\right]=\gamma s_{i}+\left(1-\gamma\right)\mathbb{E}\left[\theta/z\right]$ with $\gamma=\tau_{\varepsilon}\left(\tau_{\varepsilon}+\tau\right)^{-1}$, $\mathbb{E}\left[\theta/z\right]=\beta\tau_{u}a\tau^{-1}z$ (recall that we have normalized $\overline{\theta}=0$), and $\tau=\tau_{\theta}+\beta^{2}a^{2}\tau_{u}$ from the projection theorem for Gaussian random variables. Note that $\mathbb{E}\left[\theta/s_{i},z\right]=\gamma s_{i}+hz$ where $h=\beta a\tau_{u}\left(\tau_{\varepsilon}+\tau\right)^{-1}$. Identifying coefficients with $X\left(s_{i},z\right)=b-as_{i}+cz$, we can immediately obtain

$$a=\frac{\gamma}{\lambda}=\frac{\tau_{\varepsilon}}{\lambda\left(\tau_{\varepsilon}+\tau\right)},\quad c=\frac{1-h}{\beta+\lambda}=\frac{1}{\beta+\lambda}-\frac{\beta a\tau_{u}}{\left(\beta+\lambda\right)\left(\tau_{\varepsilon}+\tau\right)},\quad\text{and}\quad b=\frac{\alpha}{\beta+\lambda}.$$

It follows that the equilibrium parameter a is determined as the unique (real), of the following cubic equations, that is positive and lies in the interval $a\in\left(0,\tau_{\varepsilon}\lambda^{-1}\left(\tau_{\theta}+\tau_{\varepsilon}\right)^{-1}\right)$:

$$a=\frac{\tau_{\varepsilon}}{\lambda\left(\tau_{\varepsilon}+\tau_{\theta}+\beta^{2}a^{2}\tau_{u}\right)}\quad\text{or}\quad\beta^{2}\tau_{u}a^{3}+\left(\tau_{\varepsilon}+\tau_{\theta}\right)a-\lambda^{-1}\tau_{\varepsilon}=0$$

and

$$c=\frac{1}{\left(\beta+\lambda\right)}-\frac{\beta\lambda\tau_{u}a^{2}}{\left(\beta+\lambda\right)\tau_{\varepsilon}}.$$

It is immediate from the preceding equality for c that $c<\left(\beta+\lambda\right)^{-1}$ (since $a\geq0$) and that $1-\beta c>0$ (since $\beta+\lambda>0$); therefore,

$$\beta c=\frac{\beta}{\beta+\lambda}-\frac{\beta^{2}a\tau_{u}}{\left(\beta+\lambda\right)\left(\tau_{\varepsilon}+\tau\right)}<1.$$

It follows that

$$X\left(s_{i},p\right)=\hat{b}-as_{i}+\hat{c}p,$$

where $\hat{b} = b(1 - \lambda\hat{c})$, $b = \alpha/(\beta + \lambda)$, and $\hat{c} = c/(1 - \beta c)$ with $1 + \beta\hat{c} > 0$. From the equilibrium expression for $c = (\beta + \lambda)^{-1}(1 - \beta\lambda\tau_u a^2 \tau_\varepsilon^{-1})$ we obtain the expression for

$$\hat{c} = (c^{-1} - \beta)^{-1}. \blacklozenge$$

Claim 1. *Linear equilibria in strategies with bounded means and with uniformly (across players) bounded variances yield linear equilibria of the schedule game for which the public statistic function is of type $P(\theta, u)$.*

Proof: If for player i we posit the strategy

$$x_i = \hat{b}_i + \hat{c}_i p - a_i s_i$$

then the aggregate action is given by

$$\tilde{x} = \int_0^1 x_i \, di = \hat{b} + \hat{c}p - a\theta - \int_0^1 a_i \varepsilon_i \, di = \hat{b} + \hat{c}p - a\theta,$$

where $\hat{b} = \int_0^1 \hat{b}_i \, di$, $\hat{c} = \int_0^1 \hat{c}_i \, di$, and $a = \int_0^1 a_i \, di$ (assuming that all terms are well-defined). Observe that, according to our convention on the average error terms of the signals, $\int_0^1 a_i \varepsilon_i \, di = 0$ a.s. provided that $\text{var}[a_i \varepsilon_i]$ is uniformly bounded across agents (since $\text{var}[\varepsilon_i] = \sigma_\varepsilon^2$, it is enough that a_i be uniformly bounded). In equilibrium, this will be the case. Therefore, if we restrict attention to candidate linear equilibria with parameters a_i uniformly bounded in i and with well-defined average parameters $\hat{b}$ and $\hat{c}$, then $\tilde{x} = \hat{b} + \hat{c}p - a\theta$ and the public statistic function is of the type $P(\theta, u)$. $\blacklozenge$

Lemma A1 (*Cournot*): *Consider the Cournot model of section 4.1 with a noisy quantity signal. Let $\tau_\varepsilon > 0$, then the market solution has a smaller response to private information than the team solution.*

Proof: It can be checked that candidate team strategies are of the same form as the market $X(s_i, z) = \dfrac{\alpha}{\beta + \lambda} - \left(as_i + \left((\beta + \lambda)^{-1} - a\right)\hat{z}\right)$ but with potentially a different response a to private information. We have that:

$$\frac{\partial \mathbb{E}[\text{TS}]}{\partial a} = \mathbb{E}\left[(p - \text{MC}(x_i)) \frac{\partial x_i}{\partial a}\Big|_{\hat{z}\, \text{ct.}} \right] + \mathbb{E}\left[(p - \text{MC}(x_i)) \left(\frac{\partial x_i}{\partial \hat{z}} \frac{\partial \hat{z}}{\partial a} \right) \right]$$

$$= \mathbb{E}\left[(p - \text{MC}(x_i)) \left(-(s_i - \hat{z}) - \left((\beta + \lambda)^{-1} - a \right) \left(\tau_\eta a \tau^{-1} \theta + z \frac{\partial (\tau_\eta a \tau^{-1})}{\partial a} \right) \right) \right]$$

since $\hat{z} = \mathbb{E}[\theta / z] = \tau_\eta a \tau^{-1} z$, $z = a\theta - \eta$. At the market solution

$\mathbb{E}\left[(p - \text{MC}(x_i)) \frac{\partial x_i}{\partial a}\Big|_{\hat{z}\, \text{ct.}} \right] = \mathbb{E}\left[(p - \text{MC}(x_i))(s_i - \hat{z}) \right] = 0$ since firms take z as given,

$\mathbb{E}\left[(p - \text{MC}(x_i)) \hat{z} \right] = \mathbb{E}\left[(p - \text{MC}(x_i)) z \right] = 0$. We have that at the market solution

$0 < a < (\beta + \lambda)^{-1}$ since $\tau_\varepsilon > 0$, and

$$\mathbb{E}\left[(p - \text{MC}(x_i))(s_i - \hat{z}) \right] = \mathbb{E}\left[(p - \text{MC}(x_i))(\theta - \hat{z}) \right] + \mathbb{E}\left[(p - \text{MC}(x_i)) \varepsilon_i \right] = 0 .$$

Therefore,

$$\mathbb{E}\left[(p - \text{MC}(x_i))(\theta - \hat{z}) \right] = \mathbb{E}\left[(p - \text{MC}(x_i)) \theta \right] = -\mathbb{E}\left[(p - \text{MC}(x_i)) \varepsilon_i \right]$$

$$= \mathbb{E}\left[\text{MC}(x_i) \varepsilon_i \right] = \mathbb{E}\left[(\theta + \lambda x_i) \varepsilon_i \right] = -\lambda a \sigma_\varepsilon^2 < 0$$

since ε_i is independent of θ . We conclude that $\mathbb{E}\left[(p - \text{MC}(x_i)) \theta \right] = -\lambda a \sigma_\varepsilon^2$ and

therefore $\partial \mathbb{E}[\text{TS}]/\partial a = \lambda \tau_\varepsilon^{-1} \left((\beta + \lambda)^{-1} - a \right) \tau_\eta a^2 \tau^{-1} > 0$. Furthermore, it can be

checked that $\mathbb{E}[\text{TS}]$ is strictly concave in a and we can conclude that the team

solution calls for a larger response to private information than the market. ◆

<u>Lemma A2</u>: *When $\beta > 0$, at the market solution* $\text{sgn}\{\beta \tau - \Delta\} = \text{sgn}\{c^*\}$.

<u>Proof</u>: When at the market solution we have that $c^* = 0$ then $\beta \tau = \Delta$. This is so since

we can check that $\Delta = \frac{\tau \tau_\varepsilon (1 - (\beta + \lambda)c)^2}{\lambda \tau_u a^2}$ and therefore $\beta \tau = \Delta$ is equivalent to $\beta = \frac{\tau_\varepsilon}{\lambda \tau_u a^2}$

when $c = 0$. The result follows since at the market equilibrium

$c = (\beta + \lambda)^{-1} \left(1 - \beta \lambda \tau_u a^2 \tau_\varepsilon^{-1} \right)$ (from Proposition 1) and therefore $1 = \beta \lambda \tau_u a^2 \tau_\varepsilon^{-1}$ when

$c^* = 0$. At the market solution, when $c^* > 0$ we have that $\beta \tau - \Delta > 0$ and when

$c^* < 0$ we have that $\beta \tau - \Delta < 0$. The reason is that when $\beta > 0$, c^* and a^* move

together and therefore when they increase $\frac{\Delta}{\tau} = \frac{\tau_\varepsilon (1 - (\beta + \lambda)c)^2}{\lambda \tau_u a^2}$ decreases. ◆

<u>Proof of Proposition 2</u>: It can be checked that $\partial^2 \mathbb{E}[\text{TS}]/\partial^2 b < 0$ and $\partial^2 \mathbb{E}[\text{TS}]/\partial^2 c < 0$ whenever $\beta + \lambda > 0$. Given that $\partial x_i / \partial b = 1$, and $\partial x_i / \partial c = z$, we can optimize with respect to b and c to obtain

$$\frac{\partial \mathbb{E}[\text{TS}]}{\partial b} = \mathbb{E}\left[\left(p - \text{MC}(x_i)\right)\right] = 0,$$

$$\frac{\partial \mathbb{E}[\text{TS}]}{\partial c} = \mathbb{E}\left[\left(p - \text{MC}(x_i)\right)z\right] = 0,$$

where $p = \alpha + u - \beta \tilde{x}$ and $\text{MC}(x_i) = \theta + \lambda x_i$. The constraint $\mathbb{E}\left[p - \text{MC}(x_i)\right] = 0$ is equivalent to $b = \alpha/(\beta + \lambda)$ and $\mathbb{E}\left[\left(p - \text{MC}(x_i)\right)z\right] = 0$ is equivalent to

$c = c(a) \equiv \dfrac{1}{\beta + \lambda} - \dfrac{\beta a \tau_u (1 - \lambda a)}{\tau(\beta + \lambda)}$. Those constraints are also fulfilled by the market

solution since the first-order condition (FOC) for player i is $\mathbb{E}\left[p - \text{MC}(x_i) \mid s_i, z\right] = 0$, from which it follows, according to the properties of Gaussian distributions, that $\mathbb{E}\left[p - \text{MC}(x_i)\right] = 0$, and $\mathbb{E}\left[\left(p - \text{MC}(x_i)\right)z\right] = 0$ (as well as $\mathbb{E}\left[\left(p - \text{MC}(x_i)\right)s_i\right] = 0$,

which is equivalent to $c = \dfrac{a(\lambda(\tau_\varepsilon + \tau_\theta) + \beta \tau_\varepsilon) - \tau_\varepsilon}{a\beta\tau_\varepsilon(\lambda + \beta)}$).

It follows that the form of the team optimal strategy is $x_i = \lambda^{-1}\left[p - \left(\gamma s_i + (1 - \gamma)\mathbb{E}[\theta \mid z]\right)\right]$ where $\gamma = \lambda a$. We have that $\tilde{x} = \lambda^{-1}\left[p - \left(\gamma \theta + (1 - \gamma)\mathbb{E}[\theta \mid z]\right)\right]$, $\tilde{x} = \left(\alpha + u - \left(\gamma \theta + (1 - \gamma)\mathbb{E}[\theta \mid z]\right)\right)/(\beta + \lambda)$ solving for p, and therefore $\tilde{x} - x^o = (1 - \gamma)\left(\theta - \mathbb{E}[\theta \mid z]\right)/(\beta + \lambda)$. Since $\tau = \left(\text{var}[\theta \mid z]\right)^{-1}$ we obtain $\mathbb{E}\left[\left(\tilde{x} - x^o\right)^2\right] = (1 - \lambda a)^2 / \left(\tau(\beta + \lambda)^2\right)$. Similarly we obtain $x_i - \tilde{x} = -\lambda^{-1}\gamma(s_i - \theta) = -\lambda^{-1}\gamma\varepsilon_i$ and, noting that $\gamma = \lambda a$ we conclude that $\mathbb{E}\left[\left(x_i - \tilde{x}\right)^2\right] = a^2\sigma_\varepsilon^2$.

Let $\text{WL} = \mathbb{E}\left[\text{TS}^o\right] - \mathbb{E}[\text{TS}]$. Similarly as in the proof of Proposition 3 in Vives (2011) we can obtain, using an exact Taylor expansion of total surplus around the full information first best allocation x^o, that

$$\mathrm{WL} = \left((\beta + \lambda) \mathbb{E}\left[(\tilde{x} - x^o)^2 \right] + \lambda \mathbb{E}\left[(x_i - \tilde{x})^2 \right] \right) \Big/ 2 \qquad . \qquad \text{It} \qquad \text{follows} \qquad \text{that}$$

$$\mathrm{WL} = \frac{1}{2} \left(\frac{(1 - \lambda a)^2}{\left(\tau_\theta + \tau_u \beta^2 a^2 \right)(\beta + \lambda)} + \frac{\lambda a^2}{\tau_\varepsilon} \right), \text{ which is easily seen strictly convex in } a \text{ and}$$

with a unique solution $\lambda^{-1} > a^T > 0$. Note that $\lambda^{-1} < a$ is dominated by $a = \lambda^{-1}$ and that $a < 0$ is dominated by $-a > 0$. Furthermore, it is immediate that with $\tau_\varepsilon > 0$, $\mathrm{WL}' < 0$ for $a = 0$ and therefore $a > 0$ at the solution.

From the expression for WL we obtain directly that $\left. \dfrac{d\mathrm{WL}}{da} \right|_{a = a^*} = \lambda a^* \sigma_\varepsilon^2 \beta c^*$. It follows that $\mathrm{sgn}\{a^* - a^T\} = \mathrm{sgn}\{\beta c^*\}$ since $a^* > 0$ when $\tau_\varepsilon > 0$ and WL is strictly convex with one minimum. Alternatively, recalling that the strategy for firm i is of the form $x_i = \lambda^{-1}\left(p - \left(\lambda a s_i + (1 - \lambda a)\hat{z} \right) \right)$, where $p = \left(\alpha \lambda + \beta \left(\gamma \theta + (1 - \gamma)\mathbb{E}[\theta \mid z] \right) \right)(\beta + \lambda)^{-1}$, $\hat{z} = \mathbb{E}[\theta / z] = \beta \tau_u a \tau^{-1} z$, and $z = \beta a \theta + u$. At the team solution we have that

$$\frac{\partial \mathbb{E}[\mathrm{TS}]}{\partial a} = \mathbb{E}\left[(p - \mathrm{MC}(x_i)) \frac{\partial x_i}{\partial a} \right] = 0 .$$

We have that

$$\frac{\partial x_i}{\partial a} = \left. \frac{\partial x_i}{\partial a} \right|_{p, \hat{z} \text{ ct.}} + \left. \frac{\partial x_i}{\partial p} \frac{\partial p}{\partial a} \right|_{\hat{z} \text{ ct.}} + \frac{\partial x_i}{\partial \hat{z}} \frac{\partial \hat{z}}{\partial a} .$$

At the market solution $\mathbb{E}\left[\left. (p - \mathrm{MC}(x_i)) \frac{\partial x_i}{\partial a} \right|_{p, \hat{z} \text{ ct.}} \right] = \mathbb{E}\left[(p - \mathrm{MC}(x_i))(s_i - \hat{z}) \right] = 0$ since $\left. \partial x_i / \partial a \right|_{p, \hat{z} \text{ ct.}} = -(s_i - \hat{z})$, and firms take z as given, $\mathbb{E}\left[(p - \mathrm{MC}(x_i))\hat{z} \right] = \mathbb{E}\left[(p - \mathrm{MC}(x_i))z \right] = 0$. From this it follows that at the market solution

$$\mathbb{E}\left[(p - \mathrm{MC}(x_i))(s_i - \hat{z}) \right] = \mathbb{E}\left[(p - \mathrm{MC}(x_i))(\theta - \hat{z}) \right] + \mathbb{E}\left[(p - \mathrm{MC}(x_i))\varepsilon_i \right] = 0$$

and therefore,

$$\mathbb{E}\big[(p-\text{MC}(x_i))(\theta-\hat{z})\big]=\mathbb{E}\big[(p-\text{MC}(x_i))\theta\big]=-\mathbb{E}\big[(p-\text{MC}(x_i))\varepsilon_i\big]$$
$$=\mathbb{E}\big[\text{MC}(x_i)\varepsilon_i\big]=\mathbb{E}\big[(\theta+\lambda x_i)\varepsilon_i\big]=-\lambda a^*\sigma_\varepsilon^2<0.$$

From $\partial x_i/\partial p=\lambda^{-1}$ and $\partial p/\partial a\big|_{\hat{z}\;\text{ct.}}=\beta\lambda(\beta+\lambda)^{-1}(\theta-\hat{z})$, we obtain the effect of the payoff externality for given public information at the market solution:

$$\mathbb{E}\left[(p-\text{MC}(x_i))\frac{\partial x_i}{\partial p}\frac{\partial p}{\partial a}\Big|_{\hat{z}\;\text{ct.}}\right]=-\beta\lambda(\beta+\lambda)^{-1}a^*\sigma_\varepsilon^2.$$

Finally we can evaluate similarly the term corresponding to the learning externality:

$$\mathbb{E}\left[(p-\text{MC}(x_i))\left(\frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a}\right)\right]=\frac{1-\lambda a^*}{\beta+\lambda}\frac{\beta^2\tau_u a^*}{\tau^*}(\lambda a^*\sigma_\varepsilon^2)$$

from

$$\frac{\partial x_i}{\partial \hat{z}}\frac{\partial \hat{z}}{\partial a}=\left(\frac{\partial x_i}{\partial p}\frac{\partial p}{\partial \hat{z}}+\frac{\partial x_i}{\partial \hat{z}}\right)\frac{\partial \hat{z}}{\partial a}=-\frac{1-\lambda a}{\beta+\lambda}\left(\frac{\beta^2\tau_u a}{\tau}+\frac{\partial(a\tau^{-1})}{\partial a}\beta\tau_u z\right).$$

We conclude that at the market solution

$$\frac{\partial\mathbb{E}[\text{TS}]}{\partial a}=-\beta\lambda(\beta+\lambda)^{-1}a^*\sigma_\varepsilon^2+\frac{1-\lambda a^*}{\beta+\lambda}\frac{\beta^2\tau_u a^*}{\tau^*}(\lambda a^*\sigma_\varepsilon^2)=-\beta c^*\lambda a^*\sigma_\varepsilon^2$$

since we know from the proof of Proposition 1 that $c^*=(\beta+\lambda)^{-1}\big(1-\beta\tau_u a^*(\tau_\varepsilon+\tau^*)^{-1}\big).$ ♦

Proof of Proposition 3. The welfare loss at the team-efficient solution is given by $\text{WL}(a^T)$, which is decreasing in τ_ε,τ_u and τ_θ since WL is decreasing in τ_ε,τ_u and τ_θ for a given a and $d\text{WL}(a^T)/da=0$. With respect to the market solution we have that

$$\frac{d\text{WL}}{d\tau_\theta}(a^*)=\frac{\partial\text{WL}}{\partial a}\frac{\partial a^*}{\partial\tau_\theta}+\frac{\partial\text{WL}}{\partial\tau_\theta},$$

where $\dfrac{\partial a^*}{\partial\tau_\theta}=-\dfrac{a}{\tau_\theta+\tau_\varepsilon+3a^2\beta^2\tau_u}$ and a^* solves $\beta^2\tau_u a^3+(\tau_\varepsilon+\tau_\theta)a-\lambda^{-1}\tau_\varepsilon=0.$

Given that

$$\mathrm{WL} = \frac{1}{2}\left(\frac{(1-\lambda a)^2}{\left(\tau_\theta + \tau_u \beta^2 a^2\right)(\beta+\lambda)} + \frac{\lambda a^2}{\tau_\varepsilon} \right),$$

it is possible to show that

$$\frac{d\mathrm{WL}}{d\tau_\theta}\left(a^*\right) < 0 \text{ if and only if } \frac{\tau_\theta + \tau_u \beta^2 a^2}{\tau_\varepsilon} > -\frac{2\beta+\lambda}{\lambda},$$

which is always true since $2\beta + \lambda > 0$. Exactly the same condition holds for $d\mathrm{WL}\left(a^*\right)/d\tau_u < 0$. Furthermore, we can show that $d\mathrm{WL}\left(a^*\right)/d\tau_\varepsilon \le 0$ if and only if

$\beta - \lambda \le \frac{\tau_u \beta^2 a^*}{\tau_\theta}\left(a^*(\beta+\lambda)+2\right) + (\beta+\lambda)\frac{\tau_\varepsilon}{\tau_\theta}$. It follows that WL will be increasing in

τ_ε for $\beta > \lambda$ and $\tau_\varepsilon/\tau_\theta$ small enough (since a^* is increasing in $\tau_\varepsilon/\tau_\theta$). ♦

<u>Proof of Proposition 4</u>: It proceeds in a parallel way to the proof of Proposition 2. Again, it can be checked first that $\partial^2 \mathbb{E}[\pi_i]/\partial^2 b < 0$ and $\partial^2 \mathbb{E}[\pi_i]/\partial^2 c < 0$ whenever $2\beta + \lambda > 0$. Given that $\pi_i = p x_i - C(x_i)$, $p = \alpha + u - \beta \tilde{x}$, $\partial x_i/\partial b = 1$, and $\partial x_i/\partial c = \partial \tilde{x}/\partial c = z$ and $\partial p/\partial \tilde{x} = -\beta$ we can optimize with respect to b and c to obtain

$$\frac{\partial \mathbb{E}[\pi_i]}{\partial b} = \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right) - \beta x_i \right] = 0,$$

$$\frac{\partial \mathbb{E}[\pi_i]}{\partial c} = \mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)z - \beta x_i z \right] = 0.$$

where $\mathrm{MC}(x_i) = \theta + \lambda x_i$. The constraint $\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right) - \beta x_i \right] = 0$ is equivalent to $b = \alpha/(2\beta+\lambda)$; we can also check that $\mathbb{E}\left[\left(p - \mathrm{MC}(x_i)\right)z - \beta x_i z \right] = 0$ is equivalent to $c = c^{\mathrm{IT}}(a)$, where

$$c^{\mathrm{IT}}(a) = \frac{1}{2\beta+\lambda} - \frac{\beta a \tau_u \left(1-(\lambda+\beta)a\right)}{\tau(2\beta+\lambda)} \quad \text{and} \quad \tau = \tau_\theta + \beta^2 \tau_u a^2.$$

Note that due to the competition payoff externality ($\partial p/\partial \tilde{x} = -\beta$) the expressions for b and for c are different than in the market solution. It follows that the form of the internal team optimal strategy is $x_i = (\lambda+\beta)^{-1}\left(p - \left(\gamma s_i + (1-\gamma)\mathbb{E}[\theta|z]\right)\right)$ where $\gamma = (\lambda+\beta)a$. We have that $\tilde{x} = (\lambda+\beta)^{-1}\left(p - \left(\gamma \theta + (1-\gamma)\mathbb{E}[\theta|z]\right)\right)$ and that

$\tilde{x} - x^{\mathrm{M}} = (1 - \gamma)(\theta - \mathbb{E}[\theta \mid z])/(2\beta + \lambda)$ and,   since $\tau = (\mathrm{var}[\theta \mid z])^{-1}$ we   obtain

$\mathbb{E}\Big[(\tilde{x} - x^{\mathrm{M}})^2\Big] = (1 - (\lambda + \beta)a)^2 \big/ (\tau(2\beta + \lambda)^2)$. We have that $\mathbb{E}\Big[(x_i - \tilde{x})^2\Big] = a^2/\tau_\varepsilon$.

Let $\Omega = \mathbb{E}[\pi_i^{\mathrm{M}}] - \mathbb{E}[\pi_i]$.   Similarly   as   before   we   can   obtain   that

$\Omega = \Big((2\beta + \lambda)\mathbb{E}\Big[(\tilde{x} - x^{\mathrm{M}})^2\Big] + \lambda \mathbb{E}\Big[(x_i - \tilde{x})^2\Big]\Big)\Big/2$. It follows that

$$\Omega(a) = \frac{1}{2}\left(\frac{(1 - (\lambda + \beta)a)^2}{(\tau_\theta + \tau_u \beta^2 a^2)(2\beta + \lambda)} + \frac{\lambda a^2}{\tau_\varepsilon} \right),$$

which   is   easily   seen   strictly   convex   in a and   with   a   unique   solution

$(\lambda + \beta)^{-1} > a^{\mathrm{IT}} > 0$. (Note that $(\lambda + \beta)^{-1} < a$ is dominated by $a = (\lambda + \beta)^{-1}$ and that

$a < 0$ is dominated by $-a > 0$. Furthermore, it is immediate that $\Omega'(0) < 0$ and

therefore $a > 0$ at the solution.)

The impact of a on $\mathbb{E}[\pi_i]$ is easily characterized (noting that $\partial \mathbb{E}[\pi_i]/\partial c = 0$ and

therefore disregarding the indirect impact of a on $\mathbb{E}[\pi_i]$ via a change in c):

$$\frac{\partial \mathbb{E}[\pi_i]}{\partial a} = \underbrace{\mathbb{E}\left[(p - MC(x_i))\left(\frac{\partial x_i}{\partial a}\right)_{z\ ct.} \right]}_{\text{Market}} +$$

$$\underbrace{\mathbb{E}\left[(p - MC(x_i))\left(\frac{\partial x_i}{\partial z}\frac{\partial z}{\partial a}\right) \right]}_{\text{PE+LE}} +$$

$$\underbrace{\mathbb{E}\left[x_i\left(\frac{\partial p}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial a}\right) \right]}_{\text{CE}}$$

$$= \mathbb{E}\Big[(p - \mathrm{MC}(x_i))(-s_i + c\beta\theta) - \beta(c\beta - 1)\theta x_i \Big]$$

given   that $(\partial x_i/\partial a)_{z\ ct.} = -s_i$, $\partial x_i/\partial z = c$, $\partial z/\partial a = \beta\theta$, $\partial p/\partial \tilde{x} = -\beta$ and

$\partial \tilde{x}/\partial a = (c\beta - 1)\theta$. Evaluating $\partial \mathbb{E}[\pi_i]/\partial a$ at the market solution, where

$\mathbb{E}\Big[(p - \mathrm{MC}(x_i))s_i\Big] = 0$, we obtain

$$\frac{\partial \mathbb{E}[\pi_i]}{\partial a} = \beta\mathbb{E}\Big[c(p - \mathrm{MC}(x_i))\theta - (c\beta - 1)\theta x_i \Big].$$

We know that $\mathbb{E}\left[\left(p - MC(x_i)\right)\theta\right] = -\lambda a\sigma_\varepsilon^2 < 0$ and, recalling that $\bar{\theta} = 0$, it is easily checked that $\mathbb{E}\left[\theta x_i\right] = a\sigma_\theta^2\left(c\beta - 1\right)$. At the equilibrium we have therefore[22]

$$\frac{\partial \mathbb{E}\left[\pi_i\right]}{\partial a} = -\beta a^*\left(c^*\lambda\sigma_\varepsilon^2 + \left(c^*\beta - 1\right)^2\sigma_\theta^2\right).$$

Since $\mathbb{E}\left[\pi_i\right]$ is single-peaked for $a > 0$ and has a unique maximum at $a^{IT} > 0$ and $a^* > 0$, it follows that

$$\mathrm{sgn}\left\{a^{IT} - a^*\right\} = \mathrm{sgn}\left\{\left.\frac{\partial \mathbb{E}\left[\pi_i\right]}{\partial a}\right|_{a=a^*}\right\} = \mathrm{sgn}\left\{-\beta\left(c^*\lambda\sigma_\varepsilon^2 + \left(c^*\beta - 1\right)^2\sigma_\theta^2\right)\right\}. \blacklozenge$$

Proof of Proposition 5: (i) From the equation determining the responsiveness to private information a, $\beta^2\tau_u a^3 + \left(\tau_\varepsilon + \tau_\theta\right)a - \lambda^{-1}\tau_\varepsilon = 0$, it is immediate that a decreases with τ_u, τ_θ, β^2 and λ, that a increases with τ_ε. Note that $\mathrm{sgn}\left\{\partial a/\partial\beta\right\} = \mathrm{sgn}\left\{-\beta\right\}$. As τ_u ranges from 0 to ∞, a decreases from $\lambda^{-1}\tau_\varepsilon\left(\tau_\theta + \tau_\varepsilon\right)^{-1}$ to 0.

(ii) As τ_u ranges from 0 to ∞, the responsiveness to public information c goes from $\left(\beta + \lambda\right)^{-1}$ to $-\infty$ (resp. $+\infty$) if $\beta > 0$ (resp. $\beta < 0$). The result follows since, in equilibrium,

$$c = \frac{1}{\beta + \lambda} - \frac{\beta\lambda\tau_u a^2}{\left(\beta + \lambda\right)\tau_\varepsilon} = \frac{1}{\beta + \lambda} - \frac{1}{\left(\beta + \lambda\right)\beta}\left(\frac{1}{a} - \lambda\left(1 + \frac{\tau_\theta}{\tau_\varepsilon}\right)\right)$$

and $a \to 0$ as $\tau_u \to \infty$. It follows that $\mathrm{sgn}\left\{\partial c/\partial\tau_u\right\} = \mathrm{sgn}\left\{-\beta\right\}$ because $\partial a/\partial\tau_u < 0$. Similarly, from the first part of the expression for c we have $\mathrm{sgn}\left\{\partial c/\partial\tau_\theta\right\} = \mathrm{sgn}\left\{\beta\right\}$ since $\partial a/\partial\tau_\theta < 0$. Furthermore, with some work it is possible to show that, in equilibrium,

$$\frac{\partial c}{\partial\tau_\varepsilon} = \left(\beta + \lambda\right)^{-1}\lambda\beta\tau_u\tau_\varepsilon^{-1}a\left(2\frac{a\lambda - 1}{\lambda\left(\tau_\theta + \tau_\varepsilon + 3a^2\beta^2\tau_u\right)} + a\tau_\varepsilon^{-1}\right) \quad \text{and}$$

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[22]   Note also that at the equilibrium $c\beta - 1 < 0$.

$$\text{sgn}\left\{2\frac{a\lambda-1}{\lambda\left(\tau_\theta+\tau_\varepsilon+3a^2\beta^2\tau_u\right)}+a\tau_\varepsilon^{-1}\right\}=\text{sgn}\left\{a\lambda\tau_\theta-2\tau_\varepsilon+3a\lambda\tau_\varepsilon+3a^3\beta^2\lambda\tau_u\right\}$$

$$=\text{sgn}\left\{-2a\lambda\tau_\theta+\tau_\varepsilon\right\}$$

$$=\text{sgn}\left\{\beta^2\tau_u\tau_\varepsilon^2+4\lambda^2\tau_\theta^2\left(\tau_\varepsilon-\tau_\theta\right)\right\}.$$

Hence we conclude that $\text{sgn}\left\{\partial c/\partial\tau_\varepsilon\right\}=\text{sgn}\left\{\beta\left(\beta^2\tau_u\tau_\varepsilon^2+4\lambda^2\tau_\theta^2\left(\tau_\varepsilon-\tau_\theta\right)\right)\right\}$. Since

$\hat{c}=\left(c^{-1}-\beta\right)^{-1}$, it follows that $\hat{c}$ goes from λ^{-1} to $-\beta^{-1}$ as τ_u ranges from 0 to ∞,[23]

$\text{sgn}\left\{\partial\hat{c}/\partial\tau_u\right\}=\text{sgn}\left\{-\partial\hat{c}/\partial\tau_\theta\right\}=\text{sgn}\left\{-\beta\right\}$, and $\text{sgn}\left\{\partial\hat{c}/\partial\tau_\varepsilon\right\}=\text{sgn}\left\{\partial c/\partial\tau_\varepsilon\right\}$. It is then

immediate that $1+\beta\hat{c}$ is decreasing in τ_u and increasing in τ_θ.

(iii) Price informativeness $\tau=\tau_\theta+\beta^2 a^2\tau_u$ is increasing in τ_ε (since a increases with

τ_ε) and also in τ_u (since $a=\lambda^{-1}\tau_\varepsilon\left(\tau_\varepsilon+\tau\right)^{-1}$ and a decreases with τ_u). Using the

expression for $\partial a/\partial\tau_\theta$ we have that

$$\frac{\partial\tau}{\partial\tau_\theta}=1+2\beta^2\tau_u a\frac{\partial a}{\partial\tau_\theta}=1-\frac{2\beta^2 a^2\tau_u}{\tau_\theta+\tau_\varepsilon+3a^2\beta^2\tau_u}=\frac{\tau_\theta+\tau_\varepsilon+a^2\beta^2\tau_u}{\tau_\theta+\tau_\varepsilon+3a^2\beta^2\tau_u}>0.$$

Furthermore,

$$\frac{\partial\tau}{\partial\beta}=\tau_u\left(2\beta a^2+2\beta^2 a\frac{\partial a}{\partial\beta}\right)=2\beta a\tau_u\left(a-\frac{2a^4\tau_u\lambda\beta^2}{1+2a^3\tau_u\lambda\beta^2}\right)=\frac{2\beta a^2\tau_u}{1+2a^3\tau_u\lambda\beta^2},$$

and therefore $\text{sgn}\left\{\partial\tau/\partial\beta\right\}=\text{sgn}\left\{\beta\right\}$.

(iv) From $x_i=\lambda^{-1}\left[p-\mathbb{E}\left[\theta|s_i,z\right]\right]$ and $\mathbb{E}\left[\theta/s_i,z\right]=\gamma s_i+\left(1-\gamma\right)\mathbb{E}\left[\theta/z\right]$ we obtain

$x_i-\tilde{x}=\lambda^{-1}\gamma\left(s_i-\theta\right)=\lambda^{-1}\gamma\varepsilon_i$ and, noting that $\gamma=\lambda a$ we conclude that

$\mathbb{E}\left[\left(x_i-\tilde{x}\right)^2\right]=a^2\sigma_\varepsilon^2$. The results then follow from the comparative statics results for

a in (i). $\blacklozenge$

Claim 2. $\mathbb{E}\left[\theta/s_i,z\right]=\gamma s_i+hz$ with $h=\lambda\beta\tau_\varepsilon^{-1}\tau_u a^2$, $\partial|h|/\partial\tau_\theta<0$, $\partial|h|/\partial\tau_u>0$ and

$\text{sgn}\left\{\partial|h|/\partial\beta\right\}=\text{sgn}\left\{\beta\right\}$.

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[23] Note that if $\beta<0$ and $\beta+\lambda>0$ then $\lambda^{-1}<-\beta^{-1}$.

<u>Proof</u>: From $h = \beta a \tau_u (\tau_\varepsilon + \tau)^{-1}$ in the proof of Proposition 1 it is immediate that $h = \lambda \beta \tau_\varepsilon^{-1} \tau_u a^2$. We have that $\partial |h| / \partial \tau_\theta < 0$ since $\partial a / \partial \tau_\theta < 0$; $\partial |h| / \partial \tau_u > 0$ since $\partial \tau / \partial \tau_u > 0$ and therefore $\partial (\tau_u a^2) / \partial \tau_u > 0$. Finally, we have that in equilibrium

$$\frac{\partial c}{\partial \beta} = -\frac{1}{(\lambda + \beta)\tau_\varepsilon} \left(\frac{\tau_\varepsilon + \lambda^2 \tau_u a^2}{\lambda + \beta} + \frac{4a^5 \lambda^3 \tau_u^2 \beta^2}{1 + 2a^3 \tau_u \lambda \beta^2} \right) < 0 ,$$

and from $c = (1 - h)(\beta + \lambda)^{-1}$ we can obtain $\partial h / \partial \beta > 0$, and therefore, $\mathrm{sgn}\{\partial |h| / \partial \beta\} = \mathrm{sgn}\{\beta\}$. ♦

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